STABILITY ANALYSIS OF DOUBLE-HARMONIC CAVITY SYSTEM IN HEAVY BEAM LOADING WITH ITS FEEDBACK LOOPS BY A MATHEMATICAL METHOD BASED ON PEDERSEN MODEL

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Abstract

With the high beam current in storage ring, it is necessary to consider the instability problem caused by the heavy beam loading effect. It has been demonstrated that direct RF feedback (DRFB), autolevel control loop (ALC) and phase-lock loop (PLL) in the main cavity can lessen the impact of the beam effect. This paper regarded the beam, main cavity, harmonic cavity and feedback loops as double harmonic cavity system, and extended the transfer functions in the Pedersen model to this system. Some quantitative evaluations of simulation results have been got and conclusions have been drawn. In the case of a passive harmonic cavity, the optimization strategy of the controller parameters in the pre-detuning , ALC and PLL, as well as the gain and phase shift of DRFB were discussed. Meanwhile, it also involved the impact of the harmonic cavity feedback loop on the system stability at the optimum stretching condition when an active harmonic cavity was present. The research results can be used as guidelines for beam operation with beam current increasing in the future.

INTRODUCTION

The higher harmonic cavity (HHC) has been proven to improve beam life and suppress instabilities through Landau damping without affecting energy diffusion and brightness [1] [2]. However, in the fundamental mode, passing through the HHC can destabilize the beam according to the Robinson criterion, requiring detuning of the main cavity to maintain stability [3]. To study RF system instability, the influence of HHC cannot be ignored. Techniques such as direct RF feedback (DRFB) can reduce heavy beam loading and increase beam current limit by increasing cavity bandwidth and reducing RF cavity shunt impedance [4]. Autolevel control loop (ALC), and phase lock loop (PLL) in low-level RF (LLRF) systems stabilize cavity voltage while affecting overall system stability. Robinson stability criterion calculates the maximum beam current for a single cavity in the storage ring [5], and Pedersen's feedback loop model explains Robinson instability using beam and generator current modulations [6]. This paper adds HHC, DRFB, ALC, and PLL to Pedersen's model to analyze loop instability and analyze the influence of loading angle, HHC detuning, ALC and PLL controller parameters, and DRFB gain and phase shift on the maximum beam current limit. Taking the Shanghai Synchrotron Radiation Facility(SSRF) as an example, the effects of various parameters on instability were analyzed and suggestions were proposed.

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MODEL DESCRIPTION

Taking passive harmonic double cavity system as an example, the steady-state phasor diagram is shown in Fig.1.The voltage $\tilde{V}_C$in the main cavity is determined by beam current $\tilde{I}_B$, excitation source current $\tilde{I}_G$, DRFB current $\tilde{I}_F$ and cavity impedance. φ_s,φ_L and θ_L are the synchronization angle, detuning angle, and pre-detuning angle, respectively. The total voltage $\tilde{V}_T$ is the vector sum of the main cavity voltage and the passive HHC voltage $\tilde{V}_H$.

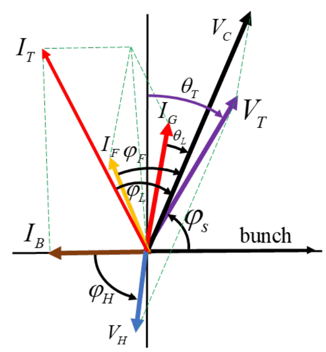

Figure 1: Phasor diagram for the steady-state case, depicting the amplitude and phase of each voltage and current of the transmitter, cavity and beam.

$Y = I_B/I_0$ is usually used to characterize the severity of the beam loading effect, where I_0 is the projection of I_T ontoV_C. Based on this, the parameter $X = I_F/I_0$ can be defined to characterize the gain of the feedback current, while the phase can be represented by φ_F. In the case of high-Q, the detuning angle φ_H can be figured out to be about 90°, and the passive HHC voltage can be calculated from $V_H = I_B \frac{r_p}{Q_p} \frac{f_{hrf}}{\Delta f}$ [7], where f_{hrf} is n times of the RF frequency, Δf is the detuning frequency. The total cavity voltage can be determined by V_T and θ_T.

$$\begin{cases} V_T = \sqrt{V_C^2 + V_H^2 - 2V_C V_H \cos(\varphi_s - \varphi_H)} \\ \theta_T = \arctan\left(-\frac{V_C \cos\varphi_s - V_H \cos\varphi_H}{V_C \sin\varphi_s - V_H \sin\varphi_H}\right) \end{cases} \quad (1)$$

The detuning angle of the main cavity can be obtained by phasor diagram, which is equal to

$$\tan\varphi_L = X\sin\varphi_F - Y\sin\varphi_s + (1 + Y\cos\varphi_s - X\cos\varphi_F)\tan\theta_L \quad (2)$$

The Pedersen model of the passive harmonic double cavity system can be deduced from the expression of the vector relationship and the impedance of the cavity, as shown in Fig.2.

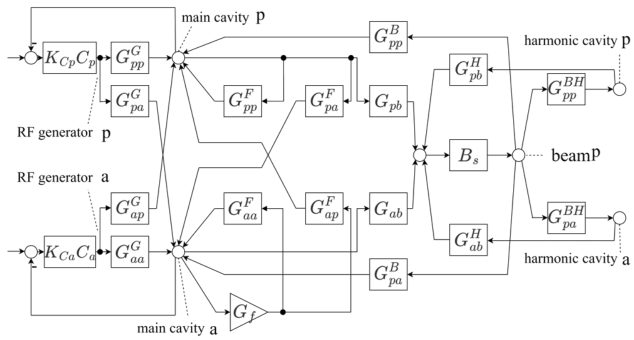

Figure 2: Expanded Pedersen model with passive HHC, ALC, PLL, and DRFB added.

Due to the slow response of tuner loop, only the ALC and PLL are considered in the simulation [8], Note that this work only includes static beam loading effects. The transfer function that relates the modulation of current excitation to the modulation of cavity voltage signal in HHC is as follows:

$$\begin{cases} G_{pa}^{BP} = \frac{\sigma_p \tan\varphi_H s}{s^2+2\sigma_p s+\sigma_p^2(1+\tan^2\varphi_H)} \\ G_{pp}^{BP} = \frac{\sigma_p^2(1+\tan^2\varphi_H)+\sigma_p s}{s^2+2\sigma_p s+\sigma_p^2(1+\tan^2\varphi_H)} \end{cases} \tag{3}$$

Where σ=$\omega_{rf}/(2Q_L)$ is the cavity damping factor, σ_p is the cavity damping factor of passive HHC. According to the vector relationship

$$\begin{cases} G_{pa}^F = \frac{I_F}{I_T}\left[G_{aa}\sin(\varphi_F-\varphi_L)+G_{pa}\cos(\varphi_F-\varphi_L)\right] \\ G_{pp}^F = \frac{I_F}{I_T}\left[G_{ap}\sin(\varphi_F-\varphi_L)+G_{pp}\cos(\varphi_F-\varphi_L)\right] \\ G_{pa}^B = \frac{I_B}{I_T}\left[-G_{aa}\sin(\varphi_s-\varphi_L)-G_{pa}\cos(\varphi_s-\varphi_L)\right] \\ G_{pp}^B = \frac{I_B}{I_T}\left[-G_{ap}\sin(\varphi_s-\varphi_L)-G_{pp}\cos(\varphi_s-\varphi_L)\right] \end{cases} \tag{4}$$

By the same token, we can get transfer functions such as G_{pa}^G,G_{pp}^G,G_{ap}^F,G_{aa}^F. $B_s = \Omega_s^2/\left(s^2+\alpha_s\cdot s+\Omega_s^2\right)$ is the transfer function from the equivalent phase modulation of the total cavity voltage to the phase modulation of the beam current, where Ω_s is the longitudinal oscillation frequency [9]. Main cavity and harmonic cavity can both affect the equivalent phase of the total cavity voltage [10], and the weight of each component is

$$\begin{cases} G_{ab} = [-\cos(\theta_T-\varphi_s)-\sin(\theta_T-\varphi_s)\tan\theta_T]\frac{V_C}{V_T} \\ G_{pb} = [-\sin(\theta_T-\varphi_s)+\cos(\theta_T-\varphi_s)\tan\theta_T]\frac{V_C}{V_T} \\ G_{ab}^p = [\cos(\theta_T-\varphi_H)+\sin(\theta_T-\varphi_H)\tan\theta_T]\frac{V_H}{V_T} \\ G_{pb}^p = [\sin(\theta_T-\varphi_H)-\cos(\theta_T-\varphi_H)\tan\theta_T]\frac{V_H}{V_T} \end{cases} \tag{5}$$

In DRFB loop, the amplifier needs to convert the voltage modulation signal into current modulation signal $G_f = X/R_L$, where R_L is the load shunt impedance in main cavity. The ALC and PLL controller is represented as low pass filter that does not include the carrier frequency portion and removes the DC component [11]. Furthermore, Where K_{Ca} and K_{Cp} represent the gain, C_a and C_p represent the bandwidth.

$$C_{a,p} = \frac{\omega_{a,p}}{s+\omega_{a,p}} \tag{6}$$

INFLUENCE OF LOOP PARAMETERS ON SYSTEM PERFORMANCE

Robinson instability calculates the maximum beam current of the bunch in a single cavity , which occurs when $\tilde{V}_G$ and $\tilde{I}_B$ in the vector diagram are in opposite phases. However, when additional loops are added, the coupling between the loops makes it difficult to intuitively demonstrate instability analysis in the vector diagram. In this case, New model can calculate the open-loop transfer function and draw the Nyquist curve. When appropriate controller parameters are set, the poles of the system will not fall in right half-plane. According to the Nyquist stability criterion, when the number of crossings of the open-loop magnitude plot (positive frequency) with the left-hand side of the (-1, 0j) point on the real axis is zero, there are no poles in the right half-plane of the closed-loop system, then the system is stable. Due to the negligible effect of the klystron's control function on beam dynamics, the loop delay time T is approximately 1-2μs [13]. The zero-mode oscillation frequency of SSRF is approximately 4.8kHz, and the delay function is approximately $e^{-sT}\approx 1-sT\approx 1$. Therefore, to simplify the model, delay is not considered.

Table 1: High-frequency parameters of SSRF [12] [13]

Energy	3.5GeV
RF frequency	499.654MHz
Harmonic number	720
Radiation loss	1.44MeV
Main RF voltage	4.5-5.4MV
r/Q of third harmonic cavity	88
Harmonic cavity number	3

The shunt impedance of the main high-frequency cavity is 28.5MΩ , the designed voltage of the main cavity is 5.4 MV. The HHC voltage is approximately 1.8 MV under optimal stretching conditions, the detuning frequency can be calculated as 22kHz. The stability can be determined by the gain margin, represented by SC for single cavity and DC for double cavity, as seen in Fig.3 Nyquist plot. Critical stability states are achieved by adjusting the gain and phase shift angle of DRFB as shown in Fig.4.

Setting the phase shift angle in the range of -180° to -300° can make the system more stable. In addition, adding HHC in the stable state will lead to a decrease in stability margin.

From Fig.5, it can be seen that the system is most unstable when the pre-detuning angle is between 0-30°. In practice, due to the poor loop control capability, there is often a deviation in the beam loading angle. To prevent it from falling into the most unstable region, the loading angle can be preset to a small negative value.

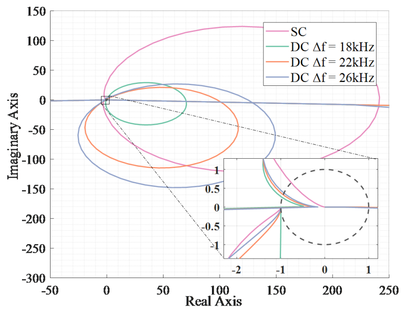

Figure 3: The beam current is 300mA. DRFB is adjusted to achieve the critical stable state, where the curve passes through (-1, 0j), and this state is extended to double cavity system with different detunings, where the system remains in the critical stable state.

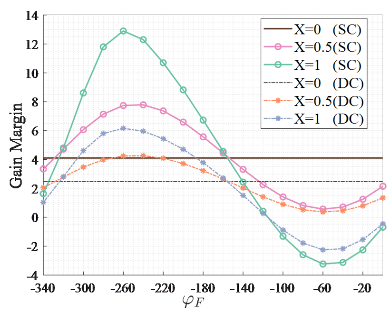

Figure 4: System gain margin versus phase shift angle, that is calculated with single cavity and double cavity, X=0-1.

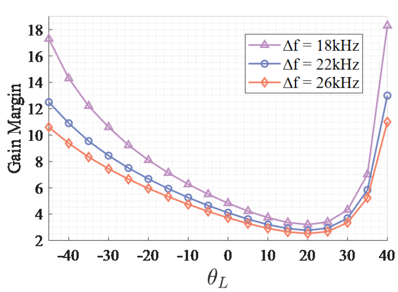

Figure 5: System gain margin versus Pre-detuning angle, that is calculated with HHC detuning frequency 18-26 kHz.

After considering ALC and PLL loops, Assume that the initial controller gain is 6 and bandwidth is 1 kHz. The effects of their gain and bandwidth on system stability are discussed separately [14].

By analyzing the Nyquist plot as fig.6, it can be concluded that the stability of various curves can be compared by the phase margin. When the ALC gain increases, the phase margin decreases. However, simulation results have confirmed that variations in PLL gain and controller bandwidth within a certain range do not affect stability margin.

After being converted to an active HHC, the optimal stretching state can be achieved at different beam intensities. Adjusting the HHC transmitter coupling coefficient

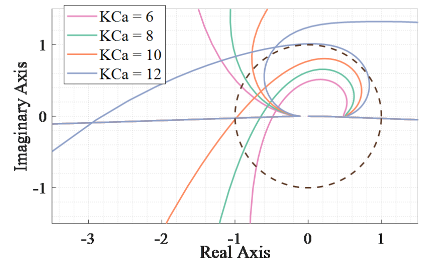

Figure 6: Nyquist plot with gradually increasing amplitude loop gain, system from stable to unstable.

can meet the optimal coupling, satisfying $\beta_{op} = 1 + P_B/P_H$ [15], and its ALC and PLL controllers are consistent with the main cavity. However, it is found that the system is prone to enter an unstable state as shown in Fig.7.

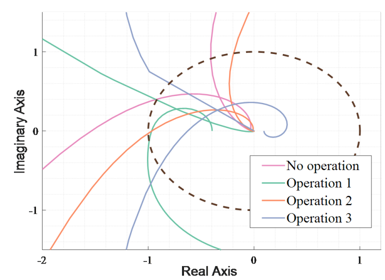

Figure 7: Nyquist plot of the system. operation1: Reduce controller gain to 2 and bandwidth to 500 Hz; operation2: DRFB $X = 1, \varphi_F = -260°$; operation3: The coupling coefficient is reduced to one tenth of the optimal coupling.

Three solutions are proposed:

- Reducing the gain and bandwidth of each controller, even if it may result in slower feedback control;
- Using DRFB, but a high feedback gain may result in a decrease in the precision of cavity voltage control or even produce self-excited oscillation in the loop;
- Decreasing the coupling coefficient of the transmitter of the HHC.

CONCLUSION

This article proposes a novel mathematical method based on the Pedersen model to analyze the stability of harmonic double cavity system for the first time. The model utilizes control theory to provide a clear description of the effects of each variable parameter on the stability of the system. Taking SSRF as an example, it is discussed that the addition of a passive HHC does not affect the maximum stable current, but it reduces the stability margin of the system in the stable state. Optimization strategies for system stability are presented by adjusting the parameters of pre-tuning angle, DRFB, ALC, and PLL. Furthermore, the model is extended to an active HHC system, and it is found that this system is prone to unstable states. Three upgrade proposals for future active harmonic systems are proposed in this paper.

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