EXAMPLE 6–4 A direct current I flows in a straight wire of length $2L$. Find the magnetic flux density $\mathbf{B}$ at a point located at a distance r from the wire in the bisecting plane: (a) by determining the vector magnetic potential $\mathbf{A}$ first, and (b) by applying Biot-Savart law.

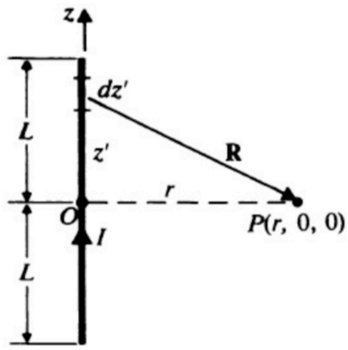


FIGURE 6–5
A current-carrying straight wire (Example 6–4).

Solution Currents exist only in closed circuits. Hence the wire in the present problem must be a part of a current-carrying loop with several straight sides. Since we do not know the rest of the circuit, Ampère's circuital law cannot be used to advantage. Refer to Fig. 6–5. The current-carrying line segment is aligned with the z-axis. A typical element on the wire is

$$d\boldsymbol{\ell}' = \mathbf{a}_z\, dz'.$$

The cylindrical coordinates of the field point P are $(r, 0, 0)$.

a) *By finding* $\mathbf{B}$ *from* $\nabla \times \mathbf{A}$. Substituting $R = \sqrt{z'^2 + r^2}$ into Eq. (6–27), we have

$$\begin{aligned}\mathbf{A} &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \int_{-L}^{L} \frac{dz'}{\sqrt{z'^2 + r^2}} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \left[\ln\left(z' + \sqrt{z'^2 + r^2}\right)\right]\Bigg|_{-L}^{L} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L}.\end{aligned} \tag{6–34}$$

Therefore,

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times (\mathbf{a}_z A_z) = \mathbf{a}_r \frac{1}{r}\frac{\partial A_z}{\partial \phi} - \mathbf{a}_\phi \frac{\partial A_z}{\partial r}.$$

Cylindrical symmetry around the wire assures that $\partial A_z/\partial\phi = 0$. Thus,

$$\begin{aligned}\mathbf{B} &= -\mathbf{a}_\phi \frac{\partial}{\partial r}\left[\frac{\mu_0 I}{4\pi}\ln\frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}\right] \\ &= \mathbf{a}_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}.\end{aligned} \tag{6-35}$$

When $r \ll L$, Eq. (6–35) reduces to

$$\mathbf{B}_\phi = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r}, \tag{6-36}$$

which is the expression for $\mathbf{B}$ at a point located at a distance r from an infinitely long, straight wire carrying current I, as given in Eq. (6–11b).

b) *By applying Biot-Savart law.* From Fig. 6–5 we see that the distance vector *from* the source element dz' *to* the field point P is

$$\mathbf{R} = \mathbf{a}_r r - \mathbf{a}_z z'$$

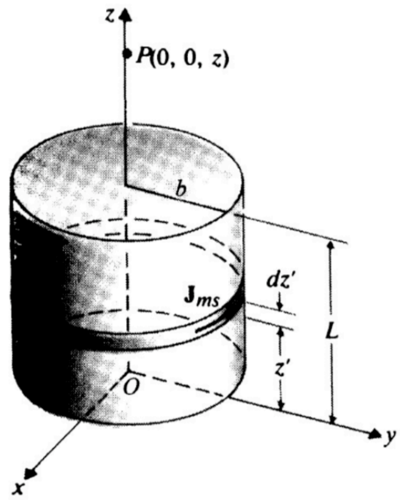
$$d\boldsymbol{\ell}' \times \mathbf{R} = \mathbf{a}_z\, dz' \times (\mathbf{a}_r r - \mathbf{a}_z z') = \mathbf{a}_\phi r\, dz'.$$

Substitution in Eq. (6–33c) gives

$$\begin{aligned}\mathbf{B} = \int d\mathbf{B} &= \mathbf{a}_\phi \frac{\mu_0 I}{4\pi}\int_{-L}^{L}\frac{r\,dz'}{(z'^2+r^2)^{3/2}} \\ &= \mathbf{a}_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2+r^2}},\end{aligned}$$

which is the same as Eq. (6–35). ▬

EXAMPLE 6–8 Determine the magnetic flux density on the axis of a uniformly magnetized circular cylinder of a magnetic material. The cylinder has a radius b, length L, and axial magnetization $\mathbf{M} = \mathbf{a}_z M_0$.



Question: **B** on z axis?

Methods:
(1) Calculate J_m and J_{ms}
(2) **B** due to J

FIGURE 6–11
A uniformly magnetized circular cylinder (Example 6–8).

Solution In this problem concerning a cylindrical bar magnet, let the axis of the magnetized cylinder coincide with the z-axis of a cylindrical coordinate system, as shown in Fig. 6–11. Since the magnetization $\mathbf{M}$ is a constant within the magnet, $\mathbf{J}_m = \nabla' \times \mathbf{M} = 0$, and there is no equivalent volume current density. The equivalent magnetization surface current density on the side wall is

$$\mathbf{J}_{ms} = \mathbf{M} \times \mathbf{a}'_n = (\mathbf{a}_z M_0) \times \mathbf{a}_r = \mathbf{a}_\phi M_0. \tag{6–64}$$

The magnet is then like a cylindrical sheet with a lineal current density of M_0 (A/m). There is no surface current on the top and bottom faces. To find $\mathbf{B}$ at $P(0, 0, z)$, we consider a differential length dz' with a current $\mathbf{a}_\phi M_0\, dz'$ and use Eq. (6–38) to obtain

Example6-6

$$\boxed{\mathbf{B} = \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}} \quad \text{(T).}}$$

$dI = J_{ms} dz'$

$$d\mathbf{B} = \mathbf{a}_z \frac{\mu_0 M_0 b^2\, dz'}{2[(z - z')^2 + b^2]^{3/2}}$$

and

$$\mathbf{B} = \int d\mathbf{B} = \mathbf{a}_z \int_0^L \frac{\mu_0 M_0 b^2\, dz'}{2[(z - z')^2 + b^2]^{3/2}} = \mathbf{a}_z \frac{\mu_0 M_0}{2} \left[\frac{z}{\sqrt{z^2 + b^2}} - \frac{z - L}{\sqrt{(z - L)^2 + b^2}} \right]. \tag{6–65}$$