From Static to dynamics

Static → From Steady states to time dependent effects → dynamics

$$\frac{\partial}{\partial t} = 0$$

$$\frac{\partial}{\partial t} \neq 0$$

Faraday's discovery and the **birth of electrical engineering**

Orsted 1820 and Faraday 1921:

Discovery of the close connection between electricity and magnetism

- Current in a wire **creates** a magnetic field (compass deviation)
- Wire carrying current in a magnetic field **manifests** action - reaction

Involves steady current

- **Changing** magnetic **creates** a changing electric field
- **Changing** electric field **creates** a changing magnetic field

Involves varying current

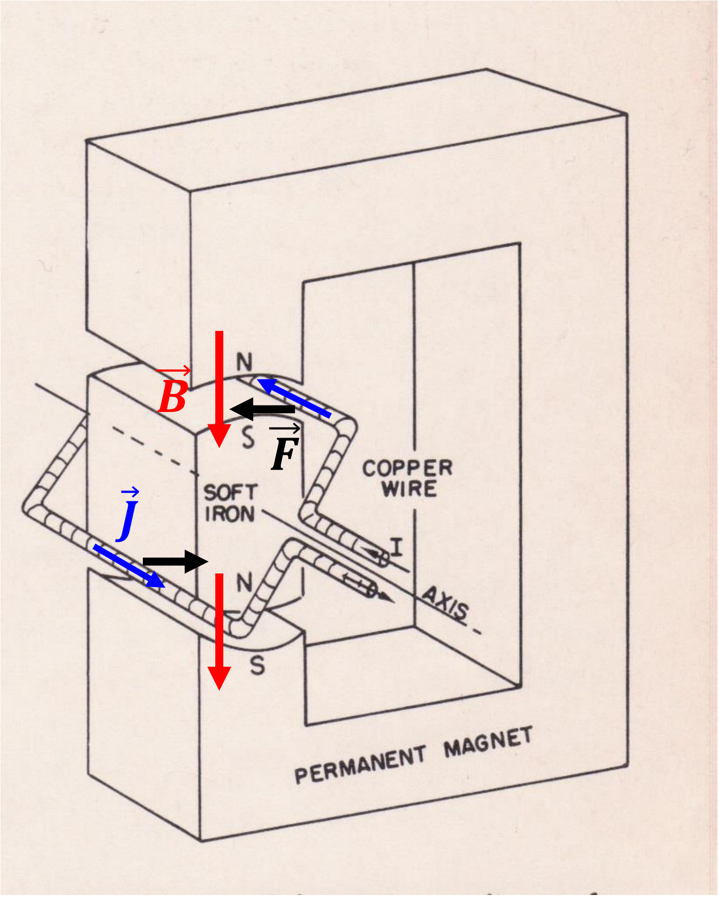
FORCE and / or **TORQUE**

Can be used to produce **WORK**

Birth of electrical engineering

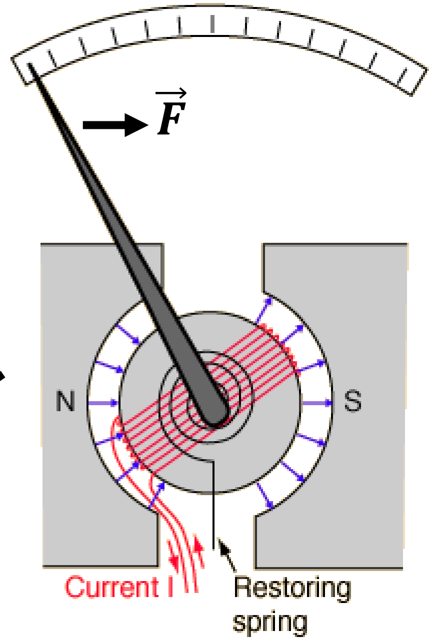
Magnetic field produces mechanical force

Application of Lorentz force



Make a motor (current reversed each half-turn)

Torque in action



Make a sensitive instrument for measuring current

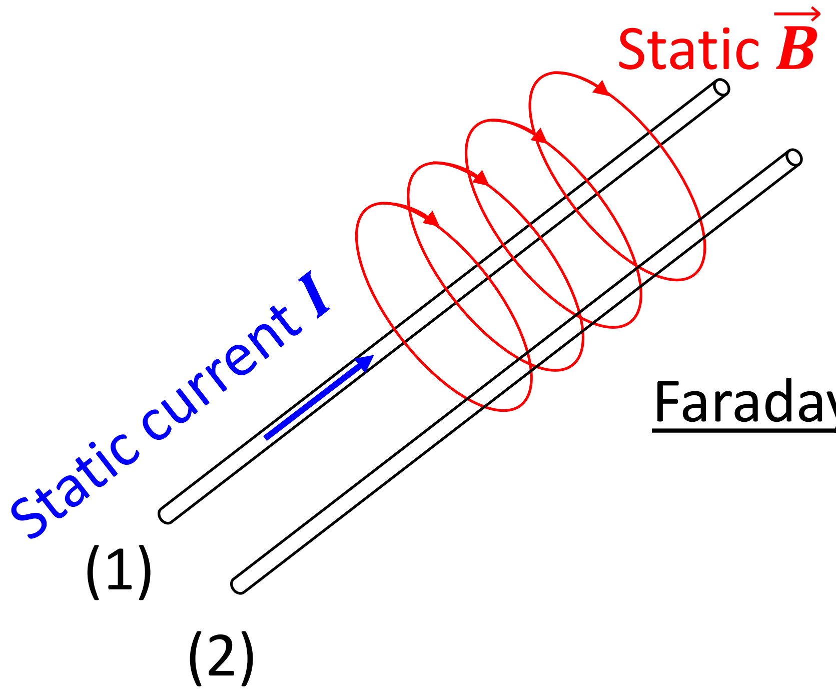
Electromagnetic Motor

Galvanometer

From Feynman lecture (Volume II)

<u>Orsted 1820</u>: Current in a wire creates magnetic field

<u>Faradav 1821</u>: Magnetic field **MAY** create an electric field ⇨ Generating thus a current in wire (2)



Nothing appears in wire (2)

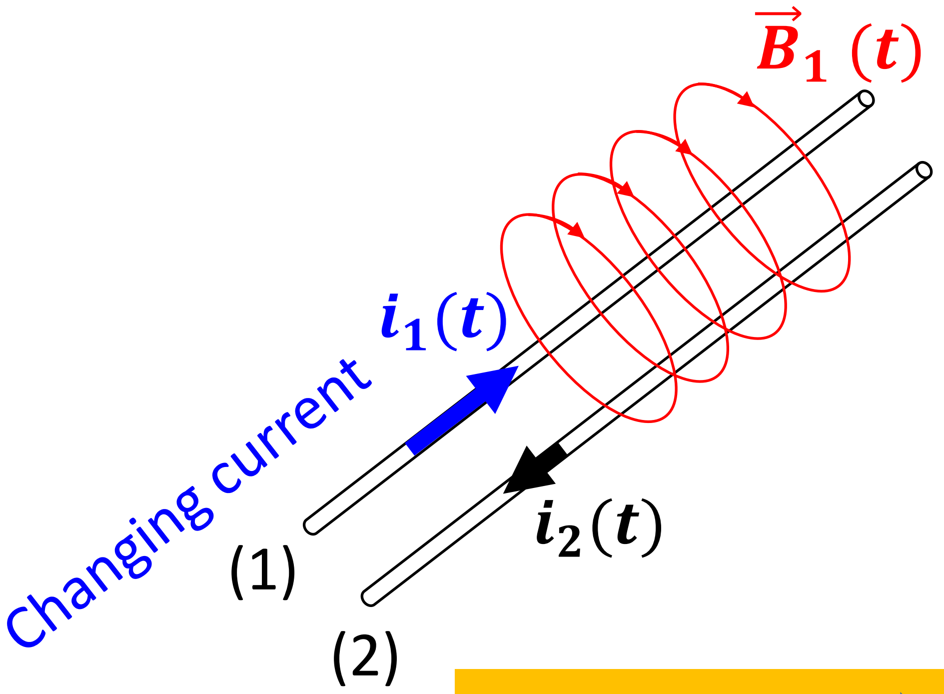
<u>Faraday's discovery</u> ⇨ From Static to **dynamics**

Requires that something is changing with **TIME**

Current induction

No Current without $\vec{E}$

A varying current in wire (1) ⇒ A varying current $\boldsymbol{i_2(t)}$ appears in wire (2)



Induction of **electric field** in wire (2) driving electrons creating thus a **current**

EMF (ElectroMotive Force)

$\boldsymbol{i_1(t)} \Rightarrow \boldsymbol{i_2(t)}$

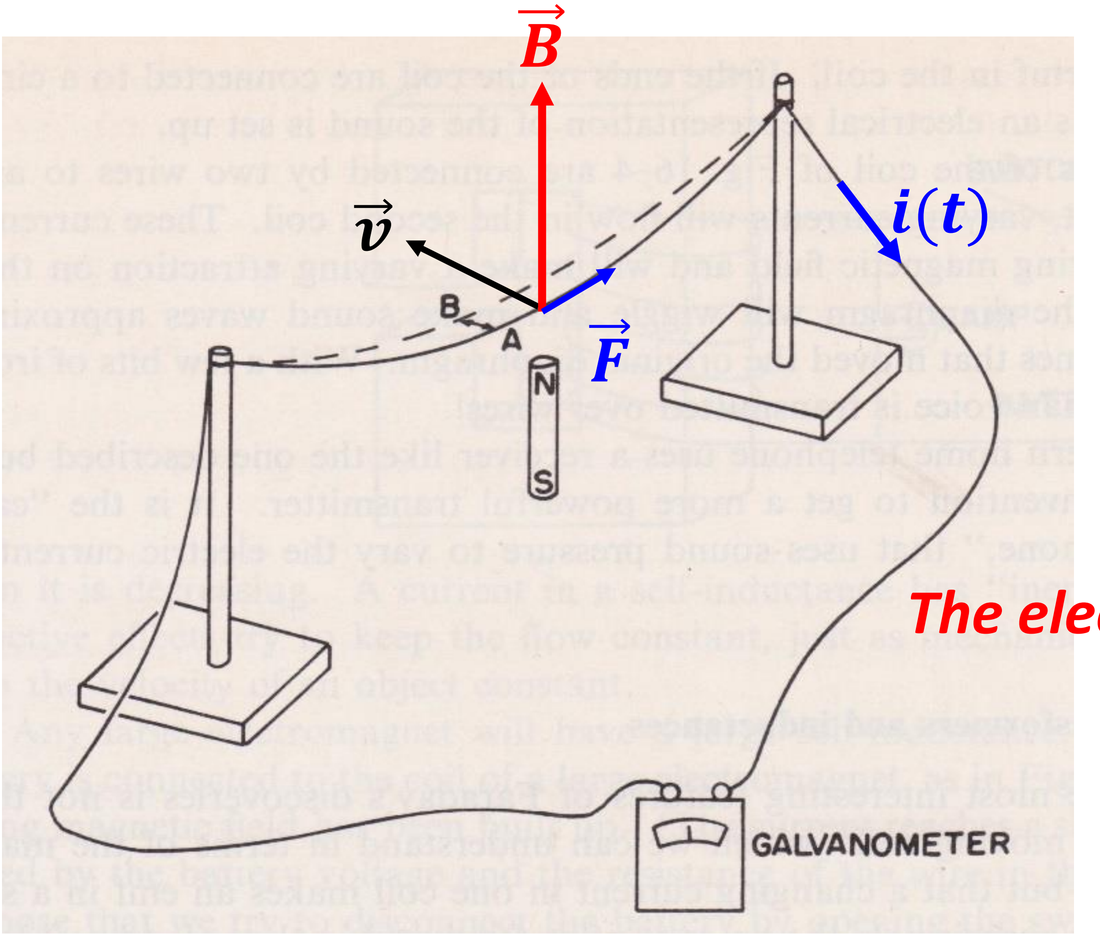
$\vec{J}_1 = -\vec{J}_2 \longrightarrow$ Opposing magnetic field

Essential feature discovered by Faraday: **Time changing processes**

Is something missing in this figure? Yes: A second magnetic field is generated by wire (2)

Current induction: Lorentz force was not known to Faraday

$$\vec{F} = q\vec{v} \times \vec{B}$$



$\vec{v}$ = **mechanical velocity**

Current induced by **Lorentz force**

Concept of motional emf

Why the galvanometer shows a current while it is far from the Lorentz force ?

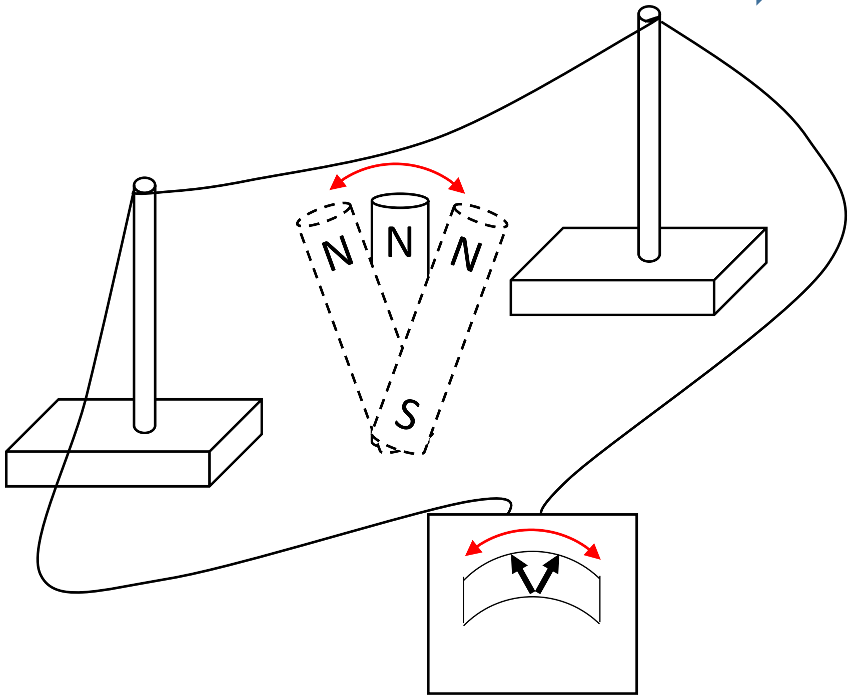
The electrons move by electric repulsion over long distances

First telegraph discovered by Gauss and Weber

From Feynman lecture (Volume II)

Another way of Current induction discovered by Faraday

No Lorentz force → **By shaking the magnet** → **Relativistic effect**



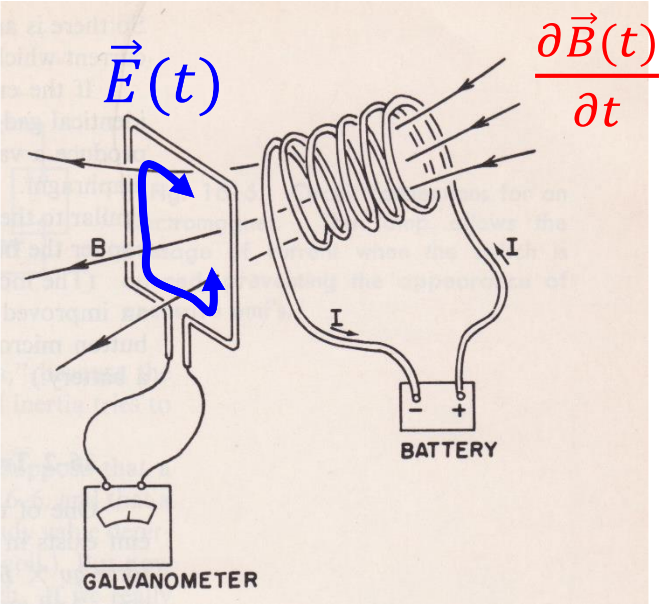
- No current induction if magnet is static
- Current induced by change in magnetic field $\frac{\partial B}{\partial t}$

Concept of **non**motional emf

Again why the galvanometer shows a current while it is far from the Lorentz force ?

There is a net integrated push around the complete circuit. This net push MUST come from an electric field which then leads to the repulsive push over long distance

Faraday's discovery: **emf**



$$\frac{\partial \vec{B}(t)}{\partial t} \longleftrightarrow \frac{\partial \Phi}{\partial t}$$

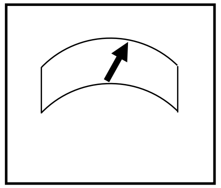
From Feynman lecture (Volume II)

Current induced by change in magnetic field $\boxed{\frac{\partial B}{\partial t}}$

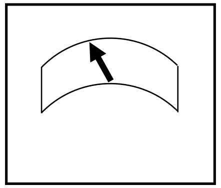
The current noticed in the loop is due to motion of electrons

- A force has pushed the electrons ⇨
- An electric field has been generated by $\boxed{\frac{\partial B}{\partial t}}$

1) We switch **ON** the battery



2) We switch **OFF** the battery

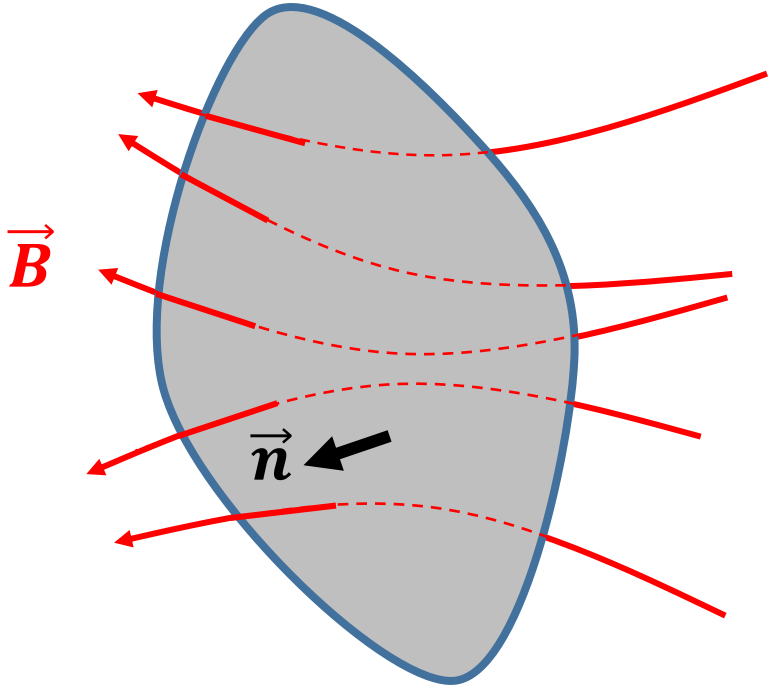


$$\boxed{\vec{E}(t) \propto \frac{\partial \vec{B}(t)}{\partial t}}$$

emf can be generated in a wire by:

- Moving mechanically the wire near a magnet or vice versa
- Changing current in a nearby wire

Faraday’s rule: Is it the changing magnetic field or changing flux that matters?



$$\frac{\partial \vec{B}(t)}{\partial t} \longleftrightarrow \frac{\partial \Phi}{\partial t}$$

Area kept unchanged

$$\Phi = \int \vec{B}.d\vec{A} = \int \boldsymbol{B}_{\perp} dA$$

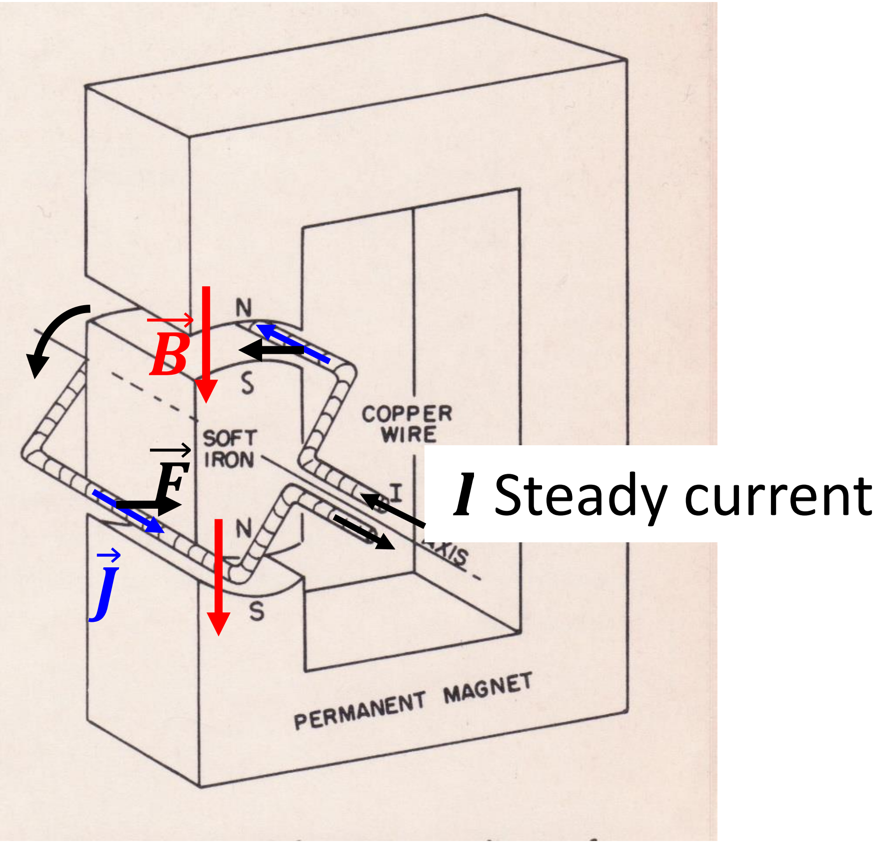
Normal component of $\vec{B}$ integrated over the whole area of the loop

emf: ONLY and **ONLY IF** the flux is changing with time

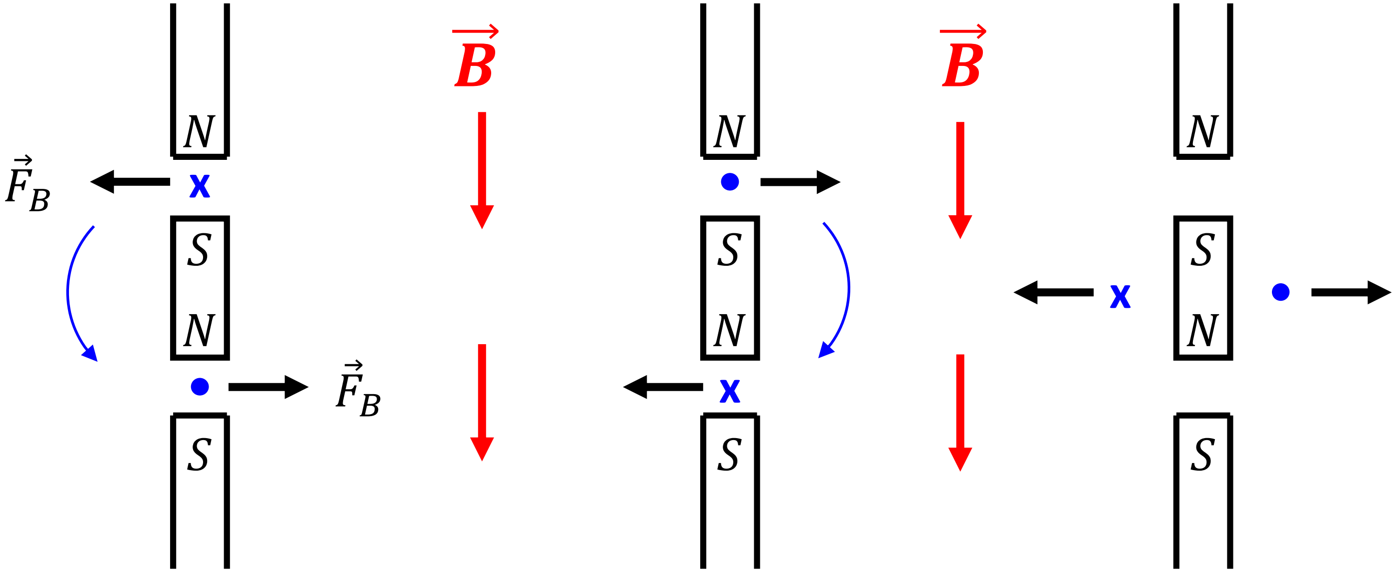
- Changing $\vec{B}$
- Changing $\vec{A}$
- Changing both $\vec{A}$ and $\vec{B}$

Motor

The loop rotates due to **Lorentz force**



From Feynman lecture (Volume II)



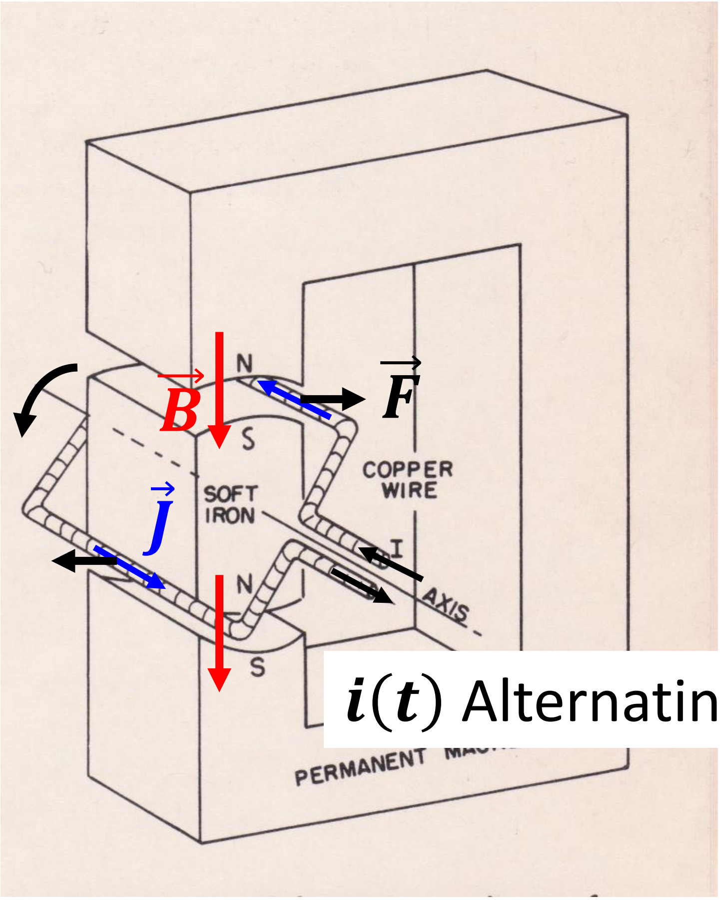
Forces acting along the same line

Current **MUST** be reversed each half-turn <u>otherwise the loop stops rotating</u>

From steady to variable current $I \rightarrow i(t)$

Alternating current generator

The loop rotates **mechanically** → Induction of an emf: **E**lectro**M**otive **F**orce



$\boldsymbol{i(t)}$ Alternating current

From Feynman lecture (Volume II)

- Area of the loop is fixed
- The normal component of $\vec{\boldsymbol{B}}(\boldsymbol{t})$is changing

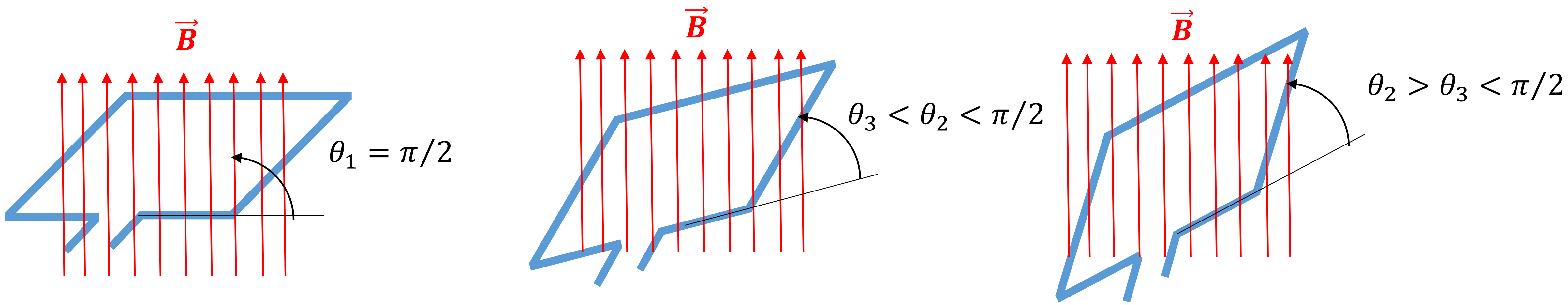
Changing flux $\Phi(t)$

Electric field generated in the wire

Circulation of $\vec{\boldsymbol{E}}(\boldsymbol{t})$ along the loop

Generation of an alternating current $i(t)$ with frequency = frequency to the mechanical rotation

Emf = rate of change of magnetic flux through the loop



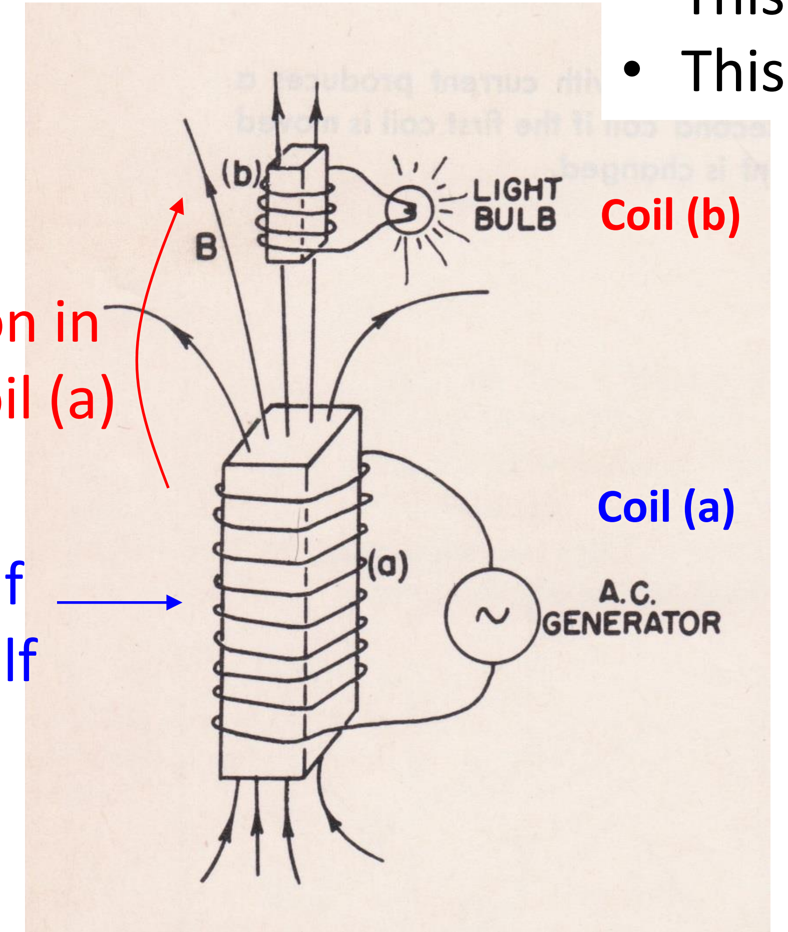
$\boldsymbol{\Phi_1}$ > $\boldsymbol{\Phi_2}$ > $\boldsymbol{\Phi_3}$

What matters in the flux is the **normal component of the field** along the unit vector of the area

From $\boldsymbol{\Phi_1}$ to $\boldsymbol{\Phi_3}$ $\quad \boxed{\dfrac{\partial \Phi}{\partial t} \rightarrow E(t) \rightarrow emf \rightarrow i(t)}$

Lenz's rule

- The induced emf creates a current
- This current generates a magnetic field
- This magnetic field tends to oppose the original one



Coil (b)

Emf induction in coil (b) by coil (a)

Coil (a)

Self-induced emf in coil (a) by itself

From Feynman lecture (Volume II)

The polarity of the induced emf is **ALWAYS** such as to oppose the change of the flux of the original magnetic field

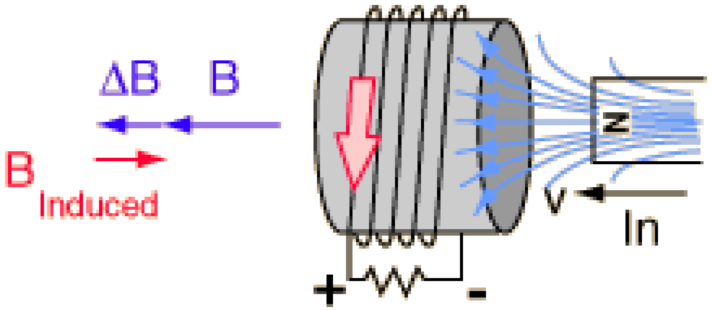
$$emf = -N\frac{d\Phi}{dt}$$

N = # of turns

Φ = Flux of B through area A

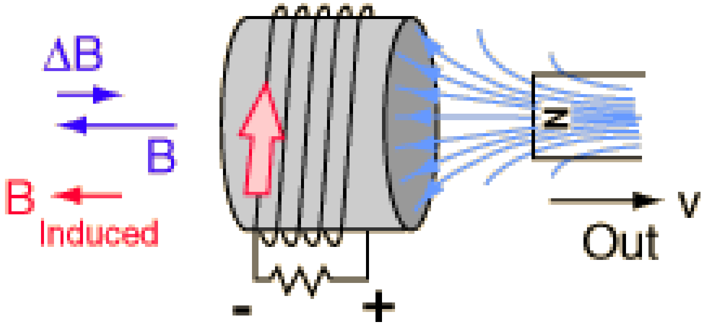
Lenz's law

Lenz's law



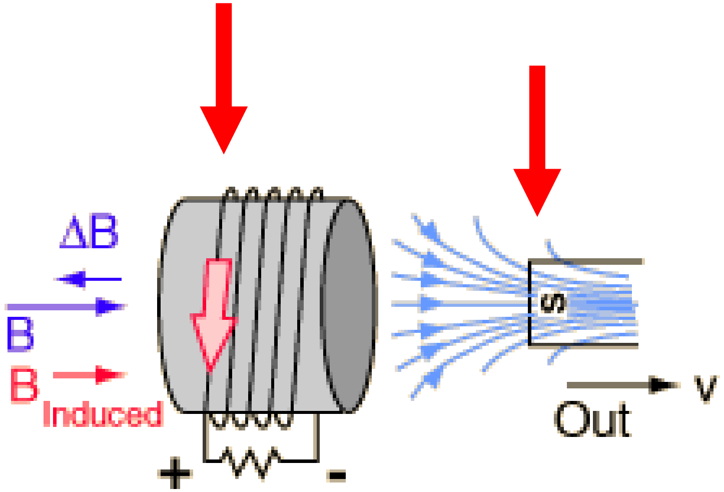
⇨ Flux increases ⇨ B increases by ΔB

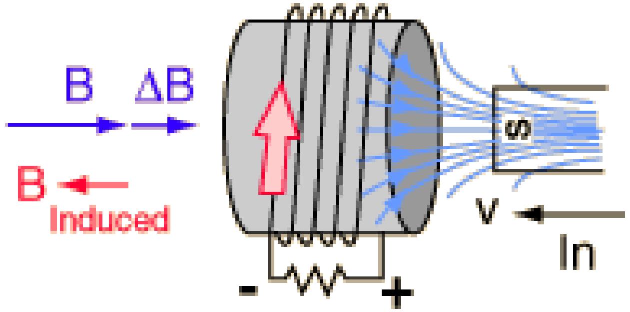
The current generated by the emf goes in a direction such as to induce a magnetic field in the opposite direction



⇨ When the flux is decreased the emf is reversed

Caution! Here the arrows are reversed as it is the south pole of the magnet that is moving in and out





Towards time dependent Maxwell's equations

$$\vec{\nabla}.\vec{E}(t) = \frac{\rho(t)}{\varepsilon_0}$$

Gauss' law

$$\vec{\nabla}.\vec{B}(t) = 0$$

No magnetic charges

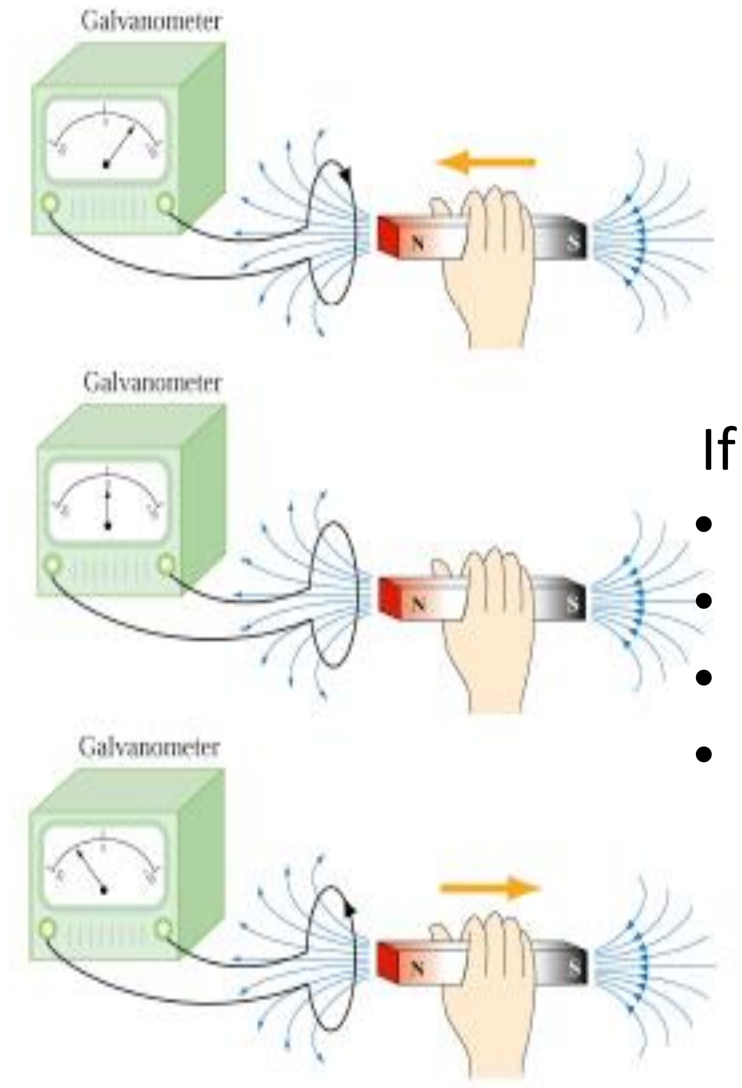
$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law ($\vec{E}$ is no longer conservative)

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell

Faraday’s law of induction



Relative motion induces current in the loop

There **MUST** be an induced electric field Along the loop

$$\oint \vec{E}_{nc}.d\vec{l} \neq 0$$

$$\rightarrow \boldsymbol{i(t)}$$

If no motion of the loop and magnet

- **NO** change of the flux
- **NO** electric force
- **NO** motion of the free charges
- **NO** current in the loop

Electrostatic ⇨ Conservative field

$$\oint \vec{E}_{c}.d\vec{l} = 0$$

Faraday's law: electromotive force

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Motional emf

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}).d\vec{l} = -\frac{d\Phi_B}{dt}$$

Non motional emf

Faraday's law

$$\mathcal{E} = \oint \vec{E}_{nc}.d\vec{l} = -\frac{d\Phi_B}{dt}$$

Non-electrostatic field

$$\oint \vec{E}_{nc}.d\vec{l} = -\frac{d\Phi_B}{dt}$$

Integral form

What is the link between these two relationships?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Differential form

Faraday's law

$$\oint \vec{E}_{nc} . d\vec{l} = -\frac{d\Phi_B}{dt} \quad + \quad \oint \vec{E}_c . d\vec{l} = 0 \quad \Rightarrow \quad \oint \underbrace{(\vec{E}_c + \vec{E}_{nc})}_{\vec{E}} . d\vec{l} = -\frac{d\Phi_B}{dt}$$

$\vec{E}$ (superposition principle)

Gauss's theorem

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \vec{B}.d\vec{A} = -\iint \frac{d\vec{B}}{dt}.d\vec{A}$$

Stoke's theorem

$$\oint \vec{E}.d\vec{l} = \iint (\vec{\nabla} \times \vec{E}).d\vec{A}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

True always

$$if\ \frac{\partial \vec{B}}{\partial t} = \vec{0} \Rightarrow \oint \underbrace{(\vec{E}_c + \vec{E}_{nc})}_{=0} . d\vec{l} = 0$$

$$\vec{\nabla} \times \vec{E} = \vec{0}$$ Electrostatic

The Heart of Faraday's law of induction

Φ_B

$\Downarrow$

$$\frac{d\Phi_B}{dt}$$

Area | Magnetic field

$$\frac{dA}{dt} \qquad\qquad \frac{dB}{dt}$$

Motional emf

Non motional emf (if we can change B without moving a magnet nor changing the area)

$\Downarrow$

Faraday's law

$$\frac{d\Phi_B}{dt} = \frac{dBA}{dt}$$

B constant

Part of the circuit is moving ⇨ $\vec{v} \times \vec{B} \neq \vec{0}$

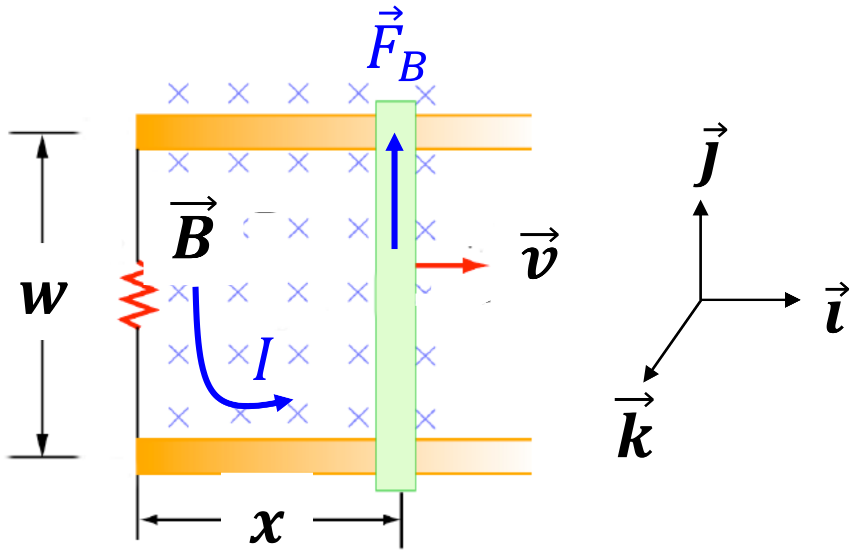
A changes ⇨ $\frac{dA}{dt} = w\frac{dx}{dt}$

A constant

Circuit is immobile ⇨ $\vec{v} \times \vec{B} = \vec{0}$

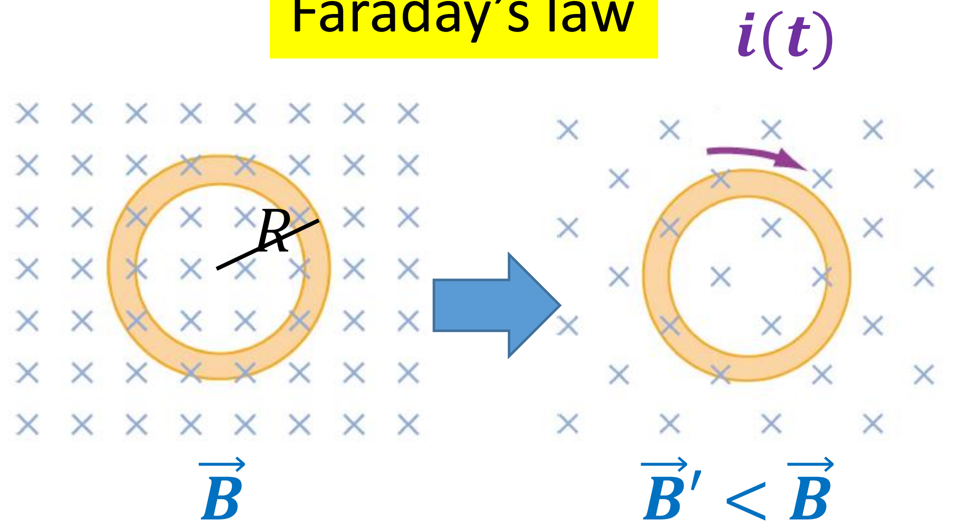
B changes ⇨ $\frac{dB}{dt} \to E(t) \to \vec{F} = q\vec{E}$

In both cases electrons move due to Coulomb force



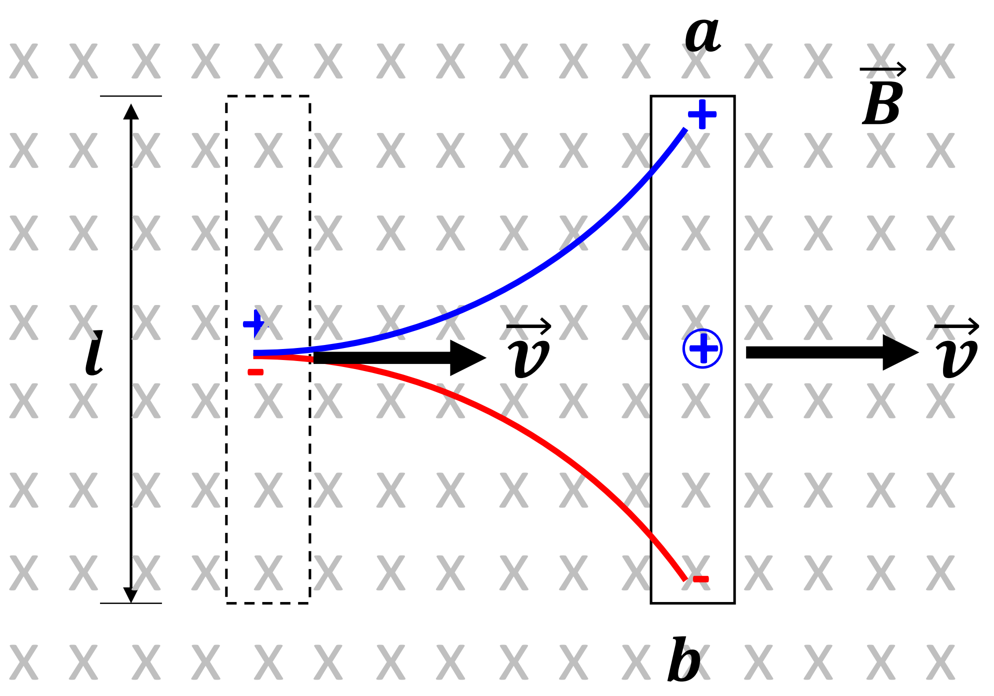
Area $A = wx$

Faraday's law

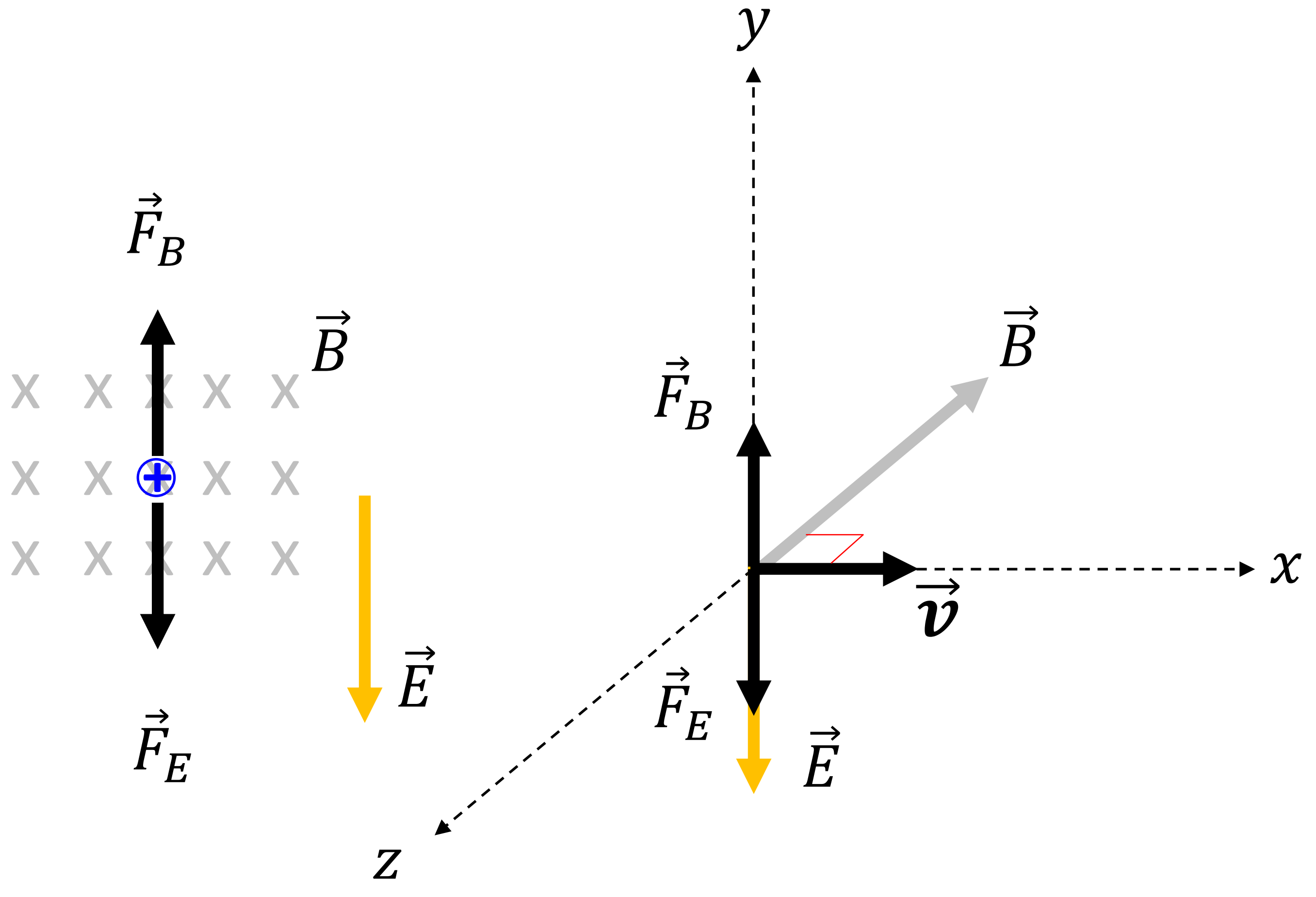


Area $A = 2\pi R$

Motional emf



An electric field is building up inside the conducting bar !



Equilibrium !

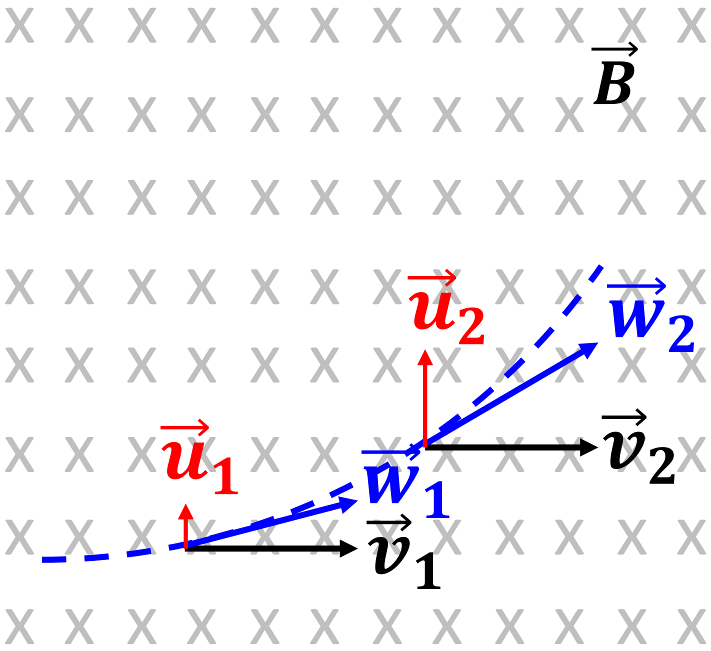
$$\vec{F}_B + \vec{F}_E = \vec{0}$$

Equilibrium $\Rightarrow$ $qvB = qE$ $\Rightarrow$ $E = vB$ $\Rightarrow$ $\varphi_{ab} = \varphi_a - \varphi_b = El$

$$\boxed{\varphi_{ab} = Blv}$$

Motion $\Rightarrow$ **Motional** emf = Potential difference $\Rightarrow$ $\boldsymbol{\mathcal{E} = \varphi_{ab} = Blv}$

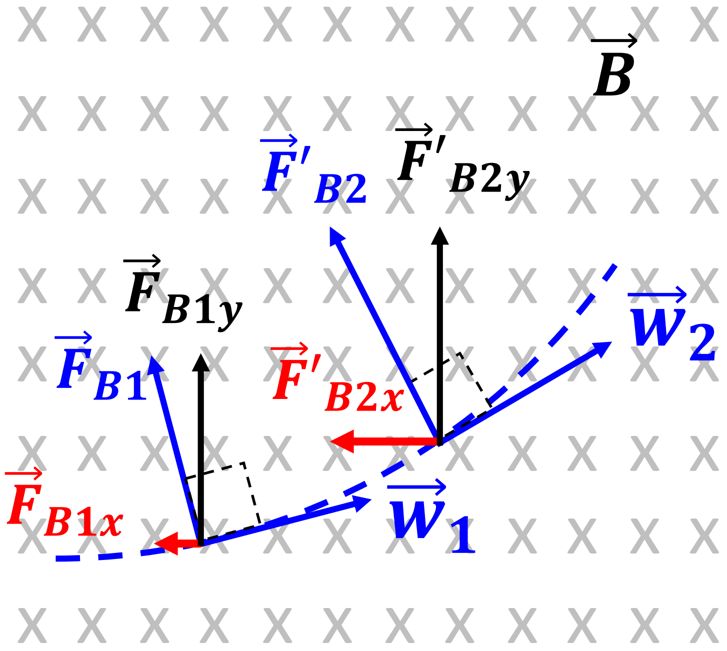
For **Motional** emf both $\boldsymbol{B}$ and $\boldsymbol{v}$ are required to get a potential difference



$$\vec{F}_B \xrightarrow{\perp} \vec{w}$$

$$\vec{F}_{By} \xrightarrow{\perp} \vec{v}$$

$$\vec{F}_{Bx} \xrightarrow{\perp} \vec{u}$$



None of these forces do work on the charge

BUT a force $\vec{F}_{Bx}$ opposing the motion of the bar is building-up

Work **MUST** be done against this force to keep the bar moving at constant velocity $\vec{v}$

Opposite force $\vec{F}_{Bx} = -quB\vec{\imath}$

Work done per unit charge against this force must be $\frac{\Delta W}{q} = -\int_{x}^{x+\Delta x} -uB\vec{\imath}.\,dx\vec{\imath}$

$$\frac{\Delta W}{q} = -\Delta\varphi = \varphi_{ab} = -\int_{x}^{x+\Delta x} -uBdx = uB\Delta x$$

$\Delta x = v\Delta t$

$\Delta l = u\Delta t$

$\Rightarrow$ $\Delta x = \frac{v\Delta l}{u}$ $\Rightarrow$

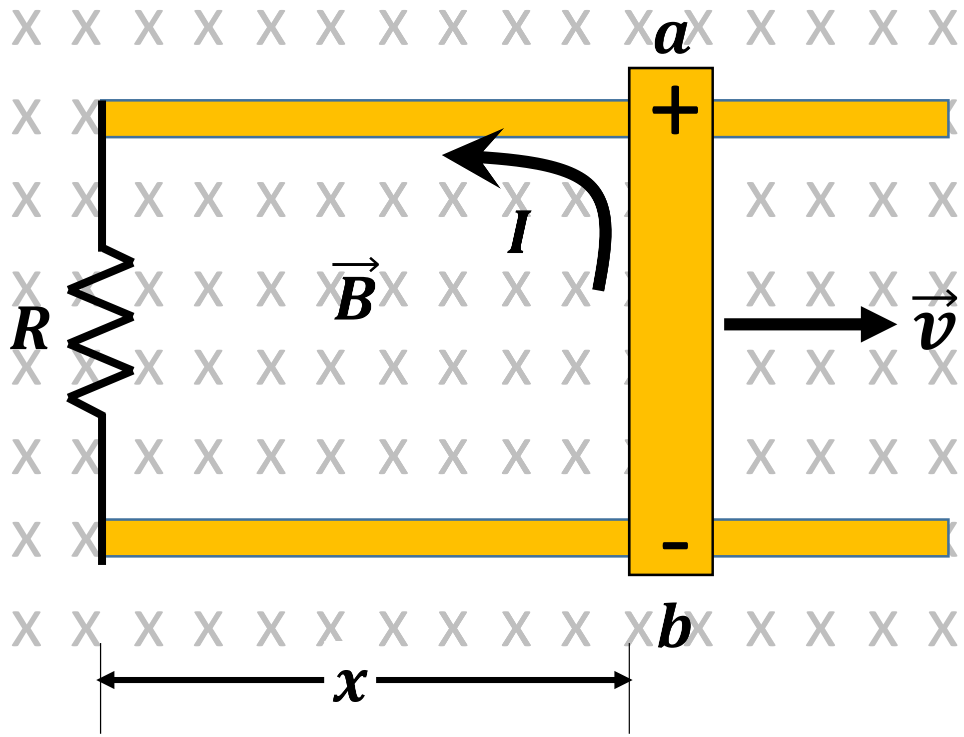
$$\boxed{\frac{\Delta W}{q} = -\Delta\varphi = \varphi_{ab} = \mathcal{E} = \boldsymbol{Blv}}$$

Motional emf found in slide # 22

This work is useless because the circuit is **not complete !**

The circuit is complete and the bar moves at constant velocity

The charges will **no longer accumulate** at the ends of the bar and a current is generated
Counterclockwise: <u>The motion of the charges in the circuit is due to coulomb repulsion</u>

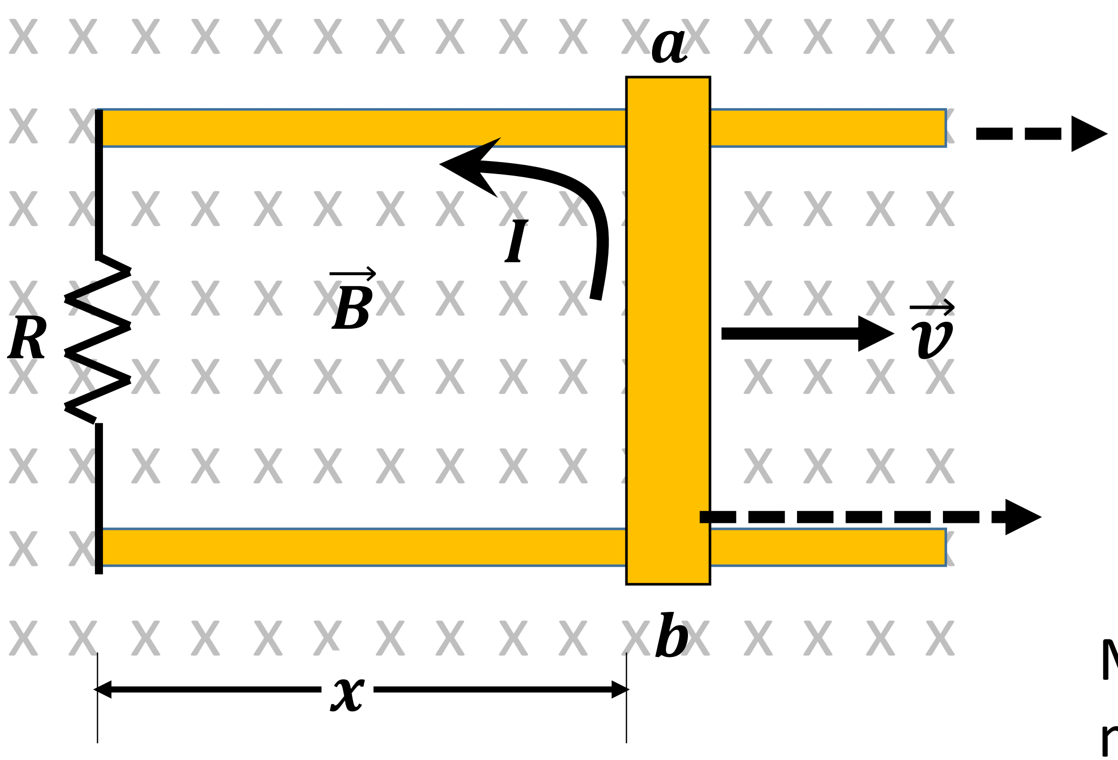


Question: Why putting a resistance R in the circuit ?

Answer: To reduce the current to a minimum in order to avoid generating an important magnetic field which may then disturb the external one

Remember: In electrostatic the test charge must be weak enough to avoid perturbing the source

The circuit is complete and the bar moves at constant velocity



Stationary U-shaped conductor

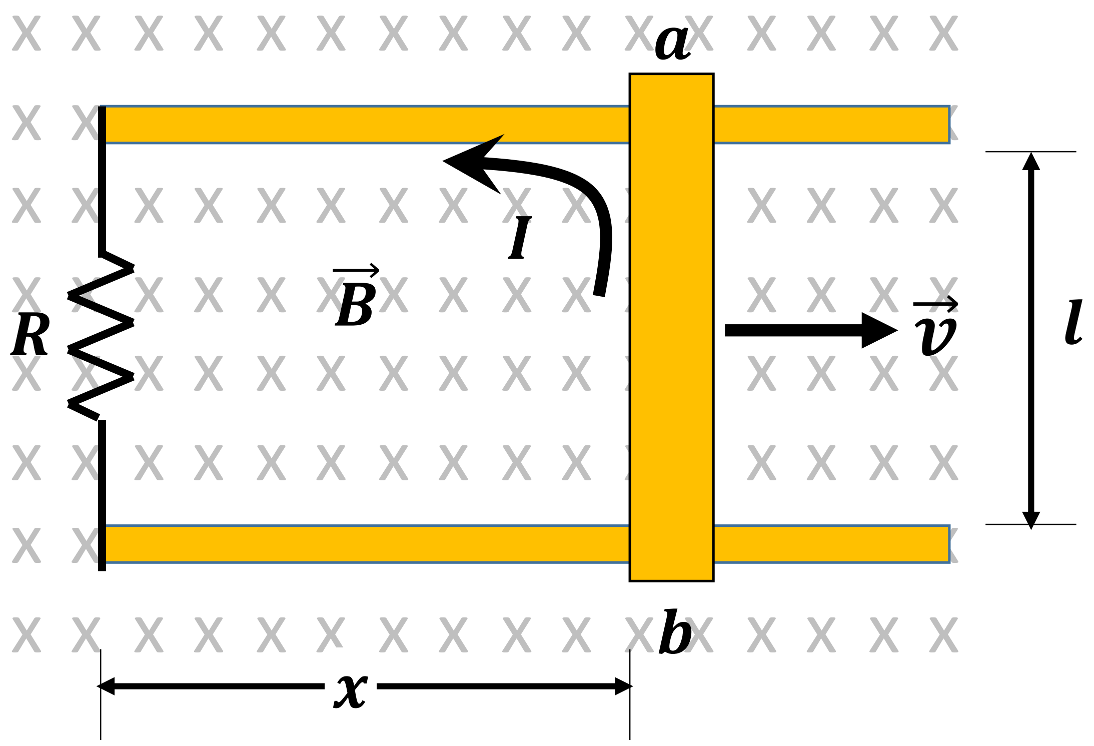
Moving conductor rod

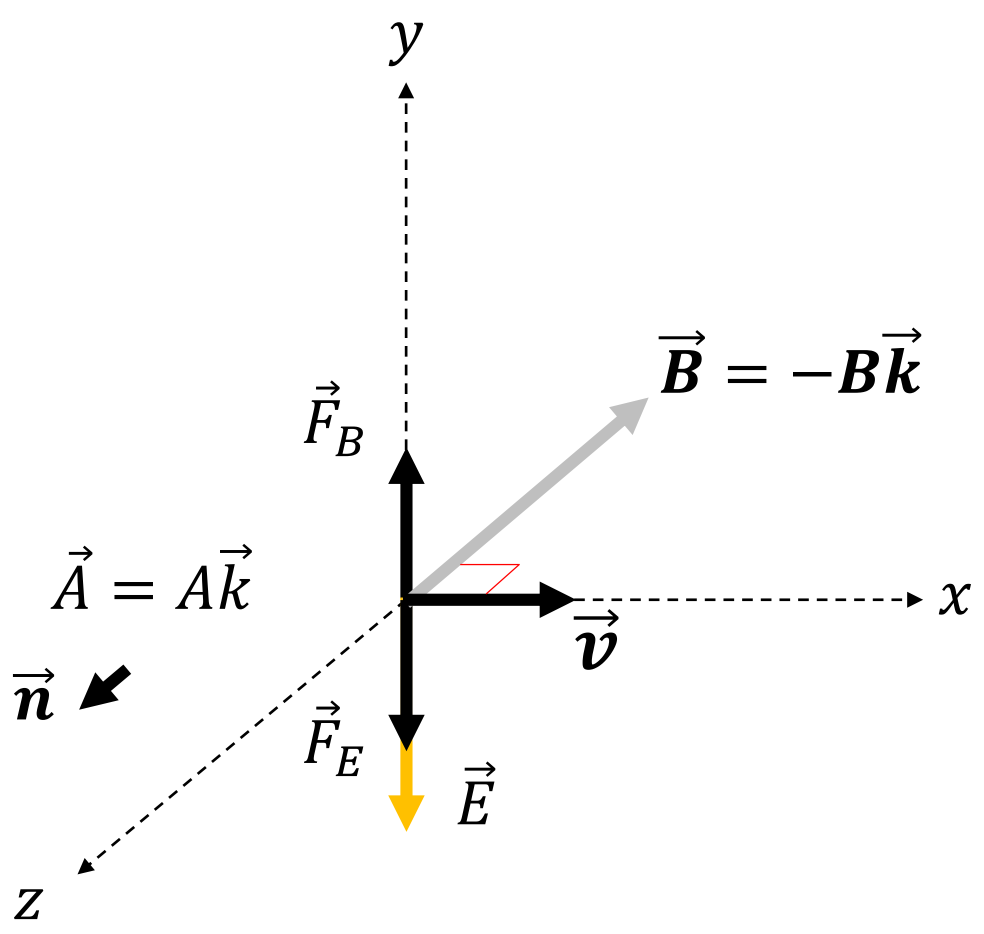
Magnetic force drives charges to the ends of the moving rod from which they redistribute in the stationary U-shaped part

Moving rod = source of electromotive force

Faraday's law: electromotive force

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



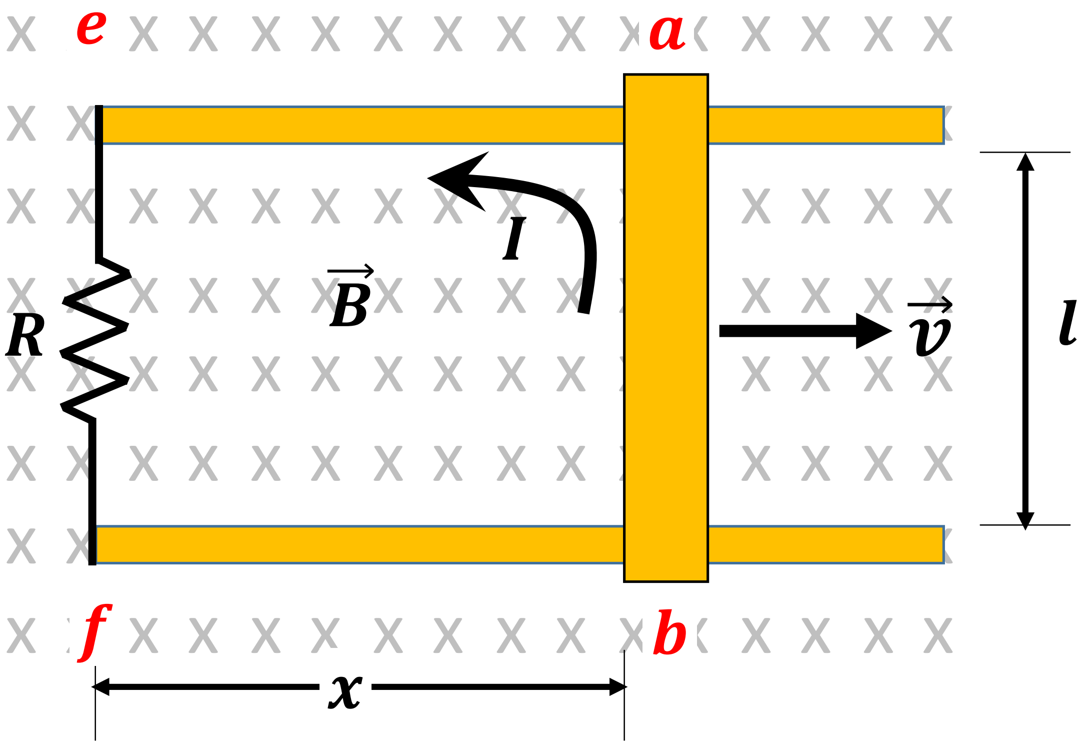


The magnetic flux changes because the area of the circuit is changing

$$\Phi_B = \vec{B}.\vec{A} = -Blx$$

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(-Blx) = -Bl\frac{dx}{dt}$$

$$\boxed{\frac{d\Phi_B}{dt} = -Blv}$$



$$\frac{d\boldsymbol{\Phi}_B}{dt} < 0$$

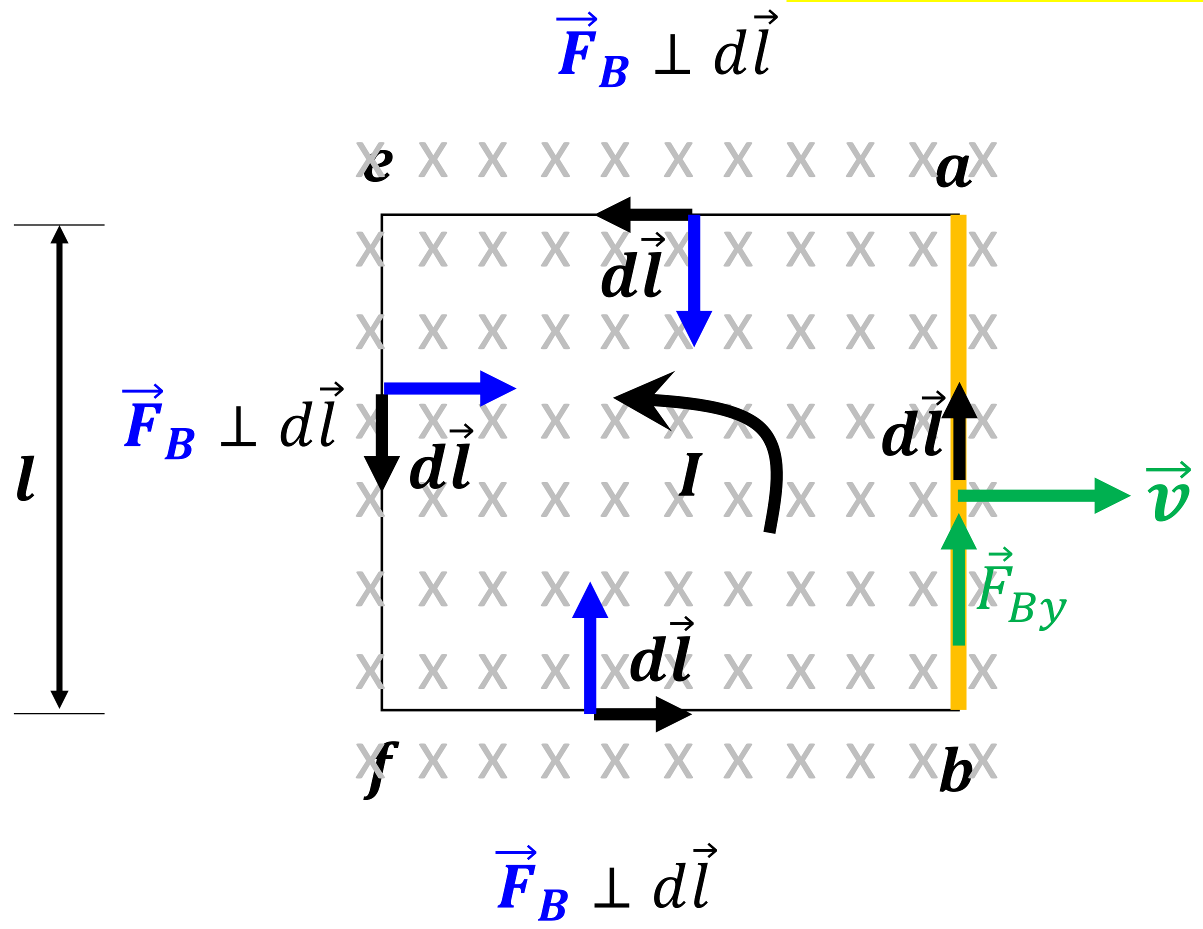
The closed path for the charges is ***aefb***

From slides # 17

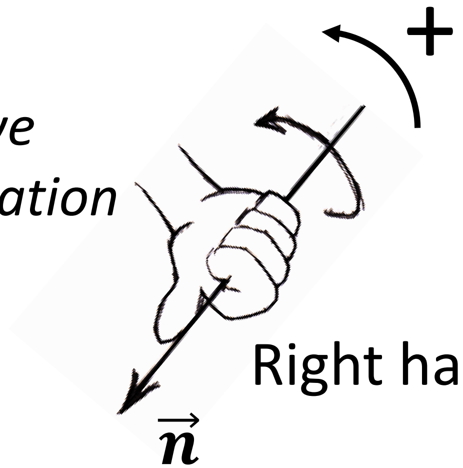
$$\boxed{\mathcal{E} = -\frac{d\boldsymbol{\Phi}_B}{dt}}$$

The magnetic field generated by the induced current opposes the original

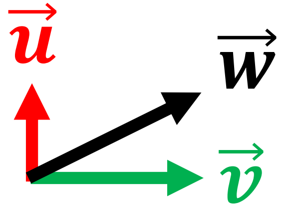
The electromotive force



Conventional positive trigonometry orientation



Right hand rule



$$\vec{F}_{By} = q\vec{v} \times \vec{B} = qv\vec{\imath} \times B(-\vec{k}) = qvB\vec{\jmath}$$

$\vec{F}_B \parallel d\vec{l}$ along the moving rod

Slide #17

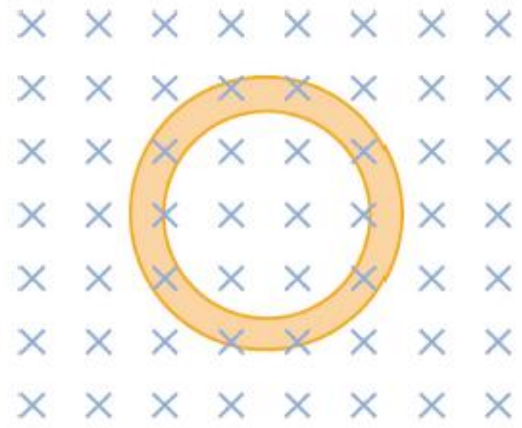
The only contribution is along the moving bar

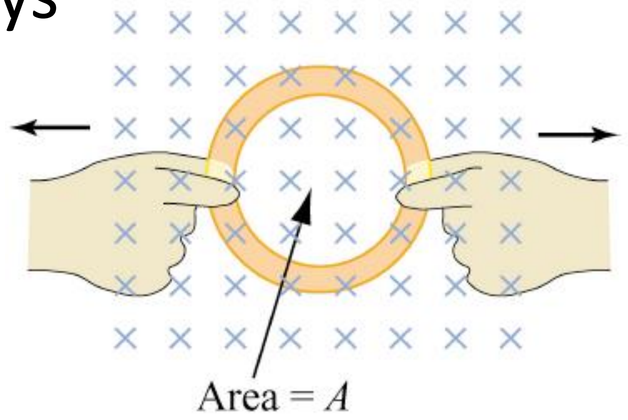
$$\mathcal{E} = \oint \frac{\vec{F}_{By}}{q} . d\vec{l} = \oint vB\vec{\jmath} . dy\vec{\jmath}$$

$$\boxed{\mathcal{E} = Blv}$$

Stationary closed conducting path (coil)

The flux can change following two ways

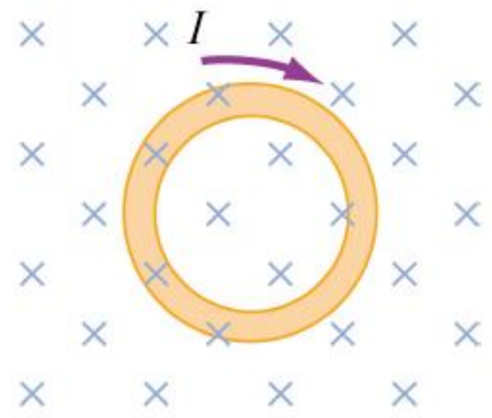


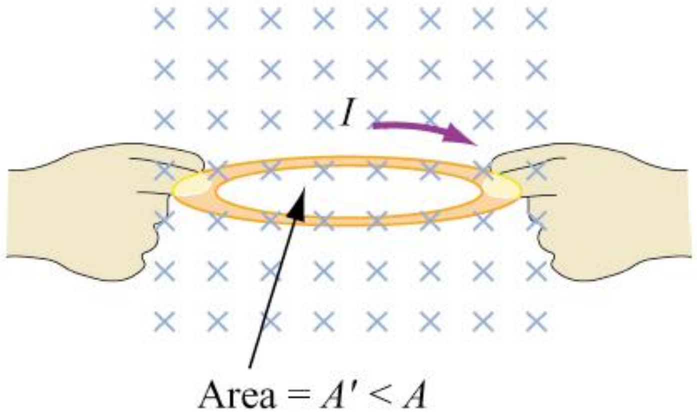


$$\boldsymbol{\Phi_B = \vec{B}.\vec{A}}$$

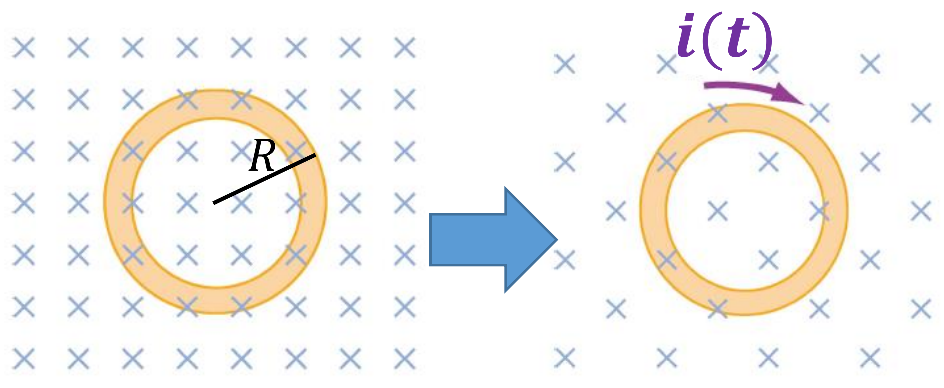
$\vec{B}$ changes

$\vec{A}$ changes





Genius conjecture of Faraday



$\vec{B}$ $\qquad$ $\boxed{\dfrac{d\Phi_B}{dt} < 0}$ $\qquad$ $B' < B$

The circuit is immobile ⇨ $\vec{v} = \vec{0}$ ⇨ $\vec{F}_B = \vec{0}$

$$\boxed{\vec{F} = q(\vec{E} + \underbrace{\vec{v} \times \vec{B}}_{\| \atop \vec{0}})}$$

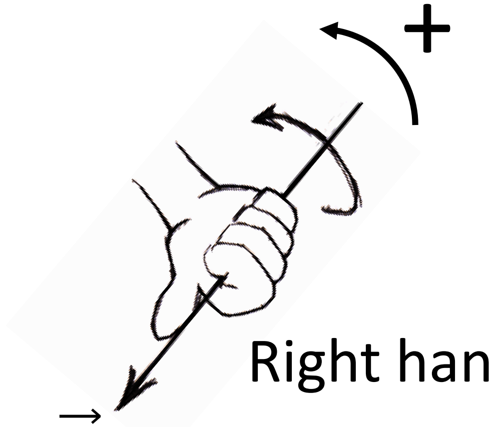
$$\frac{d\Phi_B}{dt} \longrightarrow \mathcal{E}(\text{emf}) = \oint_{\Gamma} \vec{E}_{nc}.d\vec{l}$$

Slide #17

Faraday's Conjecture

$\vec{E} \perp \vec{B}$

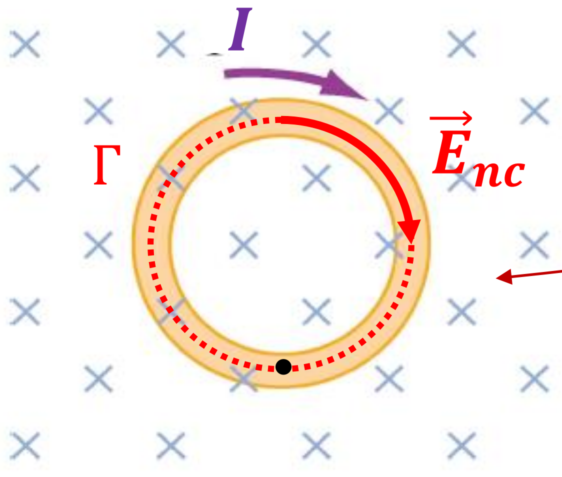
$\vec{E}_{nc}$ circulates tangentially to the loop



$\vec{n}$

Right hand rule

Unit vector of area



Electromotive force for both motional and non motional case

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}).d\vec{l}$$

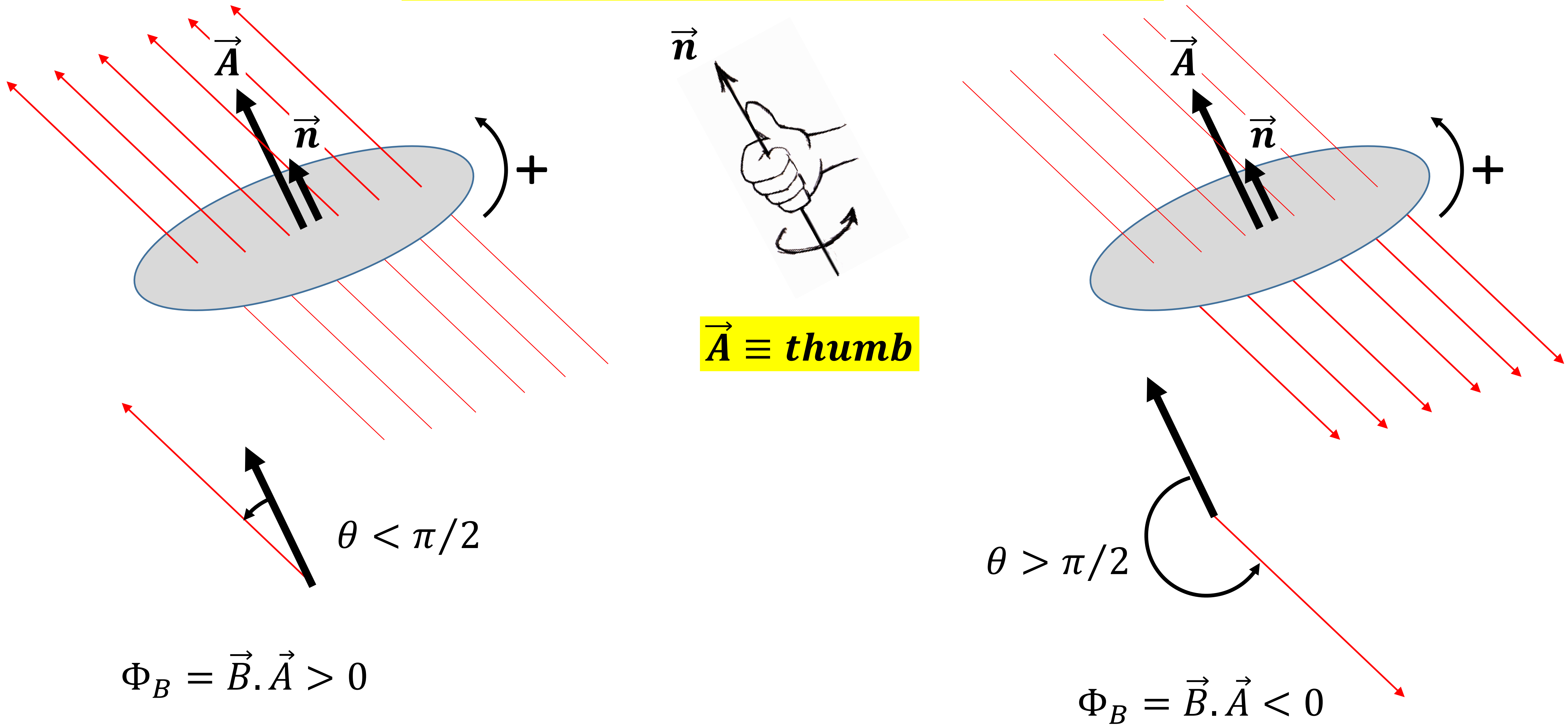
For motional electromotive force

$$\mathcal{E} = -\frac{\boldsymbol{d\Phi_B}}{\boldsymbol{dt}} = \oint_{\Gamma} \vec{E}_{nc}.d\vec{l}$$

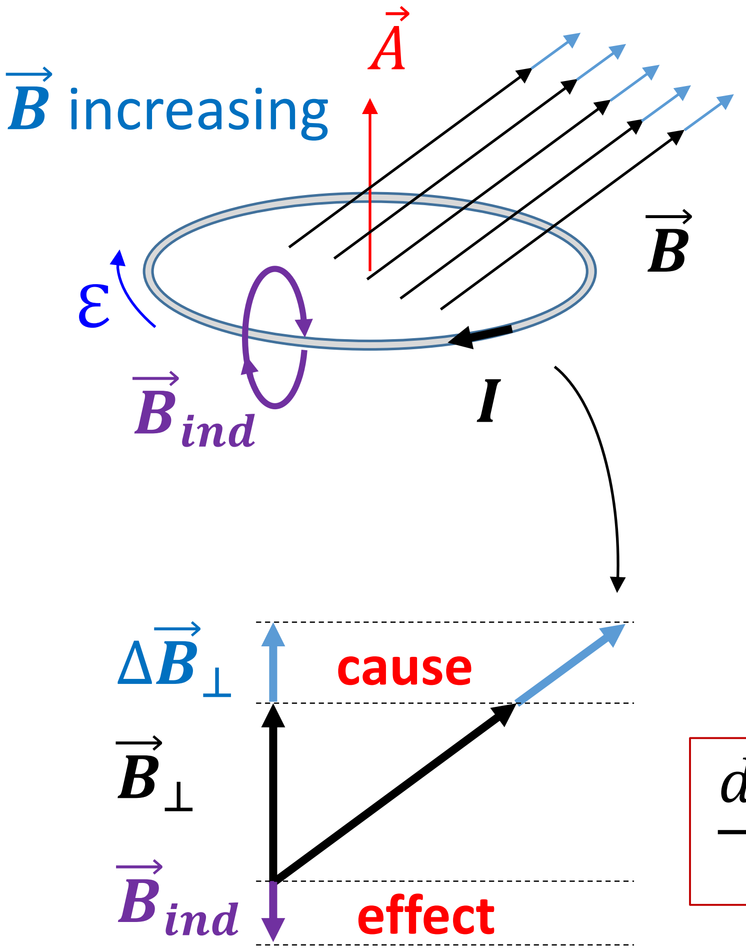
For non-motional electromotive force: Faraday's law

Lenz’s law

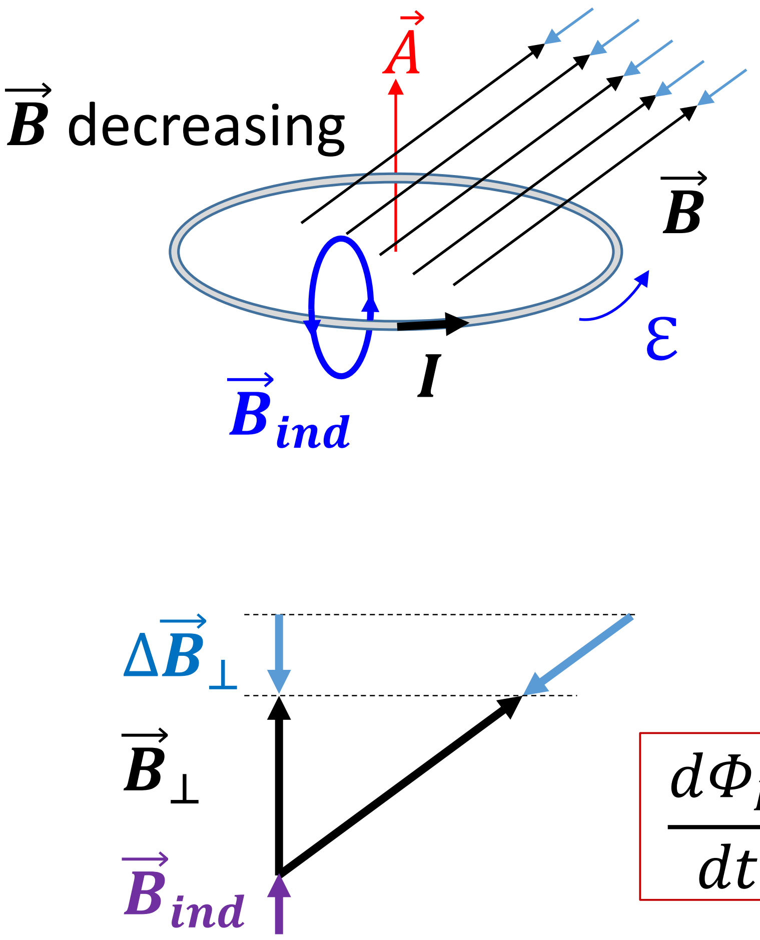
Lenz's law and sign convention for emf



$$\cos\theta > 0$$
$$\Phi_B > 0$$



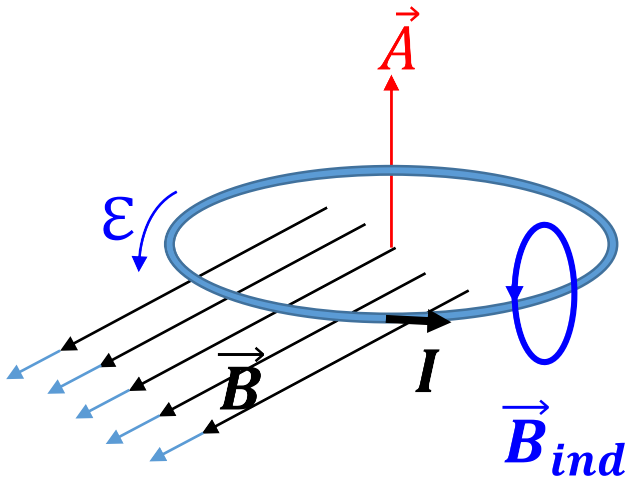
$$\frac{d\Phi_B}{dt} > 0$$

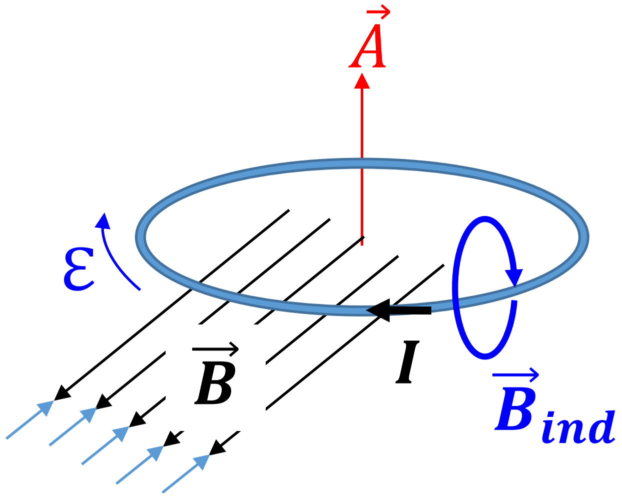


$$\frac{d\Phi_B}{dt} < 0$$

Induced emf ⇨ induces current ⇨ induces $\vec{B}_{ind}$ ⇨ opposing the **cause of the effect**

$$\boldsymbol{cos\theta < 0}$$
$$\boldsymbol{\Phi_B < 0}$$





$\vec{\boldsymbol{B}}$ increasing

$\vec{\boldsymbol{B}}$ decreasing

$$\boxed{\frac{d\Phi_B}{dt} < 0}$$

$$\boxed{\frac{d\Phi_B}{dt} > 0}$$

$\vec{B}_\perp$ parallel $\vec{A}$

$$\boldsymbol{\Phi_B = \vec{B}.\vec{A} > 0}$$
$$\theta < \pi/2$$

$$\frac{d\Phi_B}{dt} > 0 \qquad \frac{d\Phi_B}{dt} < 0$$

$\vec{B}_\perp$ anti-parallel $\vec{A}$

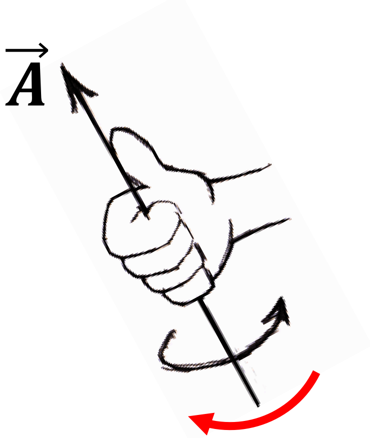
$$\boldsymbol{\Phi_B = \vec{B}.\vec{A} < 0}$$
$$\theta > \pi/2$$

$$\frac{d\Phi_B}{dt} > 0 \qquad \frac{d\Phi_B}{dt} < 0$$

$$\varepsilon < 0$$

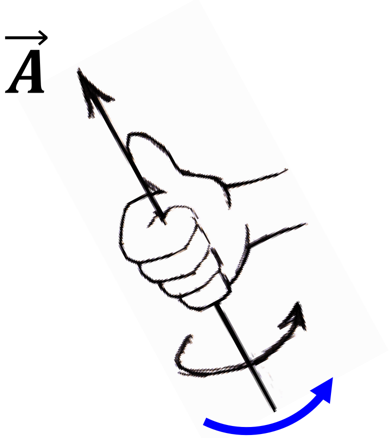
$$\varepsilon > 0$$

$\vec{n}, \vec{A}$



$\vec{A} \equiv thumb$

$\vec{n}, \vec{A}$



$\vec{A} \equiv thumb$

Summarizing Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{E}_{nc}.\,d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \underbrace{(\vec{E}_c + \vec{E}_{nc})}_{\vec{E}}.\,d\vec{l} = -\frac{d\Phi_B}{dt}$$

Stoke's theorem + Gauss's theorem

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Faraday

Varying **magnetic field** gives rise to an induced **electric field**

Maxwell

Remarkable symmetry of nature

Varying **electric field** field gives rise to an induced **magnetic field**

Varying **electric field** field gives rise to an induced **magnetic field**

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J}$$

Ampere's law

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \boxed{\frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}}$$

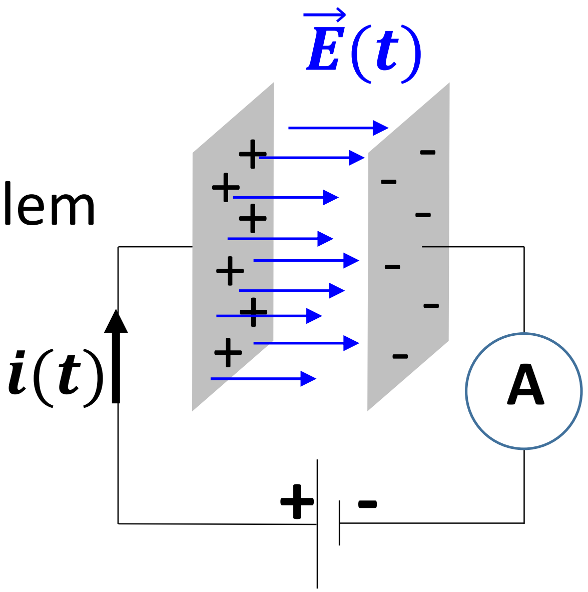
Maxwell's law

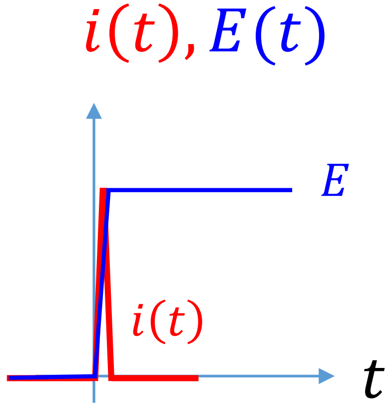
The term that brings symmetry

Displacement current

Displacement current or how Maxwell saved Ampere's law

The capacitor problem





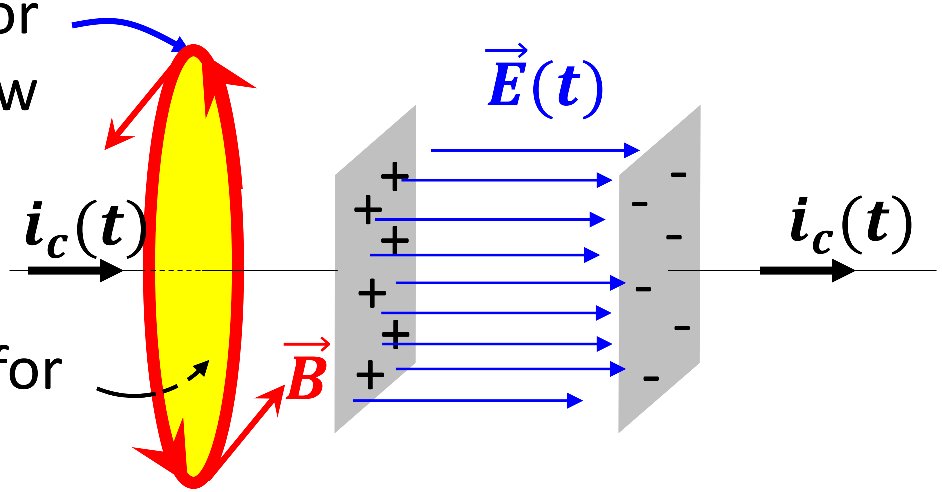
This is not a closed loop **BUT** a current **i(t)** flows in the circuit

Clearly no electron is jumping from one plate to the other !

Displacement current or how Maxwell saved Ampere’s paradoxical law

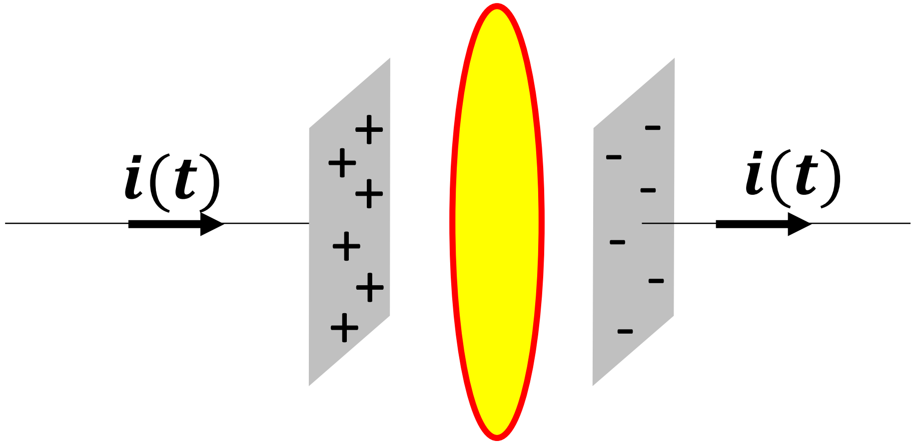
Loop path for Ampere’s law

Plane surface for Ampere’s law



Disk crossed by the wire

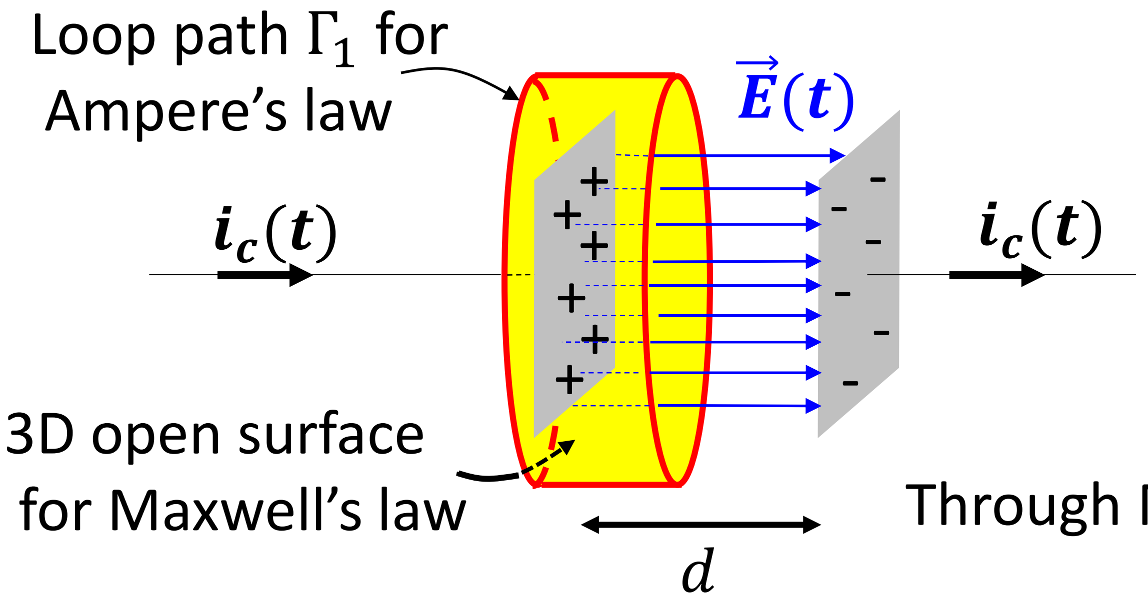
$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0 \boldsymbol{i_c}(\boldsymbol{t})_{\boldsymbol{encl}}$$



Disk **NOT** crossed by the wire

No electron moving through the surface thus **NO** magnetic field ?

$$\oint \vec{B}(t).d\vec{l} = 0$$



Cylinder open on the left hand side = Maxwell's surface.

<u>The wire does not cross Maxwell's surface</u>

Through Maxwell's surface a flux $\boldsymbol{\Phi_E(t)}$ is building-up

$$Q(t) = CV(t) = \frac{\varepsilon_0 A}{d} E(t) d = \varepsilon_0 AE(t) = \varepsilon_0 \boldsymbol{\Phi_E(t)}$$

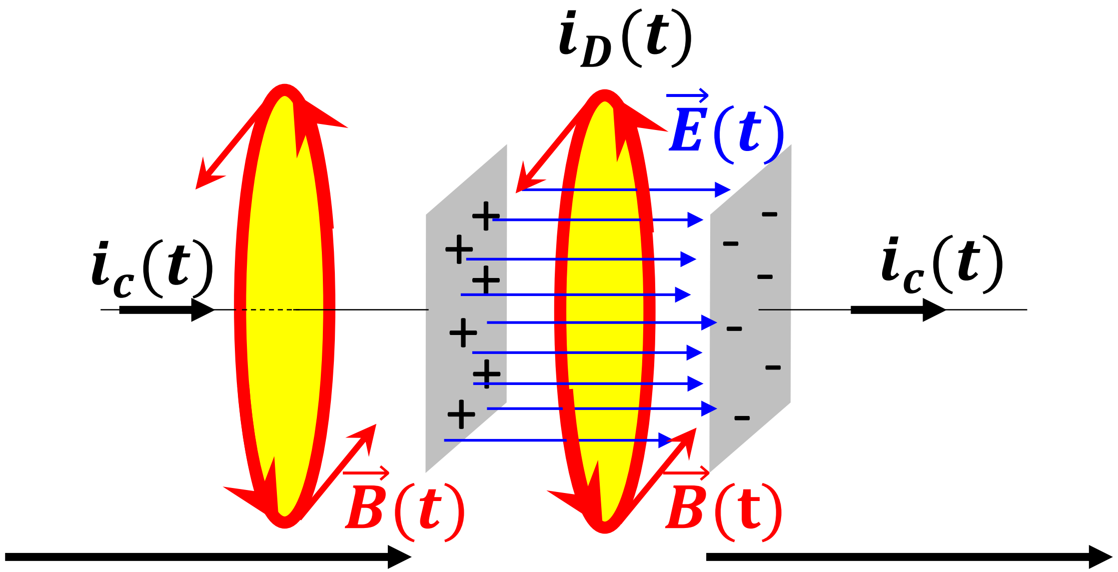
$$\boldsymbol{i(t)} = \frac{\boldsymbol{d}Q(t)}{\boldsymbol{dt}} = \varepsilon_0 \frac{\boldsymbol{d\Phi_E(t)}}{\boldsymbol{dt}}$$

$i_D(t)$

Maxwell's genius idea $\Rightarrow$ $i_D(t) = \varepsilon_0 \dfrac{d\Phi_E(t)}{dt}$

Final step: Fourth Maxwell's equation

$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0[i_c(t) + i_D(t)]_{encl}$$



$i_c(t) \neq 0$	$i_c(t) = 0$	$i_c(t) \neq 0$
$i_D(t) = 0$	$i_D(t) \neq 0$	$i_D(t) = 0$
$\vec{E}(t) = 0$	$\vec{E}(t) \neq 0$	$\vec{E}(t) = 0$

Continuity

What is the major outcome of Maxwell's contribution?

Time-varying field of either kind induces a field of the other kind in neighboring region

Waves

Towards the prediction of electromagnetic **disturbances** consisting of $\vec{E}$ and $\vec{B}$ and propagating in free space without requiring any medium

How did Maxwell's correct Ampere?

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law …

… completed by Maxwell

$$\oint \vec{B}(t).d\vec{l} = \mu_0 I_{encl} = \mu_0[i_c(t) + i_D(t)]_{encl} \Rightarrow \oint \vec{B}(t).d\vec{l} - \mu_0 i_D(t)_{encl} = \mu_0 i_c(t)_{encl}$$

$$i_D(t) = \varepsilon_0 \frac{d\Phi_E(t)}{dt} \Rightarrow \oint \vec{B}(t).d\vec{l} - \mu_0\varepsilon_0 \frac{d\Phi_E(t)}{dt} = \mu_0 i_c(t)_{encl}$$

Stoke's theorem + Gauss's theorem

$$\oint \vec{B}.d\vec{l} = \iint (\vec{\nabla} \times \vec{B}).d\vec{A}$$

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{d}{dt}\iint \vec{E}.d\vec{A} = \mu_0\varepsilon_0 \iint \frac{d\vec{E}}{dt}.d\vec{A}$$

$$\boxed{\vec{\nabla} \times \vec{B} - \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}}$$

This is all the story with Maxwell's equations

$$\vec{\nabla}.\vec{E}(t) = \frac{\rho(t)}{\varepsilon_0}$$

Gauss' law

$$\vec{\nabla}.\vec{B}(t) = 0$$

No magnetic charges

$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell

In electric field, the stored energy is:

$$U = \frac{\varepsilon_0}{2} \int E^2 dV$$

All space

$$\boxed{\frac{\varepsilon_0}{2} E^2 = energy\ density}$$

Slide #29 X_Lecture ***not given****_Current resistance & Electromotive force*

In a magnetic field, the stored energy stored:

$$U = \frac{1}{2\mu_0} \int B^2 dV$$

All space

$$\boxed{\frac{1}{2\mu_0} B^2 = energy\ density}$$