Magnetostatic: What next beyond Biot & Savart law?

The two Maxwell equations for magnetostatic

Biot & Savart and Ampere's law ⇨ $\vec{B}$

$$\vec{\nabla}.\vec{B} = 0$$

Magnetic monopole does not exist *

$\vec{\nabla} \times \vec{B}_{Biot\&Savart}$*

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

If there is no source of current (mobile charges)

$$\boxed{\vec{\nabla} \times \vec{B} = \vec{0}}$$

Waiting Maxwell's correction

* See I_Lecture 15&16_Magnetostatic

Coulomb versus Biot & Savart law

Electrostatic

$$\vec{E} = \frac{1}{4\pi\boldsymbol{\varepsilon_0}}\frac{q}{r^2}\vec{e}_r$$

Derived from experiments

Magnetostatic

$$\vec{B} = \frac{\boldsymbol{\mu_0}}{4\pi}\frac{q}{r^2}\vec{v}\times\vec{e}_r$$

Constant $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$

Deduced from measurements involving

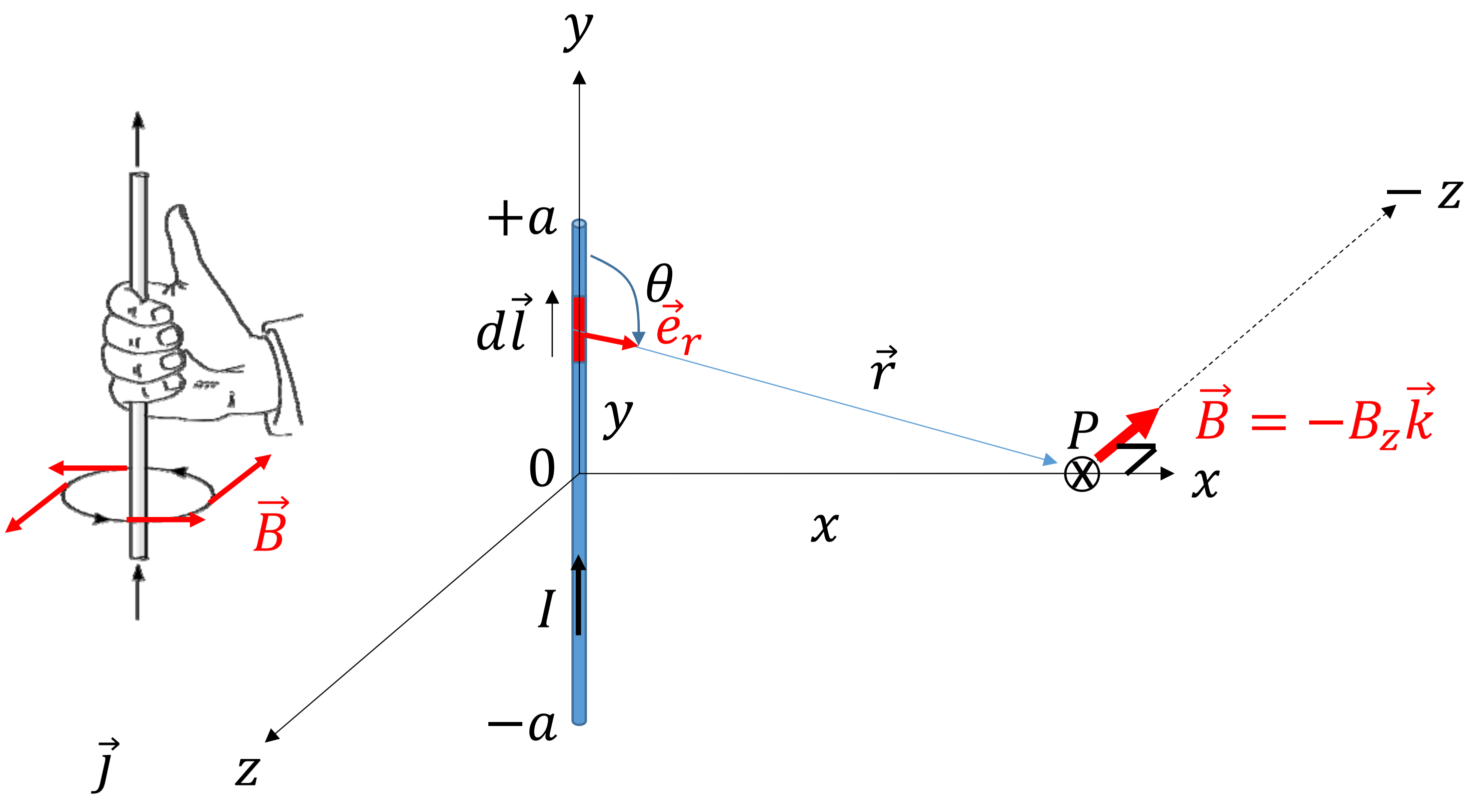
- Charged spheres
- Batteries
- wires

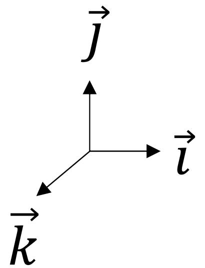
*Measurements having nothing to do with **light** and **electromagnetic waves***

Maxwell $\frac{1}{c^2} = \varepsilon_0\mu_0$

Applications of Biot & Savart law

Magnetic field of a wire $2a$ long





Right hand rule ⇨ $B_y = 0$

$\vec{B}$ lies in the plan zOx

At point P, $\vec{B} \perp x$ −axis

$B_x = 0$

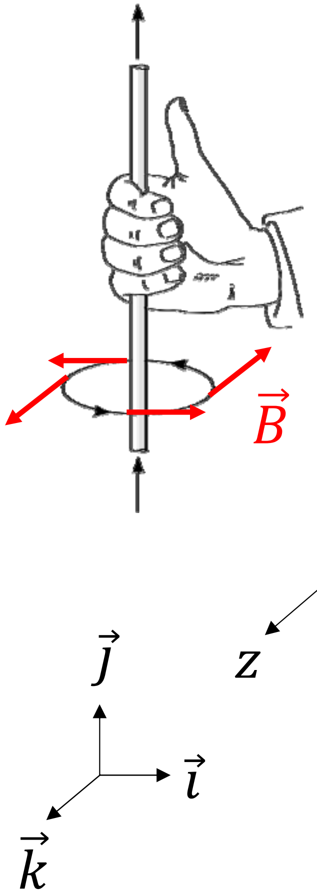
$$\vec{\boldsymbol{B}} = -\boldsymbol{B_z}\vec{\boldsymbol{k}}$$

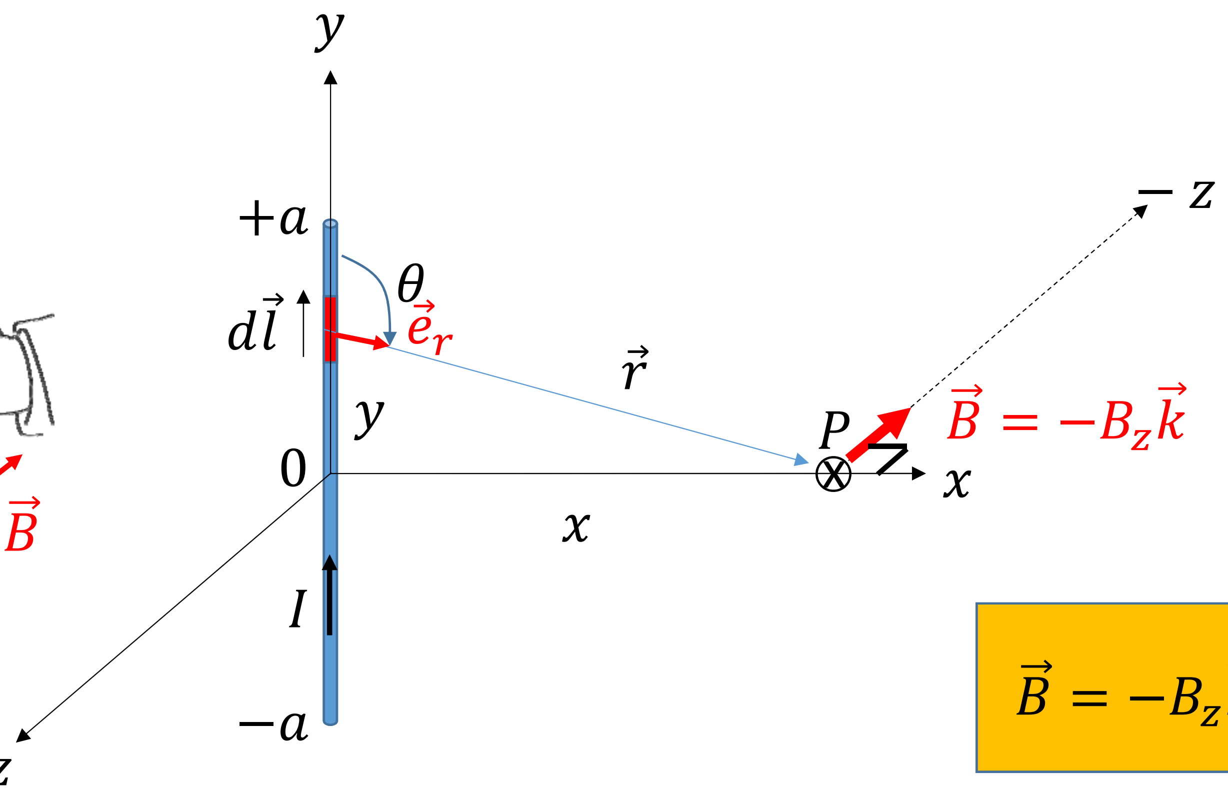
$$d\vec{l} = 0\vec{i} + \mathrm{dy}\vec{j} + 0\vec{k}$$

$$\vec{e}_r = \sin(\pi - \theta)\vec{i} + cos\theta\vec{j} + 0\vec{k}$$

Cross product

Magnetic field of a wire $2a$ long





$$d\vec{l} \times \vec{e}_r = -\sin\theta dy\vec{k}$$

$$r = \sqrt{x^2 + y^2}$$

Biot & Savart

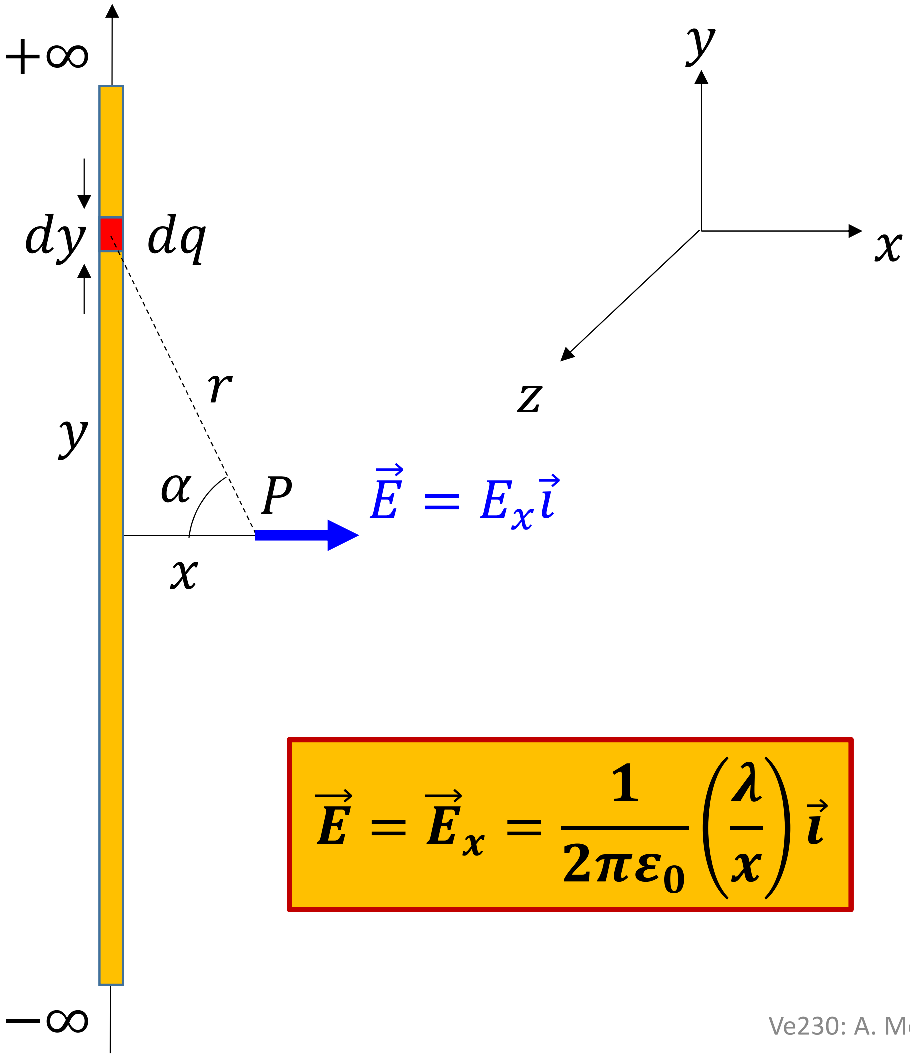
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I}{r^2} d\vec{l} \times \vec{e}_r$$

$$\vec{B} = -B_z\vec{k} \Rightarrow \vec{B} = -\frac{\mu_0 I}{4\pi x}\frac{2a}{\sqrt{x^2+a^2}}\vec{k}$$

$\boldsymbol{2a \gg x}$

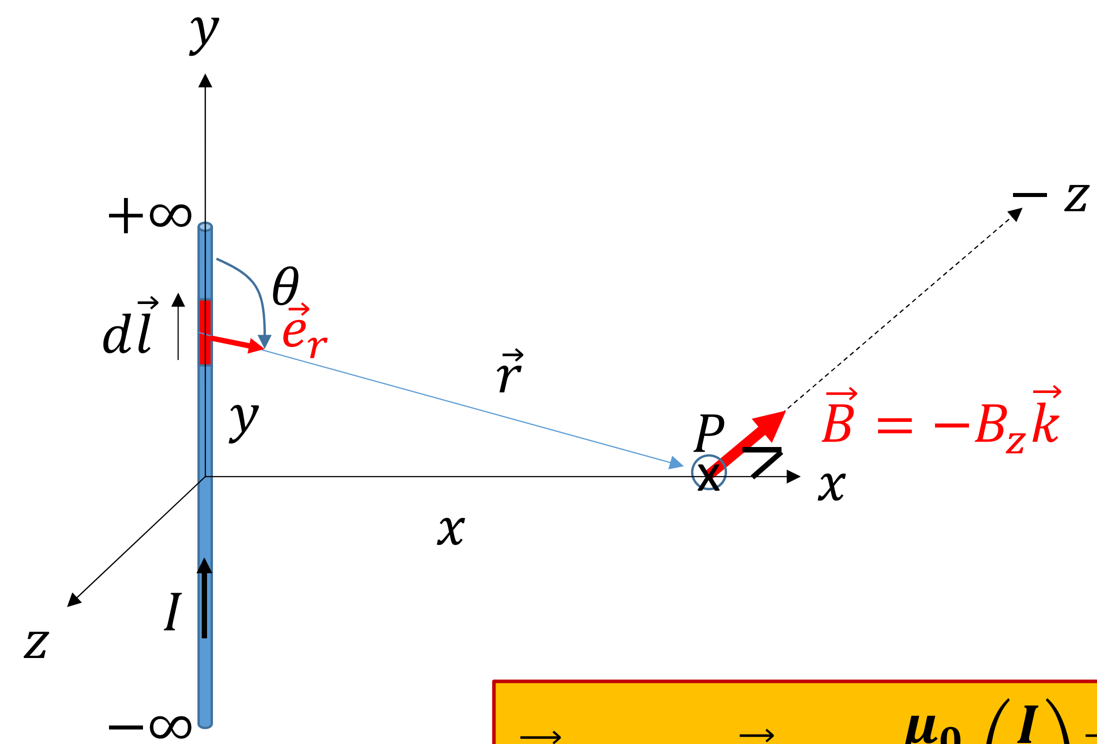
$$B = B_z(P) = \frac{\mu_0 I}{2\pi x}$$

Static charges along an infinite wire



$$\boxed{\vec{E} = \vec{E}_x = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{x}\right)\vec{\imath}}$$

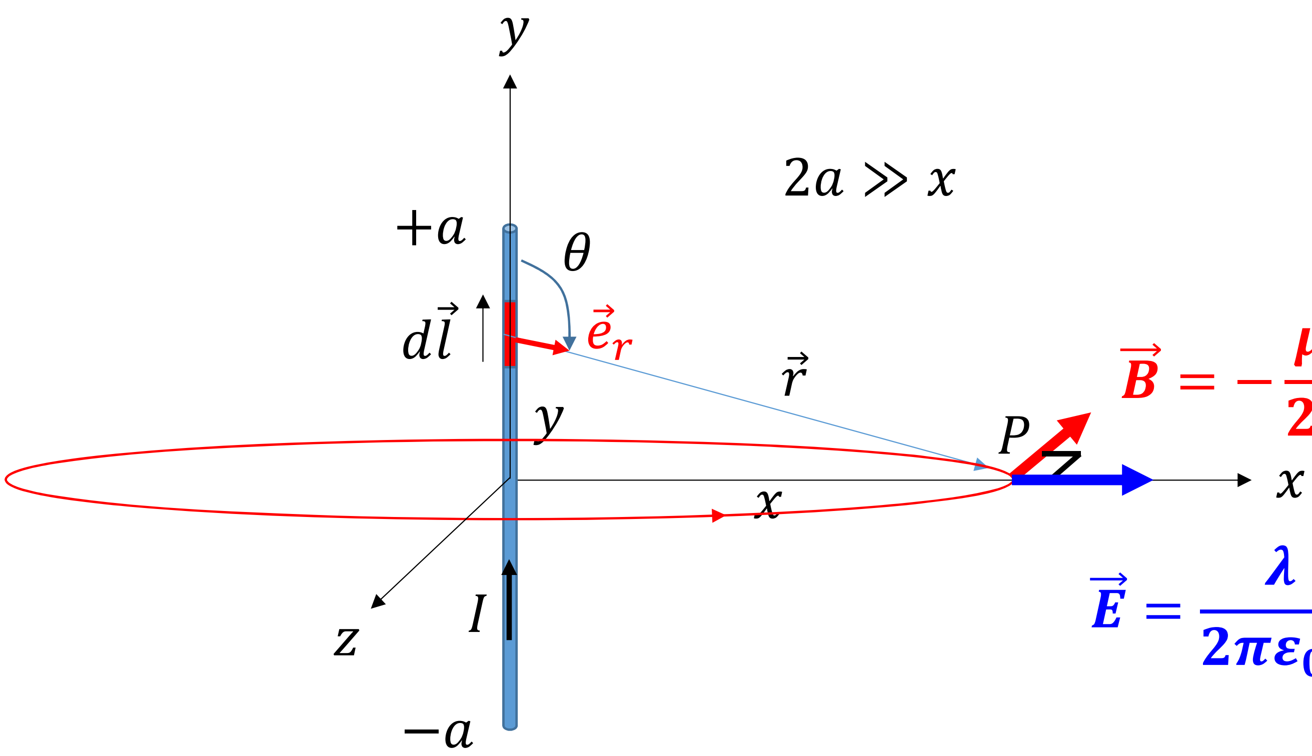
Steady moving charges along an infinite wire



$$\boxed{\vec{B} = -B_z\vec{k} = -\frac{\mu_0}{2\pi}\left(\frac{I}{x}\right)\vec{k}}$$

Electrostatic and Magnetostatic at once

Charges moving upwards along a conducting wire

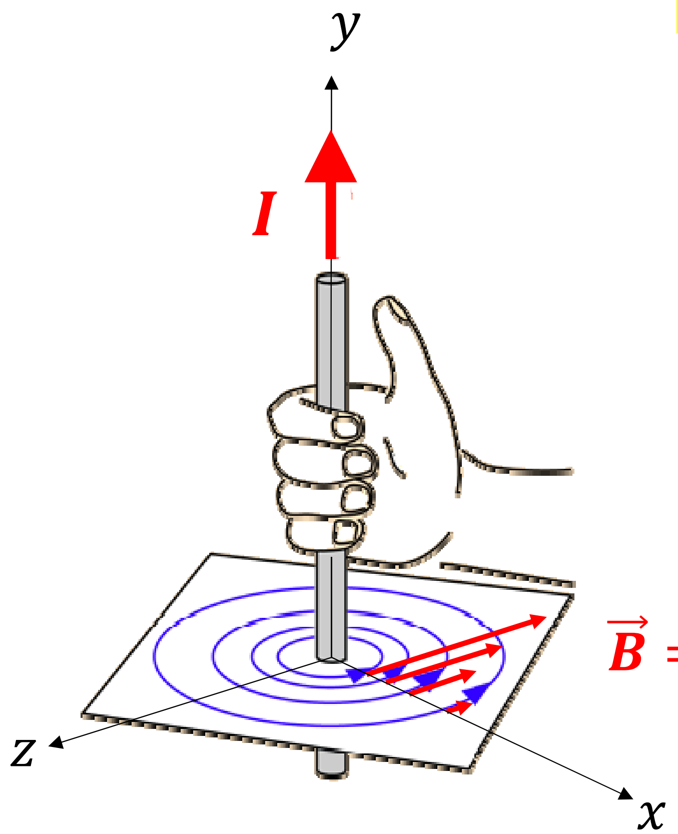


$$\vec{B} = -\frac{\mu_0 I}{2\pi x}\vec{k}$$

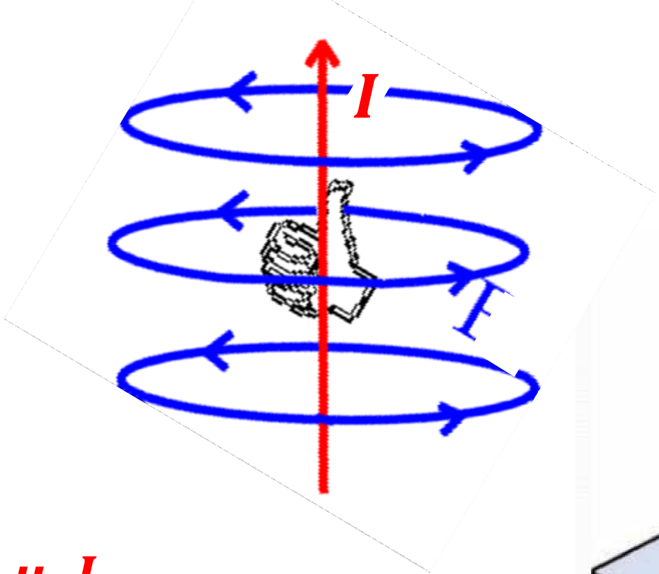
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}\vec{i}$$

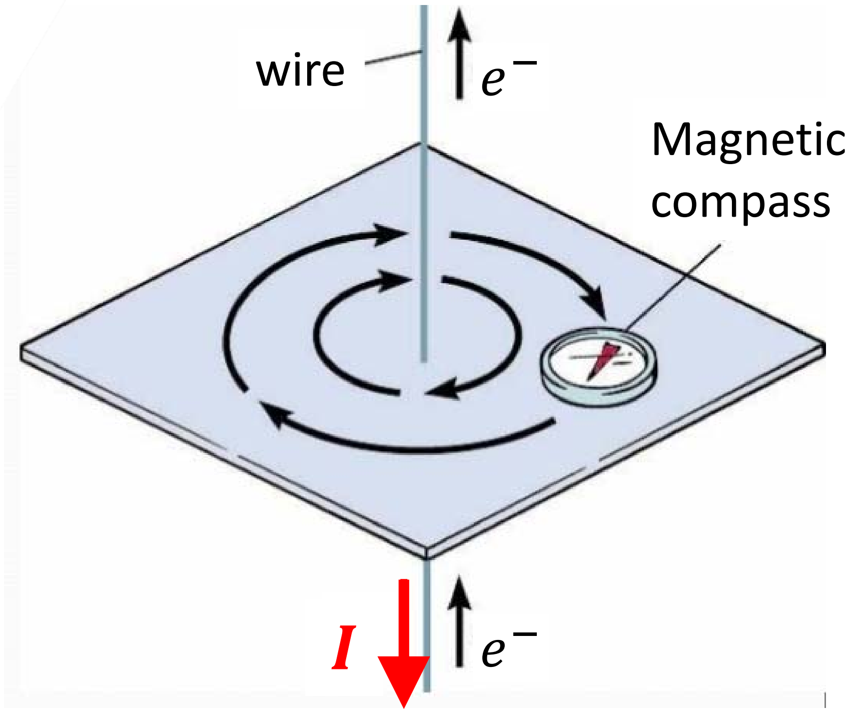
$$\vec{E} \perp \vec{B}$$

How the field lines look like ?

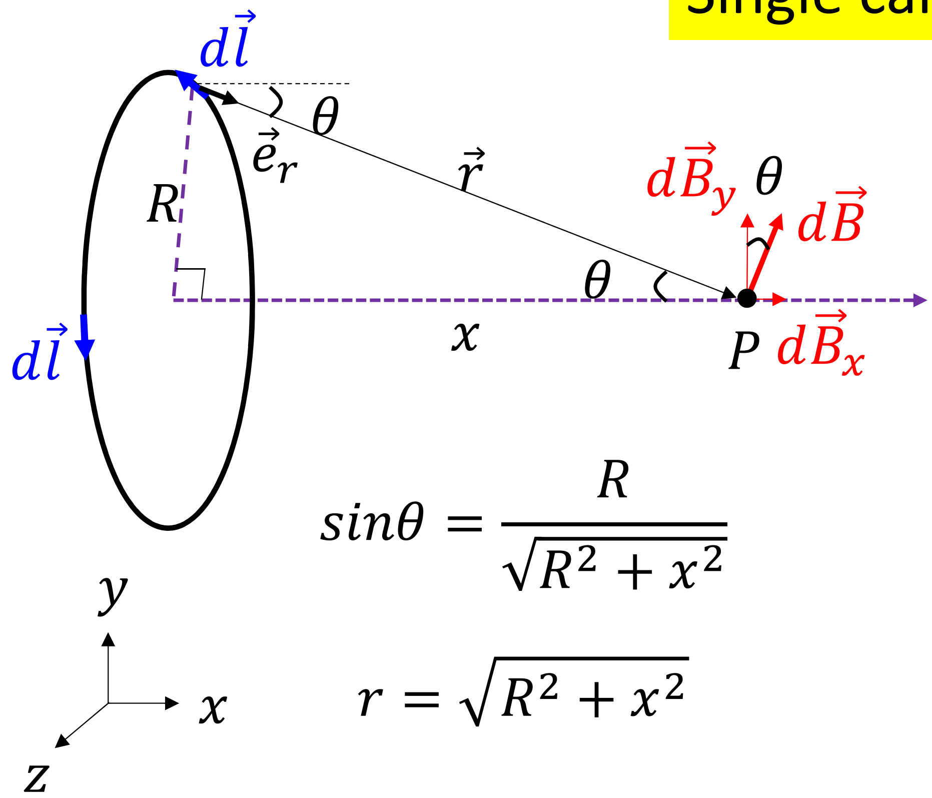


$$\vec{B} = -\frac{\mu_0 I}{2\pi x}\vec{k}$$





Single carrying current loop



$$\sin\theta = \frac{R}{\sqrt{R^2 + x^2}}$$

$$r = \sqrt{R^2 + x^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2} d\vec{l} \times \vec{e}_r \qquad d\vec{l} \perp \vec{e}_r$$

$$d\vec{B} = d\vec{B}_x + d\vec{B}_y$$

By symmetry component along y –axis cancels

$$dB_x = dB\sin(\theta) \Rightarrow B_x = \frac{\mu_0}{4\pi}\oint \frac{I}{r^2} dl\sin\theta$$

$$\Gamma = 2\pi R$$

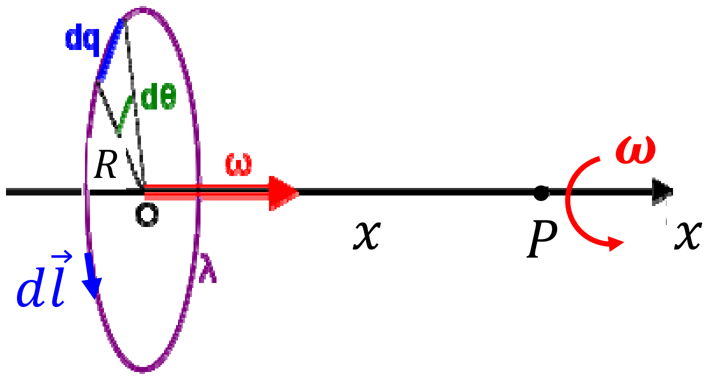
Magnetic field along x –axis $\quad \vec{B}_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}\vec{i}$

At the center of the loop $(x = 0)$

$$\vec{B}_x = \frac{\mu_0 I}{2R}\vec{i}$$

Charged loop: mechanical rotation

A closed loop of radius $\boldsymbol{R}$ rotates mechanically around the z-axis at constant angular speed $\boldsymbol{\omega}$. The loop is charged with a uniform linear charge $\boldsymbol{\lambda}$. **What is the magnetic field along x ?**



The key issue is to connect $\boldsymbol{I}$ to $\boldsymbol{\omega}$

$$\left.\begin{aligned} I &= \frac{dq}{dt} \\ dq &= \lambda dl = \lambda \underbrace{R d\theta}_{dl} \end{aligned}\right] \quad \boldsymbol{I} = \frac{dq}{dt} = \lambda R \frac{d\theta}{dt} = \boldsymbol{\lambda R \omega}$$

$$\vec{B}_x = \frac{\mu_0 \boldsymbol{I} R^2}{2(R^2 + x^2)^{3/2}} \vec{\imath} \quad \Rightarrow \quad \boxed{\vec{B}_x = \frac{\mu_0 (\boldsymbol{\lambda \omega R})\, R^2}{2(R^2 + x^2)^{3/2}} \vec{\imath}}$$

Interaction of wires conducting electric current with magnetic field

Lorentz force
(single charge q)

$$\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz force
(charge element dq)

$$d\vec{F} = dq\vec{v} \times \vec{B}$$

$$dq\vec{v} = nvAd\vec{l} = Id\vec{l}$$

$$dq = nAdl$$

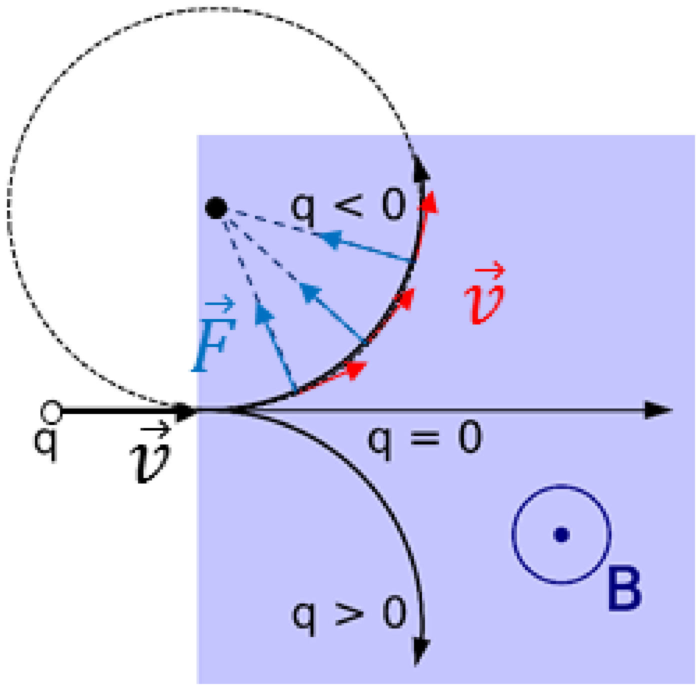
charges/unit volume

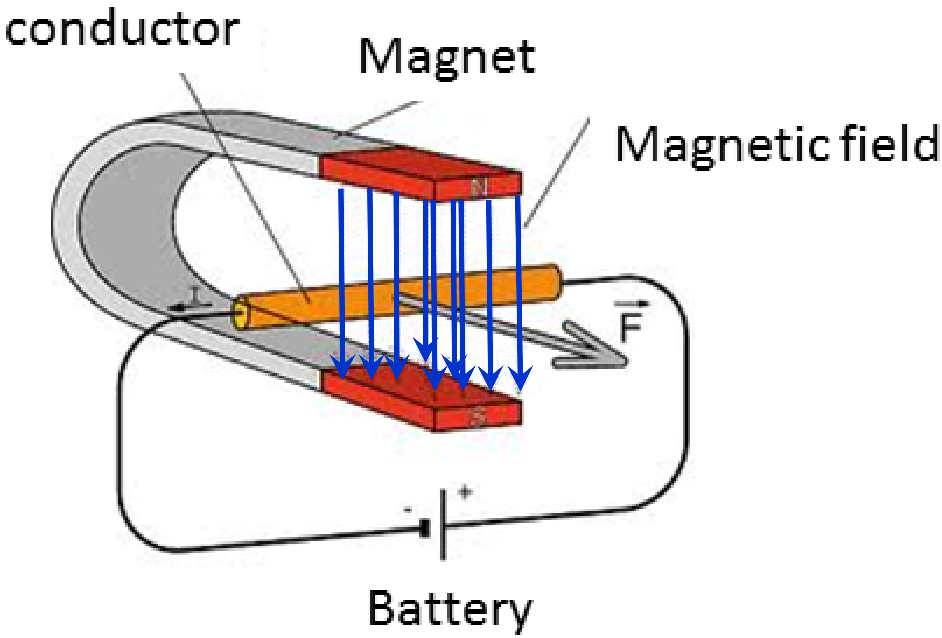
Laplace force
(wire carrying current)

$$\boxed{d\vec{F} = Id\vec{l} \times \vec{B}}$$

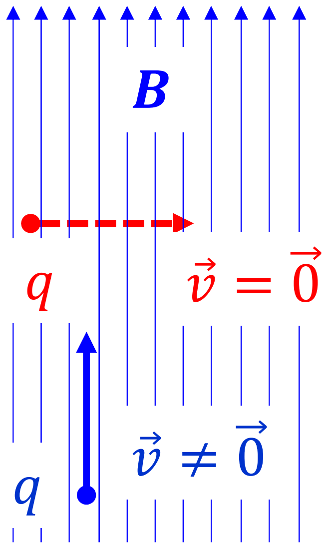
Lorentz force: $\vec{F} = q\vec{v} \times \vec{B}$

Laplace force: $\vec{F} = I\vec{l} \times \vec{B}$





Case where Lorentz and Laplace forces are zero

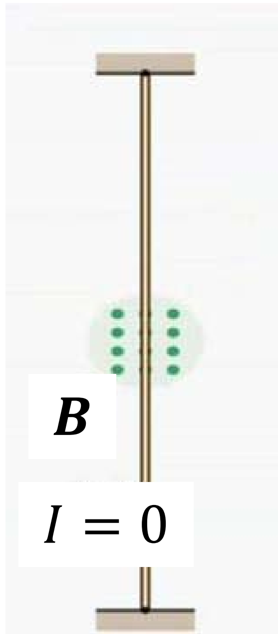


$$\vec{F} = q\vec{v} \times \vec{B} = \vec{0}$$

$\vec{v} = \vec{0}$

or

$\vec{v} \neq \vec{0}$ but $\vec{v} \parallel \vec{B}$



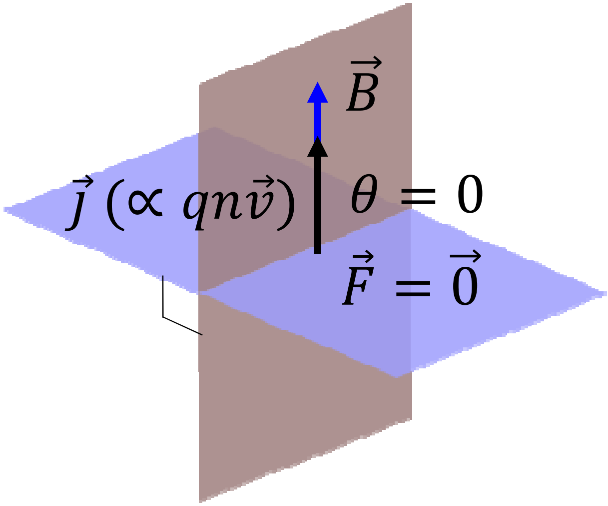
$$\vec{F} = I\vec{l} \times \vec{B} = \vec{0}$$

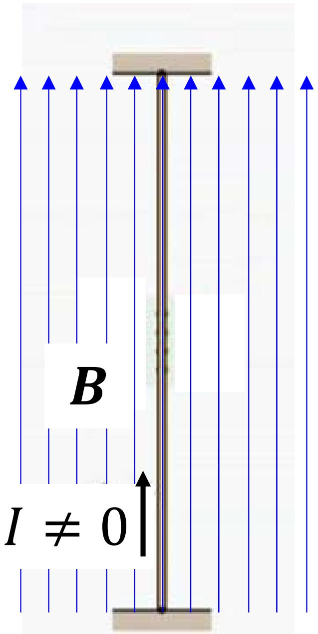
$I = 0$

or

$I \neq 0$ but

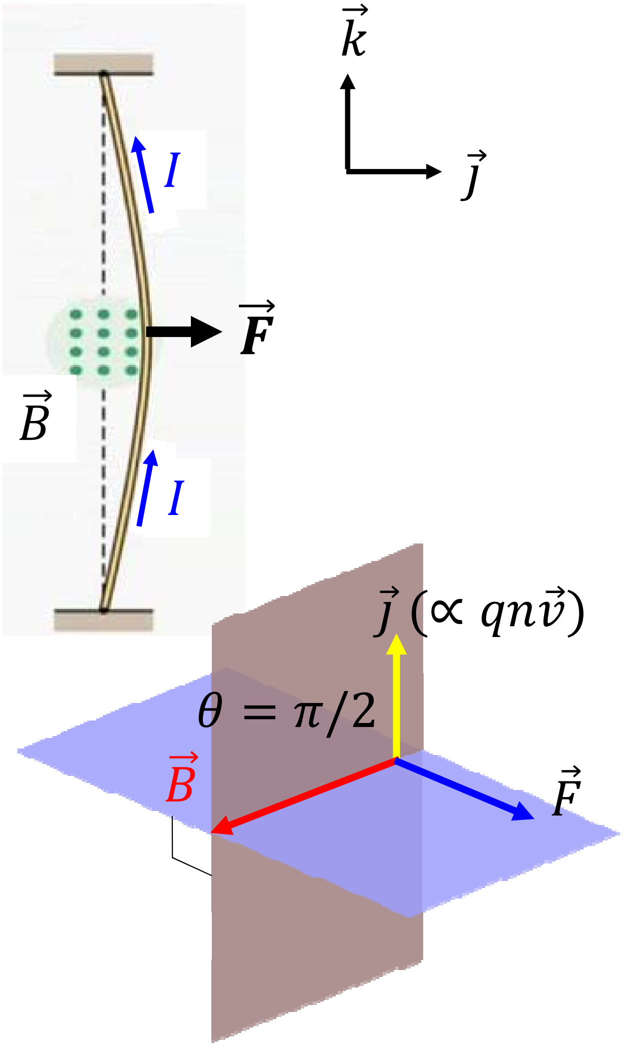
$\vec{l} \parallel \vec{B}$





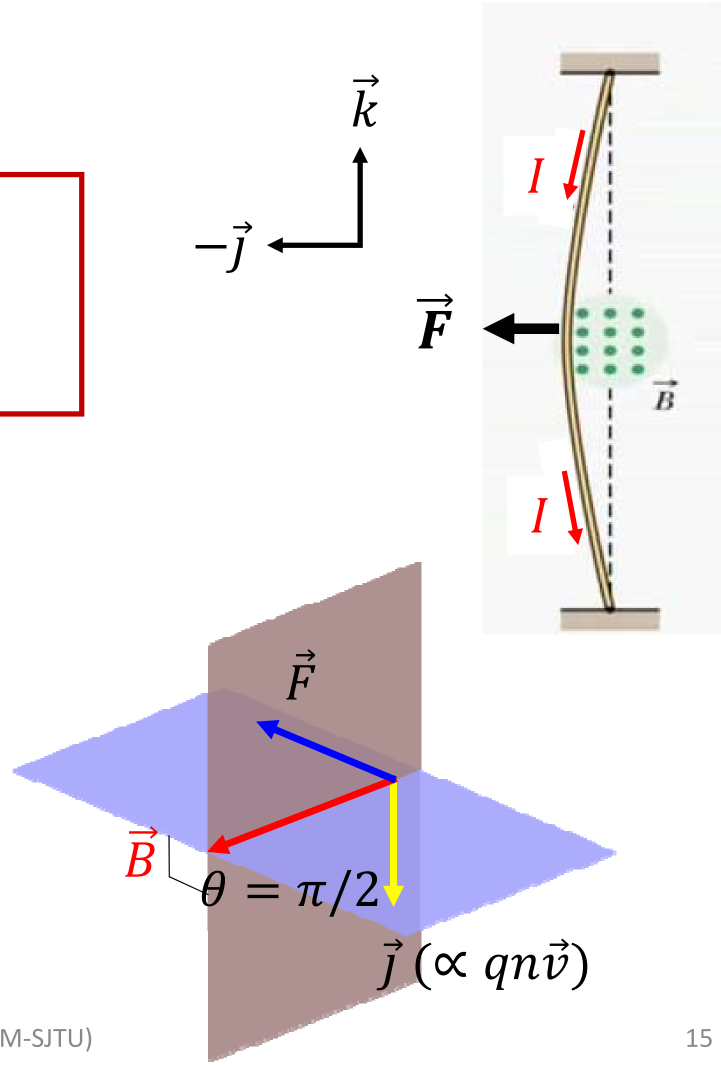
From University of physics (11rd edition)

Magnetic force on current NOT parallel to $\vec{B}$

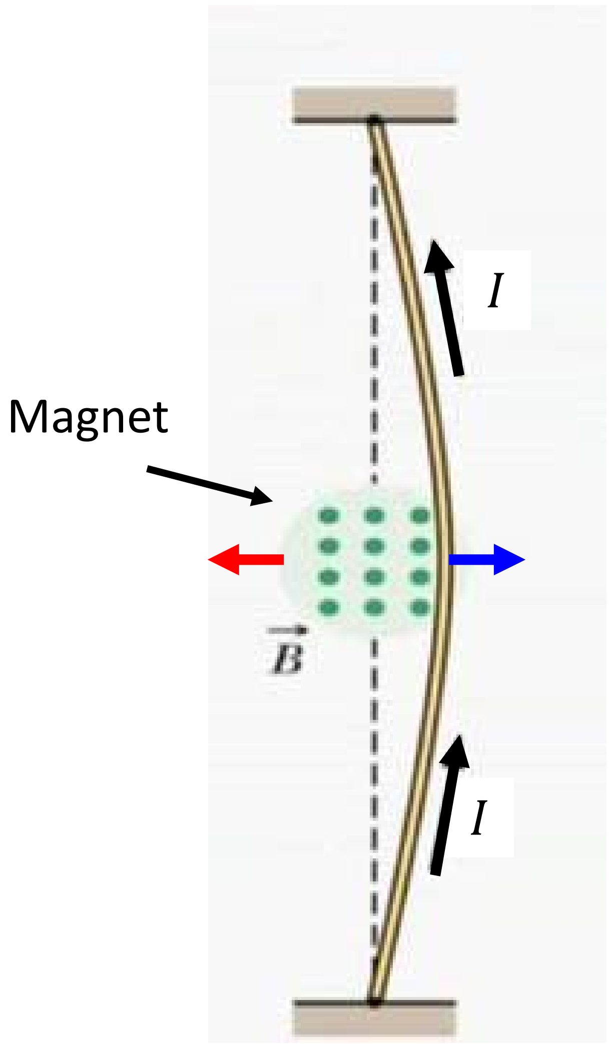


Laplace force
(wire carrying current)

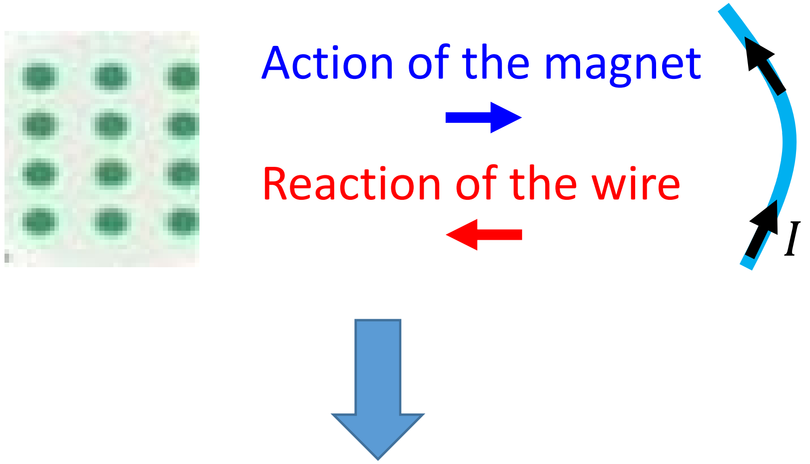
$$d\vec{F} = I d\vec{l} \times \vec{B}$$



The action – reaction principle



Action of the magnet on the wire where charges are flowing

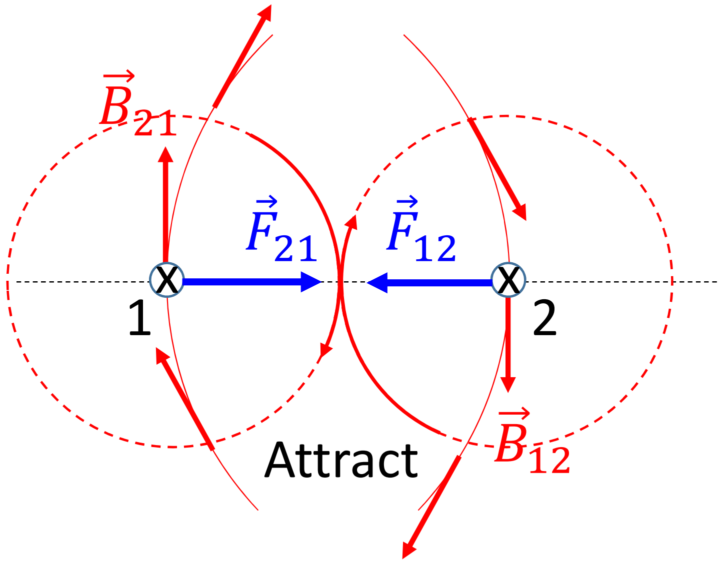


Two magnetic fields

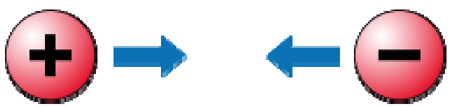
- One due to the magnet
- One due to the current in the wire

Two parallel wires carrying currents

Currents in the same direction

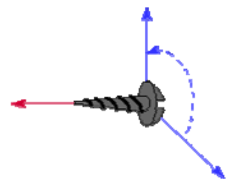


Opposite charges



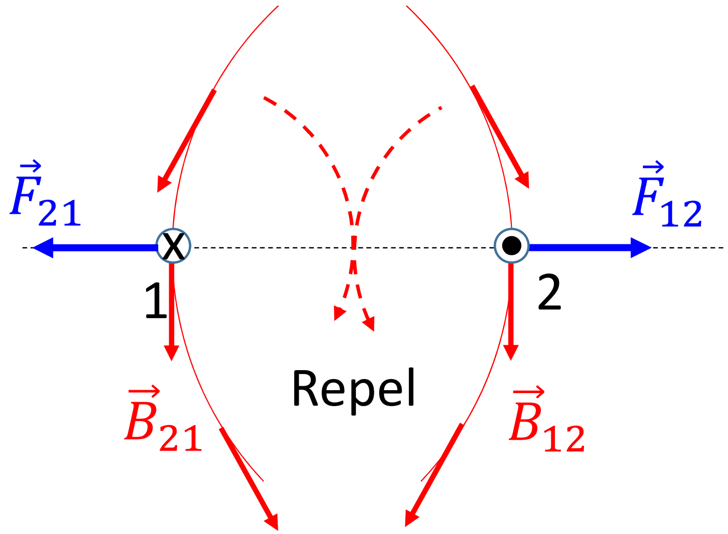
Determine the direction of $\vec{B}$

+



Determine the direction of $\vec{F}$

Currents in opposite direction



Alike charges

Ampere's law

From Biot & Savart law...

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I}{r^2} d\vec{l} \times \vec{e}_r}$$

...To ampere's law

Slides #40 I_Lecture 15&16_Magnetostatic

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\vec{\nabla} \times \left(\int \mathrm{d}\vec{l'} \times \frac{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')}{(\vec{\mathrm{r}} - \vec{\mathrm{r}}')^3}\right) \Rightarrow \boxed{\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})}$$

Slides #45-47 I_Lecture 15&16_Magnetostatic

Ampere's law is for magnetostatic what Gauss law is for electrostatic

$\Downarrow$

It exploits symmetry in relating $\vec{B}$ to source (current)

Electrostatic

Calculating electric field produced by symmetric charge distribution

Gauss Law

- Closed surface
- Flux through volume

$$\oiint \vec{E}.d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Closed surface

Gaussian surface

Perfect Symmetry

Magnetostatic

Calculating magnetic field produced by symmetric current distribution

Ampere's Law

- Closed path
- Flux through open surface

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$$

Closed path

Amperian loop

Particular case where Ampere's path is a circle

Using Biot & Savart law with a long wire

$$B = \frac{\mu_0 I}{2\pi r}$$

Slide #6

Using Ampere's law around the circle

$$\oint \vec{B}.d\vec{l} = B.\oint dl = B.2\pi r$$

Closed circle

Amperian loop

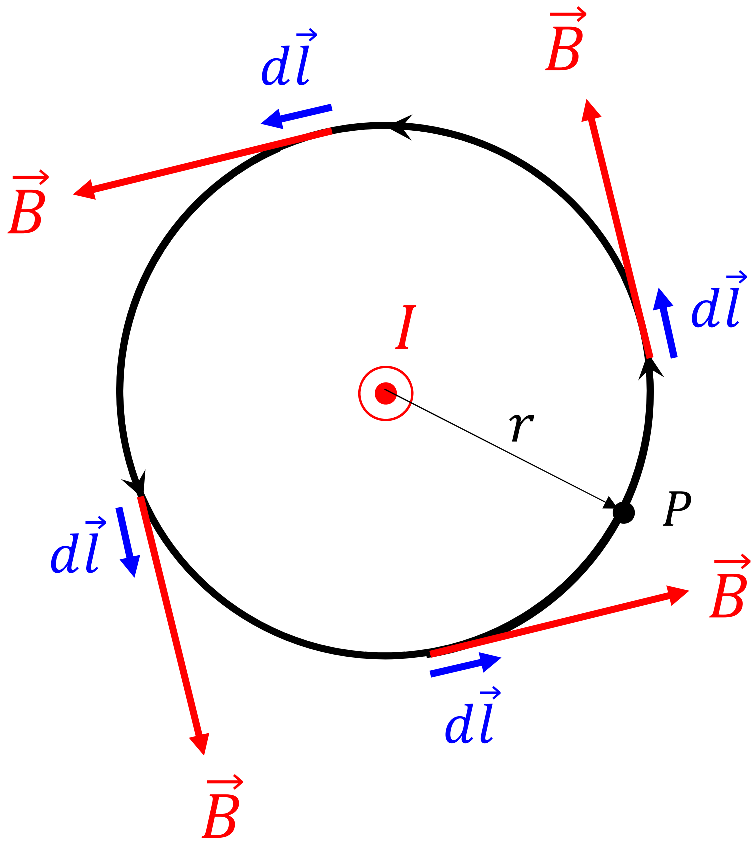
$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$$

Closed circle

Stokes theorem

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

Slide #90 D_Lectures 4-7 Coordinate system Scalar versus Vector fields Operators



Case where the closed path has any shape

$$\oint_{\text{Closed path}} \vec{B}.d\vec{l} = \oint \frac{\mu_0 I}{2\pi r}.d\vec{l}$$

$$\vec{B}.d\vec{l} = Bdlcos\phi = Brd\theta$$

Projection of $d\vec{l}$ on $\vec{B}$, see HW#2

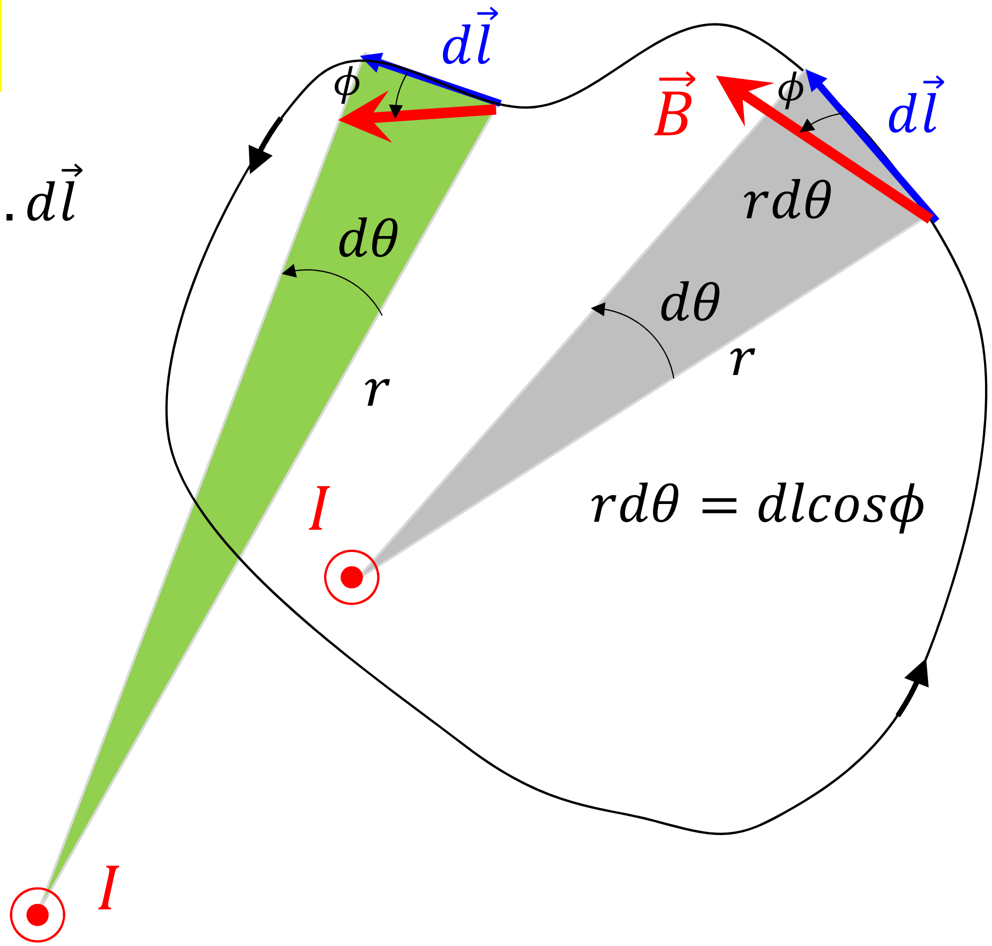
$$\oint_{\text{Closed path}} \vec{B}.d\vec{l} = \oint \frac{\mu_0 I}{2\pi r}.rd\theta = \frac{\mu_0 I}{2\pi}\boxed{\oint d\theta}$$

2π If current inside

0 If current outside

Use polar coordinate to demonstrate this

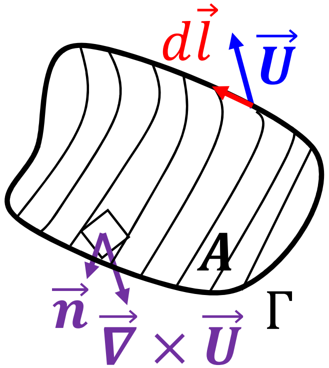
$$\boxed{\oint_{\text{Closed path}} \vec{B}.d\vec{l} = \mu_0 I}$$



Ampere's law expressed in differential form

Stoke's theorem: The integral around any closed path $\boldsymbol{\Gamma}$ of any vector $\vec{\boldsymbol{U}}$ is equal to the surface integral of the normal component of $\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{U}}$

$$\oint_{\Gamma} \vec{\boldsymbol{U}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \int_{A} (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{U}}).\vec{\boldsymbol{n}}\boldsymbol{dA}$$



$\boldsymbol{A}$ = Flux through an open surface

(Amperian surface: Not necessarily flat)

$$\oint_{\Gamma} \vec{B}.d\vec{l} = \int_{A} (\vec{\nabla} \times \vec{B}).\vec{n}dA = \int_{A} \mu_0\vec{J}.\vec{n}dA$$

Ampere's law for magnetostatic $\boxed{\vec{\nabla} \times \vec{B} = \mu_0\vec{J}}$

Caution not to misinterpret Ampere's law

$$\oint \vec{E}.d\vec{l} = 0 \quad \Rightarrow \quad \vec{E} \text{ is conservative} \qquad \boxed{\vec{E}(\mathrm{r}) = -\vec{\nabla}V(r)}$$

Closed path Γ

$$\boxed{\vec{F}_E(r) = q\vec{E}(\mathrm{r}) = -q\vec{\nabla}V(r) = -\vec{\nabla}\boldsymbol{U(r)}} \quad \Rightarrow \quad \oint \vec{E}.d\vec{l} \rightarrow \oint \boldsymbol{q}\vec{E}.d\vec{l} = \underbrace{\oint \vec{F}.d\vec{l}}_{\text{Work}} = 0$$

This **electrostatic** force does **no work** on a charge that moves around a **closed path** ⇔ returns to its the starting point

The force depends on the **position only** and derives from a potential energy $\boldsymbol{U(r)}$ which depends on **position only**

This is not necessarily the same for the magnetostatic field

$$\oint_{\text{Closed path } \Gamma} \vec{B}.d\vec{l}$$

Is it related to the question whether $\vec{B}$ is conservative?

$$\oint_{\text{Closed path } \Gamma} \vec{B}.d\vec{l} = 0$$

$$\boxed{\vec{F}_B(\vec{r},\vec{v}) = q\vec{v} \times \vec{B}}$$

$\vec{F}_B \perp \vec{v}$ and $\vec{F}_B \perp \vec{B}$

$$\oint_{\text{Closed path } \Gamma} \vec{B}.d\vec{l} \quad \not\Rightarrow \quad \oint_{\text{Closed path } \Gamma} \vec{F}_B.d\vec{l}$$

$\Rightarrow$ $\oint_{\text{Closed path } \Gamma} \vec{B}.d\vec{l}$ **IS NOT RELATED TO THE WORK DONE BY THE MAGNETIC FORCE**

Caution

$$\oint_{\text{Closed path } \Gamma} \vec{B}.d\vec{l}$$

States the Ampere's law only

*The magnetic force on moving charge is **NOT** conservative*

A conservative force depends **ONLY** on the **position** of the body on which the force is acting

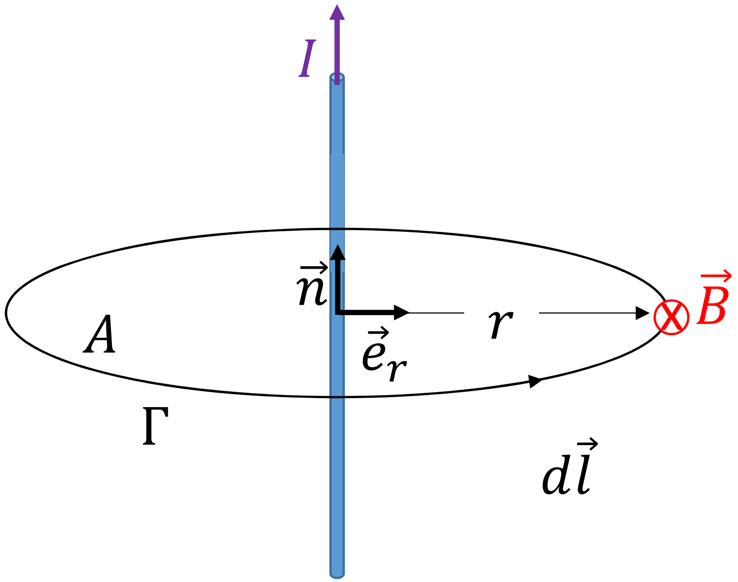
The magnetic force on moving charge depends on the **position** **BU**T also on the **velocity**

$$\vec{F}_B(\vec{r},\vec{v}) = q\vec{v} \times \vec{B}$$

The magnetic vector force is NOT parallel to the vector magnetic field !

Applications of Ampere's law

Magnetic field of a straight wire



Ampere's law $$\oint_{\Gamma} \vec{B}.d\vec{l} = B.2\pi r = \mu_0 I_{enclosed}$$

$$I_{enclosed} = I$$

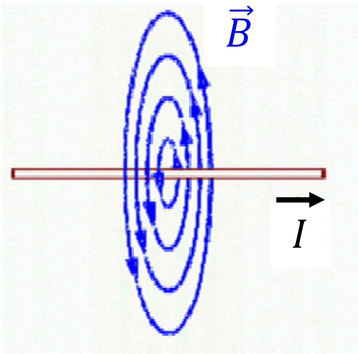
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\vec{n} \times \vec{e}_r}$$

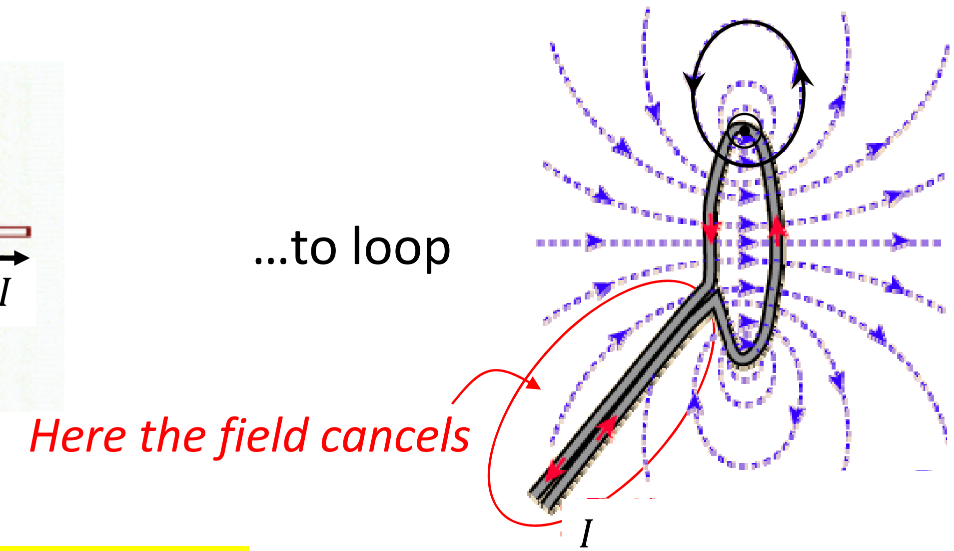
$|\vec{n} \times \vec{e}_r| = 1$

Much easier than using Biot & Savart law: see slide # 5 and 6

From straight wire…

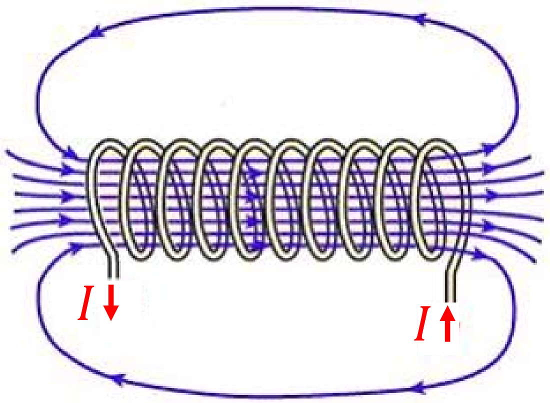


…to loop

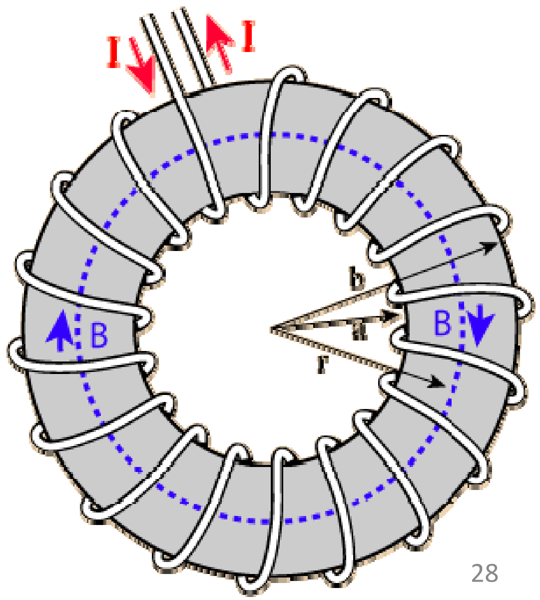


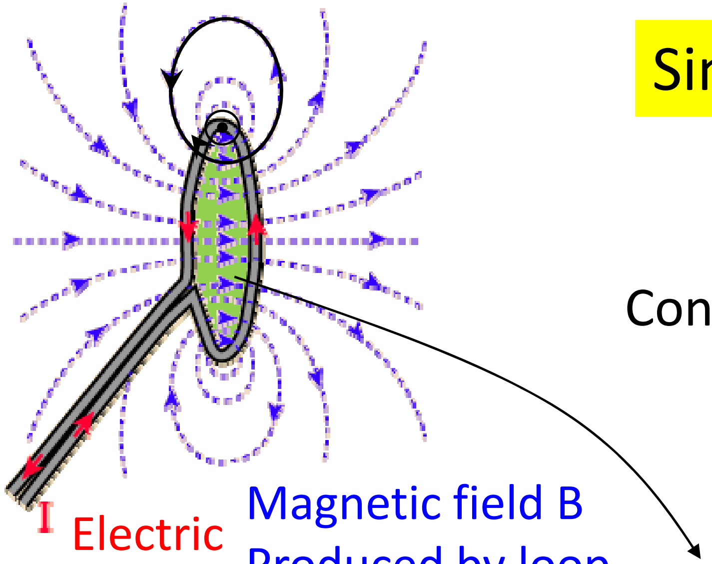
Conservation of B lines ⇨ field lines distort

… to straight solenoid



… to toroidal solenoid





Single carrying current loop

Conservation of # $\vec{B}$ lines ➡ Inside the loop lines are closer than outside:

$$B_{in} \gg B_{out}$$

Field loops are not perfect circles

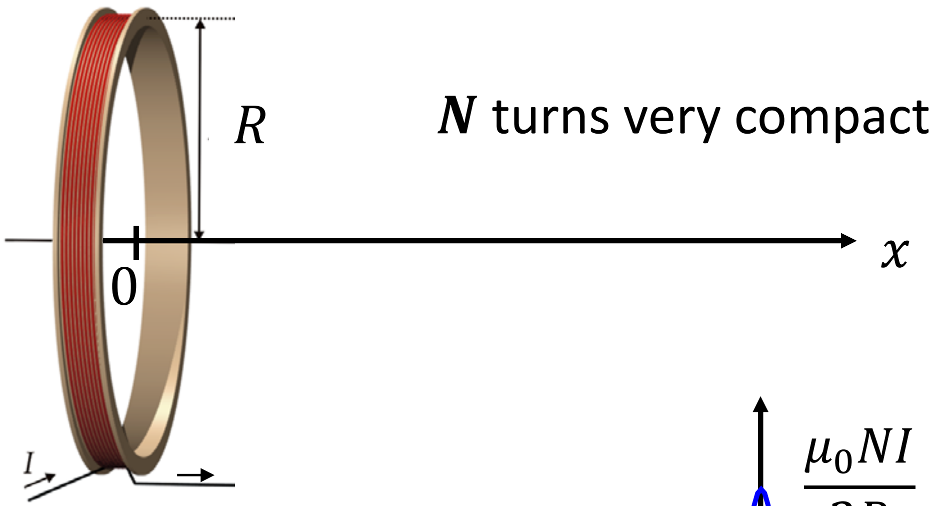
Flux through a cross-sectional area

$$\Phi = \int_A \vec{B}.d\vec{A}$$

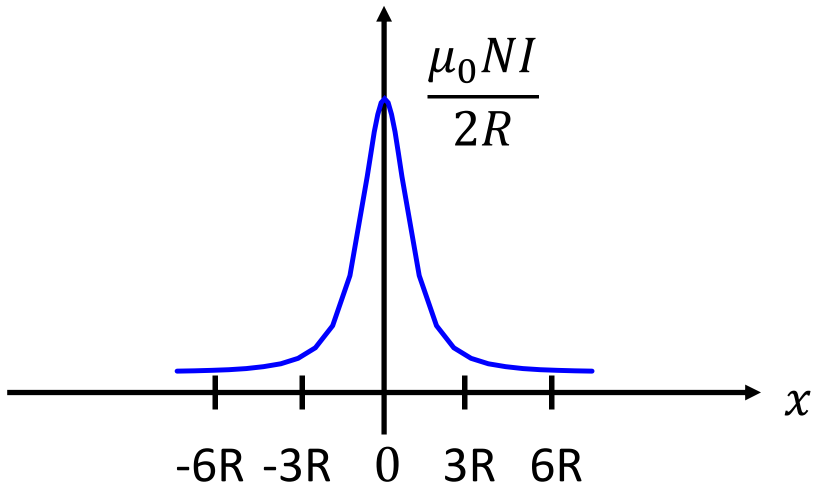
= Flux of the same lines through the rest of the area outside covering the whole universe

➡ Far outside the loop $\vec{B}$ is very weak !

Magnetic field of a coil (= short solenoid)



N turns very compact



Field produced by one loop

$$\vec{B}_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \vec{\imath}$$

Slide #10

Field produced by N loops

Superposition principle

$$\vec{B}_x = \frac{\mu_0 \boldsymbol{N} I R^2}{2(R^2 + x^2)^{3/2}} \vec{\imath}$$

Magnetic Dipole: Force and torque on a current loop

*We know from mechanics that **Torque** is a master piece in physics*

We know from electrostatics that **Dipole** subject to uniform force

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Electric Dipole

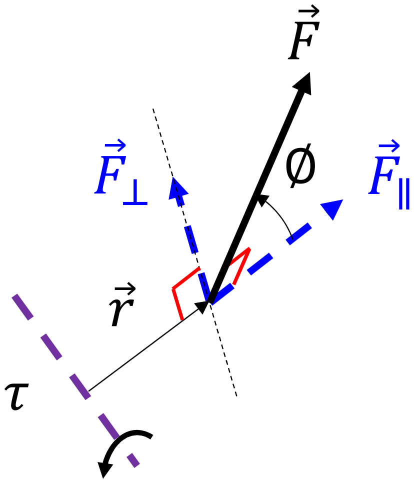
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Magnetic Dipole

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

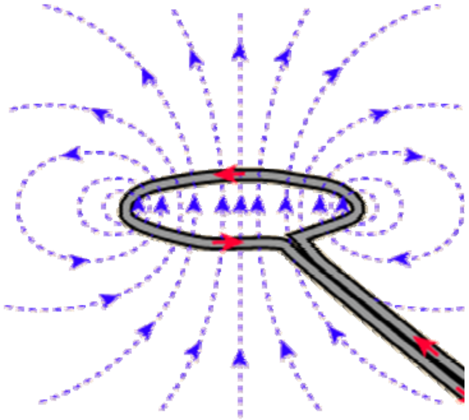
Alignment by rotation

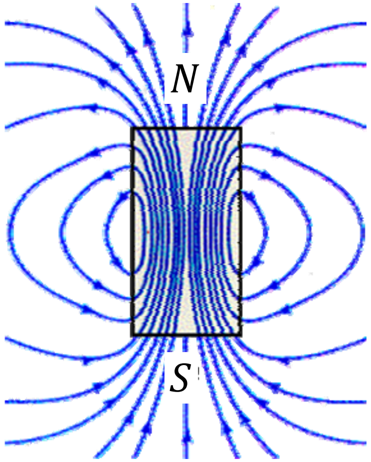
Torque τ

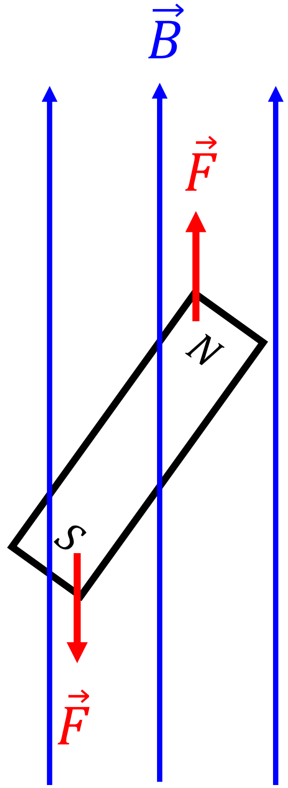


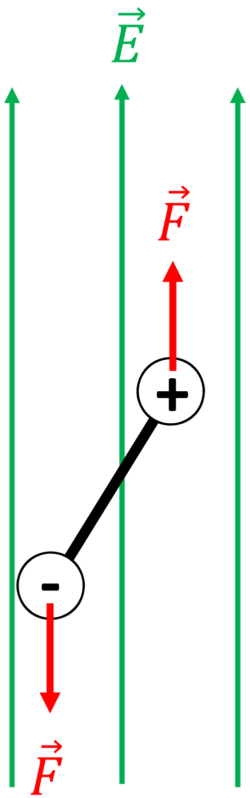
Axis of rotation

Current loop produces a dipole





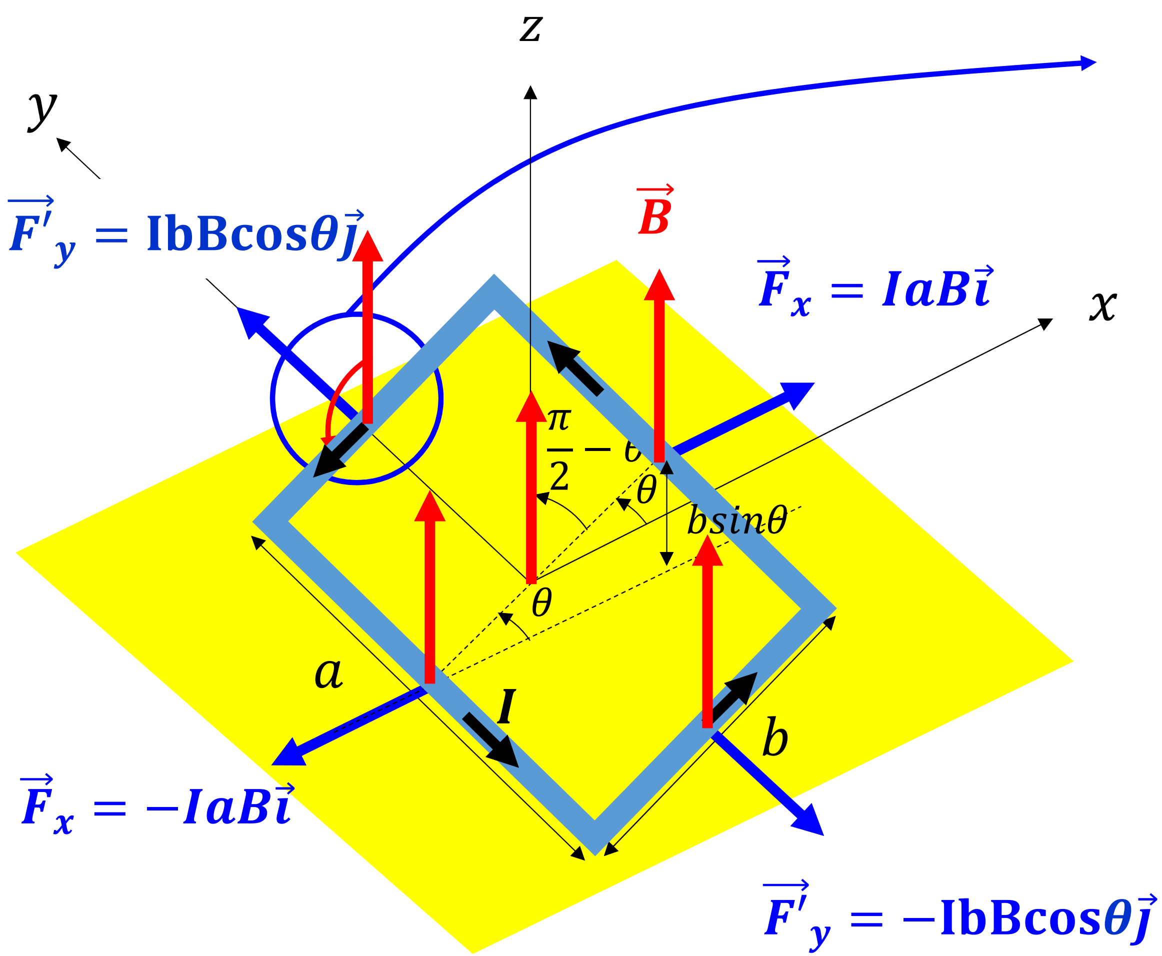


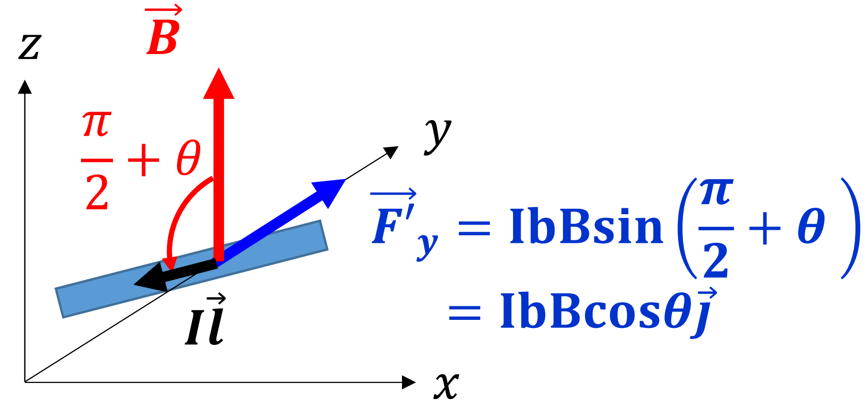


Alignment by rotation

Uniform magnetic field

Laplace force: $\vec{F} = I\vec{l} \times \vec{B}$

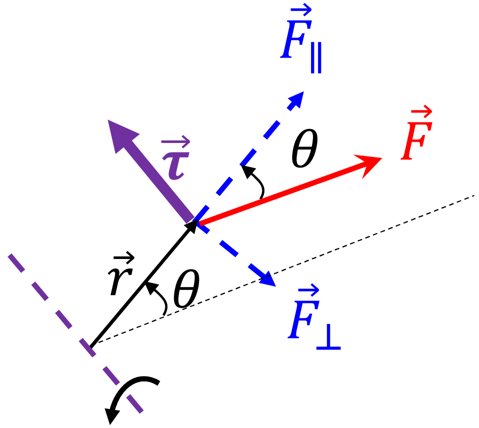




TOTAL NET FORCE = 0

WHAT ABOUT THE NET TORQUE ?

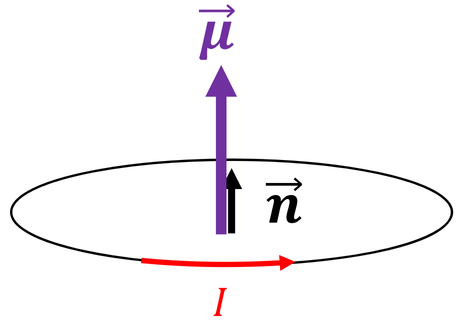
Torque τ



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \vec{r} \times \vec{F}_{\perp} = rFsin\theta\vec{j}$$

Axis of rotation (y − axis)



$ab = area$

$\mu = IA$

$$\tau = \left[\left(\frac{b}{2}\right) IaBsin\theta\right] \times 2$$

$$\tau = \underbrace{Iab}_{\alpha}\underbrace{Bsin\theta}_{\beta}$$

Magnetic dipole moment or magnetic moment

× 2 because both $\vec{F}$ and $-\vec{F}$ give rise to the torque

$\gamma = \alpha\beta sin\phi \Rightarrow \vec{\gamma} = \vec{\alpha} \times \vec{\beta} \longrightarrow$ $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\theta = 0 \Leftrightarrow$ **Stable** equilibrium position

$\theta = \pi \Leftrightarrow$ **Unstable** equilibrium position

$\vec{\mu} = I\vec{A}$

$\vec{A} = \vec{n}A$

Electrostatic

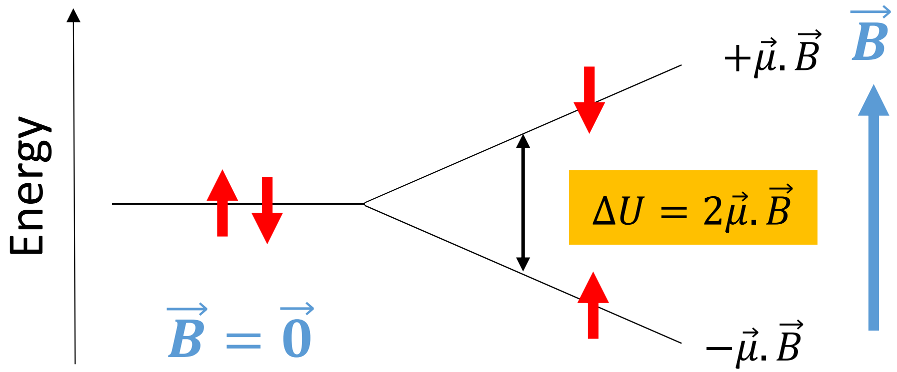
$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{p} = q\vec{d}$$

Mgnetostatic

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\mu} = I\vec{A}$$



Work done on a dipole

$$dW = -dU = \tau . d\theta$$

$$dU = -\tau . d\theta = -(-\mu B \sin\theta) d\theta$$

Because $(\vec{\mu}, \vec{B}) = -\theta$

θ is counterclockwise and $(\vec{\mu}, \vec{B})$ is clockwise

$$dU = \mu B \sin\theta \, d\theta$$

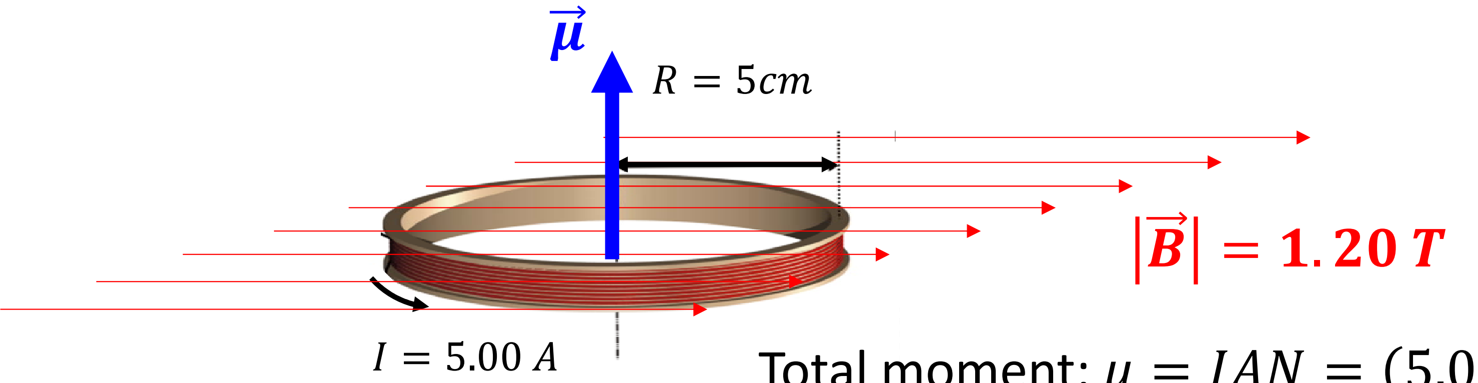
$$U = -\mu B \cos(\theta) + constant$$

Potential energy is minimum when the dipole is aligned with the field

$$\Delta U = -\vec{\mu}.\vec{B}$$

Coil in a magnetic field: Magnetic moment and torque

Find the direction of the moment of the torque…. And its magnitude



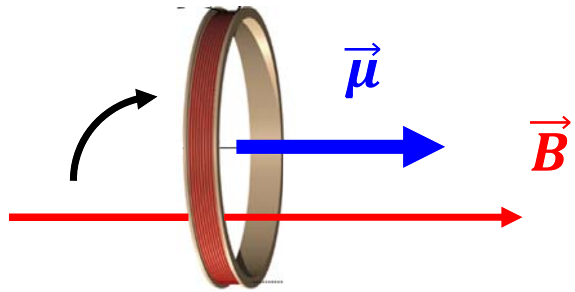
For one loop

$\vec{\mu} = I\vec{A}$

Total moment: $\mu = IAN = (5.00)(7.85 \times 10^{-3}) = 1.18\ A.m^2$

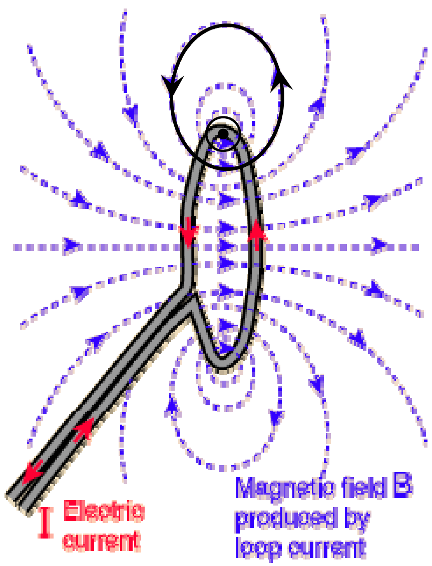
Torque: $\tau = \mu B sin\theta = (1.18)(1.20) sin\,\pi/2 = 1.41\ N.m$

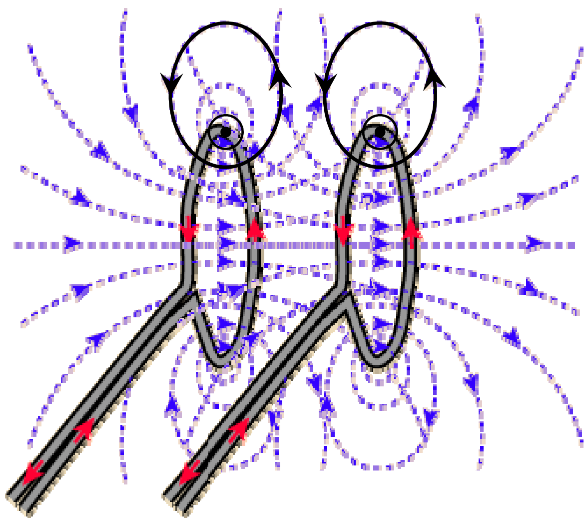
What is the most stable position of the coil?



Magnetic field of a solenoid

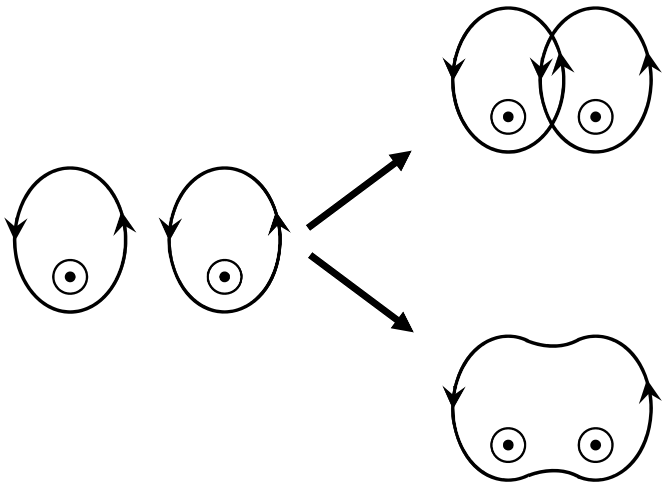
From single loop …   to   … two neighboring loops…

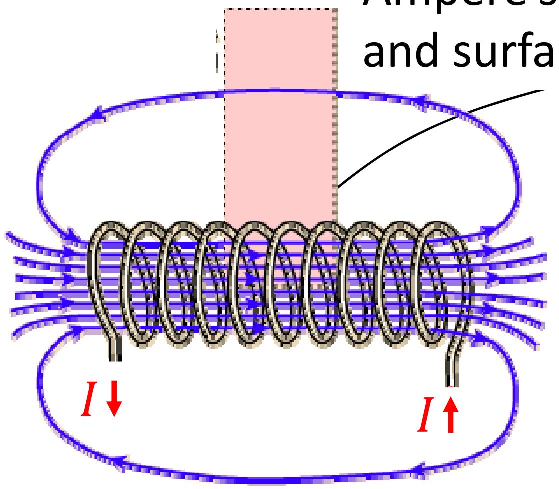


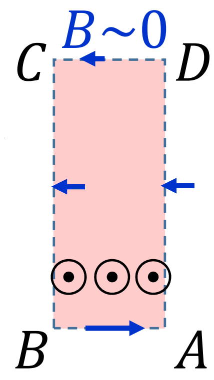


This configuration is forbidden

Field lines emerging from different sources **cannot** cross







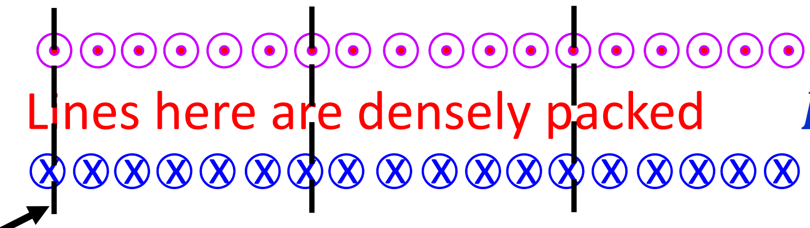
$B{\sim}0$

$\Gamma = ABCD$

$B{\sim}0$

Lines here are very far apart

Lines conservation



$\boldsymbol{B\ Max}$

Lines here are very far apart

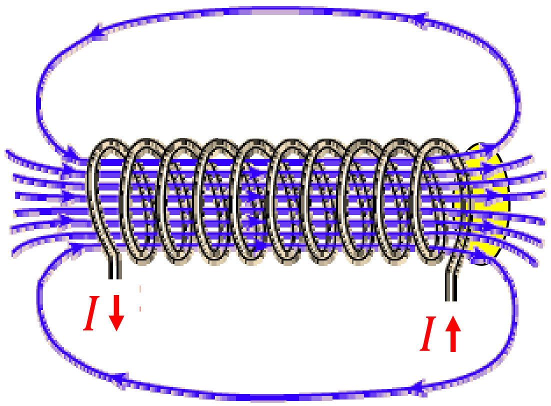
the loops do not lie in a single plane

$B{\sim}0$

Why is the field almost zero outside the solenoid ?

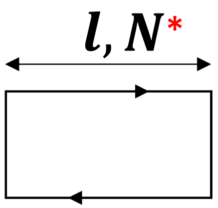
- Conservation of # field lines: The # of lines inside the solenoid is equal to the # lines outside
- Flux through a cross-sectional area inside $\Phi = \int \vec{B}.d\vec{A}$ = Flux of the same lines through the rest of the area outside covering the whole universe

A

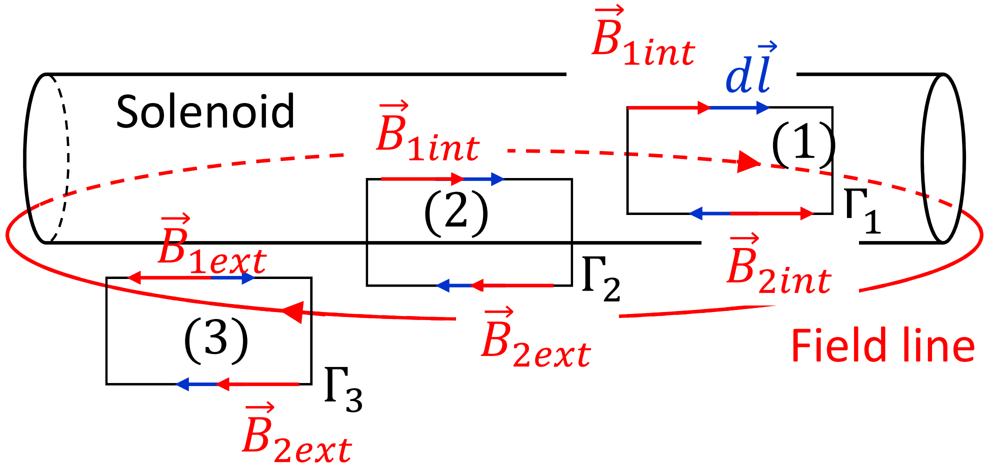


The lines outside are **MUCH** more dispersed while highly confined inside

$\vec{B}$ outside = $\vec{0}$



Ampere's law applied in 3 different situations



3 Ampere's paths

Path Γ_1 $\quad I_{enclosed} = 0$

Path Γ_2 $\quad \boldsymbol{I_{enclosed} = NI^*}$

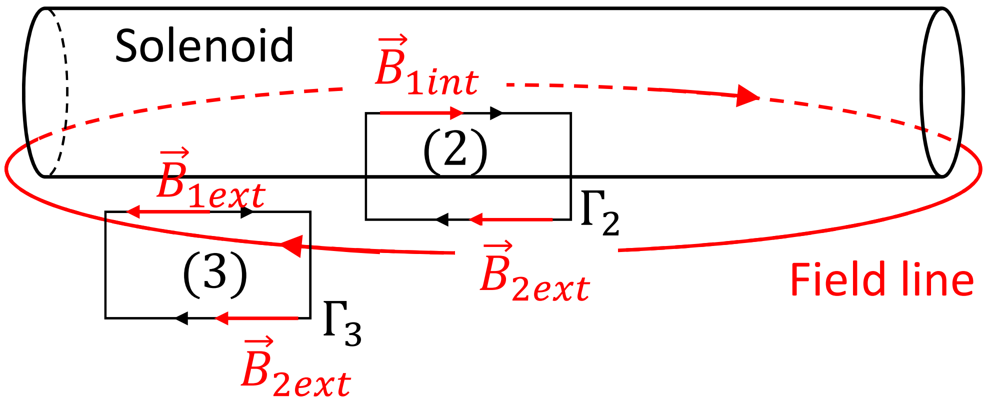
Path Γ_3 $\quad I_{enclosed} = 0$

$$\oint_{\Gamma_1} \vec{B}.d\vec{l} = 0 \Rightarrow (B_{1int} - B_{2int})l = 0 \Rightarrow \boxed{B_{1int} = B_{2int}}$$

Field uniform everywhere in the solenoid

* N # turns in the length l

Ampere's law applied in 3 different situations



Path Γ_3 $\quad I_{enclosed} = 0$

Path Γ_2 $\quad I_{enclosed} = NI$

$n = N/l$ (# turns/unit length)

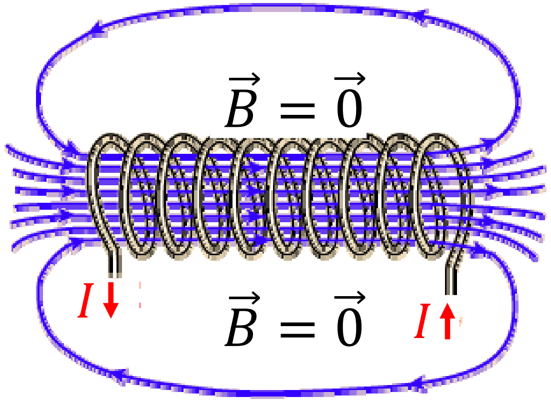
$$\oint_{\Gamma_3} \vec{B}.d\vec{l} = 0 \Rightarrow (B_{1ext} - B_{2ext})l = 0 \Rightarrow B_{1ext} = B_{2ext}$$

But $B(\infty) = 0$

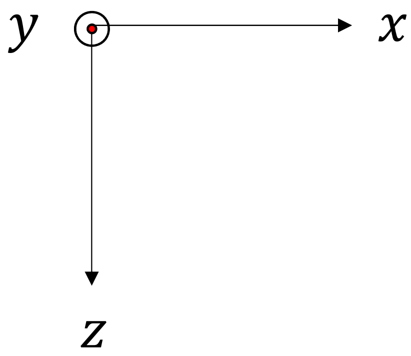
Field outside solenoid = 0

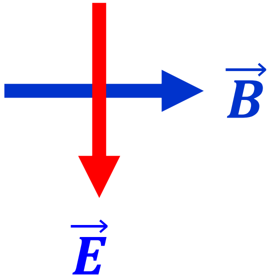
$$\oint_{\Gamma_2} \vec{B}.d\vec{l} = \mu_0 NI \Rightarrow (B_{1int} - B_{2ext})l = \mu_0 NI \Rightarrow \boxed{B_{1int} = B = \mu_0 nI}$$

Long solenoid

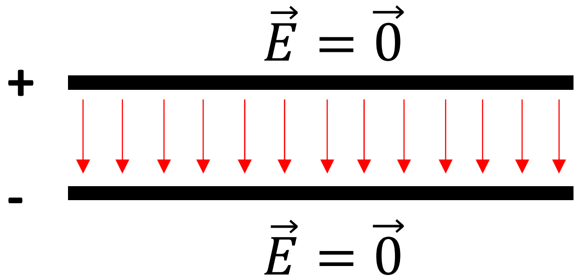


$\vec{B} = \mu_0 n I \vec{\imath}$





Long capacitor



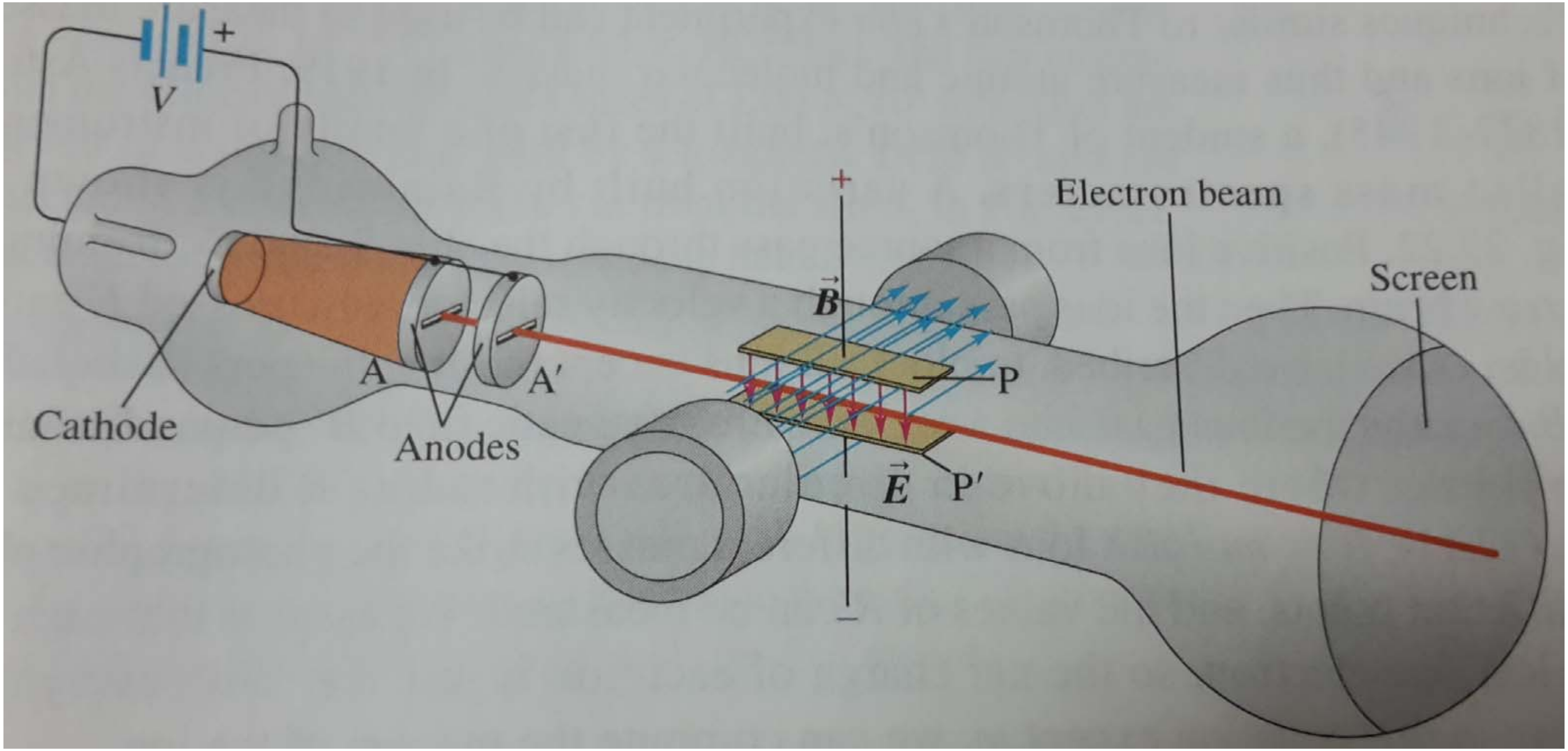
$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{k}$

In both cases the fields are zero outside

Why should the field lines go parallel inside the solenoid?

Because $\vec{\nabla}.\vec{B} = 0$

Thomson's e/m Experiment



From University of physics (11rd edition)

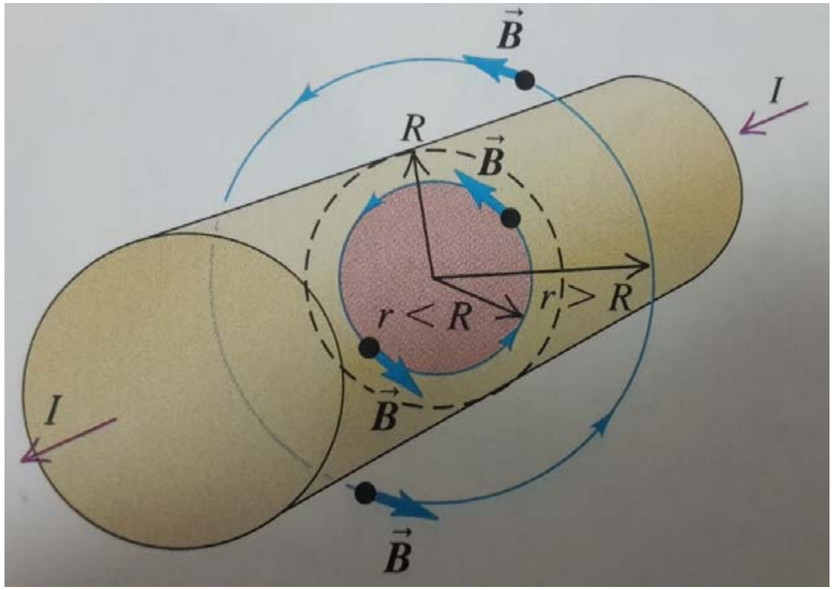
Field inside and outside a long cylinder conductor

A cylindrical conductor of radius R carries a current I. The current is uniformly distributed over the cross-sectional area of the conductor. **Find the magnetic field as a function of distance r for points inside and outside the conductor**

Circular symmetry

Field inside ⇔ ampere's path (circle) with radius $r < R$

Field outside ⇔ ampere's path (circle) with radius $r > R$



From University of physics (11rd edition)

Current per unit area

$$J = I/\pi R^2$$

Inside

$$I_{encl} = J(\pi r^2) = Ir^2/R^2$$

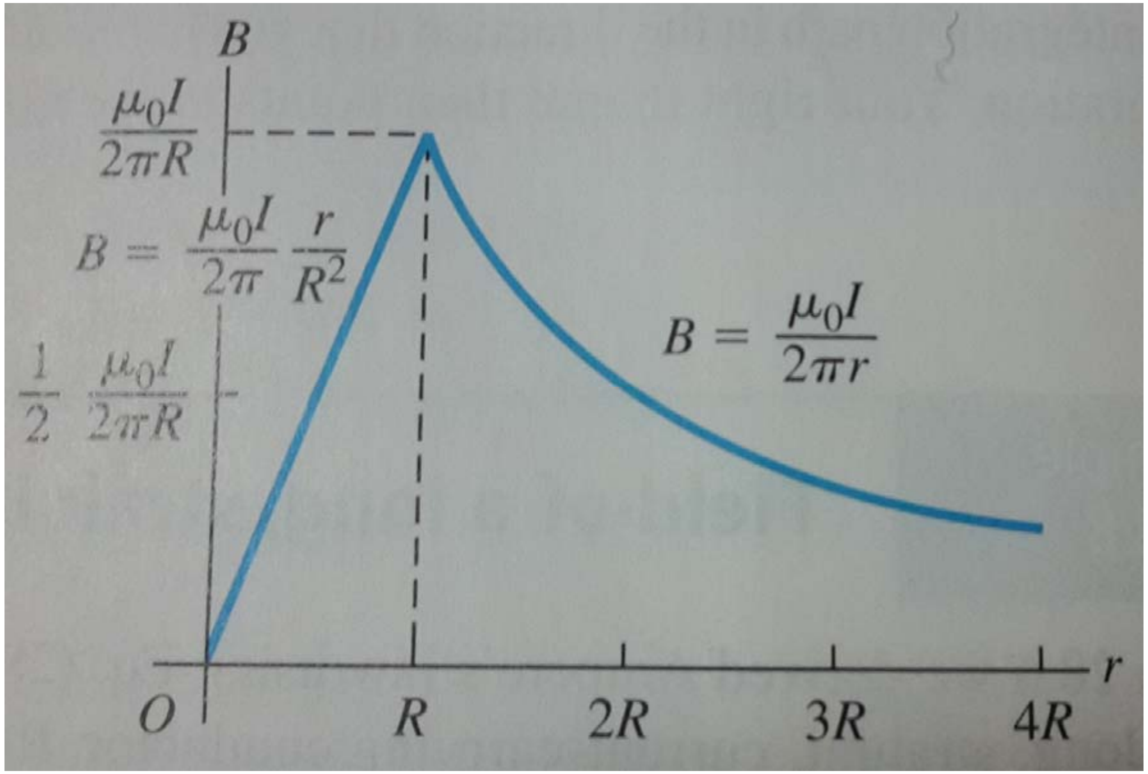
$$\boxed{B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}}$$

Outside

$$I_{encl} = I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Any comment?



From University of physics (11rd edition)

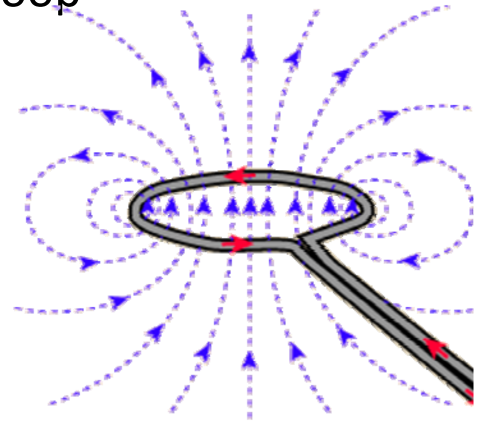
Field outside is the same irrespective of R: Cylinder, Rod or Wire

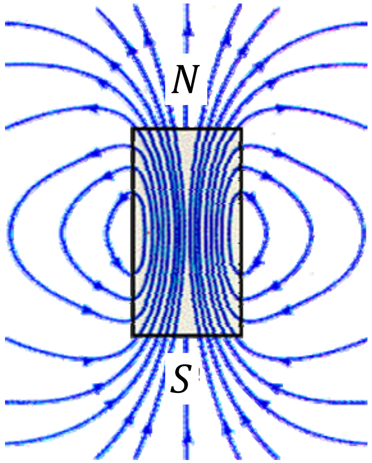
Slide #78 E_Lectures 8&9 Gauss law in Electrostatics

What is a magnet

The current is clearly responsible for the magnetic field in a loop

- BUT what about magnet?
- Where does the current come from in a magnet?





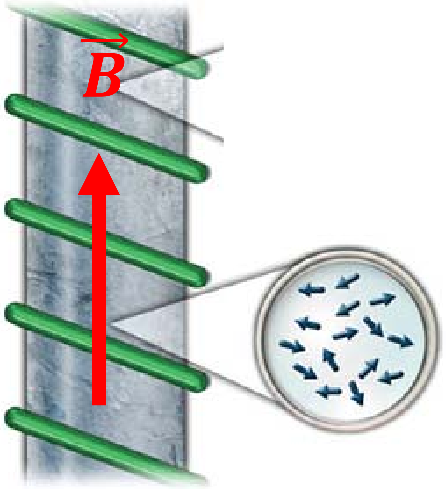
<u>Origin of magnetism in matter</u>: electron produces two dipoles

- Orbiting around nucleus
- Spin (quantum effect)

In magnets the current is for free

What happens if a iron bar is inserted in a solenoid?

Magnetic moment



Before switching on the current in the solenoid

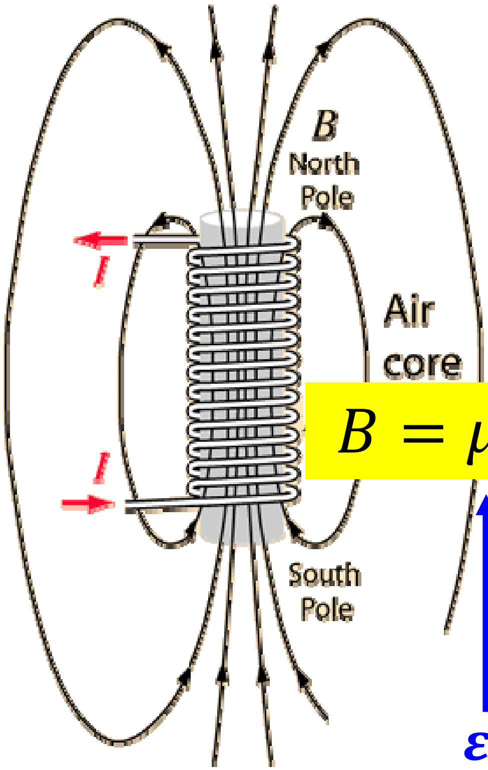
Enhances the field inside iron

!

In the case of a dielectric, the field inside is **reduced**

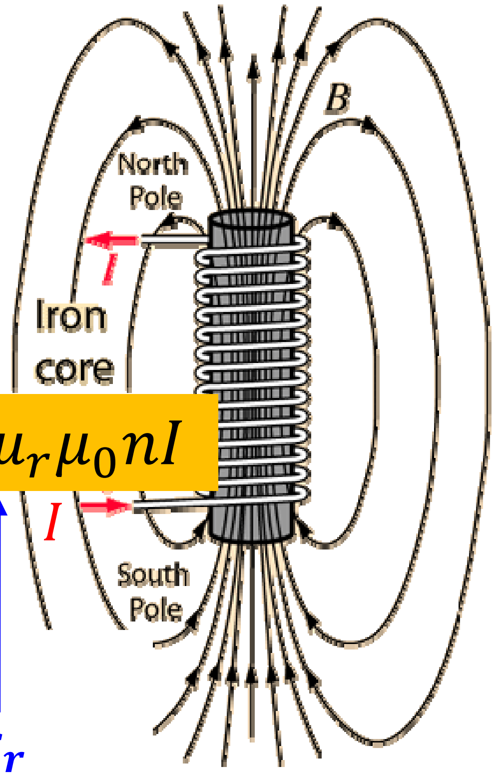
Any clue of how to make magnet?

The iron core is for a solenoid what a dielectric is for a capacitor



$$B = \mu_0 n I$$

$\boldsymbol{\varepsilon_0}$

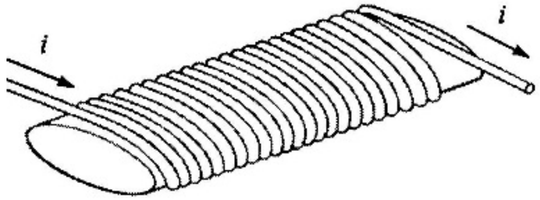


$$B = \mu_r \mu_0 n I$$

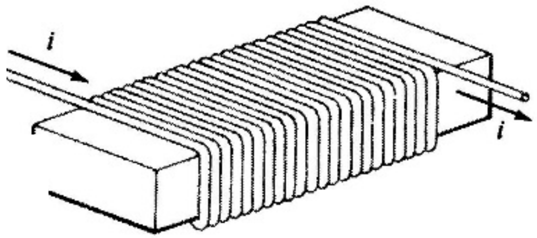
$\boldsymbol{\varepsilon_r}$

Similarity with capacitor plates separated by vacuum or dielectric

except that the net field inside the dielectric is smaller than the applied field



(a)

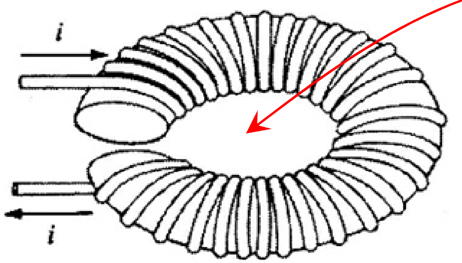


(b)

Does the internal field depend on the cross-sectional shape of the solenoid ?

No because $B = \mu_0 n I$

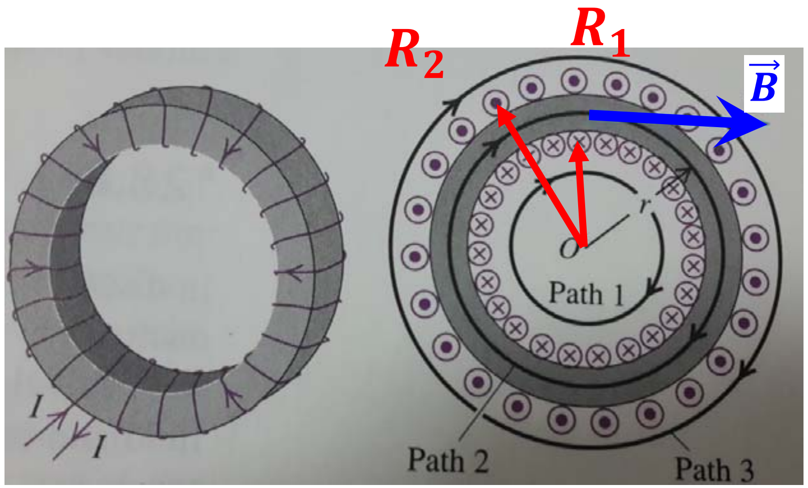
Irrespective of the shape of the solenoid



What is the field inside ?

Field of a toroidal solenoid

We consider a toroidal solenoid **(toroid)**. N turns very tight carrying a current I.
Find the magnetic field every where



From University of physics (11rd edition)

Circular symmetry

Field inside toroid ⇔ ampere's path 1 (circle) $r < R_1$

Field inside turns ⇔ ampere's path 2 (circle) $R_1 < r < R_2$

Field outside toroid ⇔ ampere's path 3 (circle) $r > R_2$

Here the Ampere's path encloses all turns
⇨ N total number of turns

Field inside ⇔ ampere's path 1 (circle) $r < R_1$ $\quad I_{encl} = 0 \quad \oint \vec{B}.d\vec{l} = B2\pi r = 0$

Equivalence in Electrostatic: see slide #20 in E_Lectures 8&9 Gauss law in Electrostatics
slide #16 in F_Lecture 10&11_Conductors and Dipoles

Field outside ⇔ ampere's path 3 (circle) $r > R_2$ $\quad I_{encl} = 0 \quad \oint \vec{B}.d\vec{l} = B2\pi r = 0$

Field inside toroid ⇔ ampere's path 2 (circle) $R_1 < r < R_2$ $\quad I_{encl} = NI \quad \oint \vec{B}.d\vec{l} = B2\pi r = \mu_0 NI$

$R_1 < r < R_2$ $\quad \boxed{B = \frac{\mu_0 NI}{2\pi r}}$ **Any comment?**

$$B = \frac{\mu_0 NI}{2\pi r}$$

Field is **NOT** uniform over a cross section of the core

$$\frac{N}{2\pi r} = n$$

Number of turns per unit length **depends on $\boldsymbol{r}$**

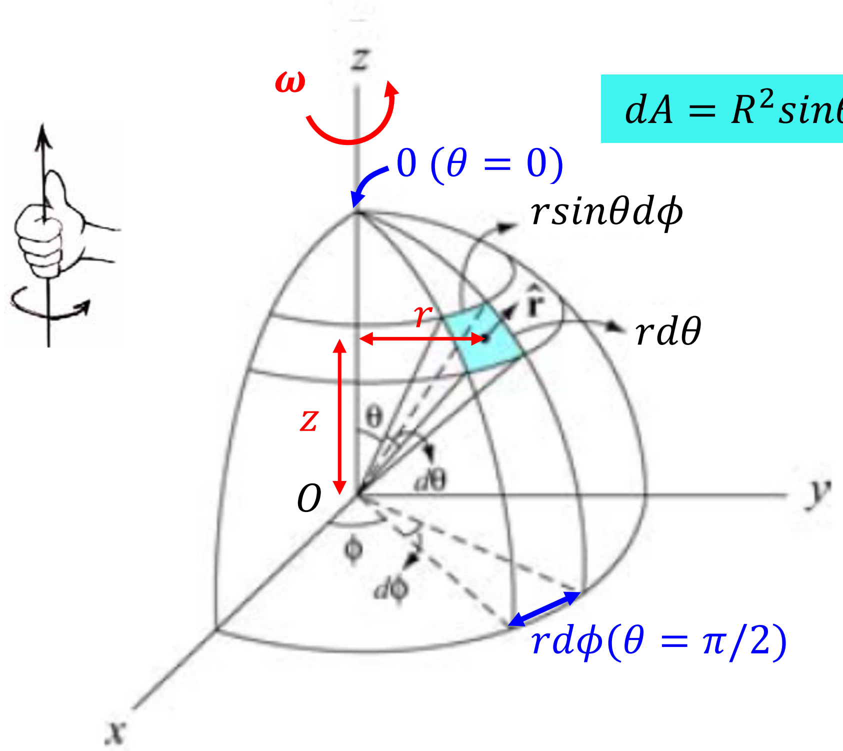
Field at the center of a straight solenoid, Slide #41

$$B = \mu_0 nI$$

Number of turns per unit length is a **constant**

Surface charged sphere: mechanical rotation

A Sphere of radius ***R*** is uniformly charged on the surface with a surface density $\boldsymbol{\sigma}$. The sphere is rotating around its $z-$ axis. What is the magnetic field at the center of the sphere?



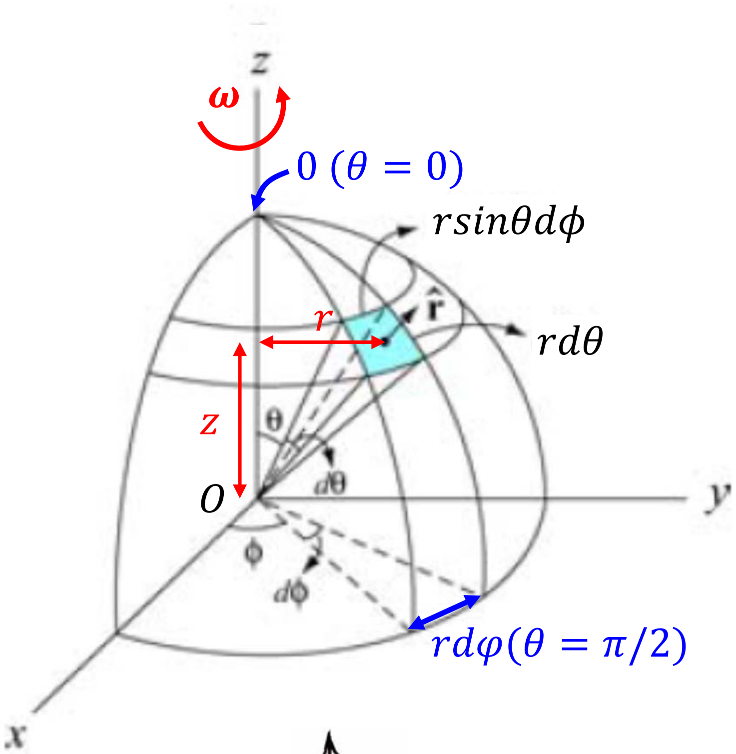
Elementary area $dA = R^2 sin\theta d\theta d\phi$

The charge on dA is $dq = R^2 \sigma sin\theta d\theta d\phi$

This elementary area in rotation behaves like a loop

$$dI = \frac{dq}{dt} = R^2 \sigma sin\theta d\theta \frac{d\phi}{dt} = R^2 \omega \sigma sin\theta d\theta$$

From slide #11 ⇨ $d\vec{B}_z(0) = \dfrac{\mu_0 r^2 dI}{2(r^2+z^2)^{3/2}} \vec{k}$



$$dI = R^2 \omega\sigma sin\theta d\theta$$

$$d\vec{B}_z(0) = \frac{\mu_0 r^2 dI}{2(r^2+z^2)^{3/2}}\vec{k}$$

$$r = Rsin\theta$$

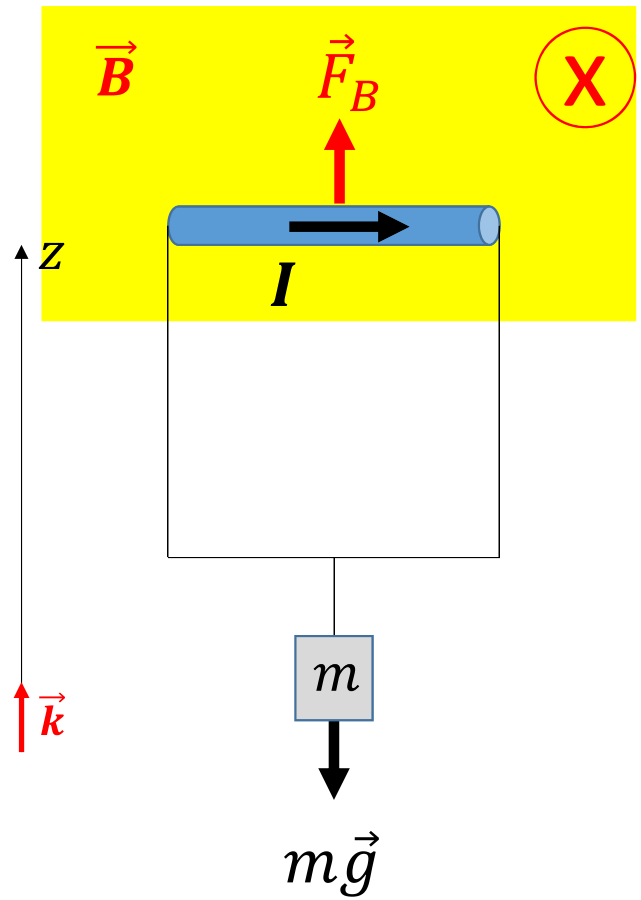
$$z = Rcos\theta$$

$$d\vec{B}_z(O) = \frac{\mu_0 \omega\sigma R sin^3(\theta)}{2}\vec{k}$$

$$\vec{B}_z(O) = -\int_0^{\pi}\frac{\mu_0\omega\sigma R}{2}[1 - cos^2(\theta)]d[cos(\theta)]\vec{k}$$

$$\boxed{\vec{B}_z(O) = \frac{2}{3}\mu_0\omega\sigma R\vec{k}}$$

What should be the direction of the magnetic field to overcome gravitation allowing thus to hang the mass in air?



For what current I in the loop would the magnetic force balance exactly the gravitational force?

(mass of the circuit negligible)

$$F_B = \int I(dl \times B) = IBa = mg \quad \Rightarrow \quad I = \frac{mg}{Ba}$$

What happens if we increase the current?

The loop rises lifting the weight

Somebody is doing work: Who?

Magnetic force?

BUT magnetic force never does work !

$$W_B = F_B h = IBah$$

$$F_B \parallel h$$

What is happening?

?

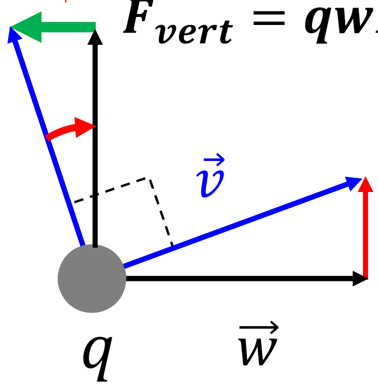
When the wire starts to rise, the charges in that wire are moving
Horizontally BUT NOT ONLY. They also move vertically !

$$\vec{v} = \vec{w} + \vec{u}$$

$F_{horiz} = quB = (\lambda a)uB$ opposes the flow of charges

$\vec{F}_B(\perp \vec{v})$
It does no work!

$F_{vert} = qwB(\perp \vec{w}) = (\lambda a)wB = IBa = F_B = mg$ (See previous slide)



While increasing the Current in the wire

$\vec{u}$ Vertical (mechanical) motion of the charges ⇨ wire

$\vec{w}$ Horizontal (electrical) motion of the charges

Steady current in the wire $I = \lambda w \ (C/s)$

The **BATTERY** must do work to keep the charges moving to the right

- In a time $\boldsymbol{dt}$, the charges move a horizontal distance $\boldsymbol{wdt}$.
- The **BATTERY does work against the horizontal force**
- **The magnetic force is passive**

$$W_{Battery} = -\int -\boldsymbol{F}_{\boldsymbol{horiz}} w dt = \int (\boldsymbol{\lambda a})\boldsymbol{uB} w dt = \boldsymbol{IBah}$$

$\boldsymbol{h} = \boldsymbol{udt}$

$\boldsymbol{I} = \boldsymbol{\lambda} w$