HW2

VE312

1. $\vec{F}\cdot\vec{W} = |\vec{F}|\cdot|\vec{W}|\cdot\cos<\vec{F},\vec{W}>$

$$\mathrm{Proj}(\vec{F}) = |\vec{F}|\cdot\cos<\vec{F},\vec{W}>\cdot\frac{\vec{W}}{|\vec{W}|}$$

$$= \frac{\vec{F}\cdot\vec{W}}{|\vec{W}|^2}\cdot\vec{W}$$

2. $\Phi(\vec{F}) = \int_0^3 2\pi r\cdot r^2\cdot dr = \frac{81}{2}\pi$

3. (a)

3. For constants a, b, c, m, consider the vector field

$$\vec{F} = (ax + by + 5z)\vec{i} + (x + cz)\vec{j} + (3y + mx)\vec{k}.$$

(a) Suppose that the flux of $\vec{F}$ through any closed surface is 0. What does this tell you about the value of the constants a, b, c and m?

(b) Suppose instead that the line integral of $\vec{F}$ around any closed curve is 0. What does this tell you about the values of the constants a, b, c and m?

Solution

(a) If the flux of $\vec{F}$ through any closed surface is 0, then by the divergence theorem, the vector field must have zero divergence.

$$\vec{\nabla}.\vec{F} = a = 0$$

This tells us that $a = 0$ but it does not tell us anything about b, c or m.

(b) If the line integral of $\vec{F}$ around any closed curve is 0, this means that the vector field has curl equal to zero everywhere.

$$\vec{\nabla}\times\vec{F} = (3-c)\vec{i} + (5-m)\vec{j} + (1-b)\vec{k}$$

This tells us that $c = 3, m = 5$ and $b = 1$. It does not tell us anything about a.

4. (a) $4\pi r^2\cdot E = \frac{Q}{\varepsilon_0}$

$$E = \frac{Q}{4\pi r^2\varepsilon_0}$$

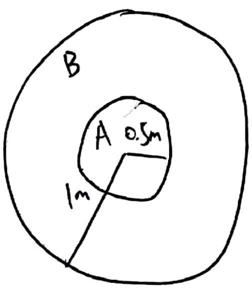
(b) 0

5. $\frac{1}{8}$

6. Total Q: $Q = \int_0^R 4\pi r^2 \rho(r)\cdot dr$

$$= \int_0^R 4a\pi r^3 dr = a\pi R^4$$

7.



$$\frac{Q_A}{\varepsilon_0} = 4\pi\cdot 0.5^2\cdot 150\cdot 0.5 = 75\pi\ (C)$$

$$\frac{Q_{A+B}}{\varepsilon_0} = 4\pi\cdot 1^2\cdot 150\cdot 1 = 600\pi\ (C)$$

$$Q_B = 525\pi\cdot\varepsilon_0 = 14.6\,nC$$

8. $\frac{Q}{\varepsilon_0} = 4\pi\cdot 0.5^2\cdot\frac{90}{0.5} = 180\pi$

$Q = 5.00\,nC$

9. Flux $= 8\ N\cdot m^2/C$

$\frac{Q}{\varepsilon_0} = 8 \quad Q = 8\varepsilon_0 = 70.8\,pC$

10. (a) $Q' = \frac{Q}{8}$

$4\pi \cdot \frac{R^2}{4} \cdot E = \frac{Q'}{\varepsilon_0}$

$E = \frac{Q}{8\pi R^2 \varepsilon_0}$

(b) $Q'' = Q$

$4\pi \cdot (\frac{3R}{2})^2 \cdot E = \frac{Q''}{\varepsilon_0}$

$E = \frac{Q}{9\pi R^2 \varepsilon_0}$

11. (a) $r = R: -Q$

$r = 2R: +Q$

(b) $4\pi r^2 \cdot E = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{4\pi r^2 \varepsilon_0}$