Section 1.4, 1.5

Predicate and Quantifier

- Predicate – a property that the subject of the statement can have
- Propositions can be of the form

P(x)

- x is a variable that comes from the domain
- P is a predicate (something we can say about x)
- Quantifiers to say something more about the domain, $\forall$, $\exists$
  - $\exists$ – "exists", existential quantifier
  - $\forall$ – "all", universal quantifier
- **Note:** Propositional functions P(·) are generalizations. They become full propositions when input a variable P(x), where x is from the specified domain.

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

DeMorgan's Law for Quantifiers

- $\neg\exists x P(x) \equiv \forall x \neg P(x)$
- $\neg\forall x P(x) \equiv \exists x \neg P(x)$

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Example: Let the domain be all students in your ICS 141 section. Let $Z(x)$ = "is currently in the Zoom call". Translate $\forall x Z(x)$ into simple English.

- Every student in section 3 is in the zoom call
- $\neg \forall x Z(x) \equiv \exists x \neg Z(x)$ (some student is not in the zoom call)

Some stuff to remember:

- Order of quantifiers matter! $\forall x \exists y P(x, y) \mathrel{!=} \exists y \forall x P(x, y)$
- Negating nested quantifiers is done in succession

Example Let the domain be the set of real numbers. Translate the following nested quantifiers into English. Then, negate the statement, and translate the negated statement into English.

$$\forall x \forall y \exists z (xy = z)$$

There are real numbers x and y so that xy = z, for some real numbers z .

$$\begin{aligned}\neg(\forall x \forall y \exists z (xy = z)) &\equiv \exists x \exists y \forall z \neg(xy = z)\\ &\equiv \exists x \exists y \forall z\, (xy \neq z)\end{aligned}$$

There are two real numbers, x and y, so that for any real number z, xy is not equal to z.

Example Let the domain be all students on campus. Let $C(x)$ ="is a student in ICS 141", $G(x)$ ="bought Gamestop stocks", and $A(x)$ ="bought AMC stocks". Express this statement using quantifiers: No one in ICS 141 bought Gamestop or AMC stocks.

No one: "There does not exists"

$$\begin{aligned}\neg \exists x\, (C(x) \rightarrow G(x) \vee A(x)) &\equiv \forall x \neg(C(x) \rightarrow G(x) \vee A(x))\\ &\equiv \forall x\, C(x) \wedge \neg(G(x) \vee A(x))\\ &\equiv \forall x\, C(x) \wedge \neg G(x) \wedge \neg A(x)\end{aligned}$$

Rules of Inference

- argument – sequence of statements that end with a conclusion
- premises – final proposition
- valid – the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises
- fallacy – an argument that is not valid
- argument form
  - a sequence of propositions involving propositional variables.
  - It is valid no matter what propositions are substituted into its propositional variables.

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Example: Use a rule of inference to arrive at a valid conclusion. Here are the given statements: "Chad is a math major or a computer science major. Chad is not a computer science major."

p q ¬q

Given: $p \vee q$ Conclusion: p - Chad is a math major

$\underline{\neg q}$

Conclusion: p Disjunctive syllogism

p

Example Use a rule of inference to arrive at a valid conclusion: "If I do all my homework, then I will do well on the midterm. If I do well on the midterm, then I will do well in the class."

q q r

Given: $p \rightarrow q$

$q \rightarrow r$

Conclusion $p \rightarrow r$ Hypothetical syllogism