Final Review - 8.6-8.10

Note: examples in the textbook and homework are helpful to understand this part.

1 Normal Incidence at a Plane Conducting Boundary

Boundary is an interface with a perfect conductor.

$$\boldsymbol{E}_i(z) = \boldsymbol{a}_x E_{i0} e^{-j\beta_1 z}$$

$$\boldsymbol{H}_i(z) = \boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

$$\boldsymbol{E}_1(z) = \boldsymbol{E}_i(z) + \boldsymbol{E}_r(z)$$

$$\boldsymbol{E}_1(0) = 0 \Rightarrow E_{r0} = -E_{i0}$$

$$\boldsymbol{H}_r(z) = \frac{1}{\eta_1}\boldsymbol{a}_{nr} \times \boldsymbol{E}_r(z) = \boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{j\beta_1 z}$$

2 Oblique Incidence at a Plane Conducting Boundary

$$\boldsymbol{a}_{ni} = \boldsymbol{a}_x \sin\theta_i + \boldsymbol{a}_z \cos\theta_i$$

$$\boldsymbol{E}_1(x,z) = \boldsymbol{E}_i(x,z) + \boldsymbol{E}_r(x,z)$$

$$\boldsymbol{H}_1(x,z) = \boldsymbol{H}_i(x,z) + \boldsymbol{H}_r(x,z)$$

2.1 Perpendicular Polarization

$$\boldsymbol{E}_i(x,z) = \boldsymbol{a}_y E_{i0} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = \frac{E_{i0}}{\eta_1}(-\boldsymbol{a}_x\cos\theta_i + \boldsymbol{a}_z\sin\theta_i) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{E}_i(x,0) + \boldsymbol{E}_r(x,0) = 0$$

$$E_{r0} = -E_{i0}, \qquad \theta_r = \theta_i$$

$$\boldsymbol{E}_r(x,z) = -\boldsymbol{a}_y E_{i0} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\boldsymbol{H}_r(x,z) = \frac{E_{i0}}{\eta_1}(-\boldsymbol{a}_x\cos\theta_i - \boldsymbol{a}_z\sin\theta_i) e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

2.2 Parallel Polarization

$$\boldsymbol{E}_i(x,z) = E_{i0}(\boldsymbol{a}_x\cos\theta_i - \boldsymbol{a}_z\sin\theta_i) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = -\boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$E_{r0} = -E_{i0}, \qquad \theta_r = \theta_i$$

$$\boldsymbol{E}_r(x,z) = -E_{i0}(\boldsymbol{a}_x\cos\theta_i - \boldsymbol{a}_z\sin\theta_i) e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\boldsymbol{H}_r(x,z) = \boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

3 Normal Incidence at a Plane Dielectric Boundary

$$\boldsymbol{E}_i(z) = \boldsymbol{a}_x E_{i0} e^{-j\beta_1 z}$$

$$\boldsymbol{H}_i(z) = \boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

Reflected Wave

$$\boldsymbol{E}_r(z) = \boldsymbol{a}_x E_{ro} e^{j\beta_1 z}$$

$$\boldsymbol{H}_r(z) = -\boldsymbol{a}_y \frac{E_{ro}}{\eta_1} e^{j\beta_1 z}$$

Transmitted Wave

$$\boldsymbol{E}_t(z) = \boldsymbol{a}_x E_{t0} e^{-j\beta_2 z}$$

$$\boldsymbol{H}_t(z) = \boldsymbol{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

Two Boundary Condition Equations:

$$\boldsymbol{E}_i(0) + \boldsymbol{E}_r(0) = \boldsymbol{E}_t(0)$$

$$\boldsymbol{H}_i(0) + \boldsymbol{H}_r(0) = \boldsymbol{H}_t(0)$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0}$$

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0}$$

Reflection coefficient

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission coefficient

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{+\eta_2 + \eta_1}$$

$$1 + \Gamma = \tau$$

4 Normal Incidence at Multiple Dielectric Interfaces

If there are three mediums, in medium 1

$$\boldsymbol{E}_1 = \boldsymbol{a}_x (E_{i0} e^{-j\beta_1 z}) + E_{r0} e^{j\beta_1 z}$$

$$\boldsymbol{H}_1 = \boldsymbol{a}_y \frac{1}{\eta_1} (E_{i0} e^{-j\beta_1 z} - E_{r0} e^{j\beta_1 z})$$

in medium 2

$$\boldsymbol{E}_2 = \boldsymbol{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z})$$

$$\boldsymbol{H}_2 = \boldsymbol{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z})$$

in medium 3

$$\boldsymbol{E}_3 = \boldsymbol{a}_x E_3^+ e^{-j\beta_3 z}$$

$$\boldsymbol{H}_3 = \boldsymbol{a}_y \frac{E_3^+}{\eta_3} e^{-j\beta_3 z}$$

At $z = 0$,

$$\boldsymbol{E}_1(0) = \boldsymbol{E}_2(0)$$

$$\boldsymbol{H}_1(0) = \boldsymbol{H}_2(0)$$

At $z = d$,

$$\boldsymbol{E}_2(d) = \boldsymbol{E}_3(d)$$

$$\boldsymbol{H}_2(d) = \boldsymbol{H}_3(d)$$

Wave impedance of the total field: the ratio of the total $\boldsymbol{E}$ to the total $\boldsymbol{H}$. For example, in medium 1,

$$Z_1(z) = \frac{E_{1x}(z)}{H_{1y}(z)} = \eta_1 \frac{e^{-j\beta-1z} + \Gamma e^{j\beta_1 z}}{e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}}$$

5 Oblique Incidence at a Plane Dielectric Boundary

$$\boldsymbol{E}_1(x,z) = \boldsymbol{E}_i(x,z) + \boldsymbol{E}_r(x,z)$$

$$\boldsymbol{E}_2(x,z) = \boldsymbol{E}_t(x,z)$$

$$\boldsymbol{H}_1(x,z) = \boldsymbol{H}_i(x,z) + \boldsymbol{H}_r(x,z)$$

$$\boldsymbol{H}_2(x,z) = \boldsymbol{H}_t(x,z)$$

Snell's law of refraction

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2}$$

$$n = \frac{c}{u_p}$$

5.1 Total Reflection

For $\epsilon_1 > \epsilon_2$, when $\theta_t = \pi/2$ the refracted wave will glaze along the interface.
Critical angle θ_c: the angle of θ_i corresponds to $\theta_t = \pi/2$.

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

If $\theta_i > \theta_c$,

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$$

$$\cos\theta_t = \pm j\sqrt{\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i - 1}$$

$$e^{-\alpha_2 z}e^{-j\beta_{2x}x}$$

$$\alpha_2 = \beta_2\sqrt{(\epsilon_1/\epsilon_2)\sin^2\theta_i - 1}$$

$$\beta_{2x} = \beta_2\sqrt{\epsilon_1/\epsilon_2}\sin\theta_i$$

5.2 Perpendicular Polarization

The incident fields

$$\boldsymbol{E}_i(x,z) = \boldsymbol{a}_y E_{i0} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = \frac{E_{i0}}{\eta_1}(-\boldsymbol{a}_x\cos\theta_i + \boldsymbol{a}_z\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The reflected fields

$$\boldsymbol{E}_r(x,z) = \boldsymbol{a}_y E_{r0} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\boldsymbol{H}_r(x,z) = \frac{E_{r0}}{\eta_1}(\boldsymbol{a}_x \cos\theta_i + \boldsymbol{a}_z \sin\theta_i)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_i)}$$

The transmitted fields

$$\boldsymbol{E}_t(x,z) = \boldsymbol{a}_y E_{t0} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\boldsymbol{H}_t(x,z) = \frac{E_{t0}}{\eta_2}(-\boldsymbol{a}_x \cos\theta_t + \boldsymbol{a}_z \sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Boundary conditions:

$$E_{iy}(x,0) + E_{ry}(x,0) = E_{ty}(x,0)$$

$$H_{ix}(x,0) + H_{rx}(x,0) = H_{tx}(x,0)$$

Snell's law of reflection

$$\theta_r = \theta_i$$

Snell's law of refraction

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2}$$

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$1 + \Gamma_\perp = \tau_\perp$$

If reflection=0,

$$\eta_2\cos\theta_{B\perp} = \eta_1\cos\theta_t$$

$$\sin\theta_{B\perp} = \frac{1}{\sqrt{1 + (\mu_1/\mu_2)}}$$

$\theta_{B\perp}$: Brewster angle of no reflection of s-polarization.

5.3 Parallel Polarization

The incident fields

$$\boldsymbol{E}_i(x,z) = E_{i0}(\boldsymbol{a}_x \cos\theta_i - \boldsymbol{a}_z \sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{H}_i(x,z) = -\boldsymbol{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

The reflected fields

$$\boldsymbol{E}_r(x,z) = E_{r0}(\boldsymbol{a}_x \cos\theta_r + \boldsymbol{a}_z \sin\theta_r)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\boldsymbol{H}_r(x,z) = -\boldsymbol{a}_y \frac{E_{r0}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

The transmitted fields

$$\boldsymbol{E}_t(x,z) = E_{t0}(\boldsymbol{a}_x \cos\theta_t - \boldsymbol{a}_z \sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\boldsymbol{H}_t(x,z) = -\boldsymbol{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Tangential $\boldsymbol{E}$ and $\boldsymbol{H}$ should be continuous at $z = 0$

$$(E_{i0} + E_{r0})\cos\theta_i = E_{t0}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{i0} - Er0) = \frac{1}{\eta_2}E_{t0}$$

$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

When reflection=0

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}$$

$$\sin^2\theta_{B\parallel} = \frac{1 - \mu_2\epsilon_1/\mu_1\epsilon_2}{1 - (\epsilon_1/\epsilon_2)^2}$$

$\theta_{B\parallel}$ is the Brewster angle of no reflection of p-polarization.