# Monte Carlo Forward Model for Photon Transport (Jacques 2011, Chapter 5)

*Research Project Report*

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# Contents

# 1 Monte Carlo Forward Model

This section describes the forward Monte Carlo simulator implemented in Python, adapted from Jacques' classic mc321.c program (Chapter 5 of [1]). The model computes the absorbed energy distribution of photons emitted from an isotropic point source in an infinite, homogeneous turbid medium. Photon interactions include absorption, scattering, and probabilistic termination. Absorbed energy is scored in spherical, cylindrical, and planar geometries, following the normalization conventions in Jacques [1].

## 1.1 Photon Launch

Each photon is initialized at the origin:

$$(x, y, z) = (0, 0, 0)$$

with initial weight $W = 1$. The emission is isotropic. The polar and azimuthal angles are sampled as:

$$\cos\theta = 2R_1 - 1, \qquad \psi = 2\pi R_2,$$

where $R_{1,2} \sim U(0, 1)$. Direction cosines:

$$u_x = \sin\theta\cos\psi, \quad u_y = \sin\theta\sin\psi, \quad u_z = \cos\theta.$$

## 1.2 Photon Step Size (HOP)

The step length $s$ is exponentially distributed:

$$s = -\frac{\ln R}{\mu_t}, \qquad \mu_t = \mu_a + \mu_s,$$

where $R \sim U(0, 1)$ is a uniform random number.

This models the mean free path between interactions. The position updates to:

$$x \leftarrow x + su_x, \quad y \leftarrow y + su_y, \quad z \leftarrow z + su_z.$$

## 1.3 Absorption (DROP)

At each step, a fraction of the weight is absorbed:

$$\Delta W = W\left(\frac{\mu_a}{\mu_a + \mu_s}\right) = W\left(\frac{\mu_a}{\mu_t}\right),$$

and the weight becomes

$$W \leftarrow W - \Delta W.$$

The absorbed energy is deposited into three geometries:

- **Spherical**: $r = \sqrt{x^2 + y^2 + z^2}$
- **Cylindrical**: $r = \sqrt{x^2 + y^2}$
- **Planar**: $r = |z|$

The corresponding bin index is:

$$i_r = \lfloor r/\Delta r \rfloor.$$

### 1.4 Scattering (SPIN)

The scattering angle $\theta$ is sampled from the Henyey–Greenstein (HG) phase function. In this project, we follow the implementation used in Jacques' reference code `mc321.c`. For anisotropy $g$, the cosine of the scattering angle is

$$\cos\theta = \begin{cases} 2R-1, & g=0, \\ \dfrac{1+g^2-\left(\frac{1-g^2}{1-g+2gR}\right)^2}{2g}, & g\neq 0, \end{cases}$$

where $R$ is a uniform random number in $[0,1]$. This is the standard HG sampling formula used throughout the Monte Carlo literature.

The printed version of Eqs. (5.43) and (5.45) in the textbook (*Jacques, 2011*, pp. 125–126) includes a denominator raised to the power 3/2. This exponent does *not* appear in the reference C code `mc321.c`, and is recognized as a typographical inconsistency in the book. Since the code implements the canonical HG sampling expression correctly, we follow the code to ensure consistency with the established formulation of the Henyey–Greenstein model.

### 1.5 Roulette (Termination)

When weight becomes small:

$$W < W_{\mathrm{th}} = 0.01,$$

Russian roulette is applied:

$$W = \begin{cases} W/p, & \text{with probability } p = 0.1, \\ 0, & \text{otherwise,} \end{cases}$$

terminating low-weight photons while preserving expected energy.

### 1.6 Fluence Calculation

After all photons are simulated, the absorbed energy arrays $C_{\mathrm{sph}}(r)$, $C_{\mathrm{cyl}}(r)$, and $C_{\mathrm{pla}}(r)$ are converted to fluence $T(r)$ following Jacques' definitions. Here, $N_{\mathrm{ph}}$ is the total number of launched photons, $\Delta r$ is the radial bin width, and $\mu_a$ is the absorption coefficient.

**Spherical fluence.**

$$T_{\mathrm{sph}}(r) = \frac{C_{\mathrm{sph}}(r)}{N_{\mathrm{ph}}\, 4\pi r^2\, \Delta r\, \mu_a}.$$

**Cylindrical fluence.**

$$T_{\mathrm{cyl}}(r) = \frac{C_{\mathrm{cyl}}(r)}{N_{\mathrm{ph}}\, 2\pi r\, \Delta r\, \mu_a}.$$

**Planar fluence.**

$$T_{\mathrm{pla}}(r) = \frac{C_{\mathrm{pla}}(r)}{N_{\mathrm{ph}}\, \Delta r\, \mu_a}.$$

These expressions correspond to spherical shells, cylindrical shells (per cm of length), and planar slabs (per cm$^2$), respectively, and match the normalization used in Jacques [1], Appendix 5.7.

## 1.7 Simulation Output

Figure 1 shows the simulated fluence $T(r)$ from an isotropic point source in an infinite homogeneous medium for three geometries: spherical, cylindrical, and planar. The optical parameters are chosen to match Jacques' example ($\mu_a = 1.673$ cm$^{-1}$, $\mu_s = 312$ cm$^{-1}$, $g = 0.90$), and the curves are plotted on a logarithmic scale as a function of distance $r$ from the source. **Note that Jacques denotes the radial fluence by $T(r)$; we follow this notation throughout.**

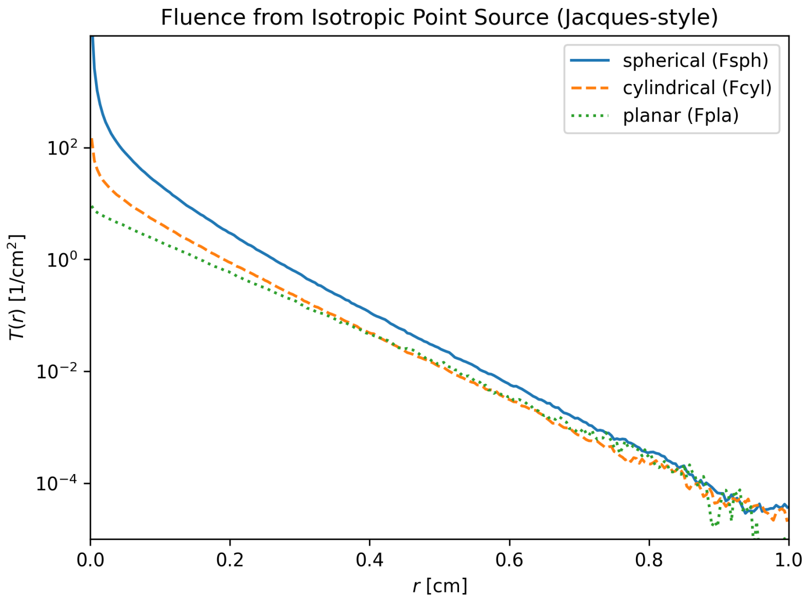


Figure 1: Fluence $T(r)$ from an isotropic point source in an infinite homogeneous medium, computed using the forward Monte Carlo simulator. The curves show spherical ($T_{\mathrm{sph}}$), cylindrical ($T_{\mathrm{cyl}}$), and planar ($T_{\mathrm{pla}}$) fluence, using the same optical parameters as Fig. 5.10 in Jacques (2011).

Near the source, the spherical fluence $T_{\mathrm{sph}}(r)$ exhibits the characteristic $1/r^2$ behaviour, while at larger radii all three curves show an approximately exponential falloff dominated by absorption.

## References

[1] S. L. Jacques, "Monte carlo modeling of light transport in tissue," in *Optical-Thermal Response of Laser-Irradiated Tissue*, pp. 109–144, Springer, 2011.