Monte Carlo Forward Model for Photon Transport

Finite Slab Geometry

Research Project Report

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1 Model Overview

This report describes the development of a forward Monte Carlo simulator for photon transport in turbid media (`mc_forward_jacques.py` v1.1). The methodology is based on **Chapter 5** of the textbook **Optical-Thermal Response of Laser-Irradiated Tissue** [1] and the standard **MCML** software [2].

The model simulates a **finite slab** of tissue with thickness d, refractive index n_t, and surroundings (air) with index n_{env}. The source is a collimated "pencil beam" incident perpendicular to the surface. This setup calculates the Green's function (impulse response) of the tissue, which can later be convolved to simulate broad beams [2].

1.1 Problem Geometry

- **Medium:** Homogeneous slab of thickness d (occupying $0 \le z \le d$).
- **Boundaries:** Air-Tissue interfaces at $z = 0$ (top) and $z = d$ (bottom).
- **Source:** Collimated pencil beam entering at $z = 0$ along the $+z$ axis.

2 Methodology

The simulation follows the standard Hop-Drop-Spin cycle, modified to handle finite boundaries.

2.1 Photon Launch

Photons are launched from the origin $(x, y, z) = (0, 0, 0)$ with direction $(0, 0, 1)$. The initial weight is adjusted for specular reflection (R_{sp}) at the first interface:

$$R_{sp} = \left(\frac{n_{env} - n_t}{n_{env} + n_t}\right)^2, \quad W = 1.0 - R_{sp}$$

2.2 Step Size and Boundary Interaction

The photon step size s is sampled from Beer's Law ($s = -\ln(R)/\mu_t$). In a finite slab, the photon may hit a boundary before completing step s. We calculate the distance to the boundary d_b along the trajectory (Eq 5.34 in [1]):

$$d_b = \begin{cases} (d - z)/u_z & \text{if } u_z > 0 \\ -z/u_z & \text{if } u_z < 0 \end{cases}$$

- If $s < d_b$: The photon moves the full step s.
- If $s > d_b$: The photon moves to the boundary (Eq 5.35). The remaining step $s \leftarrow s - d_b$ is preserved.

2.3 Fresnel Reflection (at Boundaries)

When a photon hits a boundary, the internal reflectance $R(\theta_i)$ is calculated using Fresnel's formulas (Eq 5.36 in [1]):

$$R(\theta_i) = \frac{1}{2}\left[\frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} + \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}\right]$$

where θ_t is determined by Snell's Law ($n_t \sin\theta_i = n_{env} \sin\theta_t$). In the code, this is implemented using optimized direction cosines to avoid computationally expensive trigonometric functions.

A random number ξ is drawn:

- If $\xi \leq R(\theta_i)$: The photon **reflects** ($u_z \leftarrow -u_z$) and continues.
- If $\xi > R(\theta_i)$: The photon **transmits** (escapes) and is scored as Reflectance (R_d) or Transmittance (T_t).

2.4 Scattering (Henyey-Greenstein)

The scattering phase function describes the probability density function (PDF) for the cosine of the deflection angle, $\cos\theta$. The physical definition (Eq 3.18 in [1]) involves a power of 3/2:

$$p(\cos\theta) = \frac{1-g^2}{2(1+g^2-2g\cos\theta)^{3/2}}$$

However, in the Monte Carlo simulation, we sample $\cos\theta$ by inverting the cumulative distribution function (CDF) of this PDF. The resulting sampling formula (Eq 3.19 in [1]) involves a square term rather than a 3/2 power:

$$\cos\theta = \frac{1}{2g}\left[1+g^2-\left(\frac{1-g^2}{1-g+2g\xi}\right)^2\right]$$

This derived formula is what is implemented in the 'SPIN' function of the code.

2.5 Absorption and Scoring

Absorption (DROP): At each step, weight is reduced: $\Delta W = W(\mu_a/\mu_t)$.
Scoring: Absorbed energy is accumulated in:

- **Planar Fluence $F_{pla}(z)$:** Fluence rate vs. depth z (Primary metric for slab).
- **Cylindrical Fluence $F_{cyl}(\rho)$:** Radial spread (Point Spread Function).

3 Simulation Results

Figure 1 shows the fluence profiles for a slab with thickness $d = 1.0$ cm, $\mu_a = 1.0$ cm^{-1}, $\mu_s = 100$ cm^{-1}, $g = 0.9$, and $n_t = 1.33$.

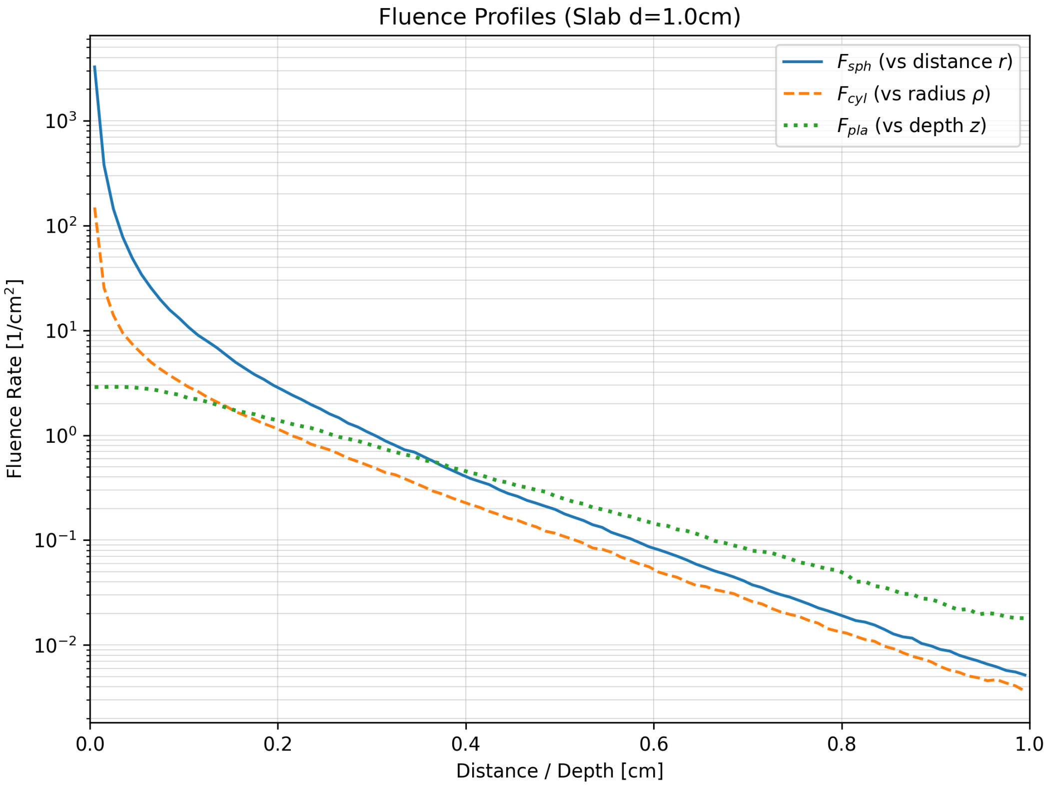

Figure 1: Fluence profiles for a 1.0 cm finite slab illuminated by a collimated pencil beam. F_{pla} (green) shows the exponential attenuation with depth z. F_{sph} (blue) shows the high intensity singularity near the source.

Macroscopic results ($N = 10,000$ photons):

- Diffuse Reflectance (R): 0.2725
- Total Transmittance (T): 0.0031
- Total Absorbed (A): 0.7244
- Energy Conservation ($R + T$): 0.2756
- Energy Conservation ($R + T + A$): 1.0000 (Verified)

The conservation sum of 1.0000 confirms the boundary conditions and Fresnel logic are correctly implemented.

4 Photon Trajectory Visualization

To provide an intuitive understanding of photon transport inside the finite tissue slab, we implemented a visualization tool that traces individual photon trajectories using the same Monte Carlo propagation rules as the forward model presented in this report. Although the visualization does not compute absorbed energy or fluence, it accurately represents the stochastic paths taken by photons, including scattering events, absorption, and Fresnel interactions at slab boundaries.

Each photon begins at the top surface ($z = 0$) as a collimated pencil beam and then propagates through the slab of thickness $d = 0.1$ cm. At each scattering event, the new propagation direction is sampled from the Henyey–Greenstein phase function using the anisotropy parameter

$g = 0.9$. Absorption is modeled through weight reduction, and roulette termination is applied for low-weight photons. Boundary interactions follow Snell's law and Fresnel reflectance, allowing photons to either reflect back toward the source or transmit through the bottom interface.

Three photon outcomes are possible:

- **Reflected** (red): the photon exits through the top surface ($z = 0$),
- **Transmitted** (green): the photon exits through the bottom surface ($z = d$),
- **Absorbed** (blue): the photon loses all remaining weight inside the slab.

Figure 2 shows a top-down view of the simulated photon trajectories, with the depth axis inverted so that $z = 0$ (incident surface) appears at the top and $z = d$ (bottom surface) appears at the bottom. The gray planes indicate the slab boundaries. As expected, many photons undergo multiple scattering events before either being absorbed or exiting the tissue.

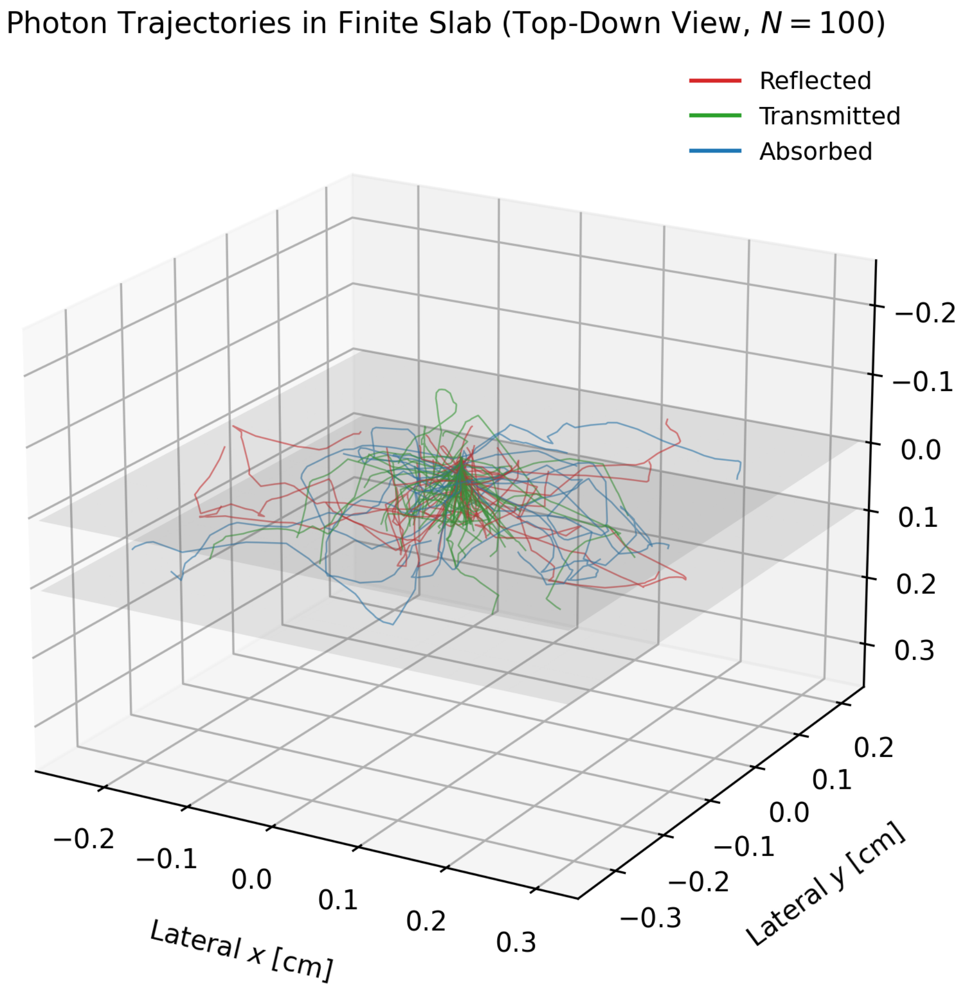

Figure 2: Top-down visualization of photon trajectories for $N = 100$. Red lines represent reflected photons, green represent transmitted photons, and blue correspond to absorbed photons. The gray planes denote the top and bottom slab boundaries.

In addition to the depth-oriented view, a side-view representation is shown in Figure 3, where the horizontal axis corresponds to depth (z), allowing a more schematic comparison of photon penetration relative to the slab boundaries. This view makes it visually clear that some photons exit through the top, some through the bottom, and others terminate internally — consistent with the expected energy partitioning of reflection, transmission, and absorption in Monte Carlo light transport.

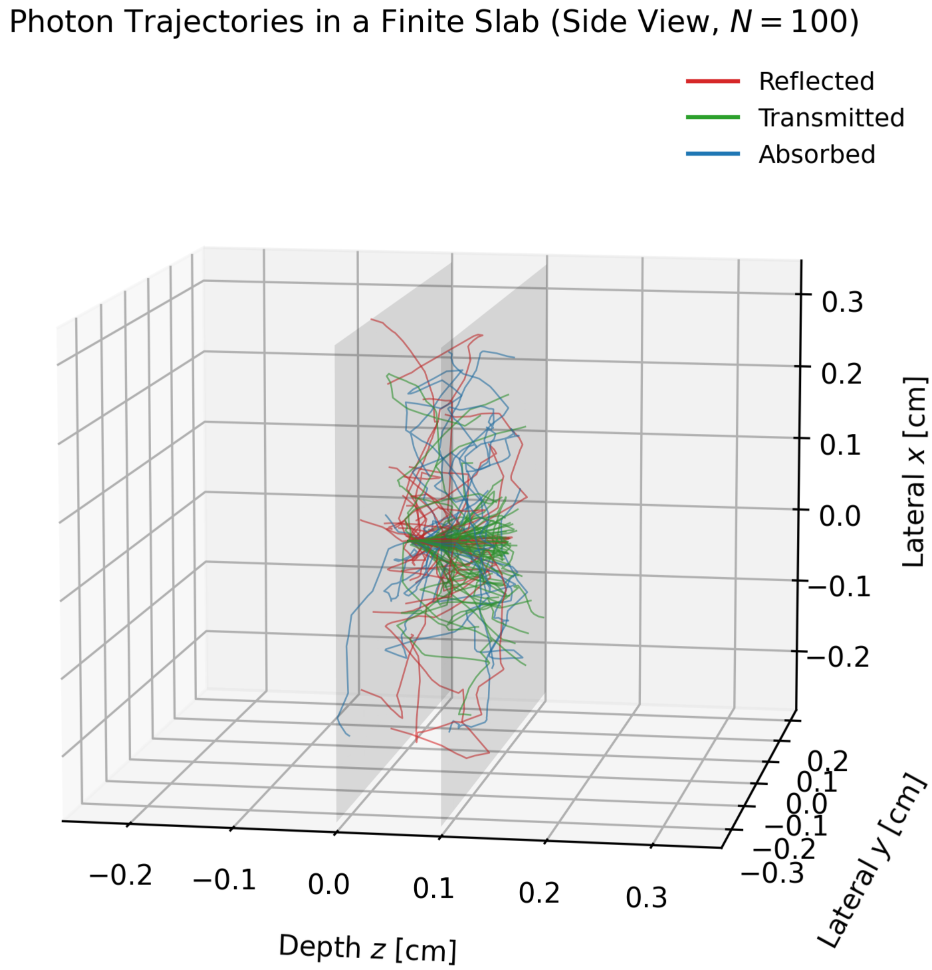

Figure 3: Side-view visualization of photon trajectories, with depth along the horizontal axis. This orientation highlights the entry point at $z = 0$, the finite slab thickness, and the branching of photon paths according to absorption, reflection, and transmission outcomes.

These visualizations confirm that the implemented forward Monte Carlo model behaves in a physically consistent manner. Photons scatter many times within the sample, a fraction emerge back through the incident surface (reflection), a fraction exit the opposite surface (transmission), and the remainder are absorbed internally. This behavior is consistent with classical MCML-based models and serves as a useful qualitative validation of the simulation.

References

[1] S. L. Jacques, "Monte carlo modeling of light transport in tissue," in *Optical-Thermal Response of Laser-Irradiated Tissue*, pp. 109–144, Springer, 2011.

[2] L. Wang and S. L. Jacques, *Monte Carlo Modeling of Light Transport in Multi-layered Tissues in Standard C*. Optical Imaging Laboratory, Oregon Medical Laser Center, US, Portland, 1992. Online; accessed 2025.