



Monte Carlo Forward Model for Photon Transport

Finite Slab Geometry

Research Project Report

Binger Yu
A01003660
gyu42@my.bcit.ca

Supervisors: *Dr. Takashi Nakamura*

School of Computing and Academic Studies
Master of Science in Applied Computing
British Columbia Institute of Technology
Burnaby, British Columbia, Canada

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1 Model Overview

This report describes the development of a forward Monte Carlo simulator for photon transport in turbid media (`mc_forward_jacques.py` v1.1). The methodology is based on **Chapter 5** of the textbook **Optical-Thermal Response of Laser-Irradiated Tissue** [1] and the standard **MCML** software [2].

The model simulates a **finite slab** of tissue with thickness d , refractive index n_t , and surroundings (air) with index n_{env} . The source is a collimated "pencil beam" incident perpendicular to the surface. This setup calculates the Green's function (impulse response) of the tissue, which can later be convolved to simulate broad beams [2].

1.1 Problem Geometry

- **Medium:** Homogeneous slab of thickness d (occupying $0 \leq z \leq d$).
- **Boundaries:** Air-Tissue interfaces at $z = 0$ (top) and $z = d$ (bottom).
- **Source:** Collimated pencil beam entering at $z = 0$ along the $+z$ axis.

2 Methodology

The simulation follows the standard Hop-Drop-Spin cycle, modified to handle finite boundaries.

2.1 Photon Launch

Photons are launched from the origin $(x, y, z) = (0, 0, 0)$ with direction $(0, 0, 1)$. The initial weight is adjusted for specular reflection (R_{sp}) at the first interface:

$$R_{sp} = \left(\frac{n_{env} - n_t}{n_{env} + n_t} \right)^2, \quad W = 1.0 - R_{sp}$$

2.2 Step Size and Boundary Interaction

The photon step size s is sampled from Beer's Law ($s = -\ln(R)/\mu_t$). In a finite slab, the photon may hit a boundary before completing step s . We calculate the distance to the boundary d_b along the trajectory (Eq 5.34 in [1]):

$$d_b = \begin{cases} (d - z)/u_z & \text{if } u_z > 0 \\ -z/u_z & \text{if } u_z < 0 \end{cases}$$

- If $s < d_b$: The photon moves the full step s .
- If $s > d_b$: The photon moves to the boundary (Eq 5.35). The remaining step $s \leftarrow s - d_b$ is preserved.

2.3 Fresnel Reflection (at Boundaries)

When a photon hits a boundary, the internal reflectance $R(\theta_i)$ is calculated using Fresnel's formulas (Eq 5.36 in [1]):

$$R(\theta_i) = \frac{1}{2} \left[\frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} + \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \right]$$

where θ_t is determined by Snell's Law ($n_t \sin \theta_i = n_{env} \sin \theta_t$). In the code, this is implemented using optimized direction cosines to avoid computationally expensive trigonometric functions.

A random number ξ is drawn:

- If $\xi \leq R(\theta_i)$: The photon **reflects** ($u_z \leftarrow -u_z$) and continues.
- If $\xi > R(\theta_i)$: The photon **transmits** (escapes) and is scored as Reflectance (R_d) or Transmittance (T_t).

2.4 Scattering (Henyey-Greenstein)

The scattering phase function describes the probability density function (PDF) for the cosine of the deflection angle, $\cos \theta$. The physical definition (Eq 3.18 in [1]) involves a power of 3/2:

$$p(\cos \theta) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos \theta)^{3/2}}$$

However, in the Monte Carlo simulation, we sample $\cos \theta$ by inverting the cumulative distribution function (CDF) of this PDF. The resulting sampling formula (Eq 3.19 in [1]) involves a square term rather than a 3/2 power:

$$\cos \theta = \frac{1}{2g} \left[1 + g^2 - \left(\frac{1 - g^2}{1 - g + 2g\xi} \right)^2 \right]$$

This derived formula is what is implemented in the ‘SPIN’ function of the code.

2.5 Absorption and Scoring

Absorption (DROP): At each step, weight is reduced: $\Delta W = W(\mu_a/\mu_t)$.

Scoring: Absorbed energy is accumulated in:

- **Planar Fluence** $F_{pla}(z)$: Fluence rate vs. depth z (Primary metric for slab).
- **Cylindrical Fluence** $F_{cyl}(\rho)$: Radial spread (Point Spread Function).

3 Simulation Results

Figure 1 shows the fluence profiles for a slab with thickness $d = 1.0$ cm, $\mu_a = 1.0$ cm $^{-1}$, $\mu_s = 100$ cm $^{-1}$, $g = 0.9$, and $n_t = 1.33$.

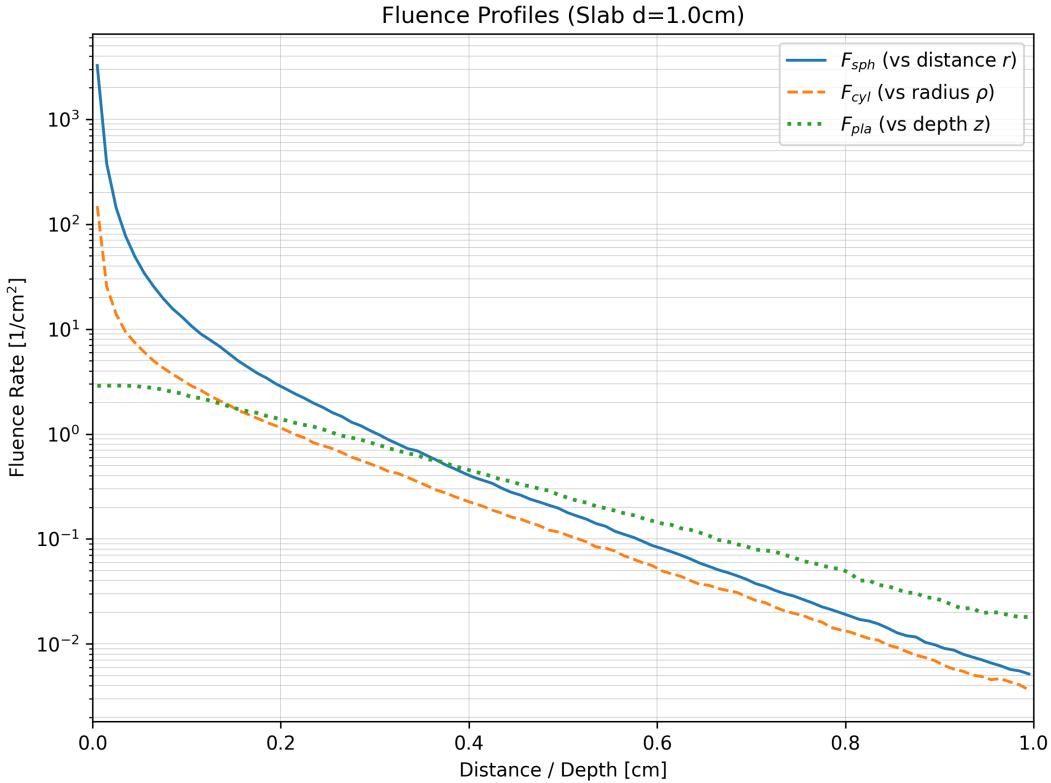


Figure 1: Fluence profiles for a 1.0 cm finite slab illuminated by a collimated pencil beam. F_{pla} (green) shows the exponential attenuation with depth z . F_{sph} (blue) shows the high intensity singularity near the source.

Macroscopic results ($N = 10,000$ photons):

- Diffuse Reflectance (R): 0.2725
- Total Transmittance (T): 0.0031
- Total Absorbed (A): 0.7244
- Energy Conservation ($R + T + A$): 1.0000 (Verified)

The conservation sum of 1.0000 confirms the boundary conditions and Fresnel logic are correctly implemented.

References

- [1] S. L. Jacques, “Monte carlo modeling of light transport in tissue,” in *Optical-Thermal Response of Laser-Irradiated Tissue*, pp. 109–144, Springer, 2011.
- [2] L. Wang and S. L. Jacques, *Monte Carlo Modeling of Light Transport in Multi-layered Tissues in Standard C*. Optical Imaging Laboratory, Oregon Medical Laser Center, US, Portland, 1992. Online; accessed 2025.