VE230 Midterm 1

Lecture 0

1. wave function $\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$

   heat function $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$

   a. both indicate that something propagates i.e. changes position with time

   b. wave function: energy is conserved, time is reversible

   c. heat function: energy dissipates, time is irreversible

2. Poisson (1813)

   $\vec{E} = -\vec{\nabla}\varphi$

   $\nabla^2 \varphi = -\frac{\rho}{\varepsilon_0}$

3. Gauss (1831)

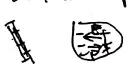
   $\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$

   $\vec{\nabla}\cdot\vec{B} = 0$

Lecture 1

1. Polarization of dielectrics

   Induce short distance separation of charges.



2. Could two neutral bodies experience a net force if brought close to each other? ○ □

   Charge distribution!!!

3. Force field is not meaningful

   Because force requires at least 2 charges

4. Uniform electric field

   a. magnitude
   b. direction

   both same. ⇉

Lecture 2

1. Lorentz force

   $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

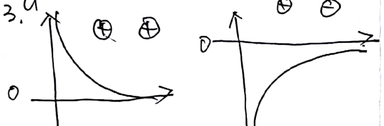
   $\vec{E}$ acts on a static and moving charge

   $\vec{B}$ acts on a moving charge only.

2. $\varphi(b) - \varphi(a) = -\int_a^b F_E dx = \frac{W_{a\to b}}{q}$

   A conservation force always acts to push the system toward lower energy

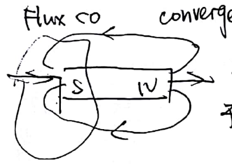
3.



4. Flux = net outward flow

   Flux > 0 divergence

   Flux < 0 convergence



   Flux = 0

   不存在磁单极

5. Maxwell first equation ⇔ Gauss

   $\text{Flux} = \frac{\text{net charge inside}}{\varepsilon_0}$

6.

   $E \cdot 4\pi R^2 = \frac{q}{\varepsilon_0}$

   $E = \frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}$

7. Maxwell second equation ⇔ Gauss

   Flux of $\vec{B}$ through closed surface = 0

8. circulation = net rotational motion

   = average tangential component · distance

   $= \int \vec{E}_{\parallel} \cdot d\vec{l}$

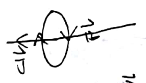
9. Maxwell third equation: Faraday

   Circulation of $\vec{E} = \frac{d}{dt}$ (flux of $\vec{B}$ through S)

   $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

10. Maxwell fourth equation: [illegible]

   $c^2 \cdot$ [circulation of $\vec{B}$] $= \frac{\text{flux of current}}{\varepsilon_0} + \frac{d(\text{flux of }\vec{E})}{dt}$



   $\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$

   $(c^2 = \frac{1}{\varepsilon_0 \mu_0})$ $(v^2 = \frac{1}{\varepsilon \mu})$

lecture 3

1. Gradient $\vec{\nabla}\varphi \rightarrow$ vector

   Divergence $\vec{\nabla}\vec{E} \rightarrow$ scalar

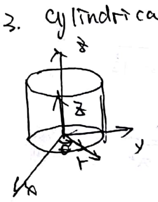
   Curl $\vec{\nabla} \times \vec{E} \rightarrow$ vector

2. $\vec{i} \times \vec{j} = \vec{k}$ $\qquad \vec{i} \times \vec{i} = 0$

   $\vec{j} \times \vec{k} = \vec{i}$ $\qquad \vdots$

   $\vec{k} \times \vec{i} = \vec{j}$

3. cylindrical coordinates



   $x = r\cos\theta$

   $y = r\sin\theta$

   $z = z$

   $dA = (R\,d\theta)\,dz$

   $dV = dr\,r\,d\theta\,dz$

4. Spherical coordinates

   $0 < \theta < \pi$

   $0 < \phi < 2\pi$

   $x = r\sin\theta\cos\phi$

   $y = r\sin\theta\sin\phi$

   $z = r\cos\theta$

   $dA = r^2\sin\theta\,d\theta\,d\phi$

   $dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

5. $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - B_y A_z)\vec{i} - (A_x B_z - A_z B_x)\vec{j} + (A_x B_y - A_y B_x)\vec{k}$

6. $\vec{A}(\vec{B} \times \vec{C}) = \vec{B}(\vec{C} \times \vec{A}) = \vec{C}(\vec{A} \times \vec{B})$

7. $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

8. $\vec{\nabla} f \perp f = c$ at point $P(x, y, z)$

9. $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$

10. $\text{div}\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

11. $\vec{\nabla} T = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$

12. $\vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

13. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) = 0$

   Theorem 1. $\vec{\nabla} \times \vec{V} = \vec{0} \Rightarrow \vec{V} = \vec{\nabla} f$

   Theorem 2. $\vec{W} = \vec{\nabla} \times \vec{V}$

   $\begin{cases} \vec{\nabla} \times \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 \end{cases}$

14. Gradient in cylindrical

$$d\vec{l} = dr\cdot\vec{e}_r + r\,d\theta\cdot\vec{e}_\theta + dz\cdot\vec{e}_z$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\vec{e}_\theta + \frac{\partial}{\partial z}\vec{e}_z\right)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{e}_r + \cdots$$

15. 度量系数

	直角	圆柱	球坐标系
h_1	1	1	1
h_2	1	r	R
h_3	1	1	$R\sin\theta$

16. Gradient in spherical

$$d\vec{l} = dr\cdot\vec{e}_r + r\,d\theta\,\vec{e}_\theta + r\sin\theta\,d\phi\,\vec{e}_\phi$$

17. Gradient 通式

$$\vec{\nabla}V = \vec{a}_{u_1}\frac{\partial V}{h_1\partial u_1} + \vec{a}_{u_2}\frac{\partial V}{h_2\partial u_2} + \vec{a}_{u_3}\frac{\partial V}{h_3\partial u_3}$$

divergence 通式

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3A_1) + \frac{\partial}{\partial u_2}(h_1h_3A_2) + \frac{\partial}{\partial u_3}(h_1h_2A_3)\right]$$

curl 通式

$$\vec{\nabla}\times\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \vec{a}_{u_1}h_1 & \vec{a}_{u_2}h_2 & \vec{a}_{u_3}h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$

18. $\vec{\nabla}(\vec{\nabla}\varphi) = \nabla^2\varphi$ Laplacian

$$\vec{\nabla}\times(\vec{\nabla}\varphi) = 0$$

$$\vec{\nabla}(\vec{\nabla}\times\vec{A}) = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

19. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

20. Gauss

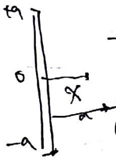
$$\oiint \vec{A}\cdot d\vec{S} = \iiint \vec{\nabla}\vec{A}\,dV$$

21. Stoke's

$$\oint_C \vec{A}\cdot d\vec{l} = \iint(\vec{\nabla}\times\vec{A})\,d\vec{S}$$

lecture 4

1. Field of line charge



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x}\frac{1}{\sqrt{x^2+a^2}}\vec{i}$$

$$\varphi_{a\to b} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_a}{r_b}$$

$$\varphi_r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r}$$

2. Field of ring.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{\frac{3}{2}}}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

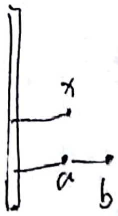
3. Field of disk

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right)$$

4. 无限大平板

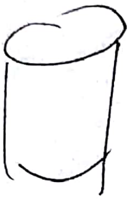
$$\vec{E} = \frac{\sigma}{2\varepsilon_0}$$

1. Field of line charge:



$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x} \cdot \frac{1}{\sqrt{x^2+a^2}} \vec{i}$$

$$\varphi_{a\to b} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_a}{r_b}$$



$$\varphi_r = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R}{r}$$

2. Field of ring

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{xQ}{(x^2+a^2)^{\frac{3}{2}}}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{x^2+a^2}}$$

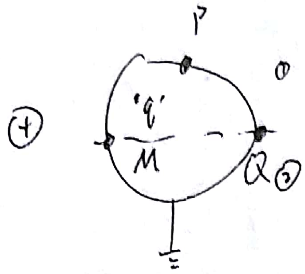
3. Field of disk

$$\bar{E} = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right)$$

4. large Plate

$$\bar{E} = \frac{\sigma}{2\varepsilon_0}$$

镜像法：



$$\left.\begin{array}{l} ① \ \varphi(P)=0 \Rightarrow \text{一个条件;} \\ ② \ \text{Then } \varphi(M)=\varphi(Q)=0 \end{array}\right\} \Rightarrow q',\ q$$

$$, \quad Q(R) = \int_0^R \rho(r)\, 4\pi r^2 \cdot dr$$
$$= \pi a R^4$$