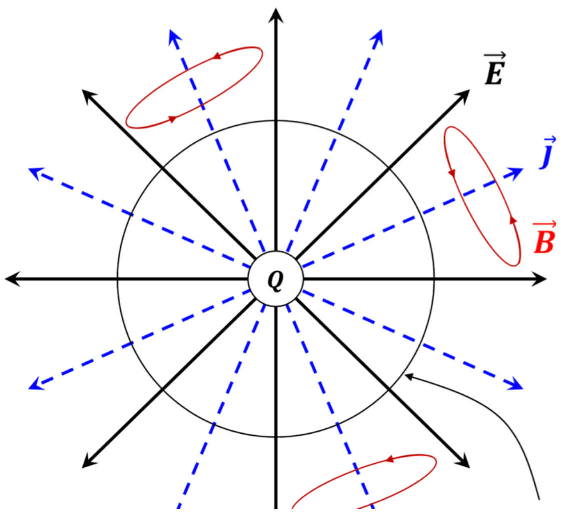
Electromagnetic Waves

I. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}} + \boxed{\dfrac{\mathbf{1}}{\boldsymbol{c}^2} \dfrac{\partial \vec{E}}{\partial t}}$

- Charge conservation law Or continuity equation $\boxed{\vec{\nabla}.\vec{\boldsymbol{j}} = -\dfrac{\partial \rho}{\partial t}}$

- Example



The charge source $Q(r, t)$ is leaky radially (symmetrically)

$$\frac{\partial Q(r,t)}{\partial t} = -4\pi r^2 j(r) \qquad E(r,t) = \frac{Q(r,t)}{4\pi\varepsilon_0 r^2}$$

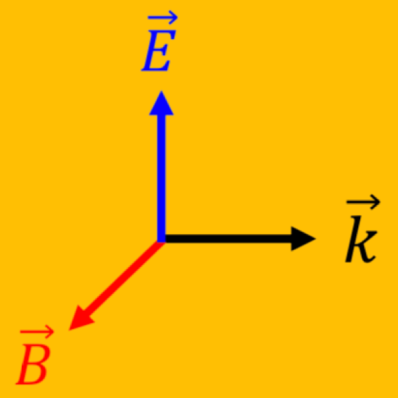
$$\frac{\partial E}{\partial t} = \frac{1}{4\pi\varepsilon_0 r^2} \frac{\partial Q(r,t)}{\partial t} = -\frac{j(r)}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}} + \frac{\mathbf{1}}{\boldsymbol{c}^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{\boldsymbol{j}} - \frac{\vec{\boldsymbol{j}}}{\varepsilon_0 \boldsymbol{c}^2} = \vec{0} \quad \Rightarrow \quad \boxed{\vec{B} = \vec{0}}$$

II. EM Waves

$$\frac{\partial B}{\partial t} \rightarrow \text{ Source of } E(x,t)$$

$$\frac{\partial E}{\partial t} \rightarrow \text{ Source of } B(x,t)$$



$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2}\vec{E}$$

$$\vec{k} \times \vec{E} = \omega\vec{B}$$

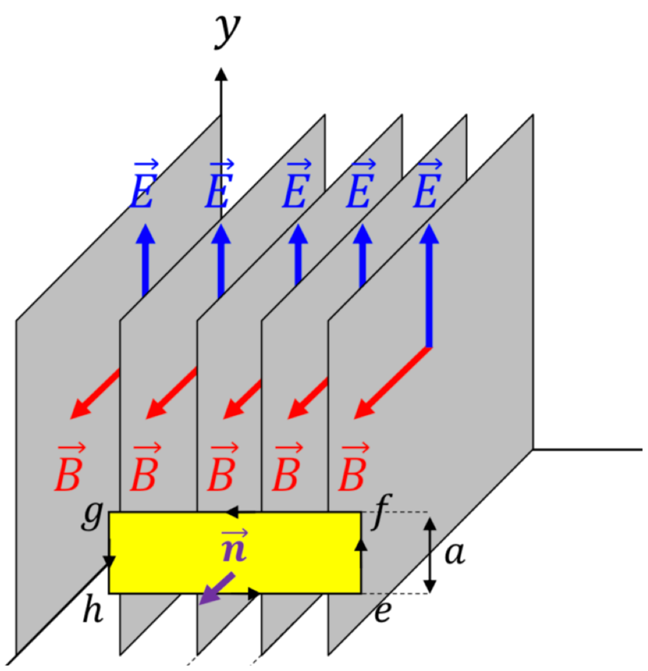
Orthogonality

$$\vec{E} = \left(E_{0x}\hat{\imath} + E_{0y}\hat{\jmath} + E_{0z}\hat{k}\right)e^{[i(k_x x + k_y y + k_z z - \omega t)]}$$

$$\vec{B} = \left(B_{0x}\hat{\imath} + B_{0y}\hat{\jmath} + B_{0z}\hat{k}\right)e^{[i(k_x x + k_y y + k_z z - \omega t)]}$$

III. One major property

$$\oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt}$$



$$\frac{d\Phi_B}{dt} = Bac$$

Along gh: $\int_g^h \vec{E}.d\vec{l} = -Ea$

$$\oint \vec{E}.d\vec{l} = -Ea = -\frac{d\Phi_B}{dt}$$

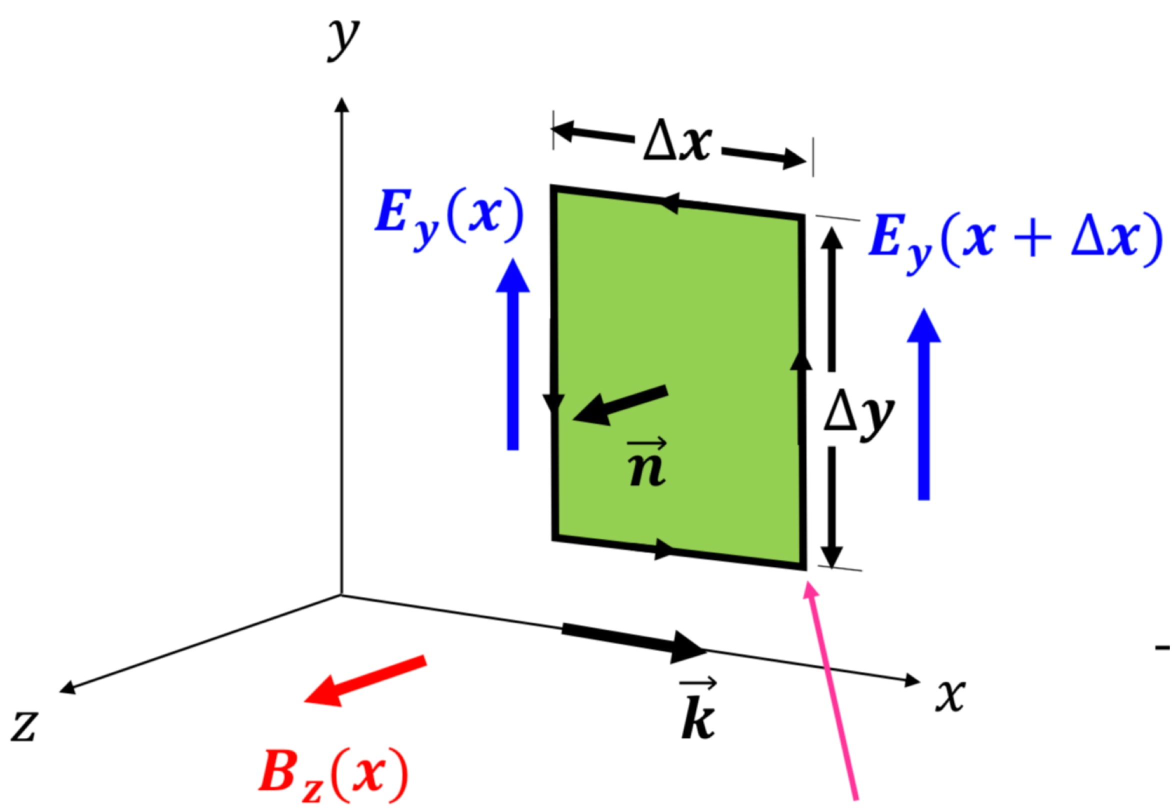
$$E = Bc$$

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\frac{F_B}{F_E} = \frac{vB}{E} = \frac{v}{c}$$

IV. Wave Equations

$$\oint \vec{E}.d\vec{l} = \boxed{-\frac{d}{dt}\iint \vec{B}.d\vec{A}}$$



$$\oint \vec{E}.d\vec{l} = E_y(x+\Delta x)\Delta y - E_y(x)\Delta y$$

$$E_y(x+\Delta x) = E_y(x) + \frac{\partial E_y}{\partial x}\Delta x + \cdots$$

$$\oint \vec{E}.d\vec{l} = \boxed{\frac{\partial E_y}{\partial x}\Delta x \Delta y}$$

$$-\frac{d}{dt}\iint \vec{B}.d\vec{A} = \boxed{-\frac{\partial B_z}{\partial t}\Delta x \Delta y}$$

-

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

Similarly, $-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial x}\right) = -\frac{\partial^2 B_z}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}}$$

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

$$-\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial x}\right)$$

$$-\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}}$$

Mechanical wave

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

V. Energy

- We already know that both E and B fields carry energy

$u =$ Total energy density

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2}$$

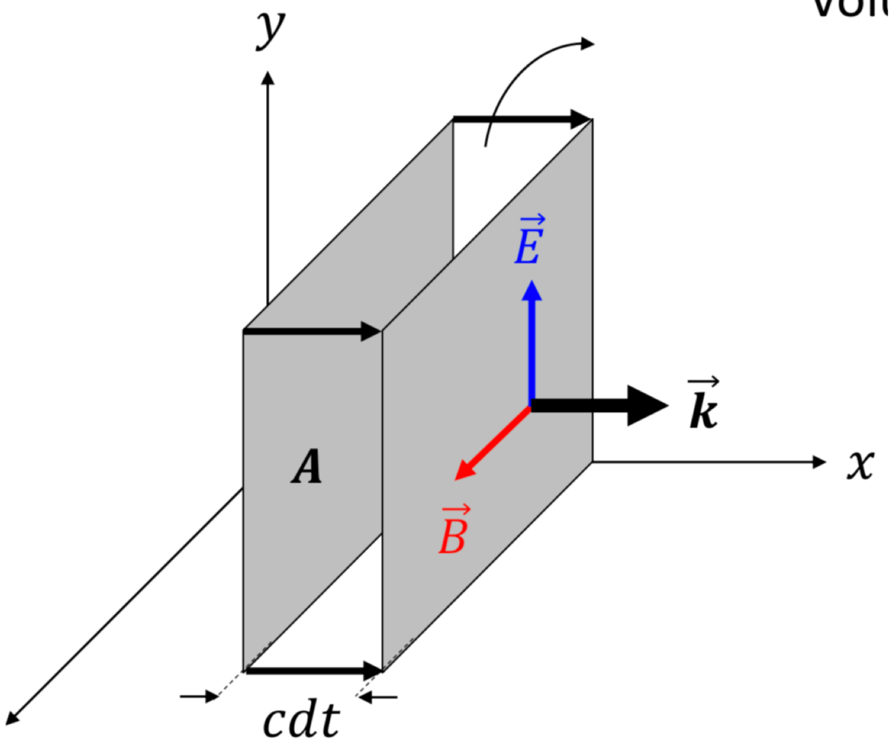
$$\boxed{u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

VI. Poynting Vector; Energy flow/unit area/unit time; Power/unit area

$$dW = \mathrm{udV} = (\varepsilon_0 E^2)(Acdt)$$

$$S = \frac{1}{A}\frac{dW}{dt} = \varepsilon_0 c E^2 = \varepsilon_0 (cE)E = \varepsilon_0 (c\mathbf{E})c\,\frac{\mathbf{E}}{\mathbf{c}} = \varepsilon_0 (c\mathbf{E})c\mathbf{B} = \varepsilon_0 c^2 \mathbf{E}\mathbf{B}$$

Volı



$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}}$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}}$$

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t) = \frac{1}{\mu_0}[\hat{j}E_{max}cos(kx - \omega t)] \times \left[\hat{k}B_{max}cos(kx - \omega t)\right]$$

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}cos^2(kx - \omega t) = \frac{E_{max}B_{max}}{2\mu_0}[1 + cos2(kx - \omega t)]$$

$$\boxed{S_{av} = \frac{E_{max}B_{max}}{2\mu_0}}$$

$$\boxed{I = S_{av} = \frac{1}{2}\varepsilon_0 c E_{max}^2}$$

Vacuum

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = u_E + u_B$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$I = S_{av} = \frac{1}{2}\varepsilon_0 c E_{max}^2$$

Dielectric (linear homogeneous isotropic)

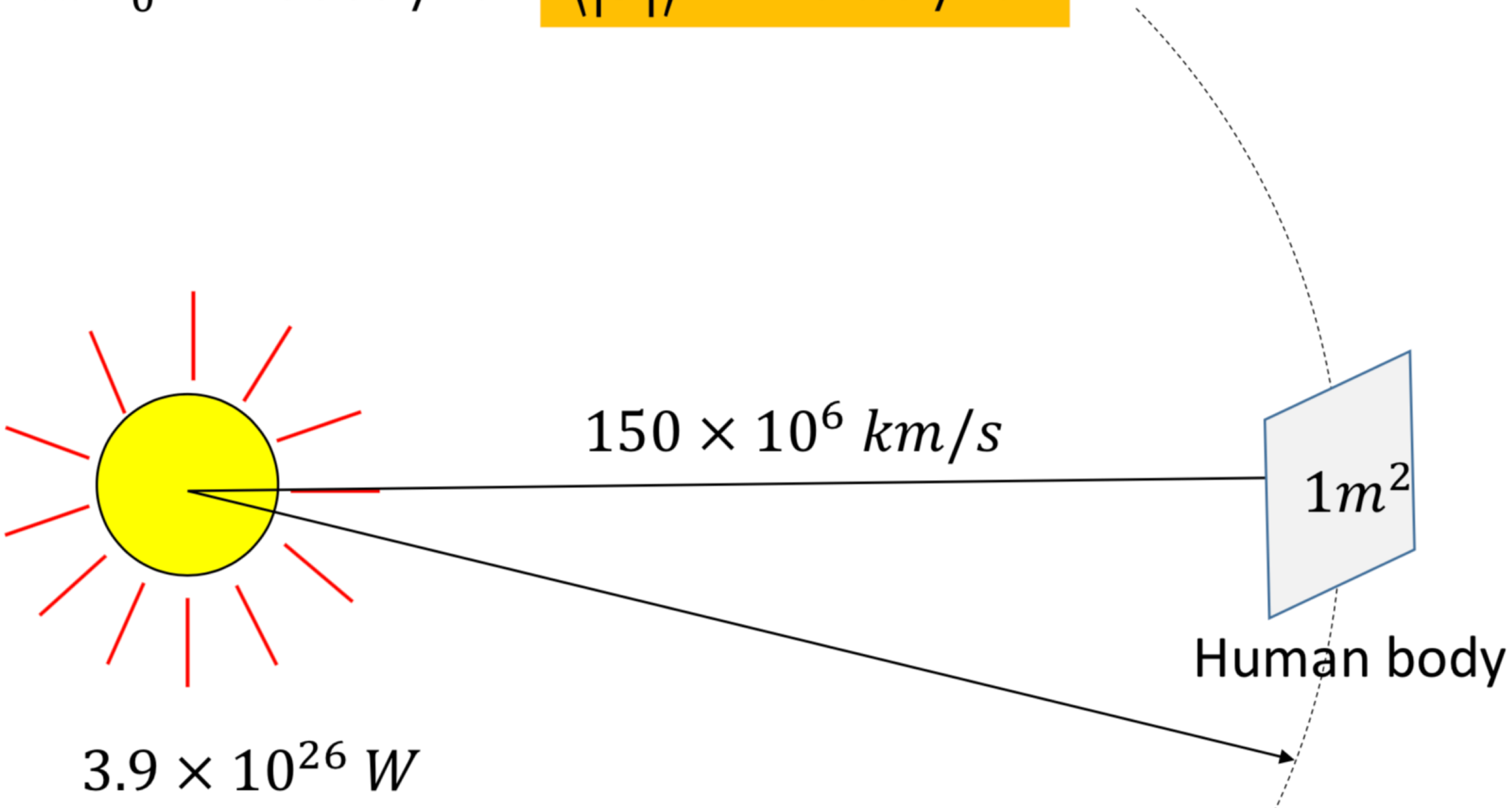
$$\varepsilon_0 \to \varepsilon = \varepsilon_0 \varepsilon_r$$

$$\mu_0 \to \mu = \mu_0 \mu_r$$

$$c \to v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad E = vB \qquad B = \varepsilon\mu v B$$

Ex: $E_0 = 1000V/m$ $\boxed{\langle|\vec{S}|\rangle = 1.3kW/m^2}$

$$\langle|\vec{S}|\rangle = \frac{3.9\times10^{26}W}{(150\times10^{9})^{2}m^{2}}$$



$\boxed{\langle|\vec{S}|\rangle = 1.4kW/m^2}$

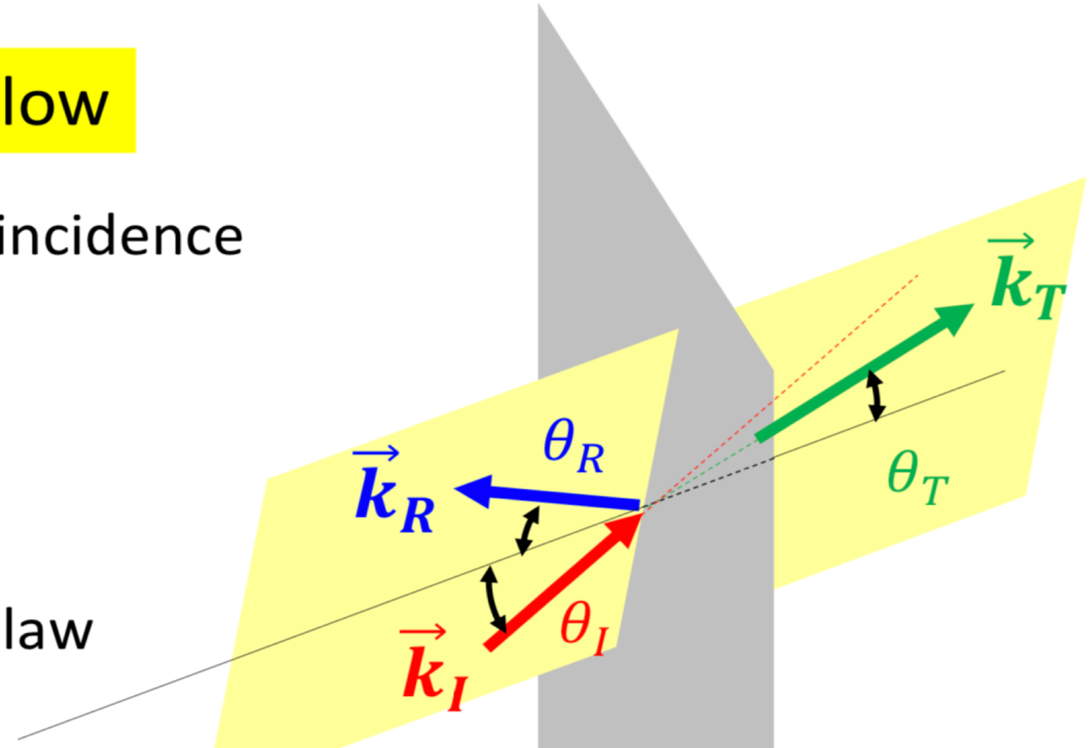
Exposing to sun rays could be very dangerous

$$\boxed{I = \frac{Power}{Area} = \frac{P_S}{4\pi r^2}}$$

VII. Incidence, Reflection and Transmission

Three laws follow

1) $\vec{k}_I, \vec{k}_R$ *and* $\vec{k}_T$ form a single plane: plane of incidence



2) $\theta_I = \theta_R$ Law of reflection or Fermat's law

3) $\dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{n_1}{n_2}$ Law of refraction