Way to Electromagnetic (EM) waves

Outline

- Last review of Maxwell's equations: <u>How all terms fit together</u>
- How does the charge conservation law became sacred !
- Orthogonality of $\vec{E}$ and $\vec{B}$ and transverse character of EM waves
- Travelling Electric and Magnetic fields: Plane wave
- Solving Maxwell's equations
- Poynting vector: Energy – Momentum
- Propagation, Polarization and incidence of EM waves on matter: conductor vs dielectric

Summarizing

Time independent $\left(\frac{\partial}{\partial t} = 0\right)$
No current or steady

Time dependent $\left(\frac{\partial}{\partial t} \neq 0\right)$
Acceleration

Electrostatics

$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ → Gauss's law → $\vec{\nabla}.\vec{E}(t) = \frac{\rho(t)}{\varepsilon_0}$ → Will be proven

$\vec{\nabla} \times \vec{E} = 0$ → Poisson equation $\vec{E} = -\vec{\nabla}.V$ → $\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$ → Faraday's law

Magnetostatics

$\vec{\nabla}.\vec{B} = 0$ → Gauss's law → $\vec{\nabla}.\vec{B}(t) = 0$

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{J}}$ → Ampere's law → $\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J}(t) + \frac{1}{c^2}\frac{\partial \vec{E}(t)}{\partial t}$ → Maxwell's law

IN WHAT FOLLOWS

$$\rho(t) \quad \vec{E}(t)$$
$$\vec{j}(t) \quad \vec{B}(t)$$

All these functions are time variable besides their spatial dependence

But for a sake of simplicity the variable t is dropped

A special attention must be payed to the forth Maxwell's equation

How the charge conservation law became sacred !

Ampere's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}}$ ➡ Maxwell's questioning

$\underbrace{\vec{\nabla}.(\vec{\nabla} \times \vec{B})}_{=0} = \mu_0 \vec{\nabla}.\vec{\boldsymbol{j}}$ ➡ $\vec{\nabla}.\vec{\boldsymbol{j}} = 0$! The flux of a current out of a closed surface is zero

COMON SENSE

The flux of a current flowing out of a closed surface

⇩

DECREASE of charge inside

⇩

So it cannot be in general zero

$$\vec{\nabla}\vec{\boldsymbol{j}} = -\frac{\partial \rho}{\partial t}$$

➡ Something is wrong with Ampere's law

Maxwell's correction $\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}} + \boxed{\frac{\mathbf{1}}{\boldsymbol{c}^2} \frac{\partial \vec{E}}{\partial t}}$

$$\vec{\boldsymbol{\nabla}}.(\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{j}} + \frac{\mathbf{1}}{\boldsymbol{c}^2} \frac{\partial \vec{\boldsymbol{\nabla}}.\vec{E}}{\partial t}$$

From Gauss law $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

$$\underbrace{\vec{\nabla}.(\vec{\nabla} \times \vec{B})}_{= 0} = \underbrace{\mu_0 \vec{\nabla}.\vec{J} + \frac{1}{\varepsilon_0 c^2} \frac{\partial \rho}{\partial t}}_{= 0} = 0$$

$$\frac{1}{\varepsilon_0 \mu_0} = c^2$$

Charge conservation law
Or continuity equation

$$\boxed{\vec{\nabla}.\vec{\boldsymbol{j}} = -\frac{\partial \rho}{\partial t}}$$

To date no one has found an experiment that disagrees with this statement

Charge conservation

$$\vec{\nabla}.\vec{\boldsymbol{j}} = -\frac{\partial \rho}{\partial t}$$

If charges are flowing out of closed surface it means that their density inside the volume bounded y this surface is decreasing **UNLESS** the difference is supplied by an external source (closed circuit)

Reminder

- Differential forms of Maxwell's equations manipulate **vectors**
- Integral forms of Maxwell's equations manipulate **scalars**

Let's look at equation (1) $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

What if things are varying with time ?

$$\boxed{\frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{E}\right) = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial t}} + \boxed{\frac{\partial \rho}{\partial t} = -\vec{\nabla}\vec{j}} \Rightarrow \varepsilon_0 \frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{E}\right) = -\vec{\nabla}\vec{j} \Rightarrow \varepsilon_0 \frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{E}\right) + \vec{\nabla}.\vec{j} = 0$$

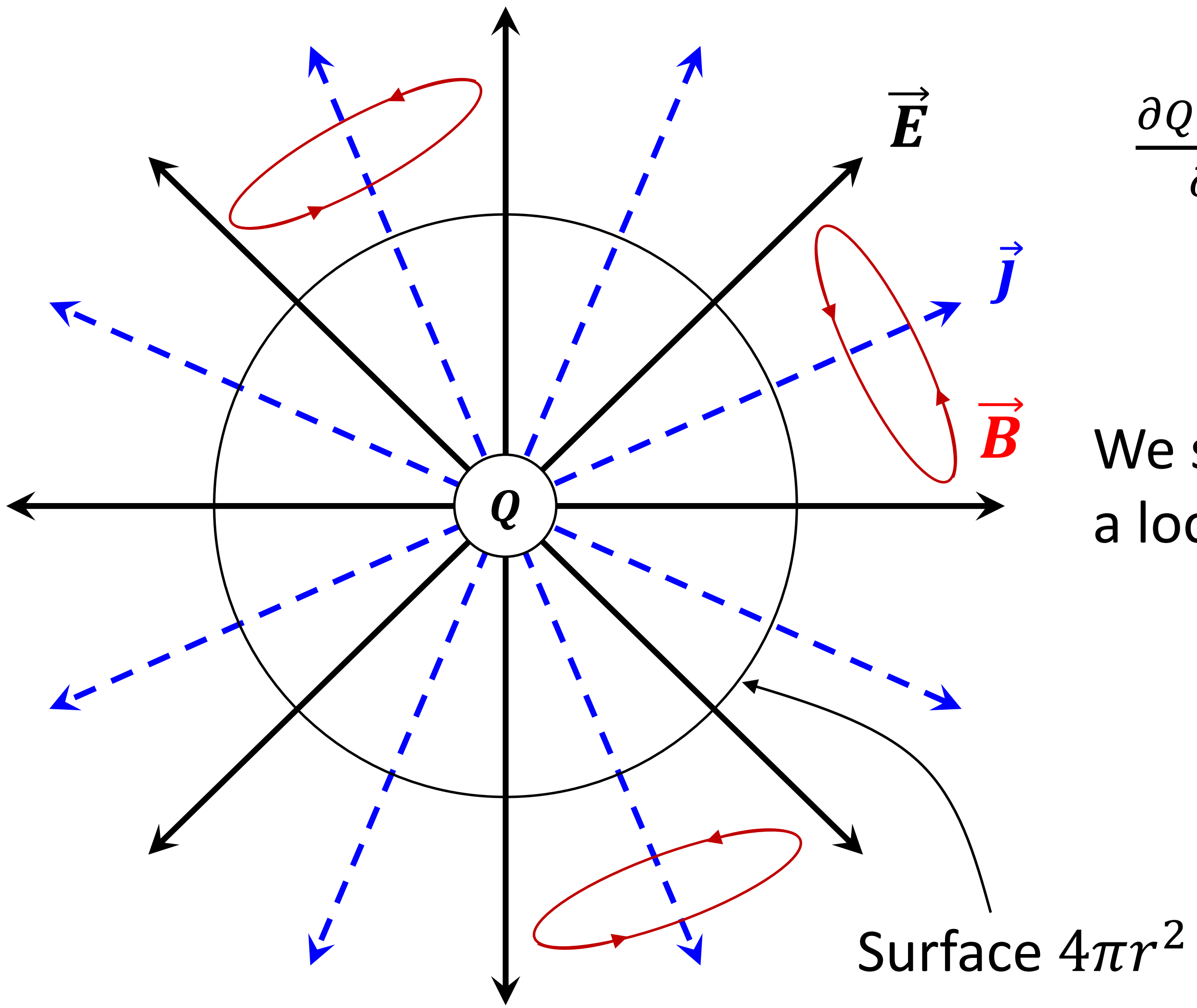
Charge conservation law

Let's now take a look at equation (4) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$

$$\underbrace{\vec{\nabla}.\left[c^2 \vec{\nabla} \times \vec{B}\right]}_{=\,0} = \frac{\vec{\nabla}\vec{j}}{\varepsilon_0} + \frac{\partial \vec{\nabla}.\vec{E}}{\partial t} \Rightarrow \varepsilon_0 \frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{E}\right) + \vec{\nabla}.\vec{j} = 0$$

$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ Valid all the time

A beautiful experiment illustrating the pertinence of Maxwell's approach



The charge source $Q(r,t)$ is leaky radially (symmetrically)

Radioactive source

$$\frac{\partial Q(r,t)}{\partial t} = -4\pi r^2 j(r)$$

Current is a scalar field

We should expect a magnetic field circulating along a loop surrounding the current density vectors

Biot & Savart's and Ampere's laws

How can a field change direction ?

Maxwell finding saves the situation

$$E(r,t) = \frac{Q(r,t)}{4\pi\varepsilon_0 r^2} \Rightarrow \frac{\partial E}{\partial t} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{\partial Q(r,t)}{\partial t} = -\frac{j(r)}{\varepsilon_0}$$

$$j(r) = \frac{I}{A}$$

$$A = 4\pi r^2$$

$$I = \frac{\partial Q(r,t)}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}} + \frac{\mathbf{1}}{\boldsymbol{c}^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{\boldsymbol{j}} - \frac{\vec{\boldsymbol{j}}}{\varepsilon_0 \boldsymbol{c}^2} = \vec{0} \Rightarrow \boxed{\vec{B} = \vec{0}}$$

Fourth Maxwell's equation

No magnetic field

Two sources of magnetic fields cancel each other out

⇨ There can be no magnetic field

Electromagnetic (EM) waves

Maxwell's theory

1865

$$\frac{\partial B}{\partial t} \rightarrow \text{Source of } E(x,t)$$

$$\frac{\partial E}{\partial t} \rightarrow \text{Source of } B(x,t)$$

+

Hertz's experiments

1888 (23 years later !)

EM waves ⇒

- Energy
- Momentum

Mechanical waves need medium to propagate

EM waves **do not** need any medium to propagate

$\}$ **BUT** both are based on the same equations

Maxwell equations and EM waves

Faraday's law ⇨ $\frac{\partial B}{\partial t} \rightarrow$ Source of $E(x,t)$ proved by emf induction

Ampere's and Maxwell's law ⇨ $\frac{\partial E}{\partial t} \rightarrow$ Source of $B(x,t)$ proved by displacement current

$$\frac{\partial E}{\partial t} \leftrightarrow \frac{\partial B}{\partial t}$$

$$\downarrow \qquad \downarrow$$

$$\underbrace{\varepsilon_0 \qquad \mu_0}$$

Characteristics of the medium

⇨ $\boxed{\frac{1}{\varepsilon_0\mu_0} = c^2}$

Maxwell motivation (1864) was to understand this extraordinary result

Static point charge → Static field $\vec{E}$

Moving point charge (steady velocity) → Static fields $\vec{E}$ and $\vec{B}$

To produce EM wave, the charge MUST accelerate

$$\frac{\partial E}{\partial t} \leftrightarrow \frac{\partial B}{\partial t}$$

Every accelerated charge radiates EM energy

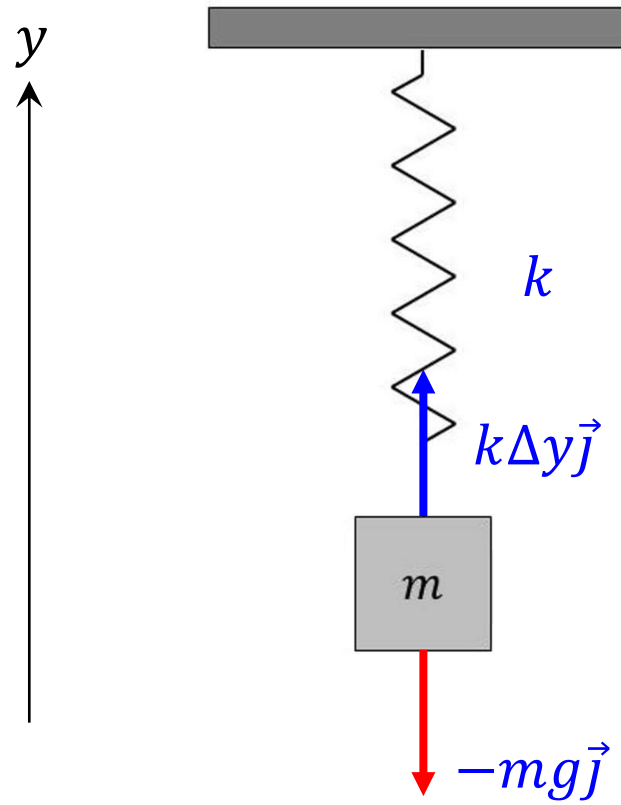
↓

- This makes classical atom unstable
- The orbiting electron has a centripetal acceleration

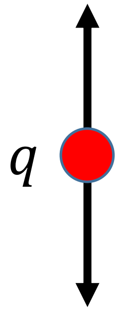
Quantum mechanics handles this issue

How can we accelerate a charge ?

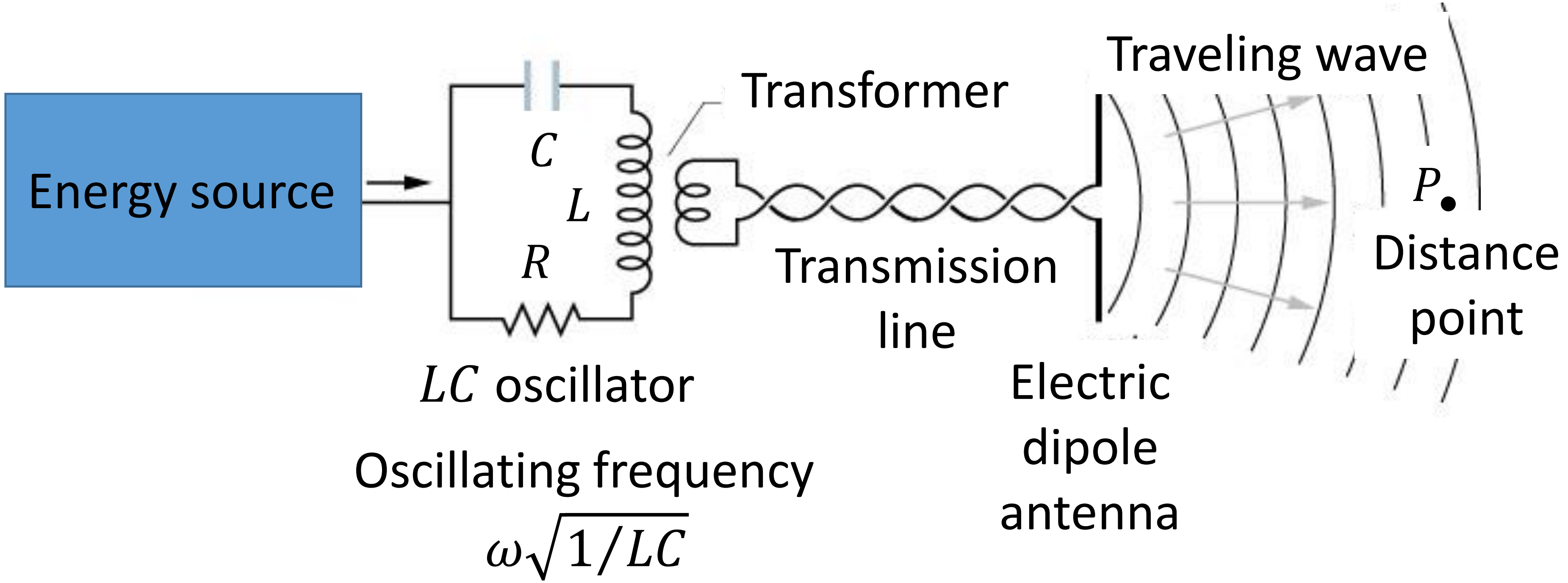
Simple harmonic oscillation



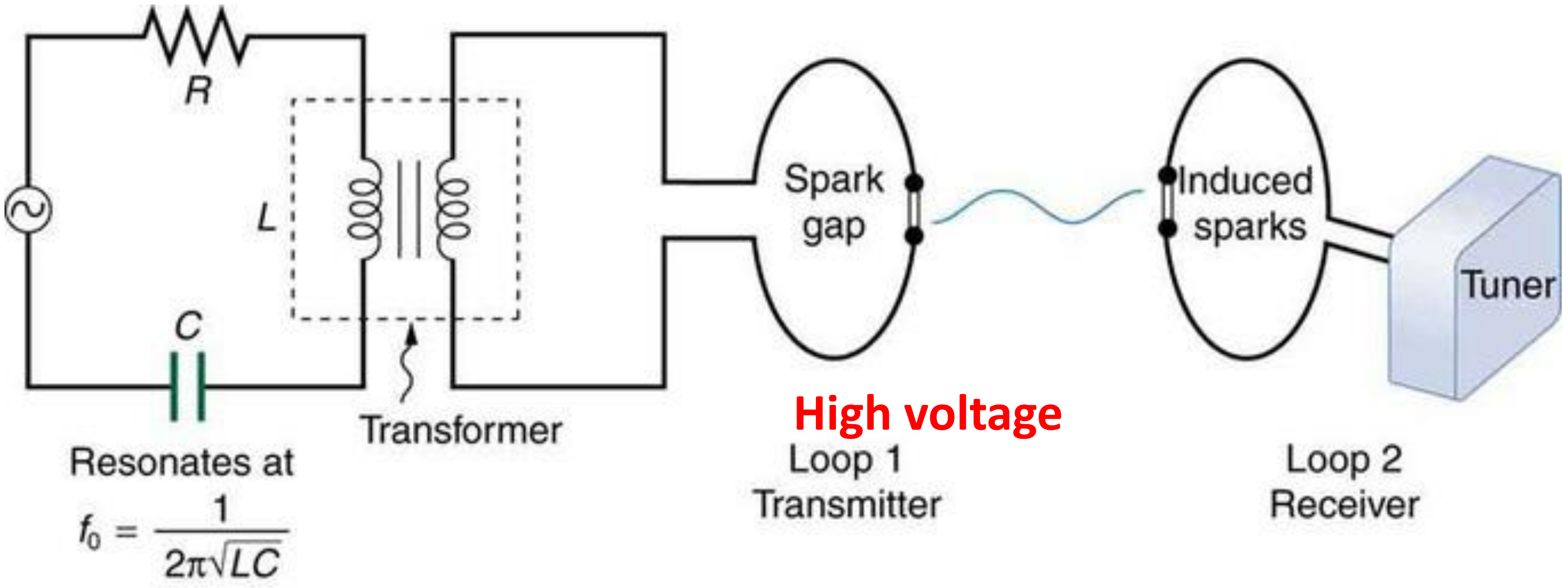
The two systems are in permanent acceleration



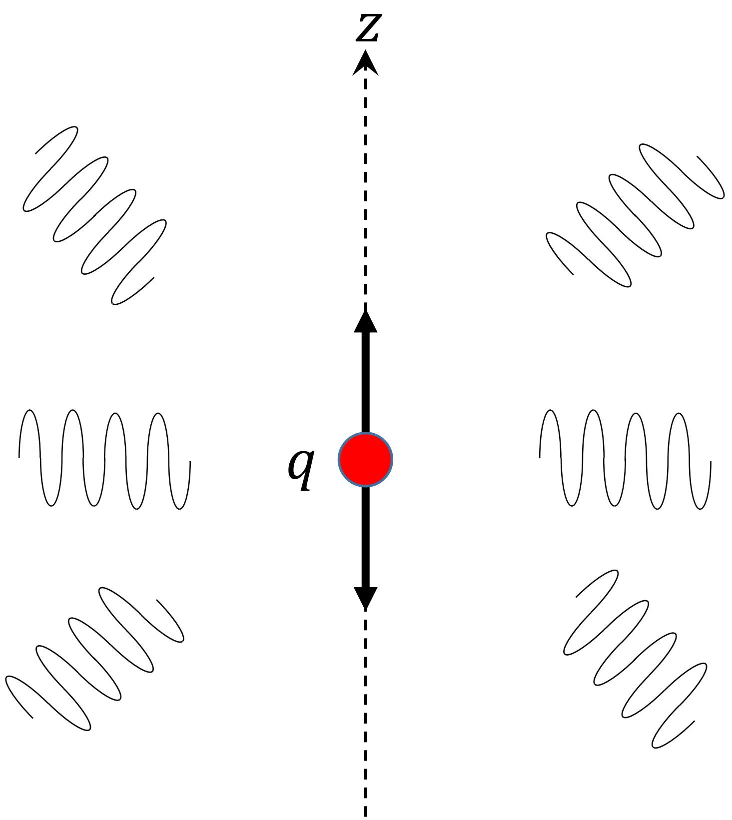
Today's circuit

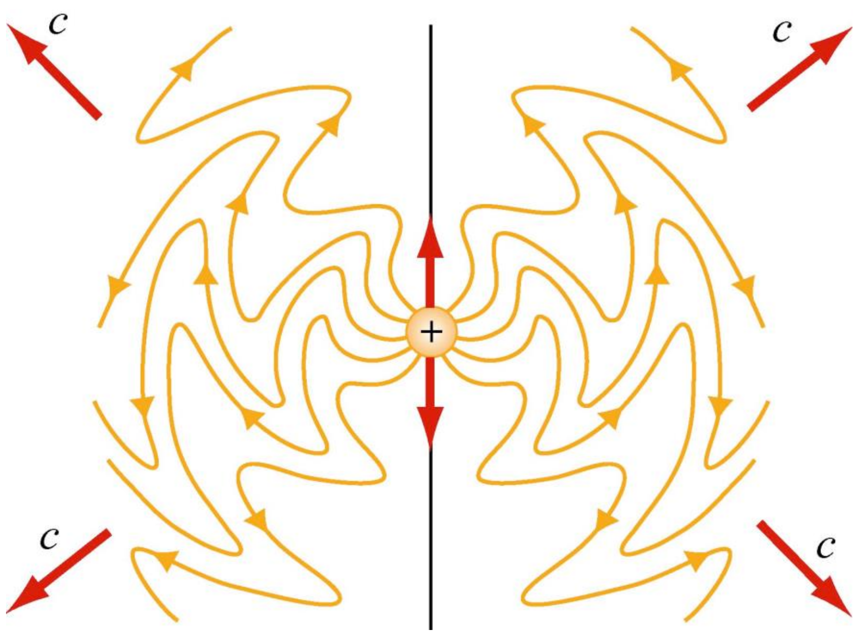


Original Hertz experiment 1888



= $\boldsymbol{RLC}$ circuit that can be tuned to resonance





Question:

1) Is it possible to have a purely Electric wave or Magnetic wave propagating through empty space ?
2) No wave along the z axis: Why?

Electrostatic

$$\vec{E} = \frac{1}{4\pi\boldsymbol{\varepsilon_0}}\frac{q}{r^2}\vec{e}_r$$

Both perpendicular to each other but **NOT ALWAYS** whether the charges are static or in uniform motion

Magnetostatic

$$\vec{B} = \frac{\boldsymbol{\mu_0}}{4\pi}\frac{q}{r^2}\vec{v} \times \vec{e}_r$$

Maxwell's equations require **ORTHOGONALITY ALL THE TIME**

ONLY ONE SOLUTION: THE CHARGES MUST ACCELERATE

<u>Orthogonality</u> of $\vec{E}$ and $\vec{B}$ and <u>transverse</u> character of EM waves

To avoid confusion between
$\vec{\boldsymbol{k}}$ as a <u>wave vector</u>
$\vec{\boldsymbol{k}}$ as a <u>unit vector</u> along z −axis

$$\left(\vec{\boldsymbol{i}}, \vec{\boldsymbol{j}}, \vec{\boldsymbol{k}}, \right) \rightarrow \left(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \widehat{\boldsymbol{k}}\right)$$

Expressing the traveling $\vec{E}$ and $\vec{B}$ in complex exponential forms

$$\vec{E} = \sum_{1}^{3} E_{0m}\hat{u}e^{i(\vec{k}.\vec{r}-\omega t)} \qquad \vec{B} = \sum_{1}^{3} B_{0m}\hat{u}e^{i(\vec{k}.\vec{r}-\omega t)}$$

$$m = (x, y, z)$$

$$\hat{u} = (\hat{\imath}, \hat{\jmath}, \hat{k})$$

$$\vec{E} = \left(E_{0x}\hat{\imath} + E_{0y}\hat{\jmath} + E_{0z}\hat{k}\right)e^{[i(k_x x + k_y y + k_z z - \omega t)]}$$

$$\vec{B} = \left(B_{0x}\hat{\imath} + B_{0y}\hat{\jmath} + B_{0z}\hat{k}\right)e^{[i(k_x x + k_y y + k_z z - \omega t)]}$$

Each component of $\vec{E}$ and $\vec{B}$ may depend on (x, y, z) !

$$\vec{k} = k_x\vec{\imath} + k_y\vec{\jmath} + k_z\vec{k}$$

$\vec{k}$ **direction of propagation)**

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\Downarrow$$

$$\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}} = \boldsymbol{k_x x} + \boldsymbol{k_y y} + \boldsymbol{k_z z}$$

Orthogonality of $\vec{E}$ and $\vec{B}$: Demonstration based on Faraday's and Maxwell's laws

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{\boldsymbol{j}} + \frac{\mathbf{1}}{\boldsymbol{c}^{\mathbf{2}}} \frac{\partial \vec{E}}{\partial t}$$

From Faraday's law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Expressing the traveling $\vec{E}$ and $\vec{B}$ in complex exponential forms

$$\vec{E} = \sum_{1}^{3} E_{0m} \hat{u} e^{i(\vec{k}.\vec{r} - \omega t)}$$

$$\vec{B} = \sum_{1}^{3} B_{0m} \hat{u} e^{i(\vec{k}.\vec{r} - \omega t)}$$

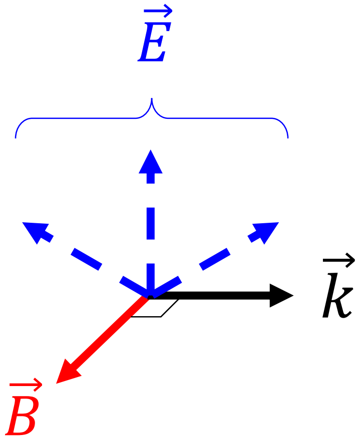
$$\Rightarrow \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \boxed{\vec{k} \times \vec{E} = \omega \vec{B}}$$

$$\Downarrow$$

$$\boxed{\vec{B} \perp \vec{k} \text{ and } \vec{B} \perp \vec{E}}$$

$$\Downarrow$$

What about $\vec{E}$ *and* $\vec{k}$?



$\vec{k}$ indicate the direction of propagation

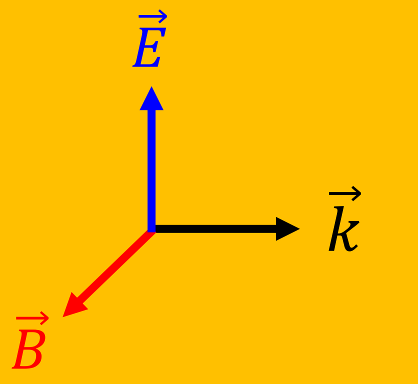
From Maxwell's (Ampere's corrected) law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{\mathbf{1}}{\boldsymbol{c^2}} \frac{\partial \vec{E}}{\partial t} \Rightarrow \text{in free space} \Rightarrow \vec{\nabla} \times \vec{B} = \frac{\mathbf{1}}{\boldsymbol{c^2}} \frac{\partial \vec{E}}{\partial t}$$

The same treatment as in previous slide

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2} \vec{E}$$

$$\vec{k} \times \vec{E} = \omega \vec{B}$$



Orthogonality

Transverse character of the EM wave: Demonstration based on Gauss's laws

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}.\vec{B} = 0$$

From Gauss's law in a charge free space $\vec{\nabla}.\vec{E} = 0$

$$\Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \Rightarrow i(k_x E_x + k_y E_y + k_z E_z) = 0$$

$$\Downarrow$$

$$\boxed{\vec{k}.\vec{E} = 0}$$

The electric field $\vec{E}$ is orthogonal to the direction of propagation

From Gauss's law applied to $\vec{B}$ $\Rightarrow$ $\boxed{\vec{k}.\vec{B} = 0}$

The magnetic field $\vec{B}$ is orthogonal to the direction of propagation

Important property of nature

Orthogonality and Transverse character of the electric and magnetic fields

Obtained from vector analysis

Expressing universality

Consequence on the relation between E and B

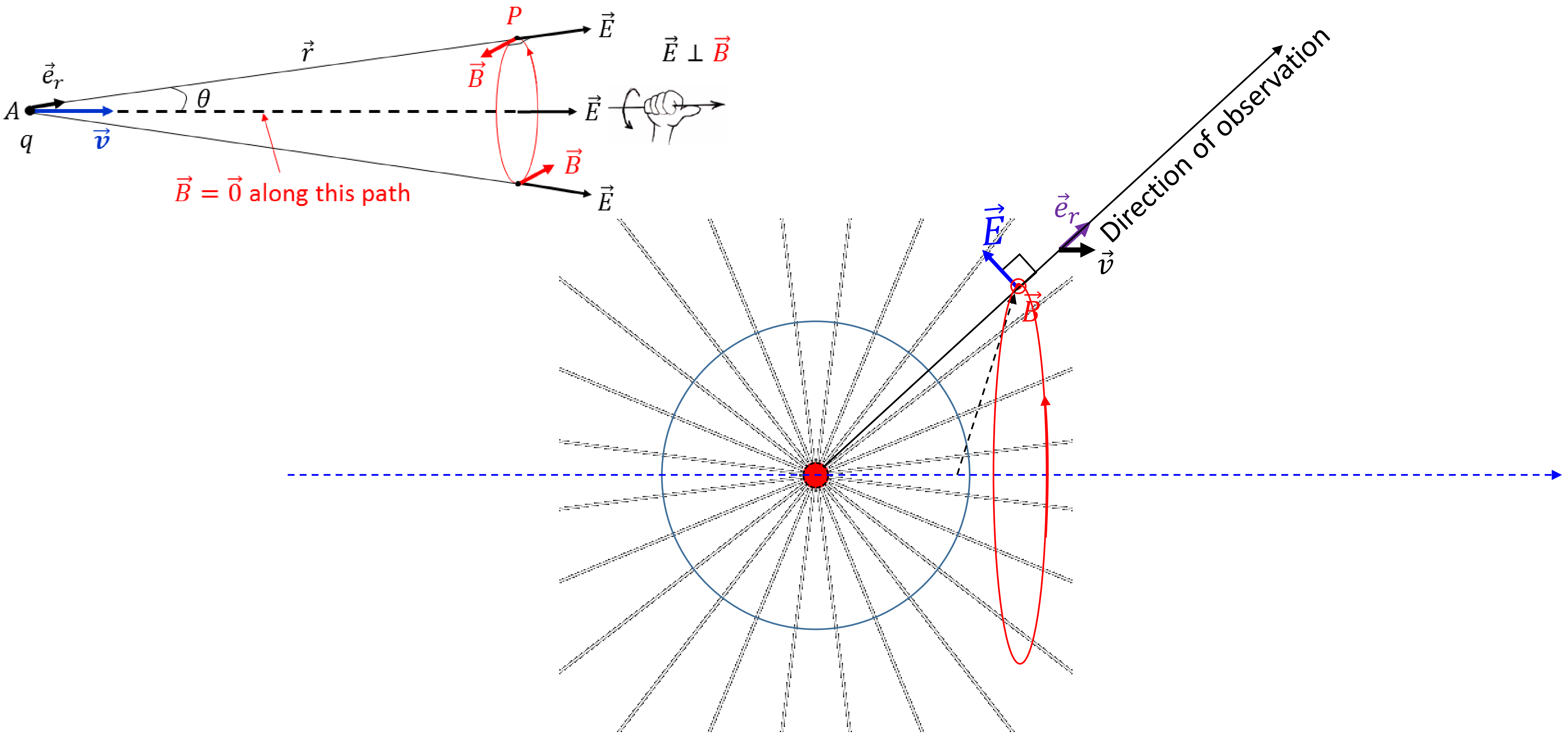
$$\vec{k} \times \vec{E} = \omega\vec{B}$$

$$kE = \omega B$$

$$E = \frac{\omega}{k}B$$ Assuming **NO** dispersion ⇔ homogeneous medium

In vacuum $E = cB$ ⇒ $B \ll E$ **In most practical cases it is E that matters**

Acceleration creates a transverse wave



Remark on vectors

In general a vector has 3 components and each component depends on four variables (x, y, z, t)

$$\vec{V} = V_x(x, y, z, t)\hat{\imath} + V_y(x, y, z, t)\hat{\jmath} + V_z\,(x, y, z, t)\hat{k}$$

If the vector has only one single component $\vec{V} = V_y(x, y, z, t)\hat{\jmath}$

BUT Still this component can be function of the four variables (x, y, z, t)

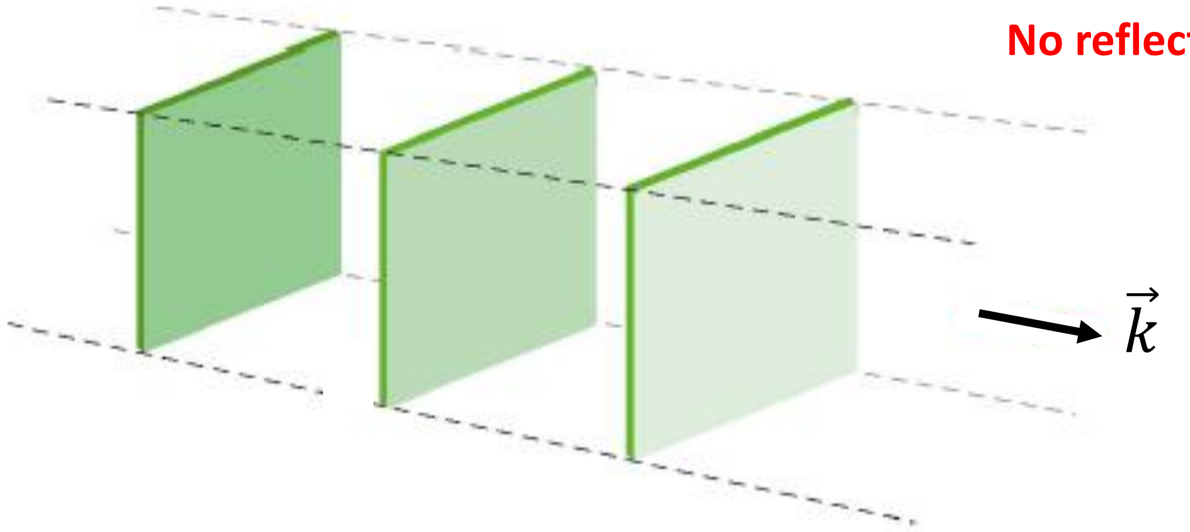
Plane wave

Assumptions

- **Homogeneous** unbounded medium (vacuum): ➡ No **absorption** and no **reflection**

- Source of EM wave consists of **an infinite plane perpendicular** to the direction of propagation
  On every plane, $\vec{E}$ and $\vec{B}$ keep the **same <u>direction</u>, same <u>magnitude</u> and same <u>phase</u>**

No reflection **No absorption** **Homogeneous**



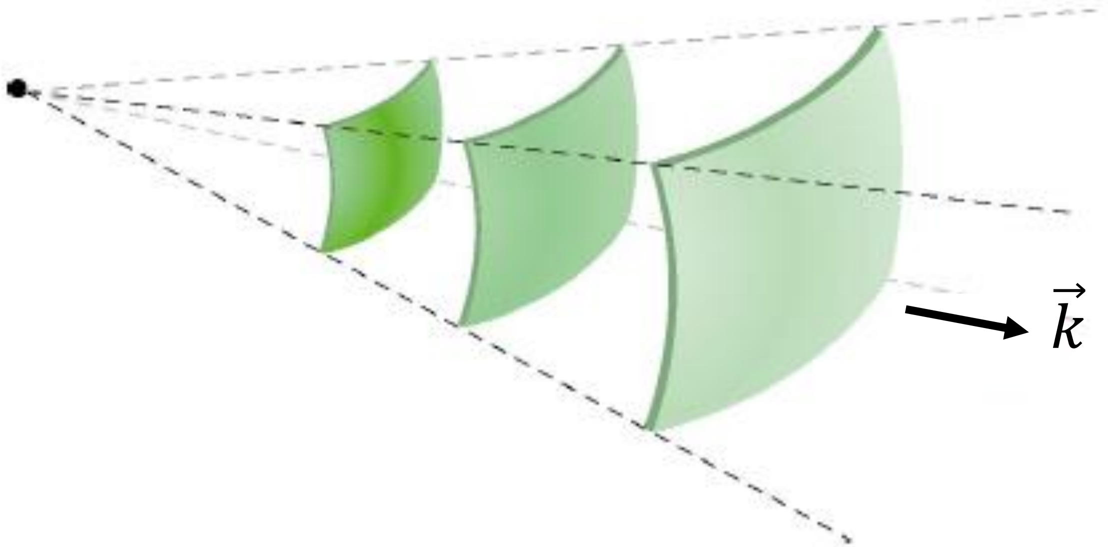
Plane wave

Not realistic because it assumes an infinite source

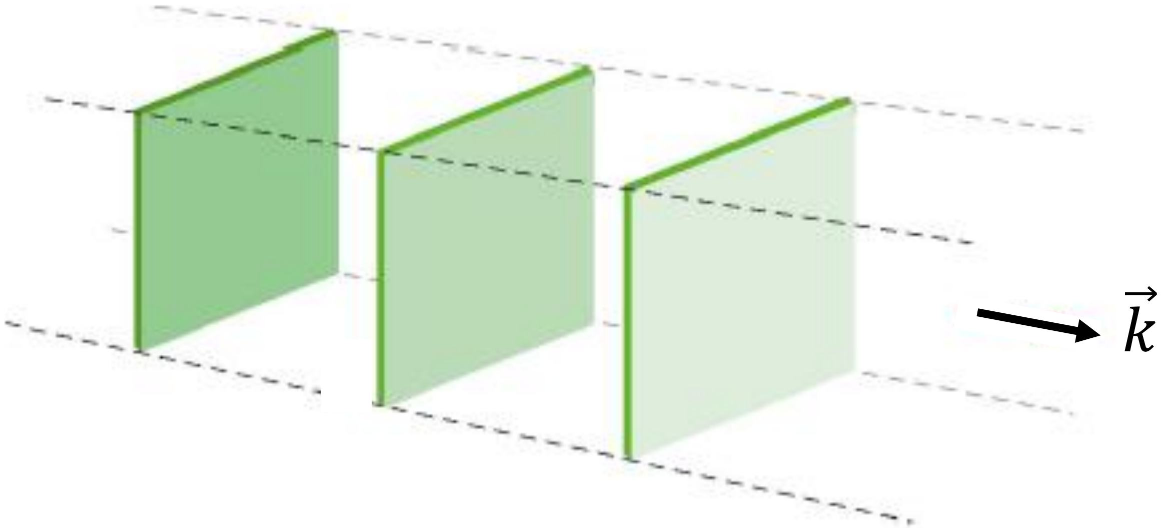
A more realistic approach to the plane wave

Away from a spatially limited source, a spherical wave is more realistic…

Spherical wave



Plane wave

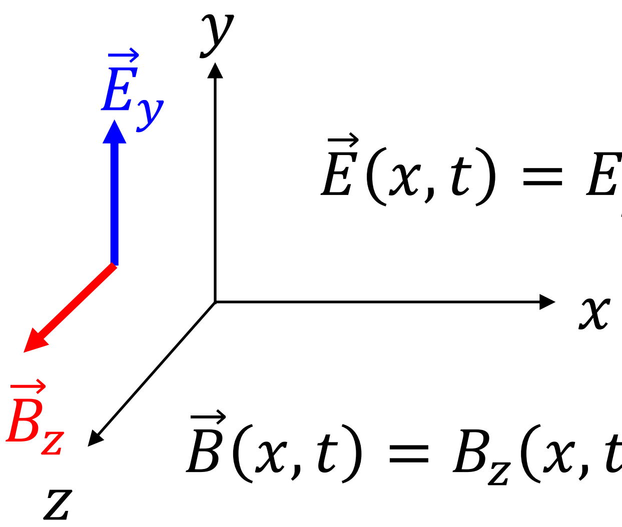


… And very far away from the source it may look like a plane wave…

Plane wave

Assumptions

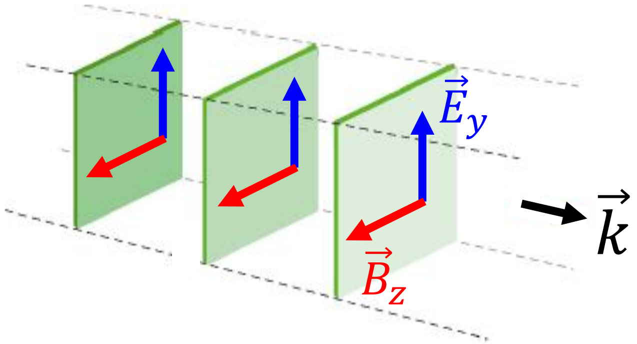
- For a <u>plane wave</u> each field has one component which depends on only two variables (x, t)



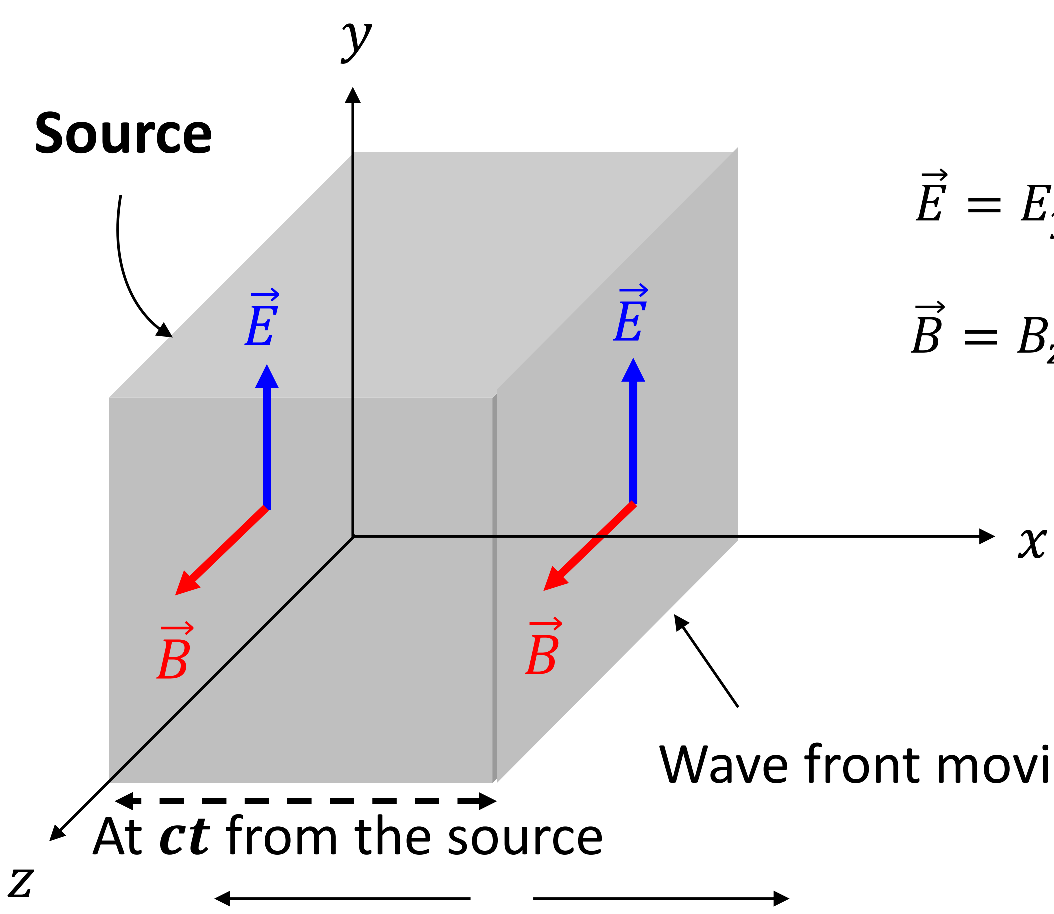
$$\vec{E}(x,t) = E_y(x,t)\hat{j}$$

$$\vec{B}(x,t) = B_z(x,t)\hat{k}$$

$$\frac{\partial E_y}{\partial y} = 0 \text{ and } \frac{\partial B_z}{\partial z} = 0$$



Plane EM wave and the relation $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$



$$\vec{E} = E_y \hat{j}$$

$$\vec{B} = B_z \hat{k}$$

As the wave moves the space is filled with Electric and magnetic field

Direction of propagation

Wave front moving to the right

At $\boldsymbol{ct}$ from the source

$$\vec{E} \neq \vec{0} \qquad \vec{E} = \vec{0}$$

$$\vec{B} \neq \vec{0} \qquad \vec{B} = \vec{0}$$

Postulating a configuration for a Plane wave

- E_y and B_z are constant in every point in a **<u>given plane at a given position $\boldsymbol{x}$</u>**
- Both fields move together in the $+\,x$-direction with a speed c (unknown)

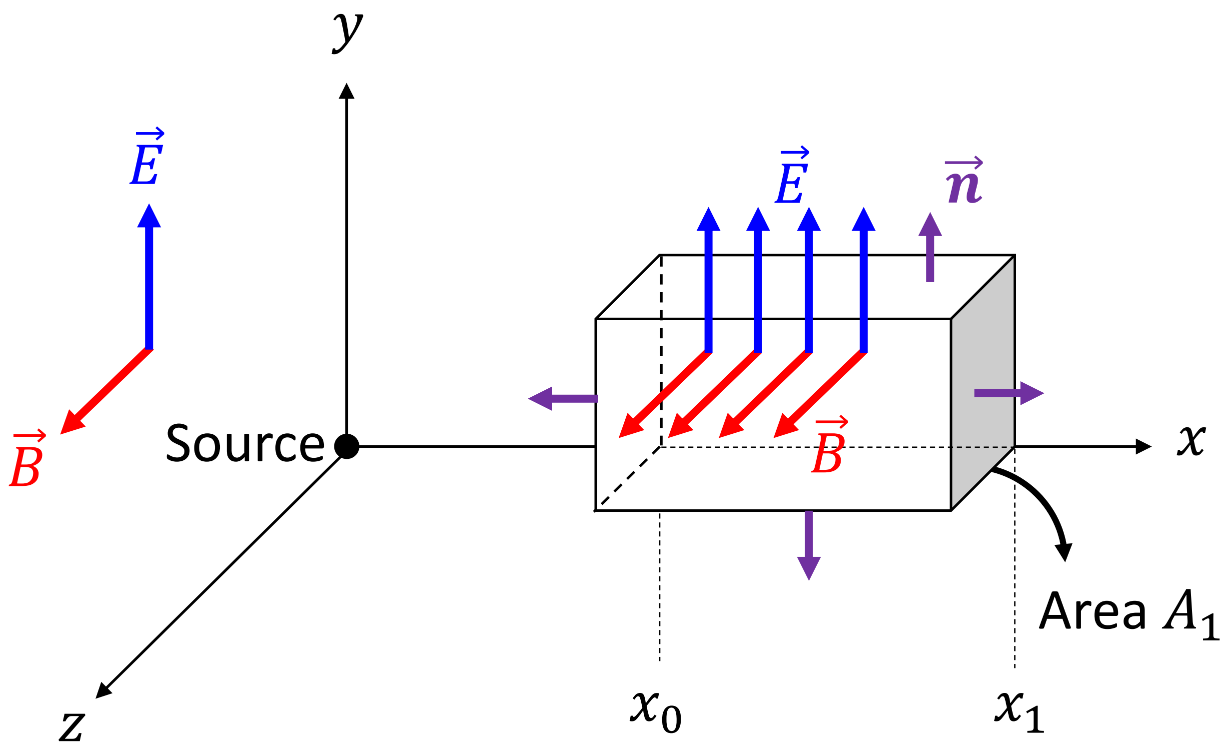
Is this configuration of a plane wave consistent with the four Maxwell's equations ?

Could $\vec{E}$ or $\vec{B}$ have an $x-$component ?

A) Gauss's law: Flux of the fields

$\vec{\nabla}.\vec{E} = 0$ when no net charges inside the Gaussian surface

$\vec{\nabla}.\vec{B} = 0$



$\vec{E} = E_y\hat{j}$

$\underbrace{E_z = 0}_{\vec{E} \perp \vec{B}}$

$\vec{B} = B_z\hat{k}$

$\underbrace{B_y = 0}_{\vec{E} \perp \vec{B}}$

What about E_x and B_x ?

Gauss's law would be violated if $\vec{E}$ and $\vec{B}$ had each a $x-$component. **Why ?**

Gauss law requires that

$$\boxed{\vec{\nabla}.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0}$$

$$\underset{0}{\parallel} \qquad \underset{0}{\parallel} \qquad \underset{0}{\parallel}$$

E_x MUST be either constant or $= 0$

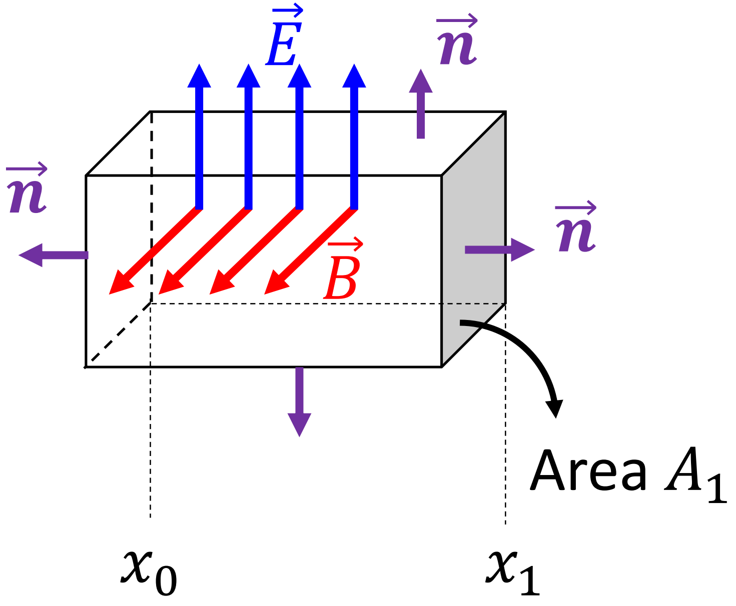
$\vec{E} \perp \vec{B} \Rightarrow E_z = 0 \Rightarrow \frac{\partial E_z}{\partial z} = 0$

E_y MUST be constant
in the whole yz plane
$\vec{E}(x,t) = E_y(x,t)\hat{j}$

See slide #32

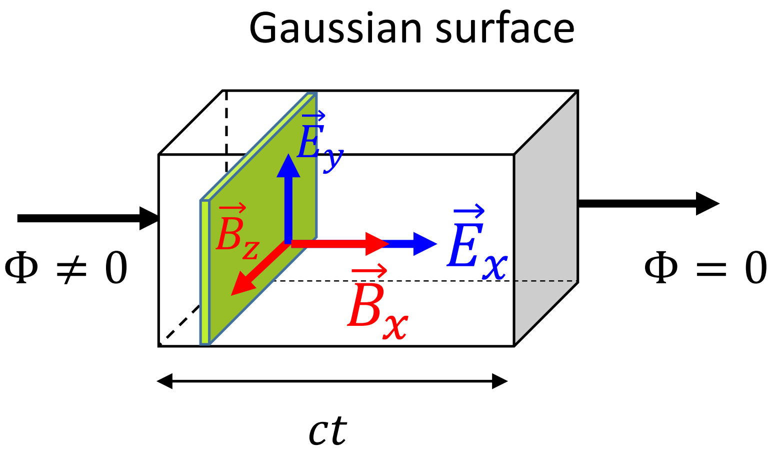
If E_x exists and because it must be constant ⇨

$$\oint_A \vec{E}.d\vec{A} = E_x(x_1)A_1 - E_x(x_0)A_1 = 0 \text{ (No charge enclosed)}$$



$E_x(x_1) = E_x(x_0)$

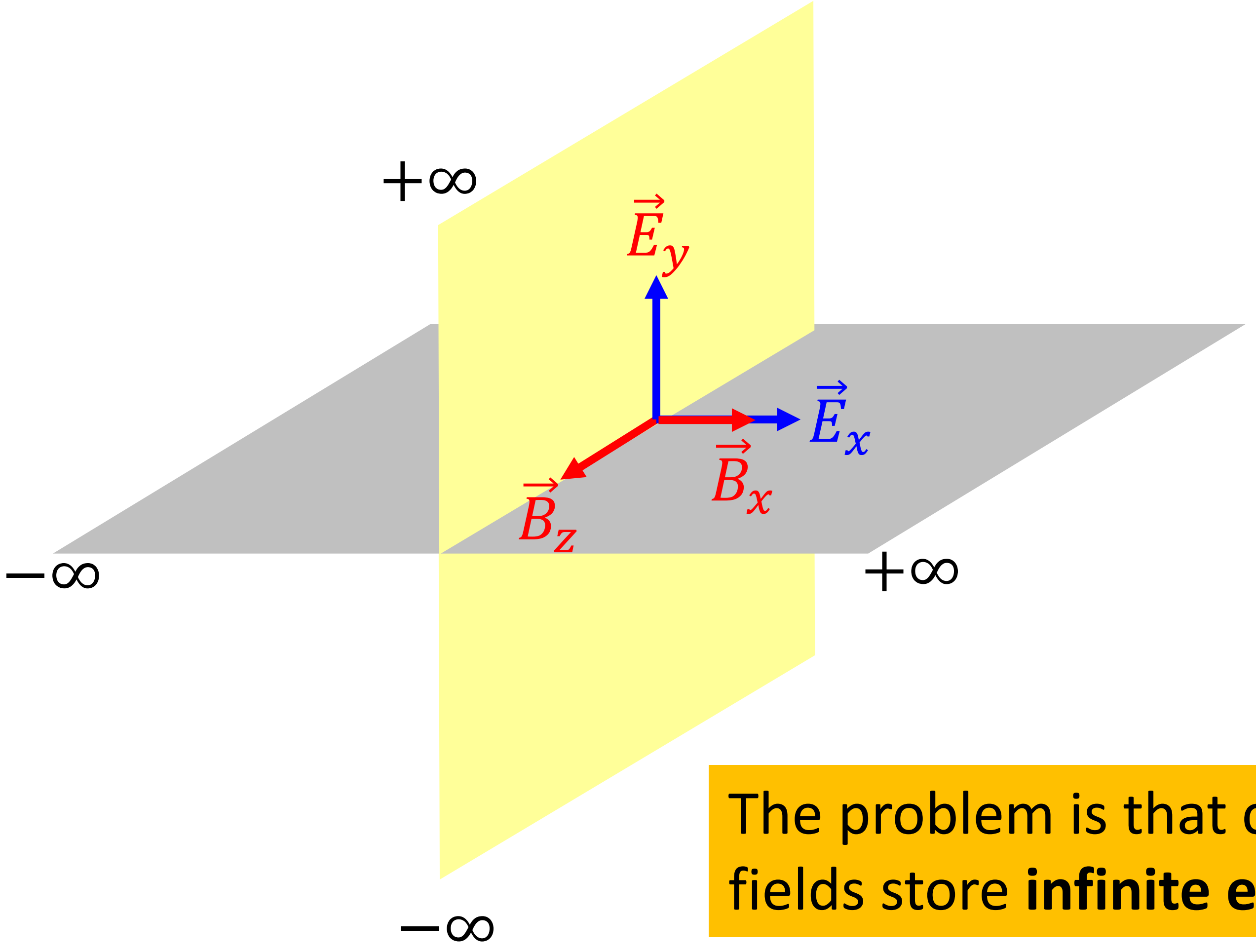
For all values of x_1 and x_0 !



Violation of Gauss law

Net flux $\neq 0$!

No charge in the box



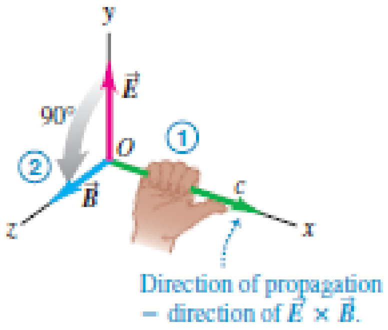
The problem is that constant fields store **infinite energy !**

$$\boldsymbol{E_x = 0}$$

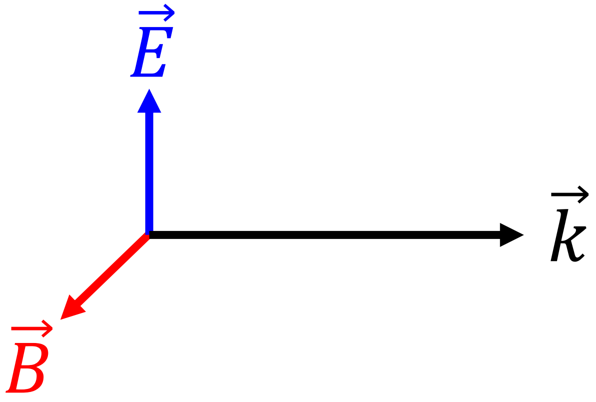
$\vec{E}$ and $\vec{B}$ MUST be perpendicular to each other
AND perpendicular to the direction of propagation

The same result holds for magnetic field

$$\boldsymbol{B_x = 0}$$



EM waves are transverse

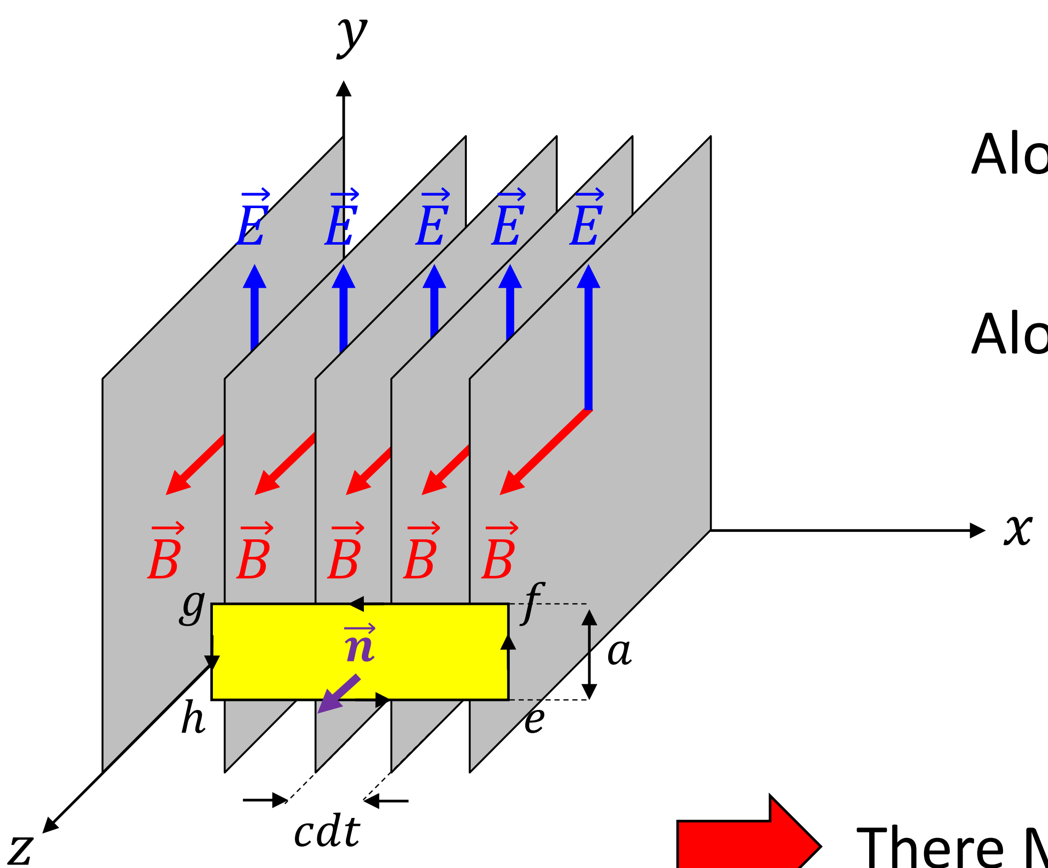


Direction of propagation

Other major properties of the fields $\vec{E}$ and $\vec{B}$ resulting from Maxwell’s equations

B) Faraday's law: circulation of the $\vec{E}$ field

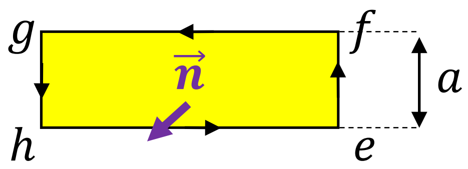
$$\oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt}$$



Along ef: the wave front still did not reach ⇨ $\int \vec{E}.d\vec{l} = 0$

Along fg and he: $\vec{E} \perp d\vec{l}$ ⇨ $\int \vec{E}.d\vec{l} = 0$

$\boxed{\text{Along } gh\text{: } \int_g^h \vec{E}.d\vec{l} = -Ea}$



➡ There MUST be a magnetic flux Φ_B through the rectangle

➡ According to right hand rule $\vec{B}$ must be along $+z$ axis and $\perp$ $\vec{E}$

$$\oint \vec{E}.d\vec{l} = -Ea = -\frac{d\Phi_B}{dt}$$

During time $\boldsymbol{dt}$ the front wave moves $\boldsymbol{cdt}$

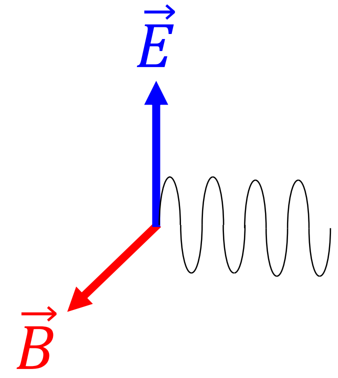
Area changes by $\boldsymbol{acdt}$ $\Rightarrow$ $\dfrac{d\Phi_B}{dt} = Bac$

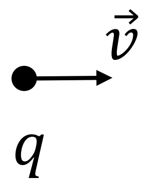
$$\boxed{E = cB}$$

Required by Faraday's law

Consequence of $E = Bc$

Consider a free charge interacting with an electromagnetic wave





$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \Rightarrow \boxed{\frac{F_B}{F_E} = \frac{vB}{E} = \frac{v}{c}}$$

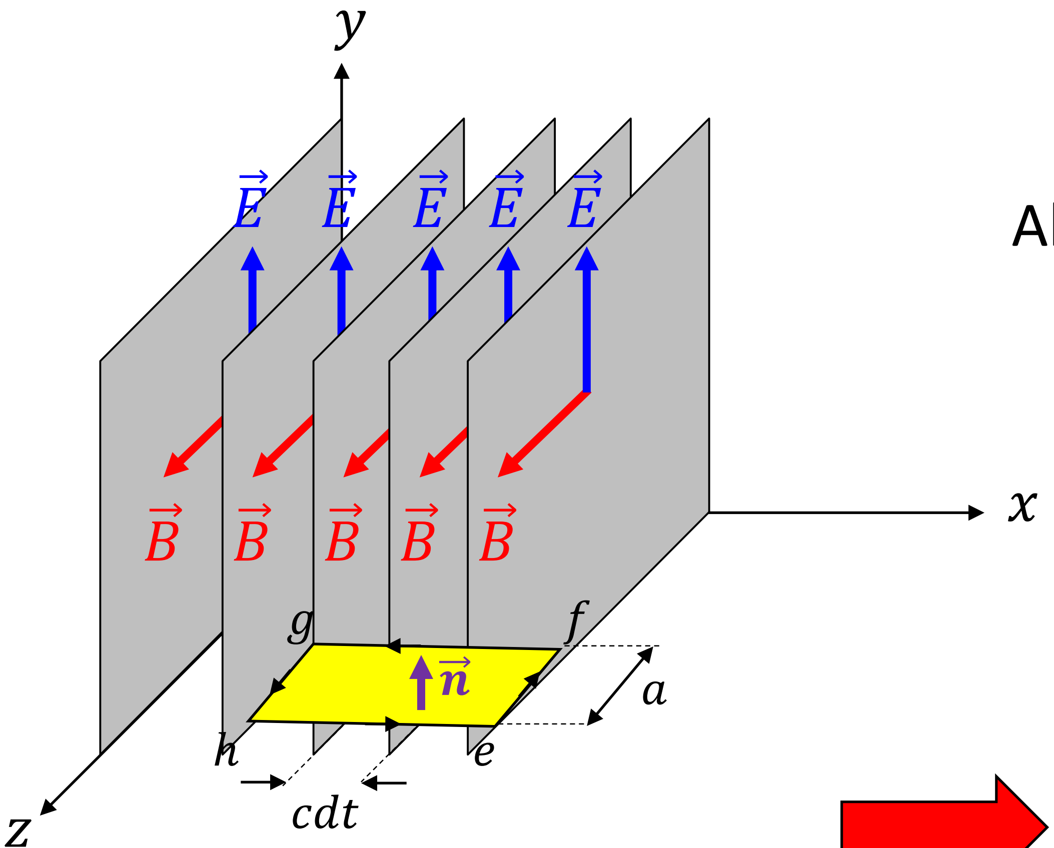
Unless the charge is relativistic $F_B << F_E$

In most situations electromagnetic waves are essentially an ***electric phenomenon***

C) Ampere's law: circulation of the $\vec{B}$ field (Maxwell's part)

$$\oint \vec{B}.d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

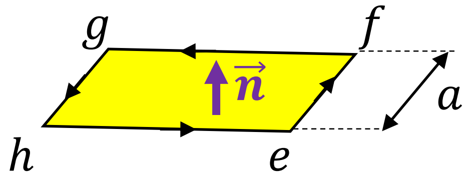
There is no current $\vec{J} = \vec{0}$



Along ef: the wave front still did not reach ⇨ $\int \vec{B}.d\vec{l} = 0$

Along fg and he: $\vec{E} \perp d\vec{l}$ ⇨ $\int \vec{B}.d\vec{l} = 0$

Along gh: $\int_g^h \vec{B}.d\vec{l} = Ba$



➡ There MUST be a electric flux Φ_E through the rectangle

➡ According to right hand rule $\vec{E}$ must be along $+y$ axis

$$\oint \vec{B}.d\vec{l} = Ba = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

During time dt the front wave moves cdt

Area changes by $acdt$ ➡ $\dfrac{d\Phi_E}{dt} = Eac$

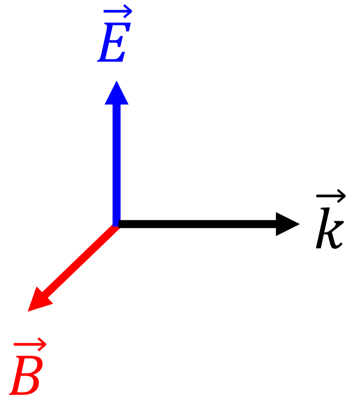
$$\boxed{B = \mu_0\varepsilon_0 cE}$$

Required by Maxwell's law

Faraday's law and Maxwell's law

$$\boxed{E = cB} \qquad \boxed{B = \mu_0\varepsilon_0 cE}$$

$$\boxed{c = \frac{1}{\mu_0\varepsilon_0 c}} \Rightarrow \boxed{c \Rightarrow \frac{1}{\sqrt{\mu_0\varepsilon_0}}}$$



Direction of propagation

$\vec{E}(x,t)$ **AND** $\vec{B}(x,t)$ **MUST** be in phase in space and time

- Unlike all the other types of waves, EM waves require **NO MEDIUM** through/along which to travel. **EM waves can travel through empty space (vacuum)!**

- Speed of light is independent of speed of observer! We could be heading toward a light beam at the speed of light, but we would still measure *c* as the speed of the beam!

$$c = 299\ 792\ 458 \text{ m/s}$$

Not intuitive at all !

Summary

Time depended Maxwell's equations

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Charge conservation

$$\vec{\nabla}\vec{\boldsymbol{j}} = -\frac{\partial \rho}{\partial t}$$

→ Concept of flow of charge

→ EM wave generation

$m = (x, y, z) \quad \vec{u} = (\vec{\imath}, \vec{\jmath}, \vec{k})$

$$\begin{cases} \vec{E} = \sum_{1}^{3} E_{0m}\hat{u}e^{i(\vec{k}.\vec{r}-\omega t)} \\ \vec{B} = \sum_{1}^{3} B_{0m}\hat{u}e^{i(\vec{k}.\vec{r}-\omega t)} \end{cases}$$

Solution not necessarily a ***sin wave***

$$f(\vec{r}, t) = g(\vec{k}.\vec{r} - \omega t) + h(\vec{k}.\vec{r} - \omega t)$$

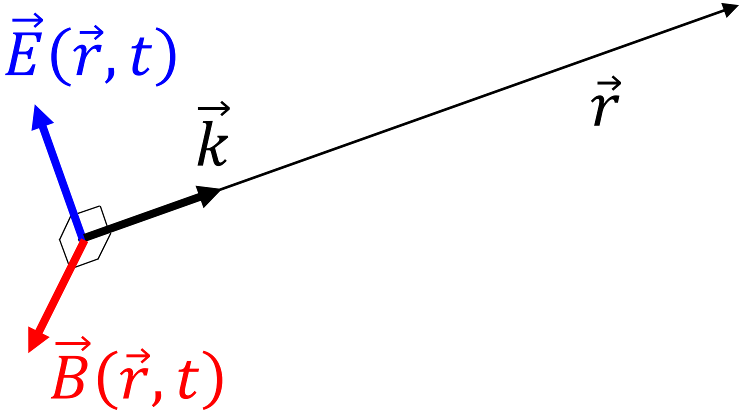
$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2}\vec{E}$$

$$\vec{k} \times \vec{E} = \omega\vec{B}$$

$$B = \frac{E}{c}$$

$$B = \mu_0\varepsilon_0 cE$$

$$\frac{1}{\varepsilon_0\mu_0} = c^2$$

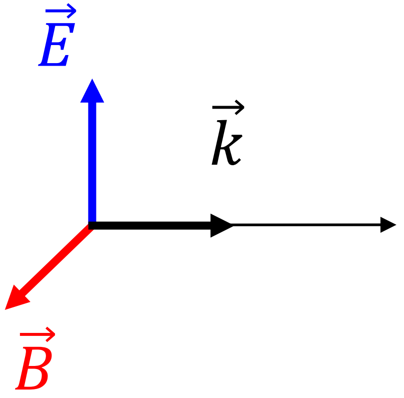


Transverse waves

With these we are now ready to obtain the wave equations !

Dimension equation

$$\vec{k}\times\vec{E} \qquad \vec{B} \; + \; \vec{k}\perp\vec{E} \;\Rightarrow\; [kE] = \frac{VL^{-1}}{L} = VL^{-2}$$



$$[B] = \left[\frac{E}{v}\right] = \frac{VL^{-1}}{LT^{-1}} = (VL^{-2})/T^{-1}$$

$$[B] = [kE]/\boldsymbol{T^{-1}}$$

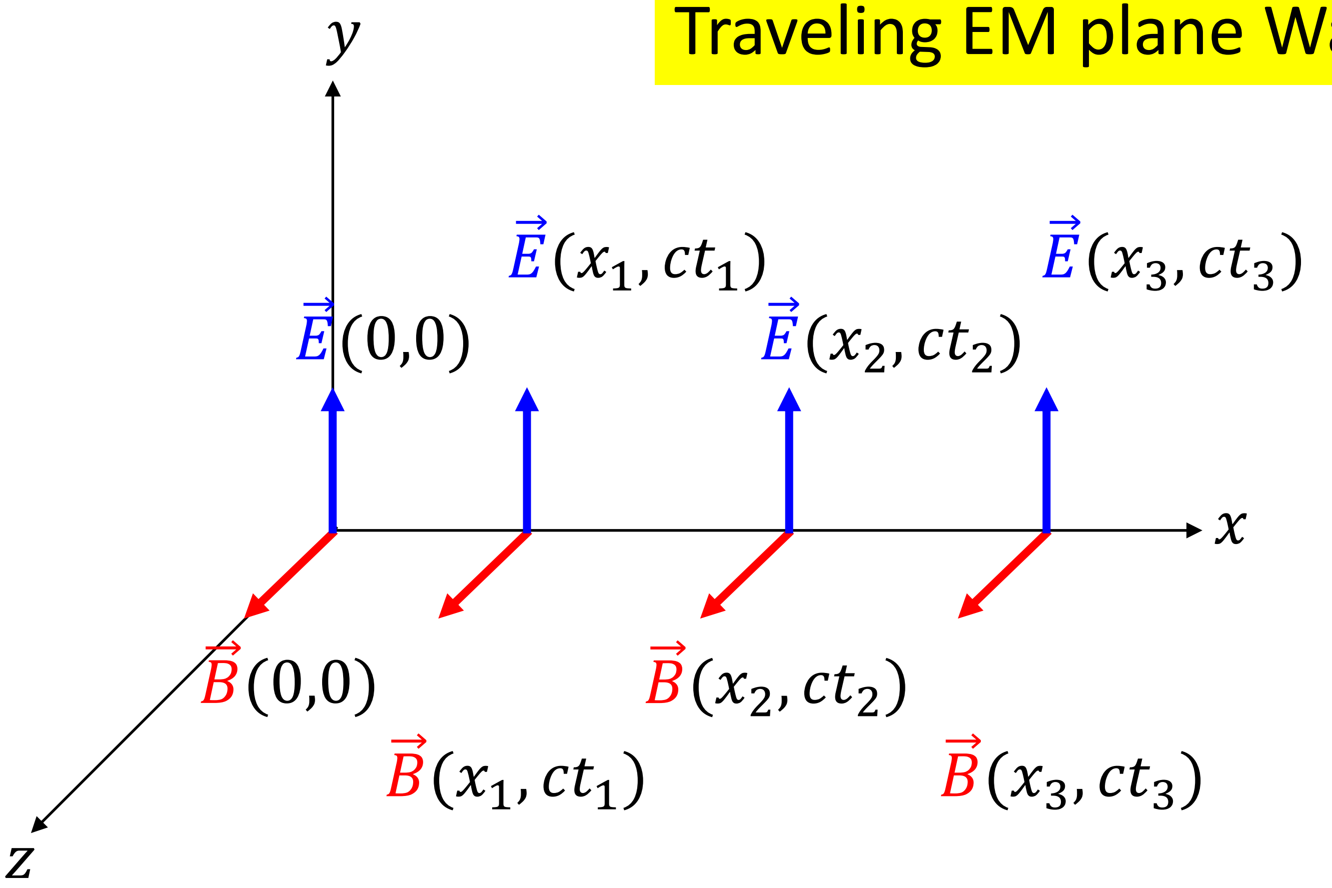
$$\boxed{\vec{k}\times\vec{E} = \boldsymbol{\omega}\vec{B}}$$

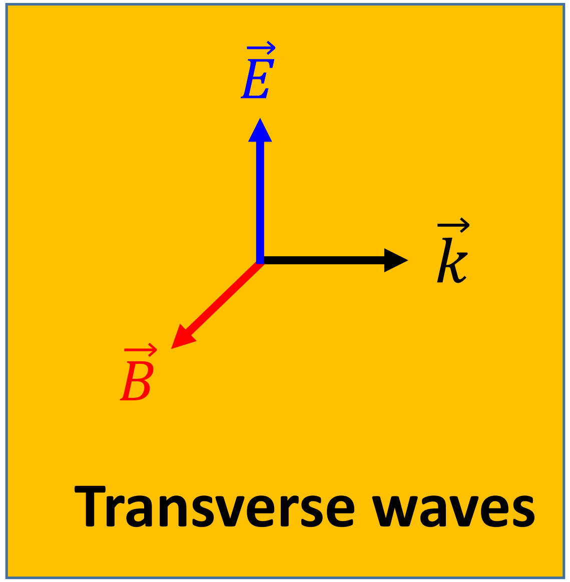
$$[kB] = [k^2E]/T^{-1} = \frac{VL^{-1}}{L^2T^{-1}} = \frac{\boldsymbol{T^{-1}}}{\boldsymbol{(LT^{-1})^2}}[E]$$

– Sign comes from the right hand rule

$$\boxed{\vec{k}\times\vec{B} = -\frac{\omega}{c^2}\vec{E}}$$

Traveling EM plane Wave equation





$$\vec{E}(x,t) = E_y(x,t)\hat{j}$$

$$\vec{B}(x,t) = B_z(x,t)\hat{k}$$

Field amplitudes are **constant** in the same plane

BUT

may **change** from plane to plane

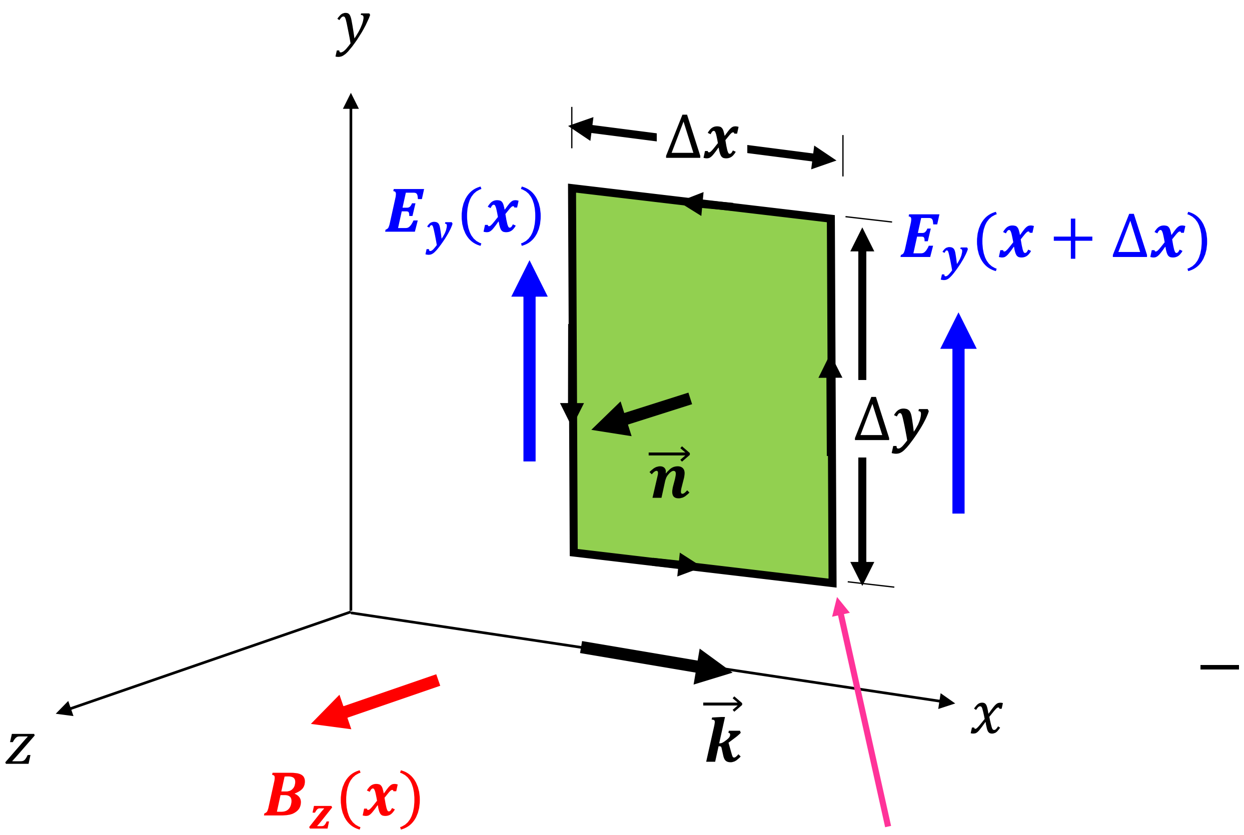
WAVE EQUATIONS

…From integral forms of Maxwell's equation…

Faraday's law: circulation of the $\vec{E}$ field

Stokes' vs Gauss's theorem

$$\oint \vec{E}.d\vec{l} = \boxed{-\frac{d}{dt}\iint \vec{B}.d\vec{A}}$$



We start here $(x+\Delta x)$ and go counterclockwise

$$\oint \vec{E}.d\vec{l} = E_y(x+\Delta x)\Delta y - E_y(x)\Delta y$$

$$E_y(x+\Delta x) = E_y(x) + \frac{\partial E_y}{\partial x}\Delta x + \cdots$$

$$\oint \vec{E}.d\vec{l} = \boxed{\frac{\partial E_y}{\partial x}\Delta x\Delta y}$$

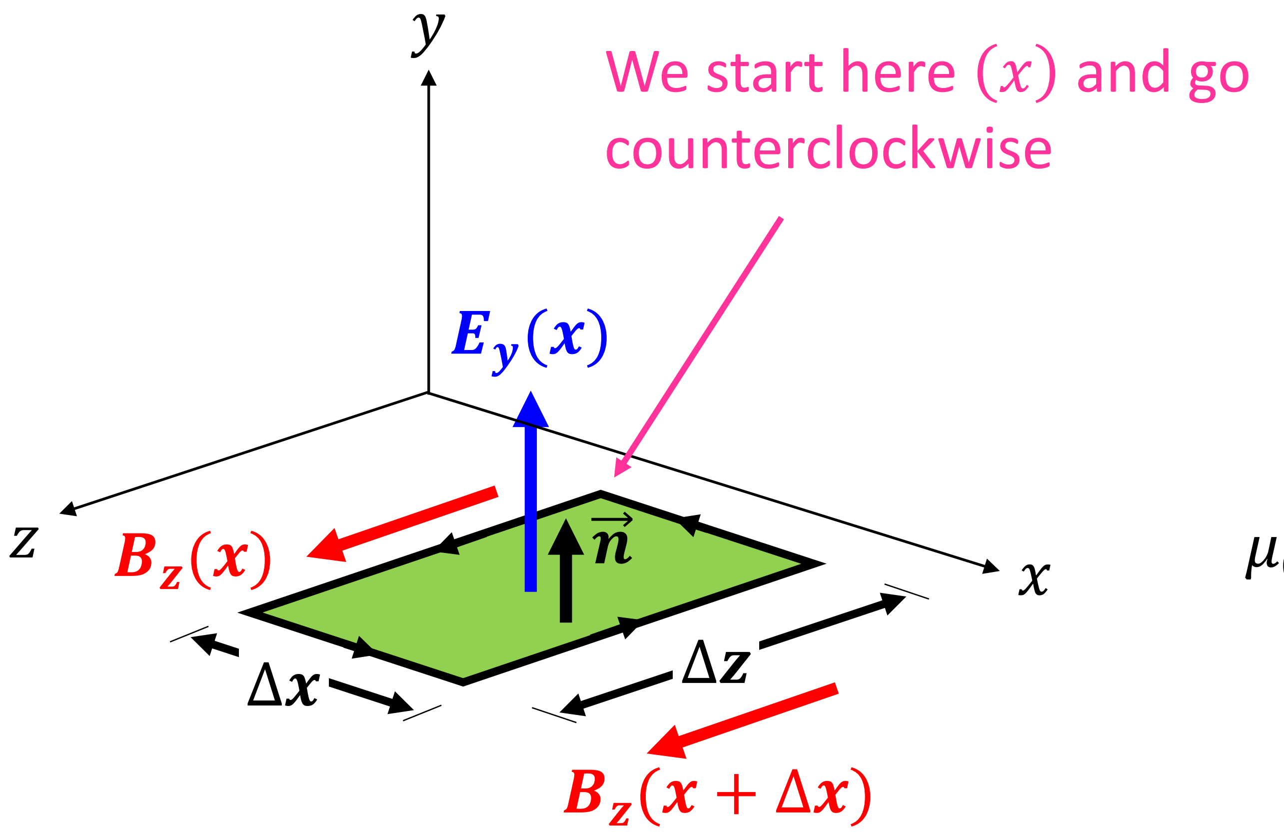
$$-\frac{d}{dt}\iint \vec{B}.d\vec{A} = \boxed{-\frac{\partial B_z}{\partial t}\Delta x\Delta y}$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

Ampere's law: circulation of the $\vec{B}$ field (Maxwell's part)

Again Stokes' vs Gauss's theorem

$$\oint \vec{B}.d\vec{l} = \boxed{\mu_0\varepsilon_0 \frac{d}{dt}\iint \vec{E}.d\vec{A}}$$



$$\oint \vec{B}.d\vec{l} = B_z(x)\Delta z - B_z(x+\Delta x)\Delta z$$

$$B_z(x+\Delta x) = B_z(x) + \frac{\partial B_z}{\partial x}\Delta x + \cdots$$

$$\oint \vec{B}.d\vec{l} = \boxed{-\frac{\partial B_z}{\partial x}\Delta x\Delta z}$$

$$\mu_0\varepsilon_0 \frac{d}{dt}\iint \vec{E}.d\vec{A} = \boxed{\mu_0\varepsilon_0 \frac{\partial E_y}{\partial t}\Delta x\Delta y}$$

$$\boxed{-\frac{\partial B_z}{\partial x} = \mu_0\varepsilon_0 \frac{\partial E_y}{\partial t}}$$

Plane traveling EM Wave equation

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

$$\left\{\begin{aligned} &\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) \\ &\frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial x}\right) = -\frac{\partial^2 B_z}{\partial t^2} \end{aligned}\right.$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\mu_0 \varepsilon_0 = \frac{1}{c^2}}$$

$$\boxed{-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}}$$

$$\left\{\begin{aligned} &-\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial x}\right) \\ &-\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \end{aligned}\right.$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}}$$

Compare these wave equations to a mechanical wave equation

Electromagnetic wave

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$

$$E = E_y(x,t)$$

$$\vec{E} = \vec{E}_y(\vec{r},t)$$

$$B = B_Z(x,t)$$

$$\vec{B} = \vec{B}_Z(\vec{r},t)$$

Mechanical wave

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

For more complex waves → Superposition principle applies

Superposing many waves → Superposing many $\vec{E}'s$ and $\vec{B}'s$

$$\vec{E} = \sum \vec{E}'\mathrm{s} \qquad \vec{B} = \sum \vec{B}'\mathrm{s}$$

Remark:

- The wave equation is dispersionless. Thus any function of the form $f(\vec{k}.\vec{r}-\omega t)$ satisfies the equation provided Maxwell's constrains apply **(cross products between $\vec{\boldsymbol{k}}$, $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$)**

$$\boxed{\vec{k}\times\vec{B}=-\frac{\omega}{c^2}\vec{E} \qquad \textbf{and} \qquad \vec{k}\times\vec{E}=\omega\vec{B}}$$

- $\vec{E}$ and $\vec{B}$ waves do not have to be sinusoidal. As the wave equation is linear the solution could be any linear combination of sinusoidal function by Fourier transform

Energy and momentum in electromagnetic waves

The Poynting vector $\vec{S}$

We already know that both $\vec{E}$ and $\vec{B}$ fields carry energy

Waves contain energy

- Microwave ovens
- Radio transmitters
- Laser for eye surgery
- Etc…

From electro and magnetostatic

$u =$ Total energy density

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

$$B = \mu_0 \varepsilon_0 c E$$

Slide #44

The energy density due to $\vec{\boldsymbol{E}}$ is equal to the energy density due to $\vec{\boldsymbol{B}}$ field

$$\boxed{u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

As both $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields vary in space and time, $\boldsymbol{u}$ also depends on space and time

⬇

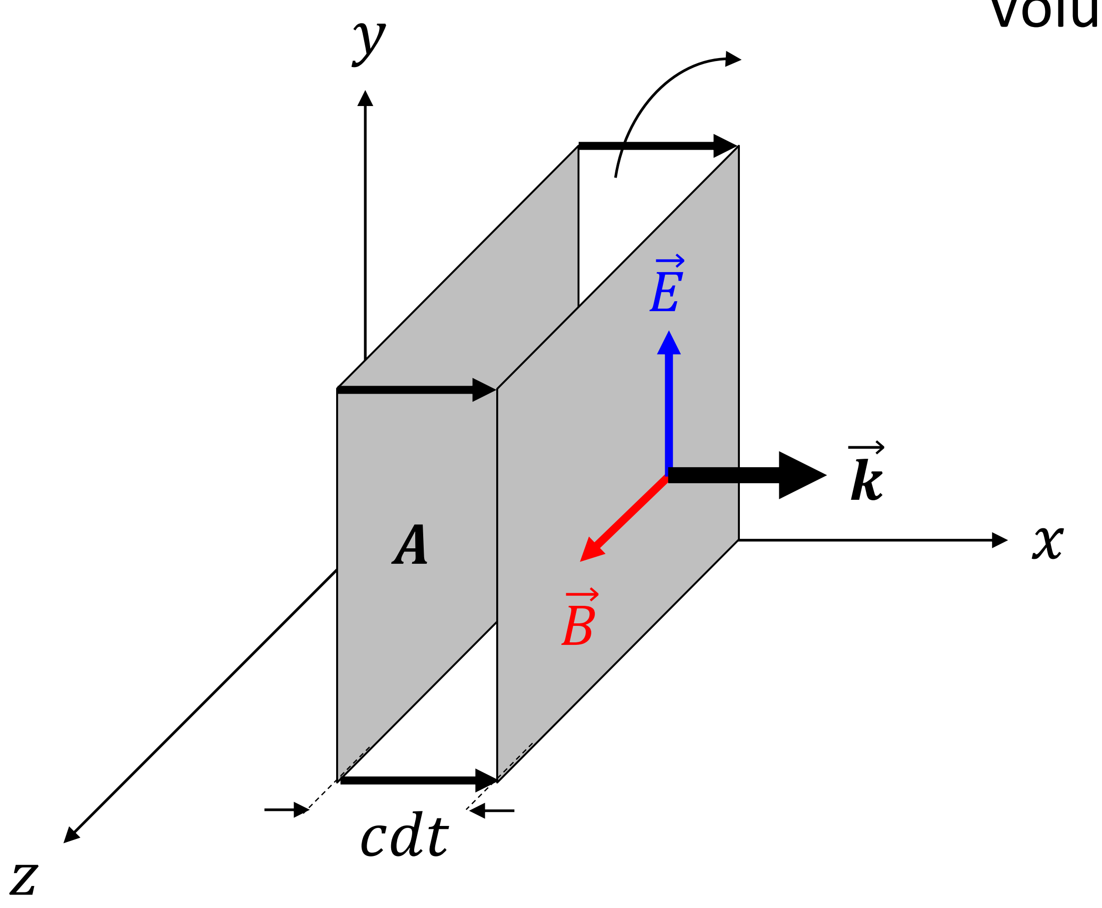
Concept of flow of <u>energy</u> and <u>momentum</u> $\equiv$ flow of <u>charges</u> $\equiv$ <u>flow of heat</u>

⬇

Poynting <u>**vector**</u> and Poynting theorem

Electromagnetic energy flow and the Poynting vector

Energy transferred / unit time / unit area = power transferred / unit area



Volume $dV \ = Acdt$

The energy contained in this volume after the wave has traveled the distance cdt

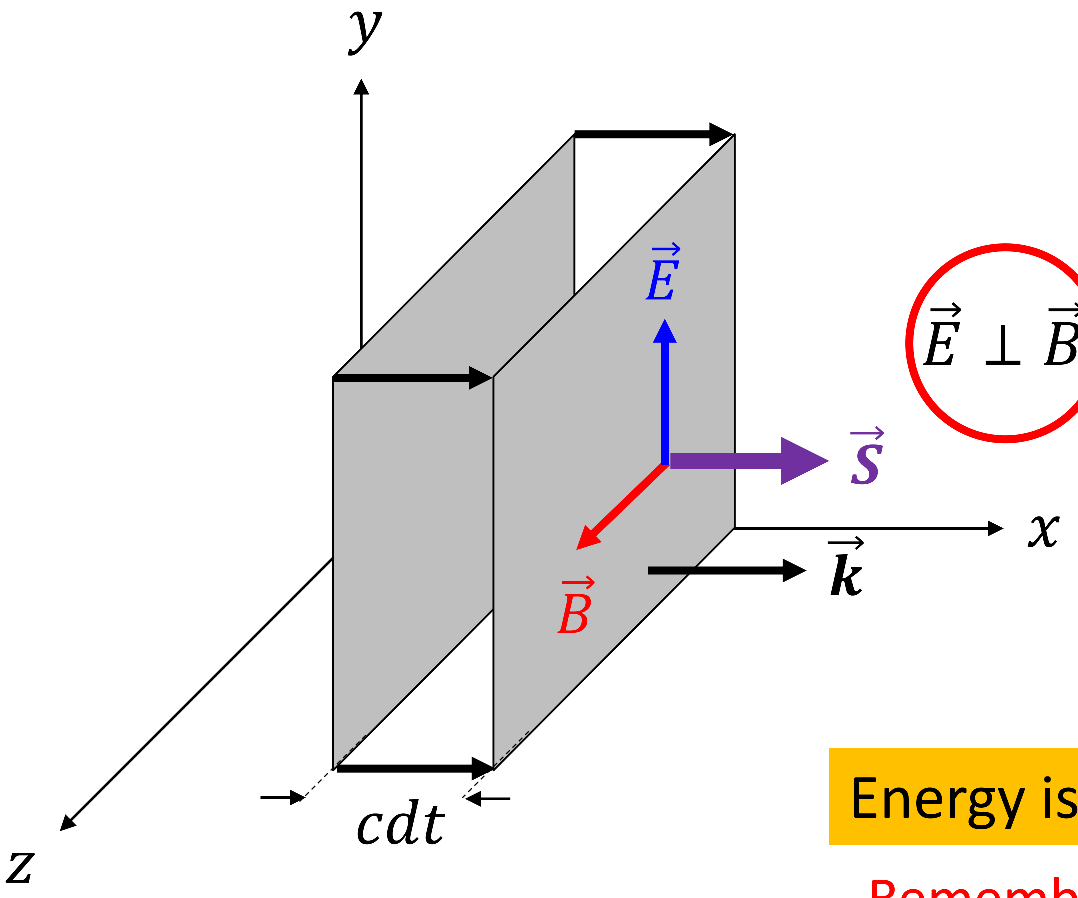
$$dW \ = \mathrm{udV} = (\varepsilon_0 E^2)(Acdt\ )$$

Energy flow /unit area / unit time or power transferred / unit area

$$S = \frac{1}{A}\frac{dW}{dt} = \varepsilon_0 c E^2$$

Concept of flow of energy

$$S = \frac{1}{A}\frac{dW}{dt} = \varepsilon_0 c E^2 = \varepsilon_0 (cE)E = \varepsilon_0 (c\mathbf{E})c\frac{\mathbf{E}}{\mathbf{c}} = \varepsilon_0 (c\mathbf{E})c\mathbf{B} = \varepsilon_0 c^2 \mathbf{E}\mathbf{B}$$



CROSS PRODUCT

$\vec{E} \perp \vec{B}$ and $\varepsilon_0 c^2 = \dfrac{1}{\mu_0}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Universal character

Poynting vector

Energy is **SCALAR BUT** the **FLOW** of energy is a **VECTOR**

Remember! Heat is a scalar BUT the flow of heat is a vector

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t) = \frac{1}{\mu_0}[\hat{\jmath}E_{max}cos(kx - \omega t)] \times \left[\hat{k}B_{max}cos(kx - \omega t)\right]$$

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}cos^2(kx - \omega t) = \frac{E_{max}B_{max}}{2\mu_0}[1 + cos2(kx - \omega t)]$$

$$\Rightarrow \quad \vec{S}_{av} = S_{av}\vec{\imath} \qquad \boxed{S_{av} = \frac{E_{max}B_{max}}{2\mu_0}}$$

This expresses the intensity of sinusoidal EM wave in vacuum

$$\boxed{I = S_{av} = \frac{1}{2}\varepsilon_0 c E_{max}^2}$$

Question: The Poynting vector **does** vary with time. Why our eyes do not see this variation when hit by light coming from a bulb?

Answer: Because the oscillation frequency is too high ! $5 \times 10^{14}\ Hz$

What about waves propagating in a dielectric?

Vacuum

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = u_E + u_B$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$I = S_{av} = \frac{1}{2}\varepsilon_0 c E_{max}^2$$

Dielectric (linear homogeneous isotropic)

$$\varepsilon_0 \rightarrow \varepsilon = \varepsilon_0 \varepsilon_r$$

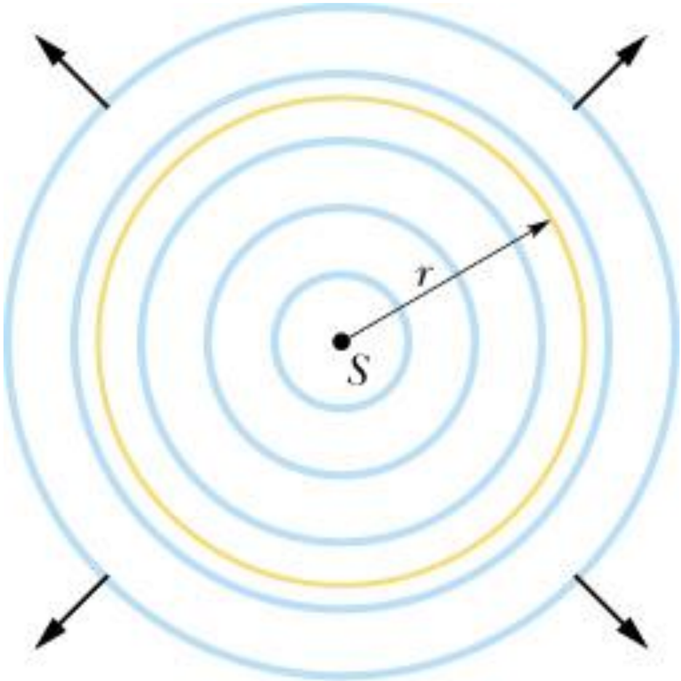
$$\mu_0 \rightarrow \mu = \mu_0 \mu_r$$

$$c \rightarrow v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad E = vB \qquad B = \varepsilon\mu v B$$

In vacuum $u_E^0 = u_B^0$ ⇒ In Dielectric $u_E^D = u_B^D$

Question: How does the intensity (power/area) change with distance r?

Consider a point source S that is emitting EM waves isotropically (equally in all directions) at a rate P_S. Assume energy of waves is conserved as they spread from source.



$$\boxed{I = \frac{Power}{Area} = \frac{P_S}{4\pi r^2}}$$

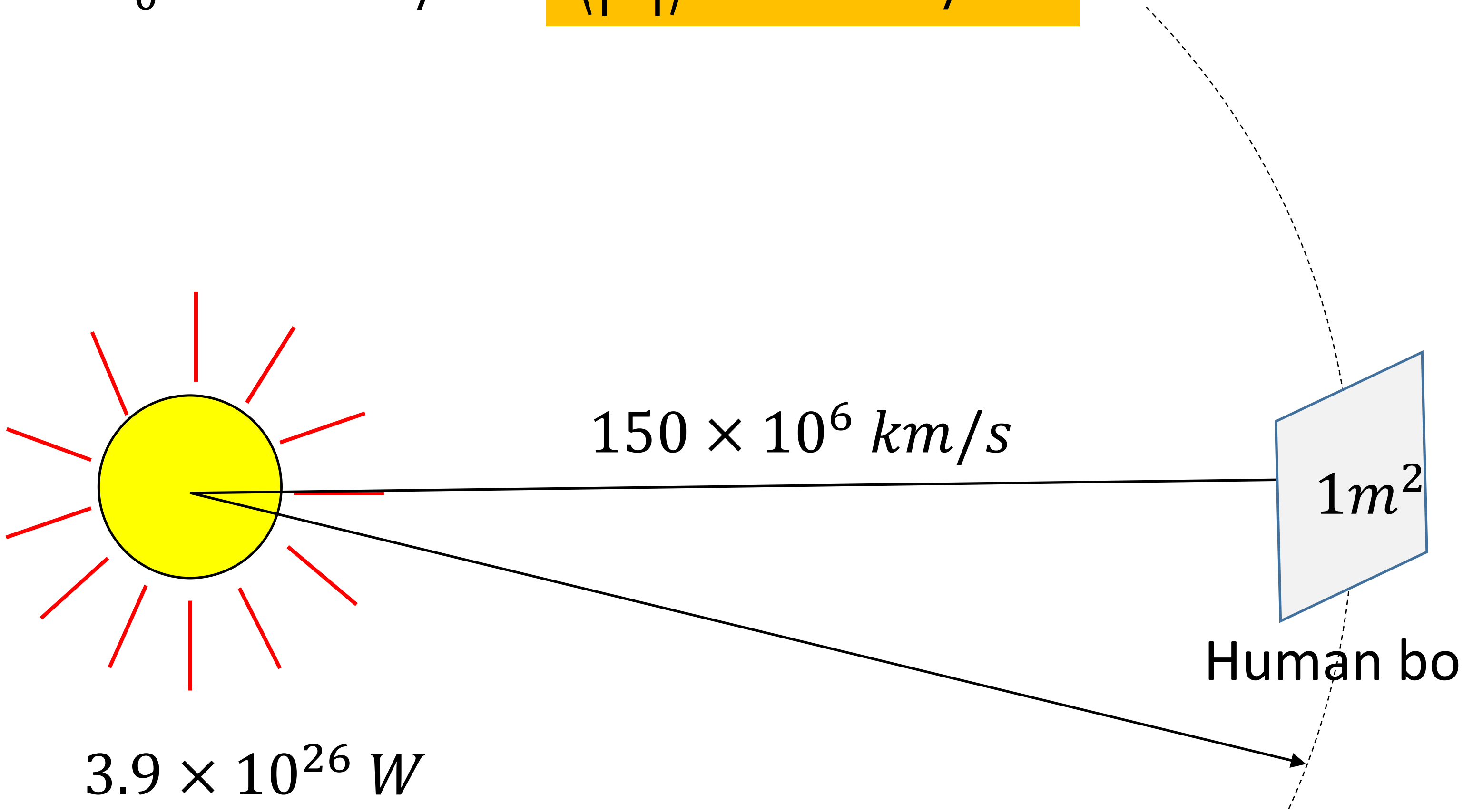
Ex: $E_0 = 100V/m$ (visible light)

$\langle \vec{S} \rangle = 13W/m^2$

This is not harmful

Ex: $E_0 = 1000V/m$ $\boxed{\langle|\vec{S}|\rangle = 1.3kW/m^2}$

$$\langle|\vec{S}|\rangle = \frac{3.9\times 10^{26}W}{(150\times 10^9)^2 m^2}$$



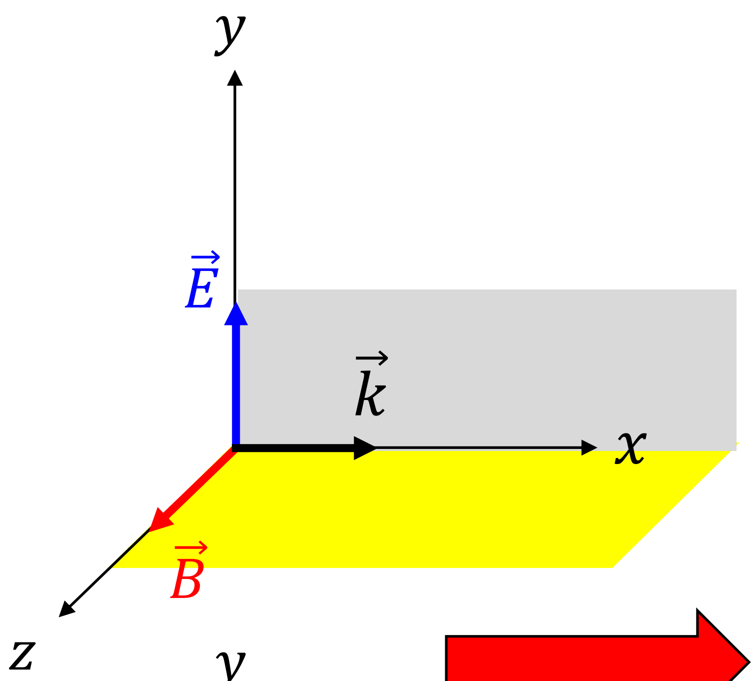
$\boxed{\langle|\vec{S}|\rangle = 1.4kW/m^2}$

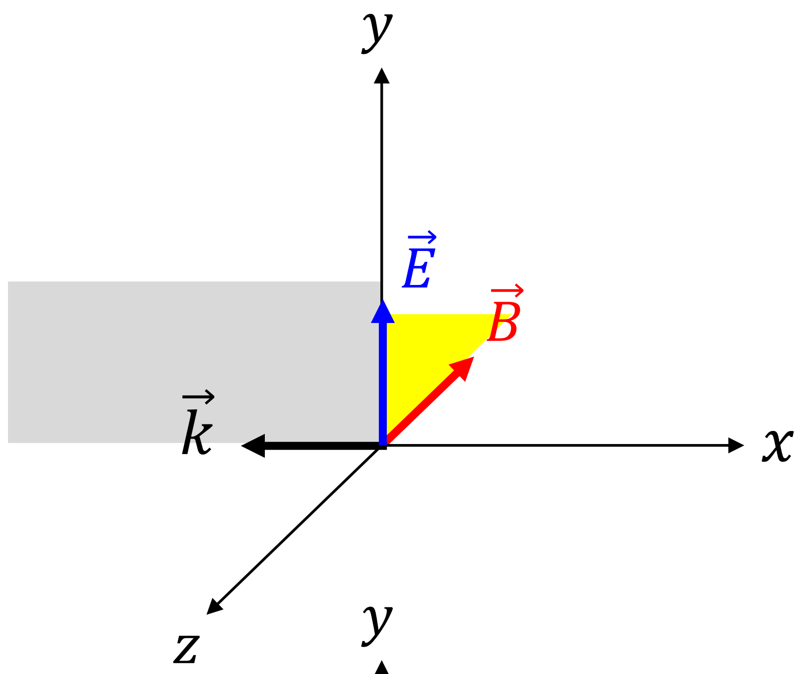
Exposing to sun rays could be very dangerous

Propagation, Polarization and incidence of EM waves on matter: conductor vs dielectric

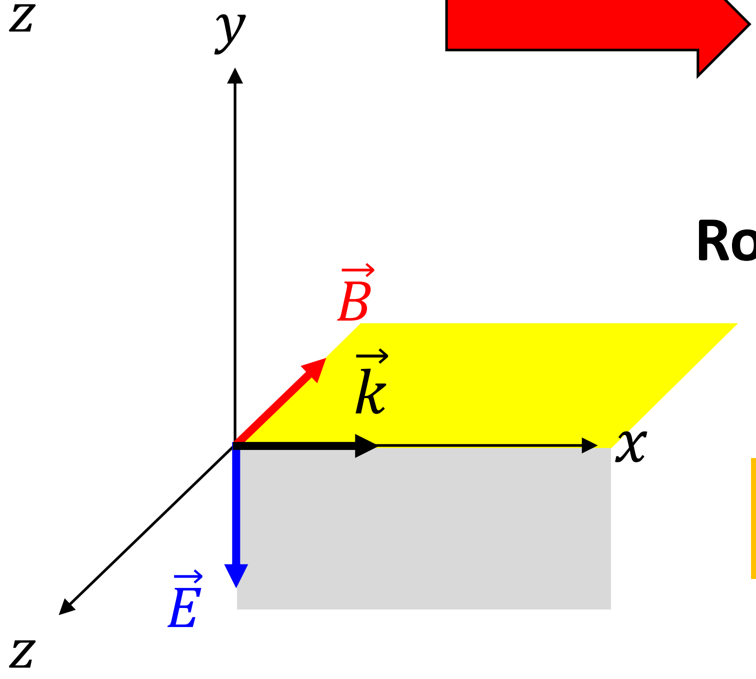
$$\vec{k}.\vec{E} = 0 \qquad \vec{k} \times \vec{E} = \omega\vec{B}$$

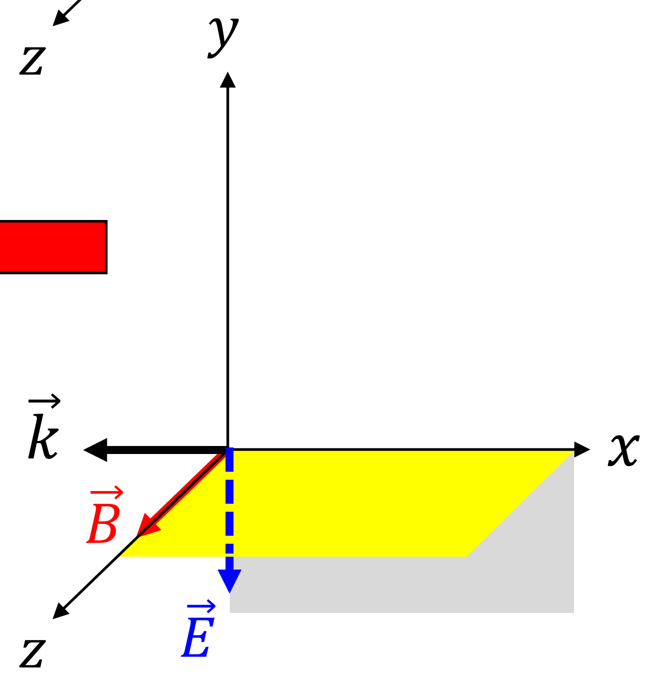
$$\vec{k}.\vec{B} = 0 \qquad \vec{k} \times \vec{B} = -\frac{\omega}{c^2}\vec{E}$$





Two opposite directions
of propagation
Rotation = π around the y −axis

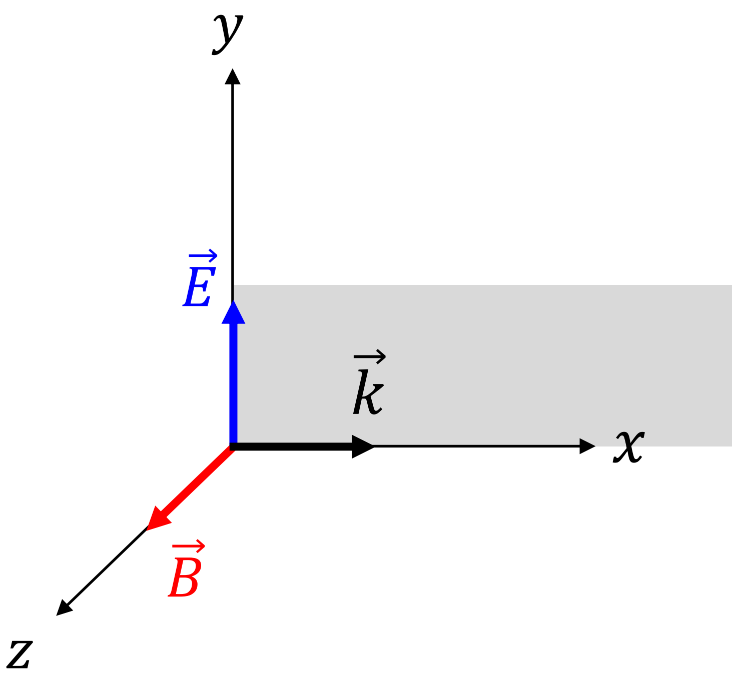




In all four cases $\vec{E}$ is vertical

Definition of polarization

The direction of the linear polarization = The axis along which the $\vec{\boldsymbol{E}}$ field points



The polarization plane is defined by the two vectors

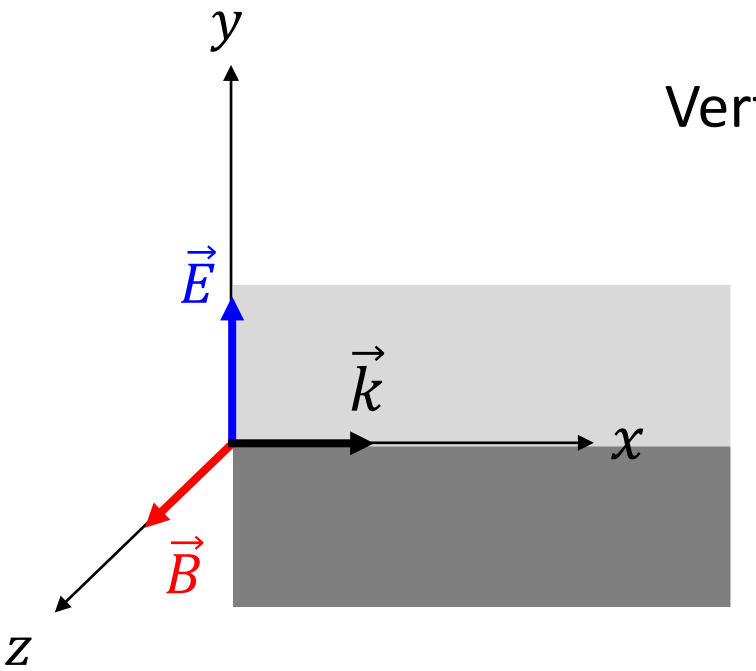
$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{k}}$

This is a linear vertical polarization

Two particular types of linear polarization

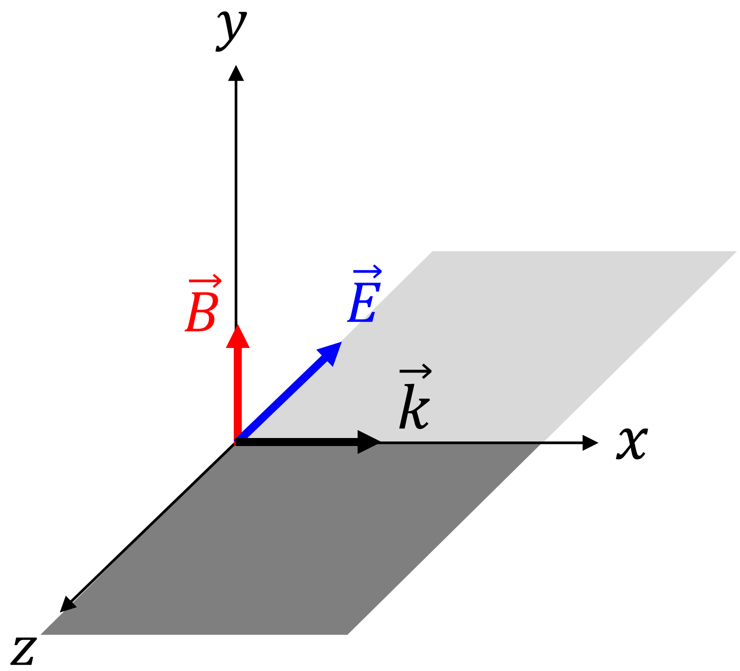
Vertical

Horizontal



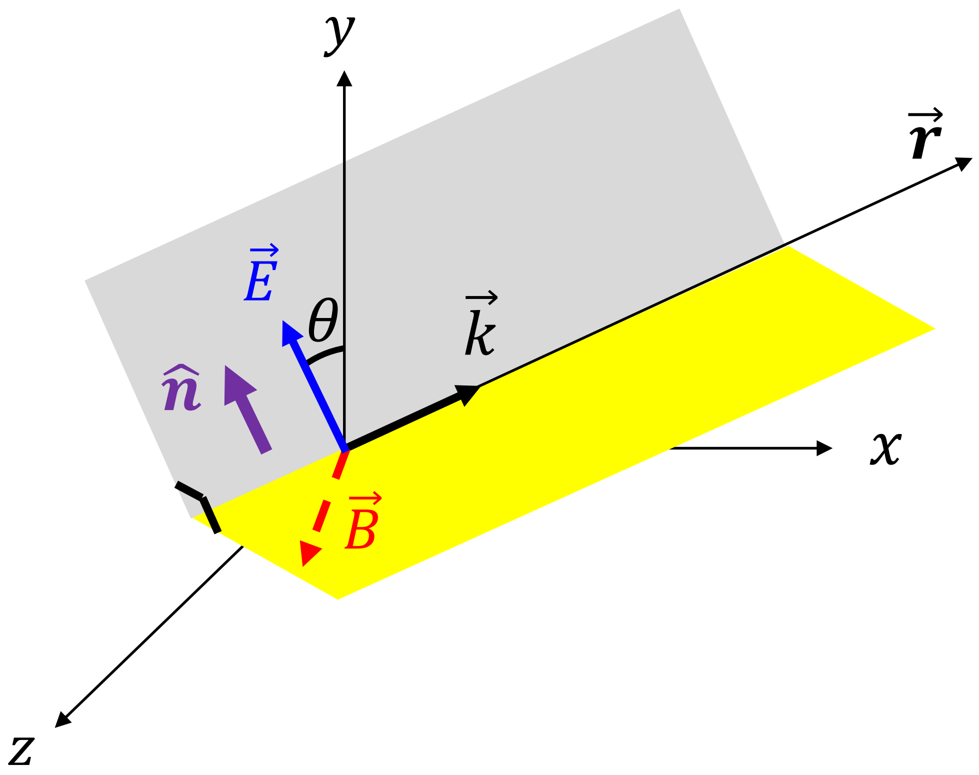
Propagation along x-axis

Rotation = $\pi/2$ arround the x −axis



Two distinct and $\perp$ planes of $\vec{E}$ vibration

Polarization along any arbitrary direction



$\boldsymbol{\hat{n}.\vec{k} = 0}$ and $\boldsymbol{\vec{k}.\vec{E} = 0}$

$\hat{n}$ and $\vec{k}$ define the plane of vibration of $\vec{E}$

$$\hat{n} = cos\theta\hat{j} + sin\theta\hat{k}$$

$$\vec{E}(\vec{r},t) = E_0 e^{i(\vec{k}.\vec{r}-\omega t)}\hat{n}$$

$$\vec{B}(\vec{r},t) = B_0 e^{i(\vec{k}.\vec{r}-\omega t)}(\vec{k} \times \hat{n})$$

$\boldsymbol{E = cB}$ $\Rightarrow$ $$\vec{B}(\vec{r},t) = \frac{E_0}{c} e^{i(\vec{k}.\vec{r}-\omega t)}(\vec{k} \times \hat{n}) = \frac{1}{c}\vec{k} \times \vec{E}$$

$\vec{E}$ field along any direction in the $xy-$plane

$\vec{\boldsymbol{E}} \perp \vec{\boldsymbol{B}}$ ⇨ and both fields have two components.

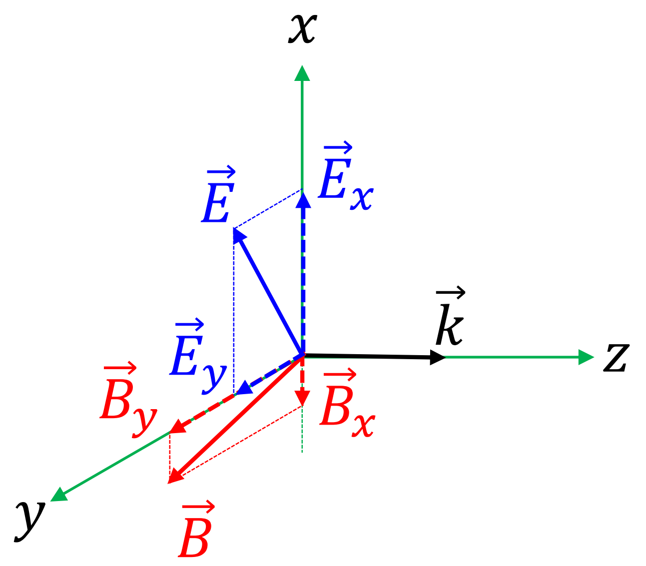
Linear combination: $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$ is also a solution of Maxwell's equation

$$\vec{E}_x = E_{0x}\cos(\omega t - kz)\hat{\imath}$$

$$\vec{E}_y = E_{0y}\cos(\omega t - kz + \delta)\hat{\jmath}$$

$$\vec{B}_x = -\frac{1}{c}E_{0x}\cos(\omega t - kz + \delta)\hat{\imath}$$

$$\vec{B}_y = \frac{1}{c}E_{0y}\cos(\omega t - kz)\hat{\jmath}$$



($\vec{E}_x$ and $\vec{B}_y$) or ($\vec{E}_y$ and $\vec{B}_x$) fields are in phase

The two components of the **SAME** field are not necessarily in phase

($\vec{E}_x$ and $\vec{E}_y$) or ($\vec{B}_x$ and $\vec{B}_y$)

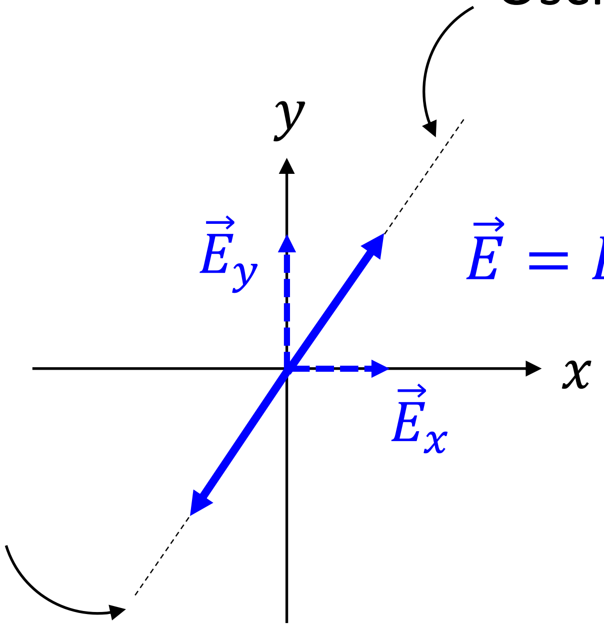
Special case #1

x – y plane

$\delta = 0$ no dephasing between $\vec{E}_x$ and $\vec{E}_y$

both reach max and min at the same time

Oscillation along this line



$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$

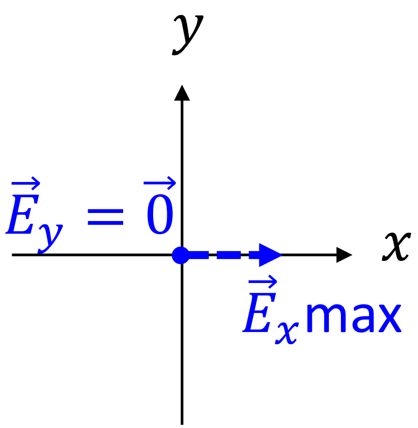
$E = \sqrt{E_x^2 + E_y^2}$

$\vec{E}$ is linearly polarized in the this direction

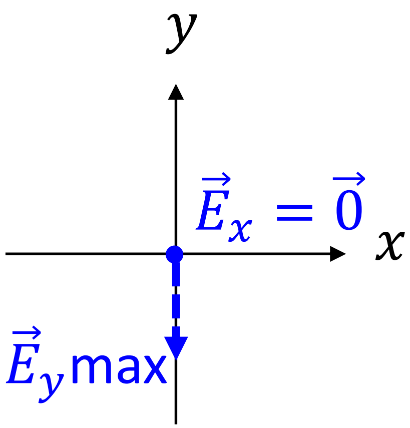
Special case #2

$\delta = \frac{\pi}{2}$ quadrature of phase between $\vec{E}_x$ and $\vec{E}_y$

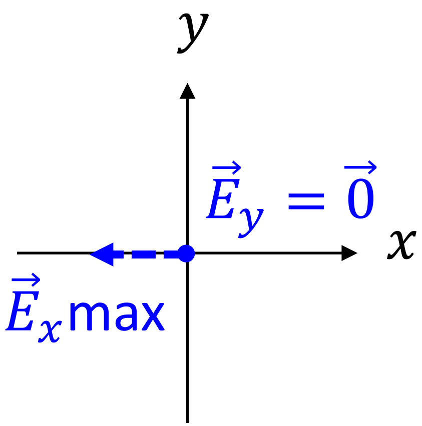
When $\boldsymbol{\vec{E}_x}$ is max, $\boldsymbol{\vec{E}_y}$ is zero and vice versa



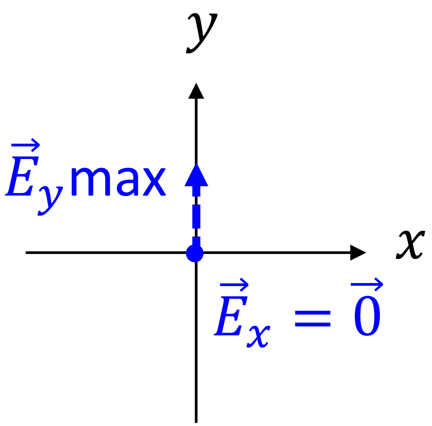
At $\frac{T}{4}$ latter ⇨



At $\frac{T}{4}$ latter ⇨

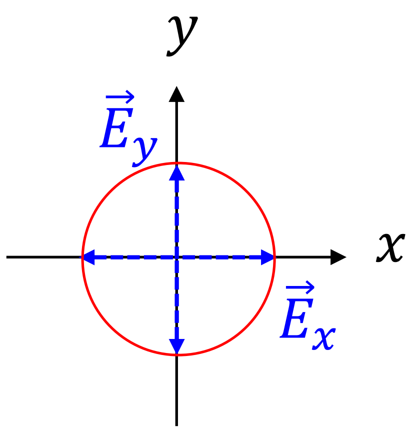


At $\frac{T}{4}$ latter ⇨



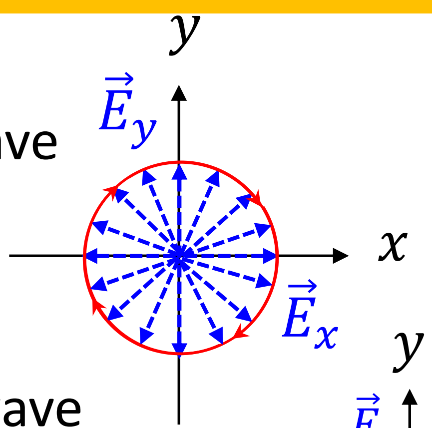
$$\vec{E} = E_{0x}\cos(\omega t - kz)\hat{\imath} + E_{0y}\cos\left(\omega t - kz + \frac{\pi}{2}\right)\hat{\jmath}$$

$$\vec{B} = -B_{0x}\cos\left(\omega t - kz + \frac{\pi}{2}\right)\hat{\imath} + B_{0y}\cos(\omega t - kz)\hat{\jmath}$$

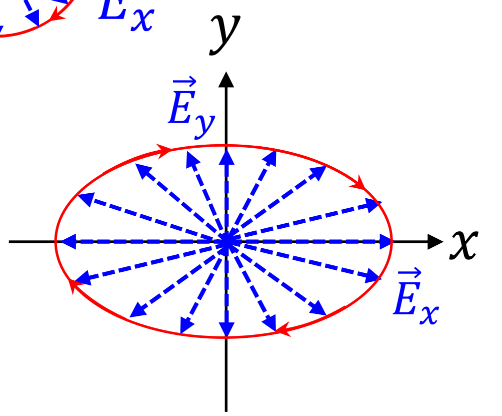


Although $\vec{E}_x$ or $\vec{E}_y$ may be zero at any moment, they are never zero at the same time: **$\vec{E}$ is never zero**

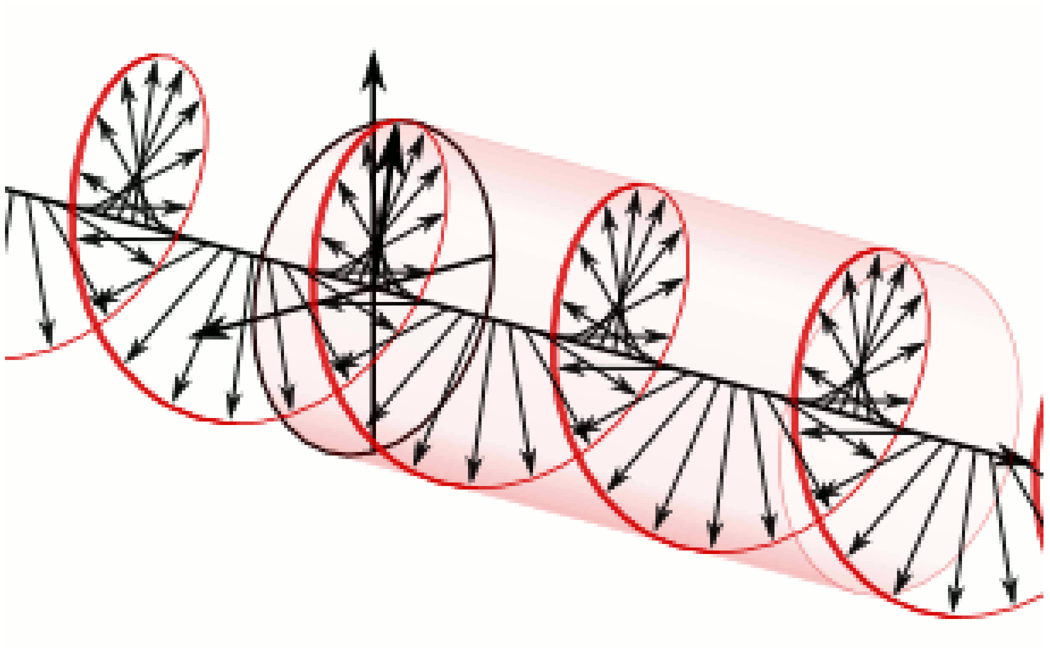
$E_{0x} = E_{0y}$ and $B_{0x} = B_{0y}$ → Clockwise circularly polarized EM wave

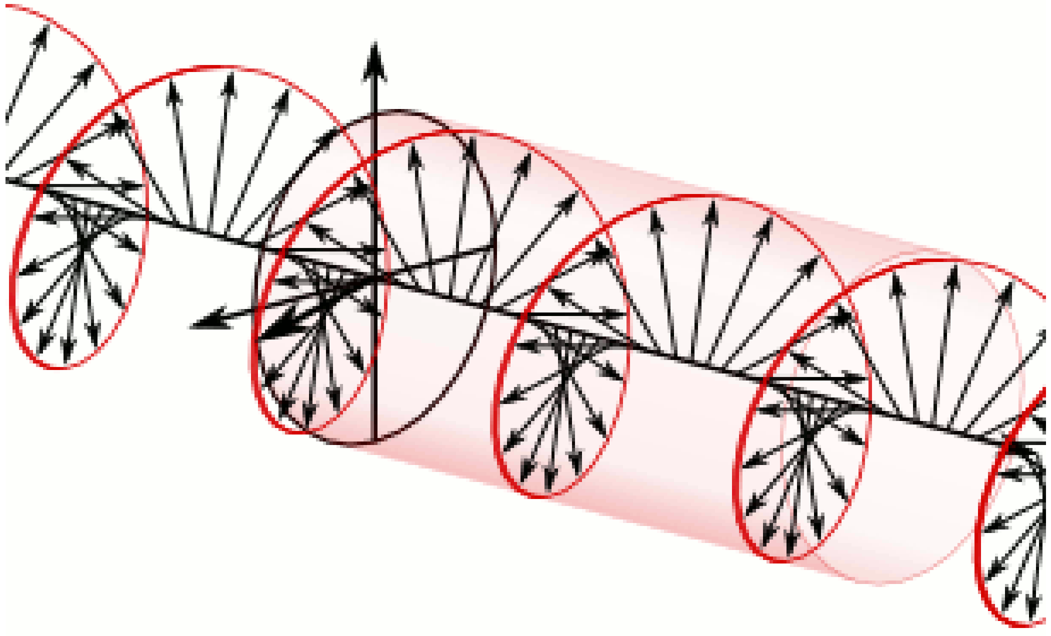


$E_{0x} \neq E_{0y}$ and $B_{0x} \neq B_{0y}$ → Clockwise elliptically polarized EM wave



$\delta = -\frac{\pi}{2}$ → Polarization counterclockwise





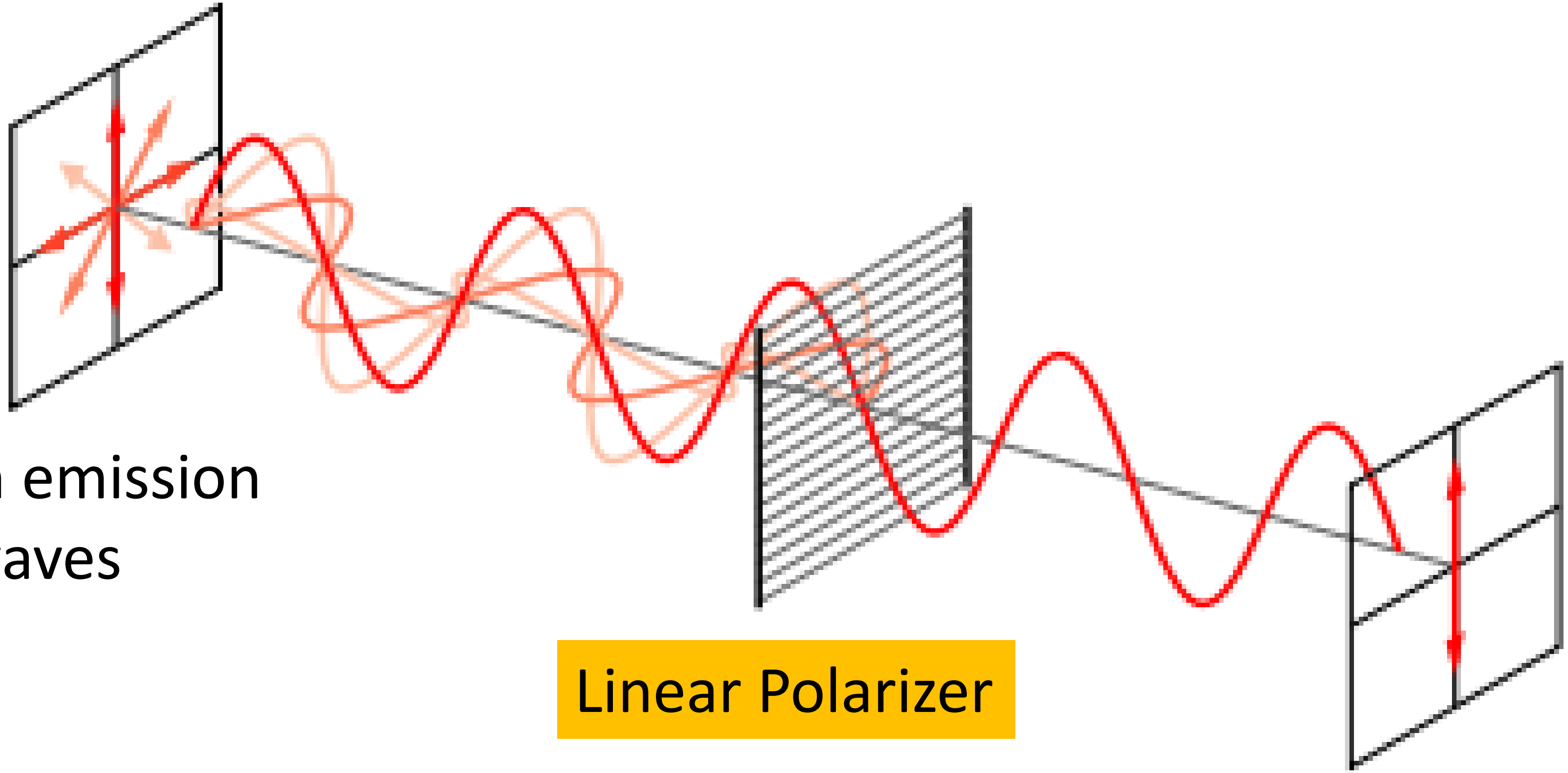
From the source: left-handed /anti-clockwise circularly polarized wave.

From the receiver: right-handed /clockwise circularly polarized wave

From the source: right-handed / clockwise circularly polarized wave.

From the receiver: left-handed / anti-clockwise circularly polarized wave

Linear polarization

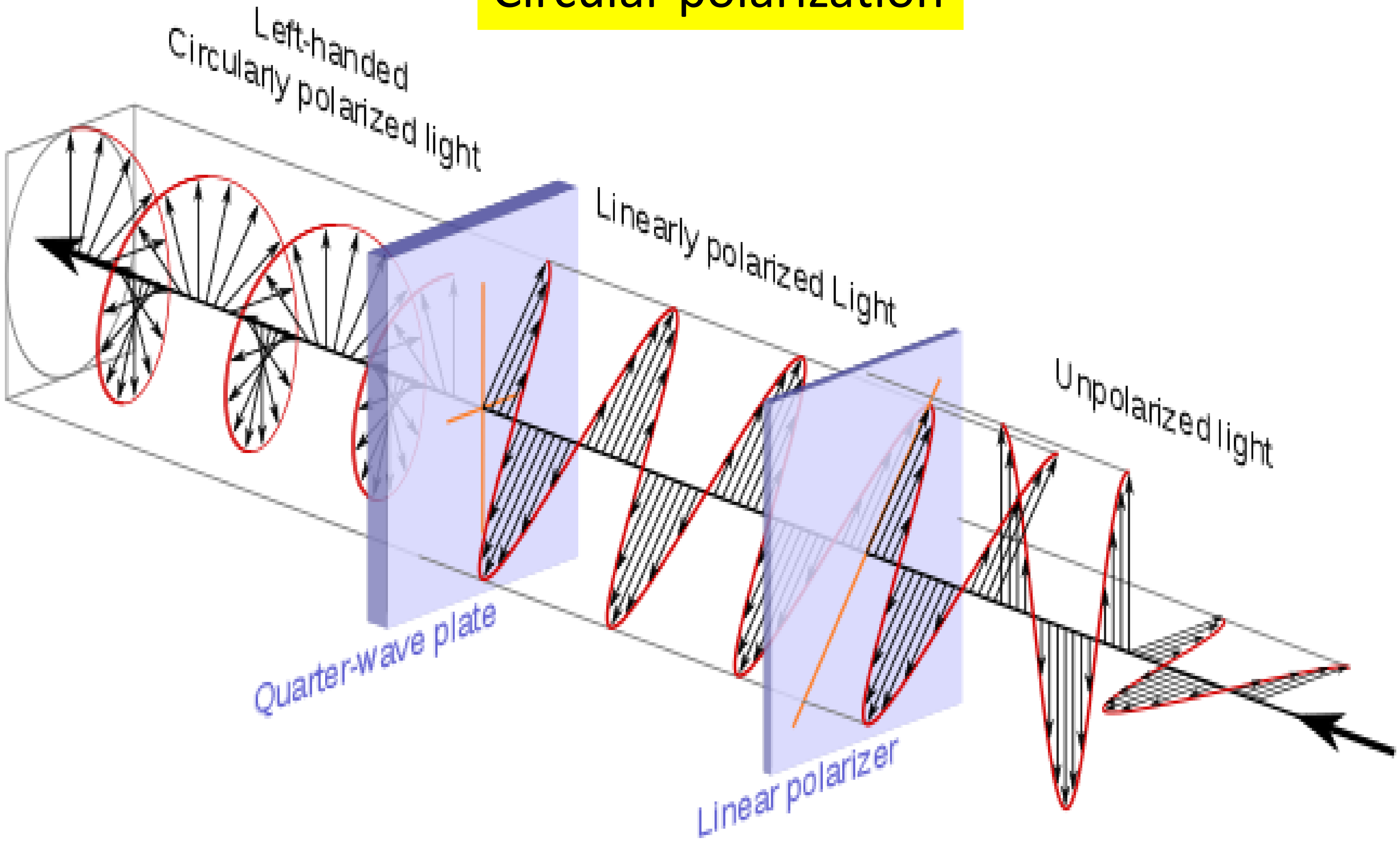


Random emission of EM waves

Linear Polarizer

Linearly polarized of EM waves

Circular polarization



Propagation and incidence of EM waves on conductors and dielectric

Normal incidence of an EM wave: The case vacuum/matter

Vacuum / Conductor

Vacuum / Dielectric

Given an EM wave propagating along a given direction

+

Interpose a medium along the direction of propagation

Boundary condition at the surface

+

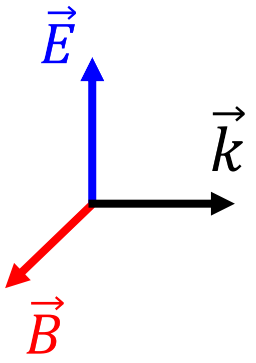
AND

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2}\vec{E}$$

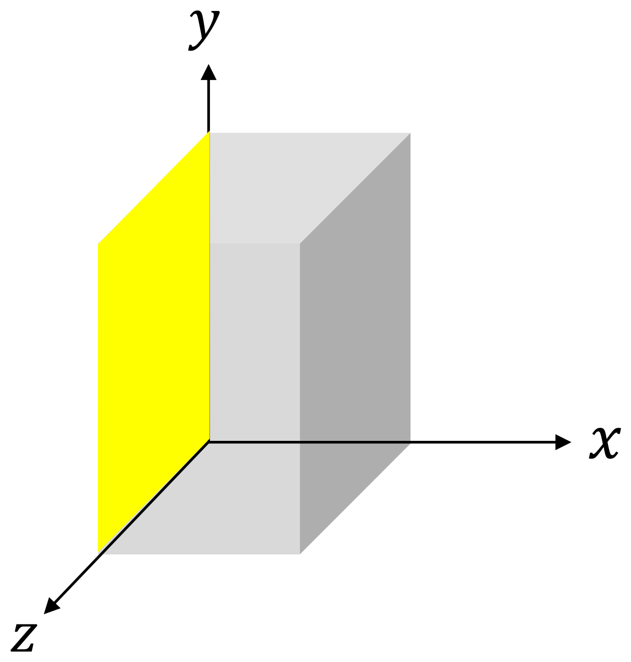
$$\vec{k} \times \vec{E} = \omega\vec{B}$$

Full description of the interaction

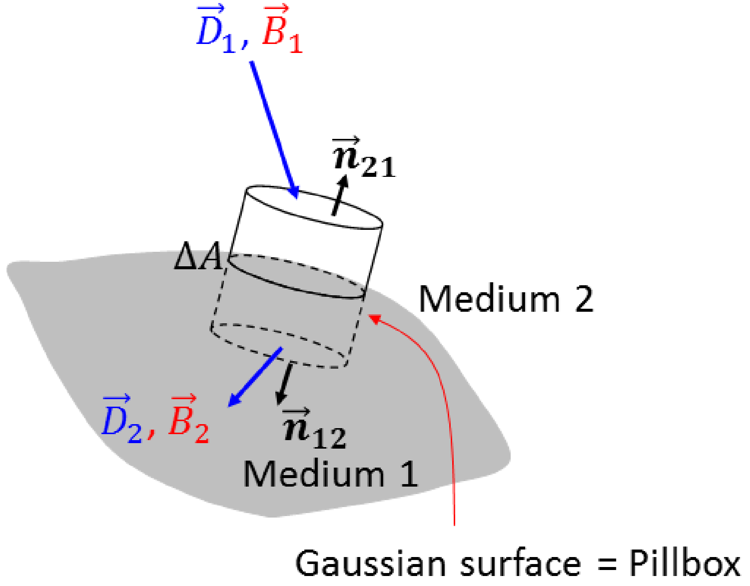
A vertical EM wave propagating along the x-direction



Boundary conditions at the surface

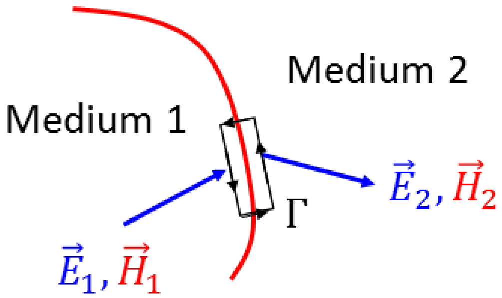


For Normal components we use Gauss theorem



$$D = \epsilon E$$

For Tangential components we use Stokes theorem



$$H = \frac{B}{\mu}$$

Boundary conditions at the surface

No free charges ($\rho_{free} = 0$) and no current $J_{free} = 0$

For electric field $D = \epsilon E$

$Reminder\ \vec{\nabla}.\vec{D} = \rho_{free}$

Gauss theorem $\vec{\nabla}.\vec{D} = 0$ $\quad D_1^{\perp} = D_2^{\perp}$

(deals with the normal components)

Stokes theorem $\oint \vec{E}.d\vec{l}$

(deals with the tangential components)

$$\boxed{\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \qquad E_1^{\parallel} = E_2^{\parallel}}$$

For magnetic field $H = \dfrac{B}{\mu}$

$Reminder\ \oint \frac{\vec{B}}{\mu}.d\vec{l} = J_{free}$

Gauss theorem $\vec{\nabla}.\vec{B} = 0$

Stokes theorem $\quad H_1^{\parallel} = H_2^{\parallel}$

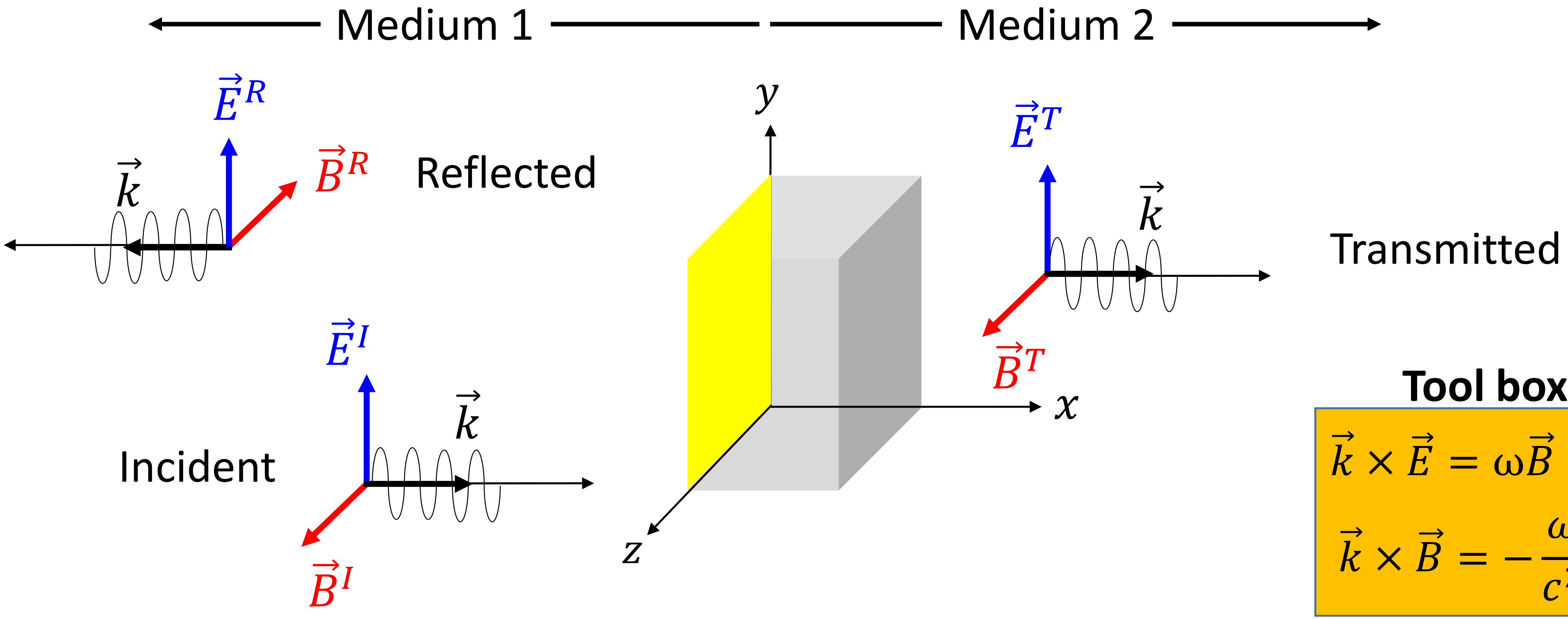
$$\boxed{B_1^{\perp} = B_2^{\perp} \qquad \frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}}$$

Normal incidence of a linear polarized EM wave: Reflection and Transmission

Normal incidence of a linear polarized EM wave: Reflection and Transmission

Do conductor and dielectric behave similarly ?

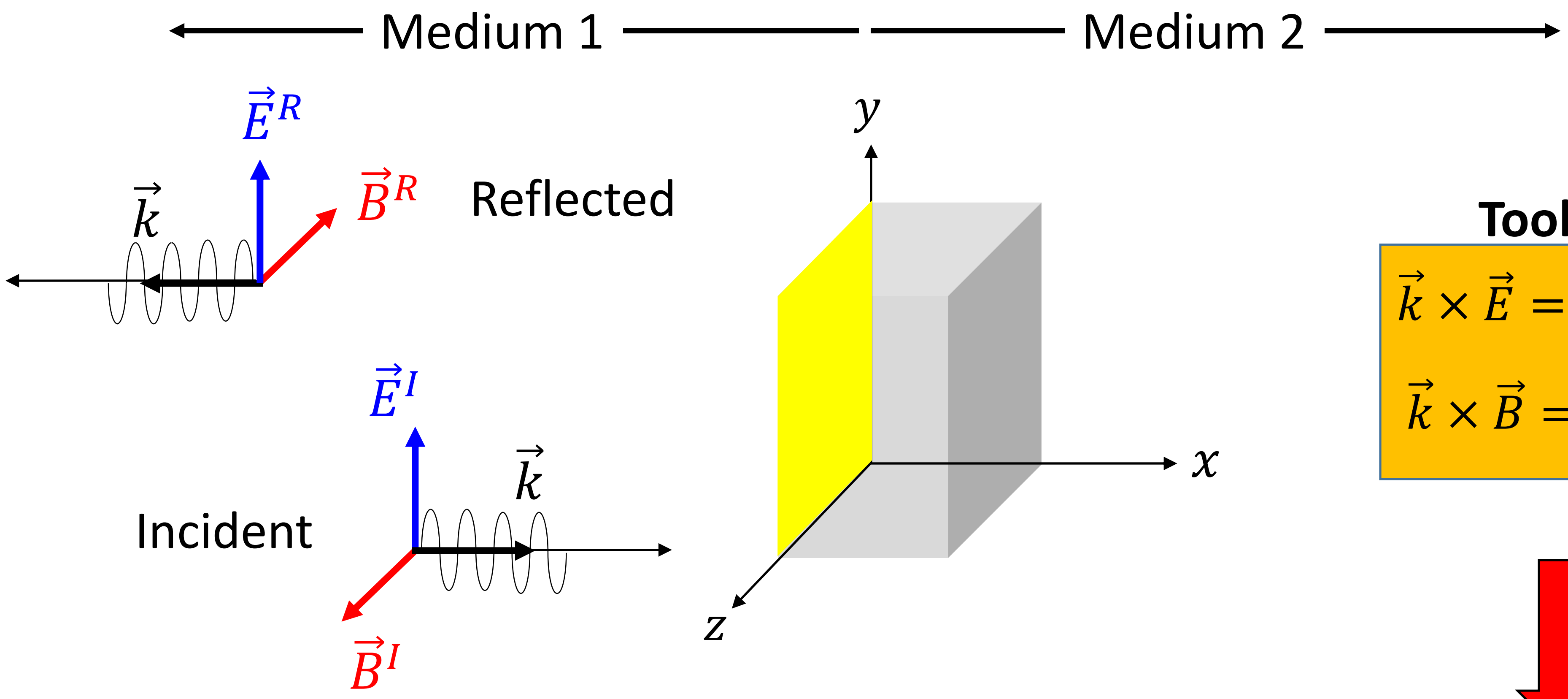
Intuitive vs deductive approaches



Tool box

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2} \vec{E}$$



Tool box

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

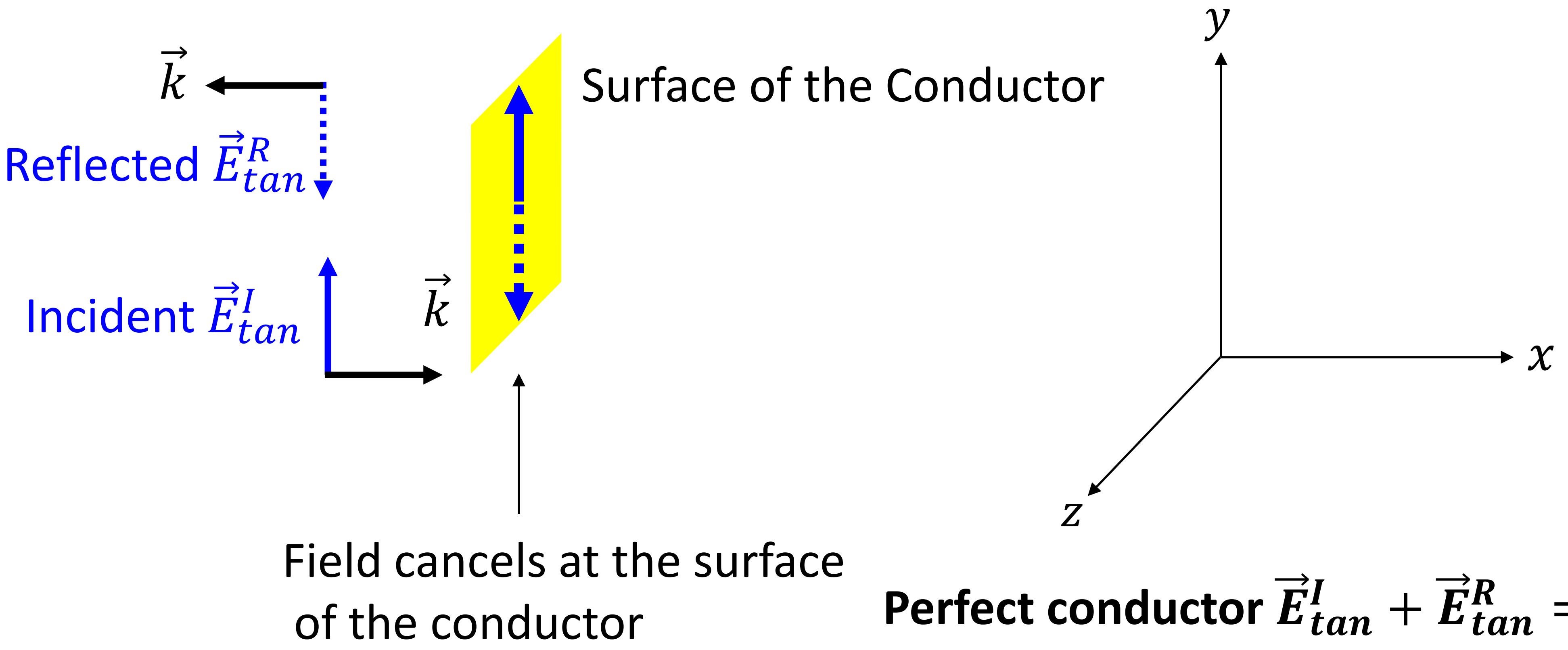
$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2} \vec{E}$$

The electric field has not changed direction

Is it the only possibility ? $\rightarrow$ $\vec{E}^R \,//\, \vec{E}^I$

Intuitive approach

The superposition principle applied to an incident and a reflected wave



Perfect conductor $\vec{E}^I_{tan} + \vec{E}^R_{tan} = \vec{0}$

$$\left|\vec{E}^I_{tan}\right| = \left|\vec{E}^R_{tan}\right|$$

Question: What produces the reflected Electric and magnetic field?

Answer:

The surface currents that must be present to make $\vec{E}$ exactly zero at the surface is the source of the magnetic field.

Deductive approach

Incidence

$$\vec{E}^I(x,t) = E_0^I e^{i(\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{j}}$$

$$\vec{B}^I(x,t) = B_0^I e^{i(\boldsymbol{k_1}x-\omega t)}\widehat{\boldsymbol{k}} = \frac{E_0^I}{\boldsymbol{v_1}} e^{i(\boldsymbol{k_1}x-\omega t)}\widehat{\boldsymbol{k}}$$

Transmission

$$\vec{E}^T(x,t) = E_0^T e^{i(\boldsymbol{k_2}x-\omega t)}\hat{\boldsymbol{j}}$$

$$\vec{B}^T(x,t) = B_0^T e^{i(k_2x-\omega t)}\widehat{\boldsymbol{k}} = \frac{E_0^T}{\boldsymbol{v_2}} e^{i(\boldsymbol{k_2}x-\omega t)}\widehat{\boldsymbol{k}}$$

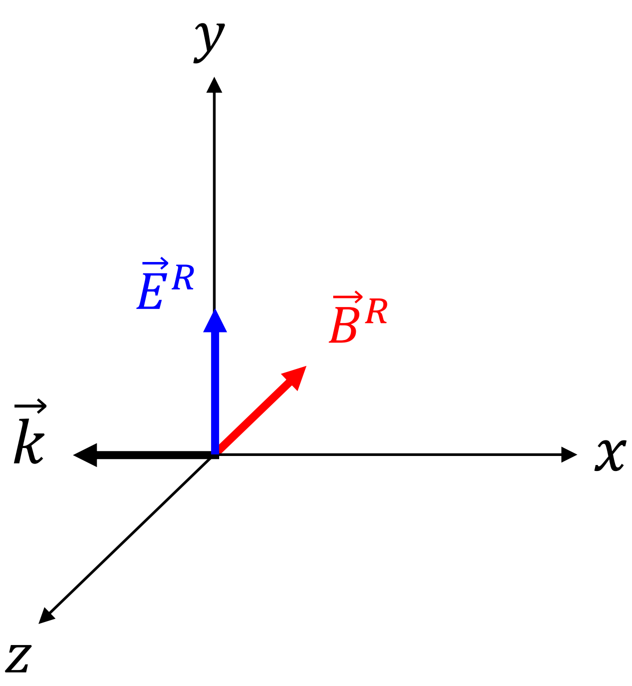
If medium 1 = vacuum $\boldsymbol{v_1 = c}$

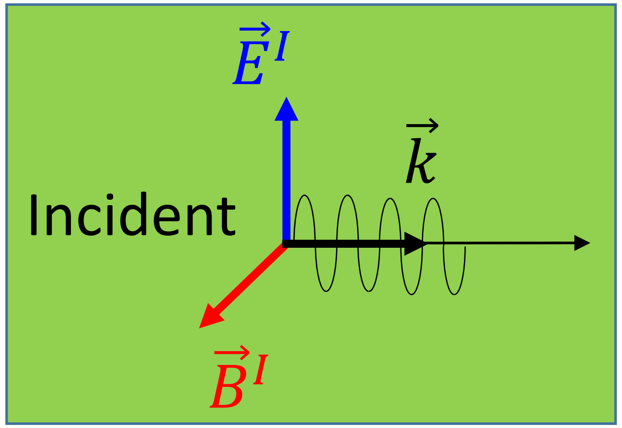
If medium 2 = conductor $\boldsymbol{v_2 = 0}$

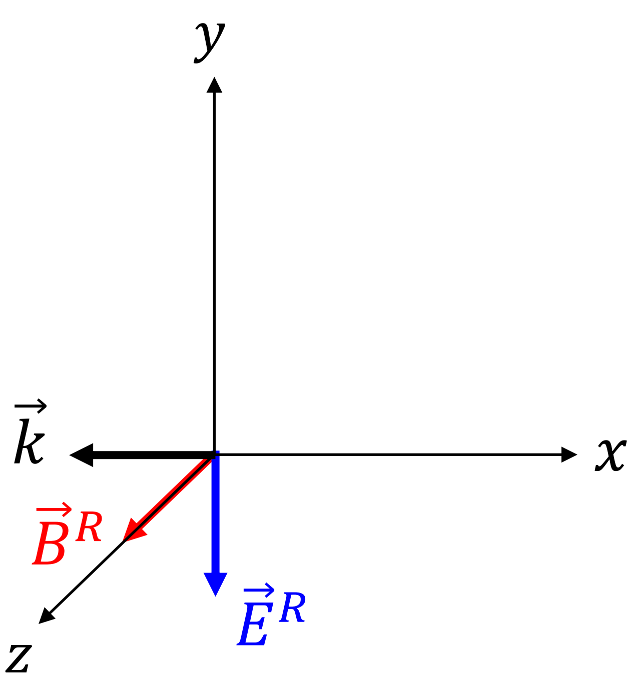
Why ?

Reflection

The wave number vector $\vec{\boldsymbol{k}}$ changes to opposite direction







$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2} \vec{E}$$

$$\vec{E}^R(x,t) = E_0^R e^{i(-\boldsymbol{k_1} x - \omega t)} \hat{\boldsymbol{j}}$$

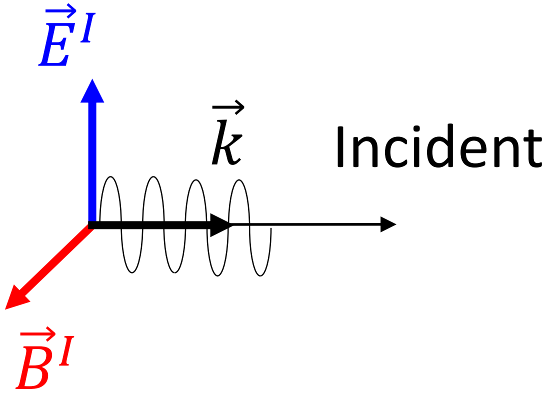
$$\vec{E}^R(x,t) = -E_0^R E_0 e^{i(-\boldsymbol{k_1} x - \omega t)} \hat{\boldsymbol{j}}$$

two possible configurations

$$\vec{B}^R(x,t) = -\frac{E_0^R}{\boldsymbol{v_1}} e^{i(-\boldsymbol{k_1} x - \omega t)} \widehat{\boldsymbol{k}}$$

$$\vec{B}^R(x,t) = \frac{E_0^R}{\boldsymbol{v_1}} e^{i(-\boldsymbol{k_1} x - \omega t)} \widehat{\boldsymbol{k}}$$

Which one of the two reflection configurations holds for a conductor and for a dielectric?



$$\vec{E}^I(x,t) = E_0^I e^{i(\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{j}}$$

$$\vec{B}^I(x,t) = \frac{E_0^I}{\boldsymbol{v_1}} e^{i(\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{k}}$$

Tool box

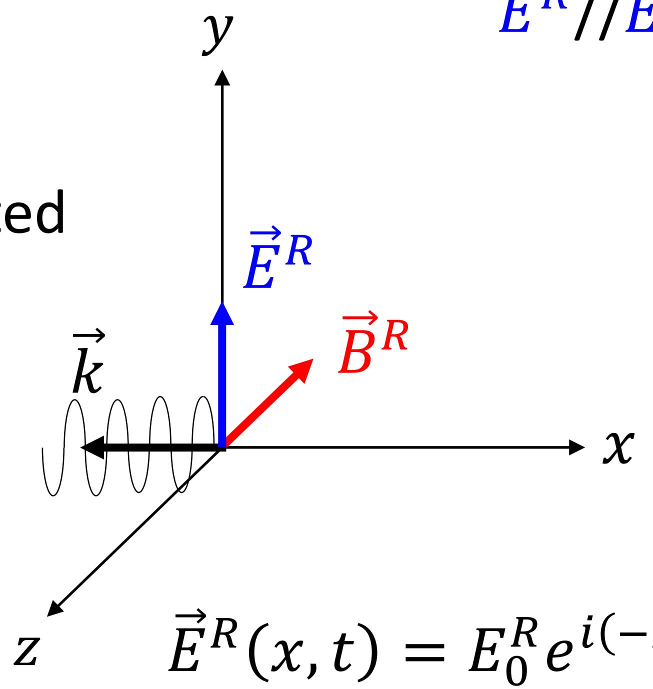
$$\vec{k}\times\vec{E} = \omega\vec{B}$$

$$\vec{k}\times\vec{B} = -\frac{\omega}{c^2}\vec{E}$$

Let's take the first configuration

$\vec{E}^R // \vec{E}^I$

Reflected



$$\vec{E}^R(x,t) = E_0^R e^{i(-\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{j}}$$

$$\vec{B}^R(x,t) = -\frac{E_0^R}{\boldsymbol{v_1}} e^{i(-\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{k}}$$

Which one of the two reflection configurations holds for a conductor and for a dielectric

Boundary conditions

In normal incidence, only tangential components matter

$$\vec{E}^I(\perp) = \vec{E}^R(\perp) = \vec{E}^T(\perp) = \vec{0}$$

$$\vec{B}^I(\perp) = \vec{B}^R(\perp) = \vec{B}^T(\perp) = \vec{0}$$

$$\Rightarrow$$

$$\vec{E}^R(x,t) = E_0^R e^{i(-k_1 x - \omega t)}\hat{\boldsymbol{j}}$$

$$\vec{B}^R(x,t) = -\frac{E_0^R}{\boldsymbol{v_1}} e^{i(-k_1 x - \omega t)}\widehat{\boldsymbol{k}}$$

$$\vec{E}^I + \vec{E}^R = \vec{E}^T \Rightarrow E^I + E^R = E^T$$

$$\vec{H}^I + \vec{H}^R = \vec{H}^T \Rightarrow \frac{1}{\mu_1}\left(\frac{E_0^I}{\boldsymbol{v_1}}\right) - \frac{1}{\mu_1}\left(\frac{E_0^R}{\boldsymbol{v_1}}\right) = \frac{1}{\mu_2}\left(\frac{E_0^T}{\boldsymbol{v_2}}\right)$$

Solving for E^R and E^T

Reflection

$$\vec{E}^R(x,t) = \left(\frac{1-\beta}{1+\beta}\right) E_0^I e^{i(-\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{j}}$$

and

$$\vec{B}^R(x,t) = -\frac{1}{\boldsymbol{v_1}}\left(\frac{1-\beta}{1+\beta}\right) E_0^I e^{i(-\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{k}}$$

$$\vec{B}^R(x,t) = -\left(\frac{1-\beta}{1+\beta}\right) B_0^I e^{i(-\boldsymbol{k_1}x-\omega t)}\hat{\boldsymbol{k}}$$

$$\boxed{\beta = \frac{\mu_1 v_1}{\mu_2 v_2}}$$

Transmission

$$\vec{E}^T(x,t) = \left(\frac{2}{1+\beta}\right) E_0^I e^{i(\boldsymbol{k_2}x-\omega t)}\hat{\boldsymbol{j}}$$

and

$$\vec{B}^T(x,t) = B_0^T e^{i(\boldsymbol{k_2}x-\omega t)}\hat{\boldsymbol{k}} = \frac{E_0^T}{\boldsymbol{v_2}} e^{i(\boldsymbol{k_2}x-\omega t)}\hat{\boldsymbol{k}}$$

$$\vec{B}^T(x,t) = \frac{\boldsymbol{v_1}}{\boldsymbol{v_2}}\left(\frac{2}{1+\beta}\right) B_0^I e^{i(\boldsymbol{k_2}x-\omega t)}\hat{\boldsymbol{k}}$$

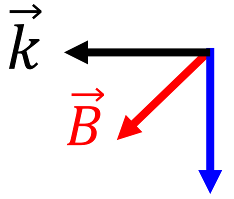
Case of vacuum / conductor

$$\beta = \frac{\mu_0 c}{\mu_2 v_2}$$

$v_2 = 0$ wave does not penetrate the conductor

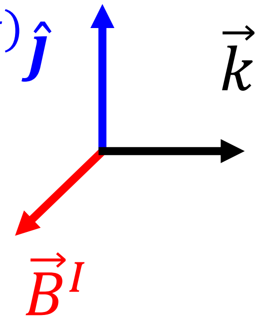
$\beta = \infty$

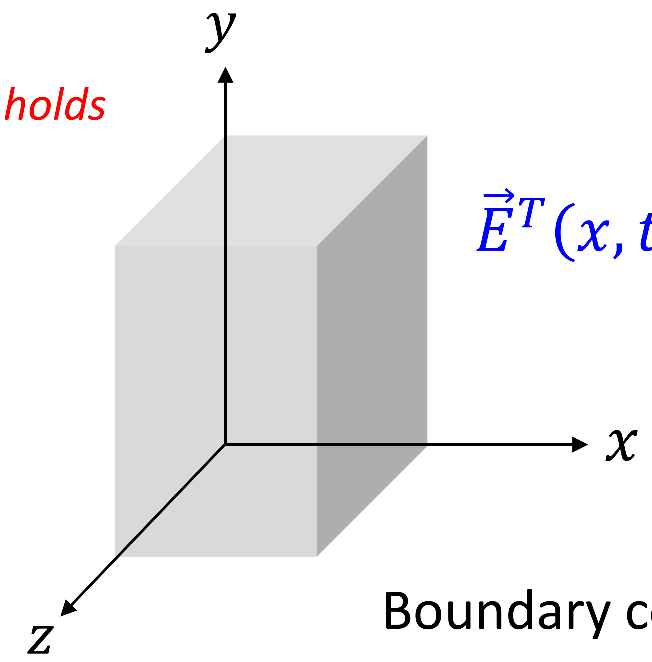
It is the second configuration of slide#85 that holds



$$\vec{E}^R(x,t) = -E_0^I e^{i(-k_1 x - \omega t)}\hat{\boldsymbol{j}}$$

$$\vec{E}^I(x,t) = E_0^I e^{i(k_1 x - \omega t)}\hat{\boldsymbol{j}}$$





$$\vec{E}^T(x,t) = 0\hat{\boldsymbol{j}}$$

Boundary conditions are necessary

The tool box

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2}\vec{E}$$

is not enough

At the surface of the **conductor** the electric field cancels as expected from electrostatic and from the intuitive approach

⇨ **The reflected electric field must be inverted**

What fraction of the incident energy is reflected and what fraction is transmitted

Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ $\Rightarrow$ $I = S_{av} = \frac{1}{2}\varepsilon v E_{max}^2$

$$R = \frac{I_R}{I_I} = \left(\frac{E_0^R}{E_0^I}\right)^2 = \left(\frac{1-\beta}{1+\beta}\right)^2$$

$$T = \frac{I_T}{I_I} = \beta\left(\frac{2}{1+\beta}\right)^2$$

$$R + T = 1$$

Energy conservation

$\beta = \infty$

$\Downarrow$

$T = 0$

$\Downarrow$

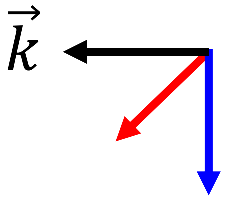
$R = 1$

Metal is a good reflector

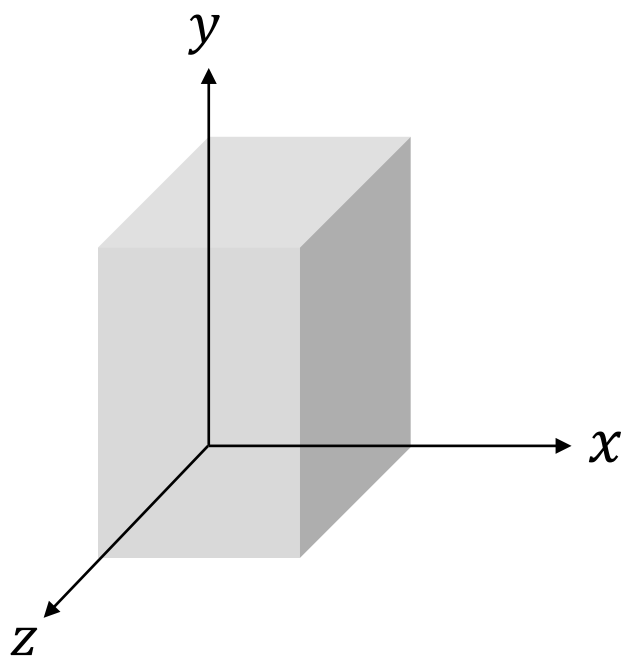
What happens to the magnetic field at the surface of the conductor?

$$\beta = \frac{\mu_0 c}{\mu_2 v_2}$$

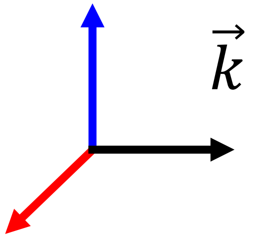
$v_2 = 0$ does not penetrate the conductor $\Rightarrow$ $\beta = \infty$



$$\vec{B}^R(x,t) = B_0^I e^{i(-k_1 x - \omega t)} \hat{\boldsymbol{k}}$$



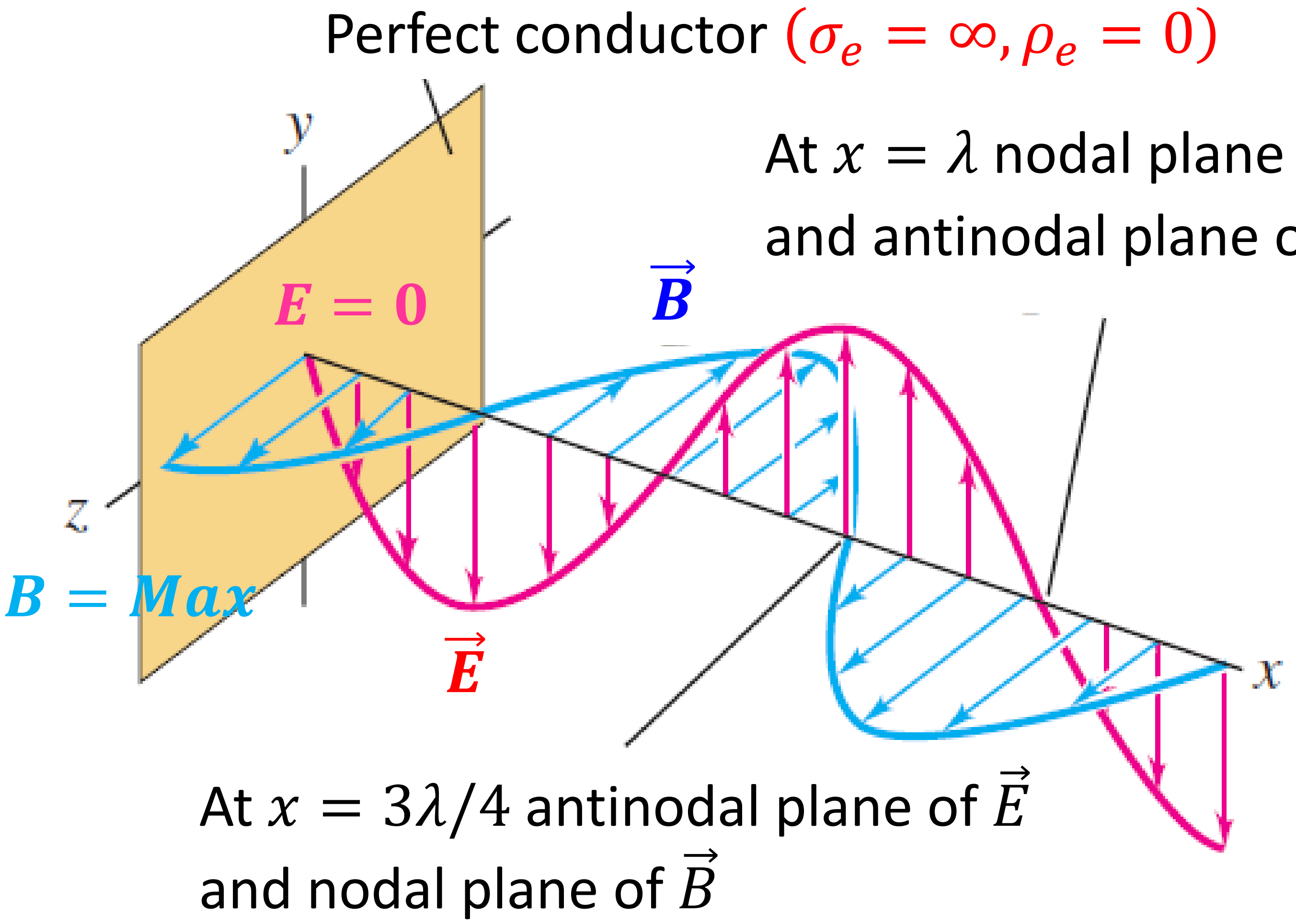
$$\vec{B}^T(x,t) = 0\hat{\boldsymbol{k}}$$



$$\vec{B}^I(x,t) = B_0^I e^{i(k_1 x - \omega t)} \hat{\boldsymbol{k}}$$

At the surface the direction of the magnetic field remains unchanged

The conductor induces a quadrature phase shift



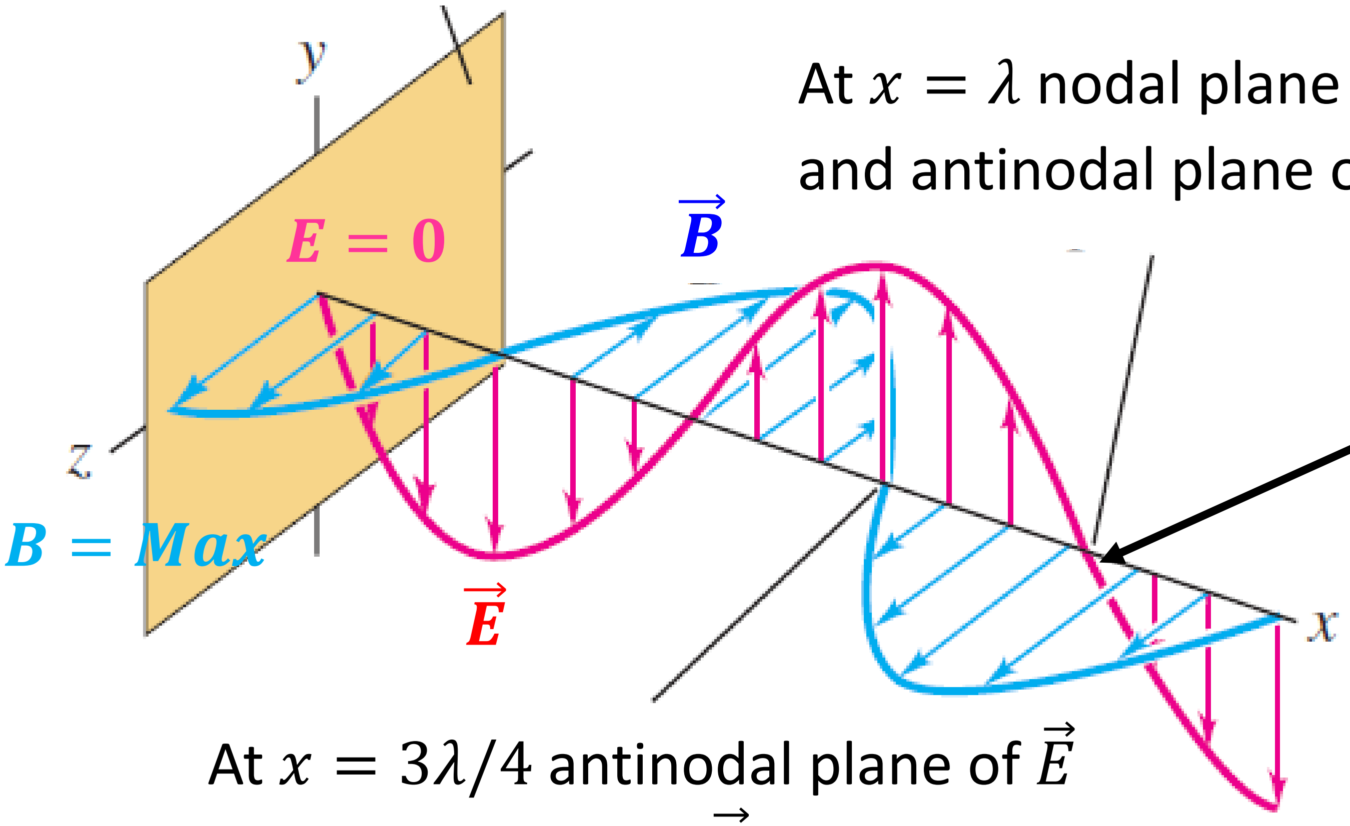
Perfect conductor ⇨ $\vec{E}_{tan} = \vec{0}$

Dephasing $\frac{\pi}{2}$

From University of physics

The conductor induces a quadrature phase shift

Perfect conductor $(\sigma_e = \infty, \rho_e = 0)$



At $x = \lambda$ nodal plane of $\vec{E}$ and antinodal plane of $\vec{B}$

At $x = 3\lambda/4$ antinodal plane of $\vec{E}$ and nodal plane of $\vec{B}$

To obtain **standing waves** the second conductor **MUST** be placed at a **nodal** plane of $\vec{E}$ like this one and parallel to the first conductor

Question: What is the energy contained in a standing wave?

From University of physics

What is the intensity in a standing wave?

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

+

$$E_y(x,t) = +2E_{max} sinkxsin\omega t$$

$$B_z(x,t) = +2B_{max} coskxcos\omega t$$

$$S_x = \frac{E_{max}B_{max} sin2kxsin2\omega t}{\mu_0}$$

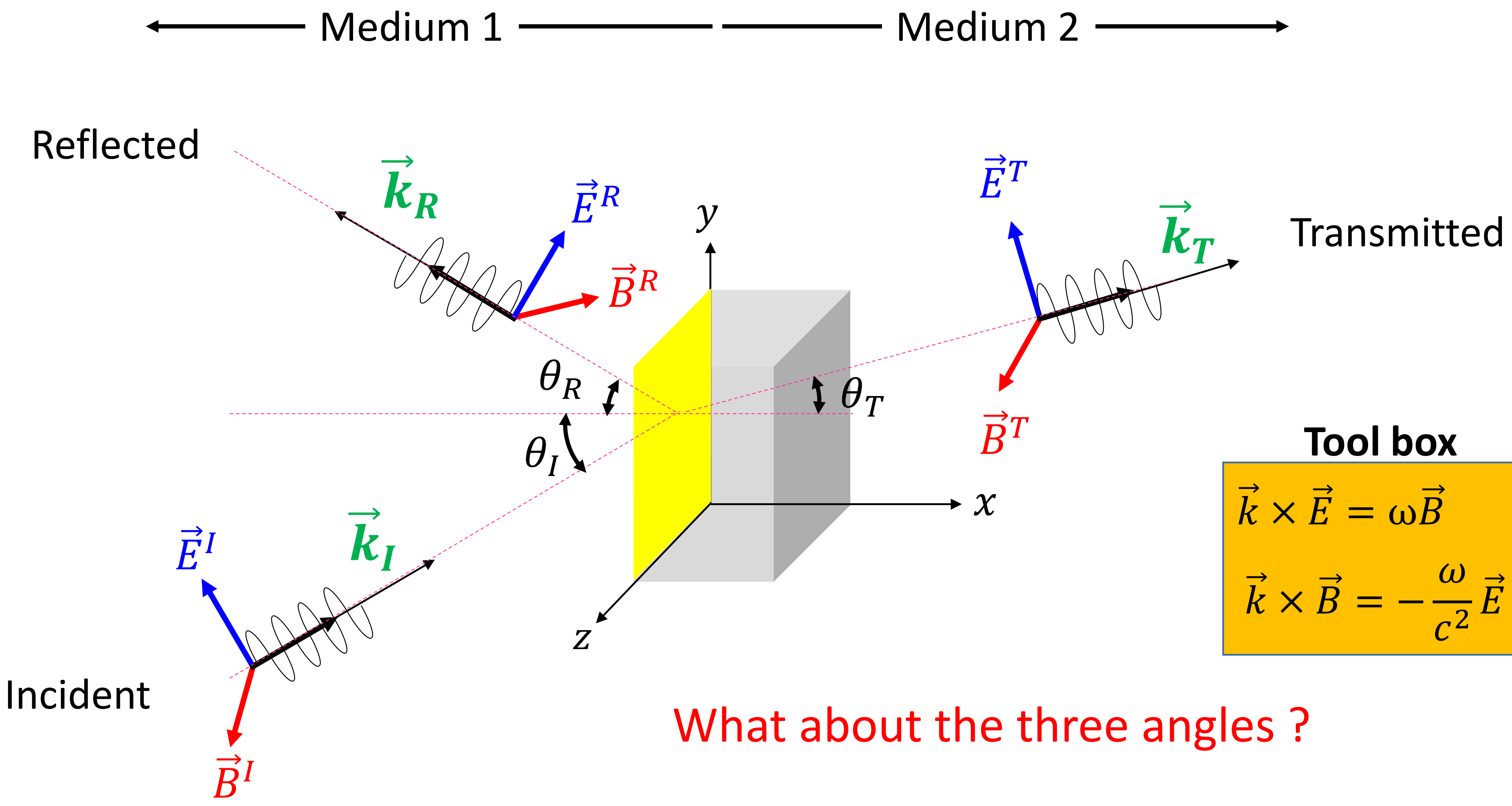
As expected from two equal waves traveling in opposite directions, each transporting energy

$$I = S_{av} = < S_x >_t = 0$$

$$\vec{S}^{\,I} + \vec{S}^{\,R} = \vec{0}$$

While using waves to transmit power, it is important to avoid reflections that give rise to standing waves

<u>Oblique incidence</u> of a linear polarized EM wave: Reflection and Transmission



Tool box

$$\vec{k} \times \vec{E} = \omega \vec{B}$$

$$\vec{k} \times \vec{B} = -\frac{\omega}{c^2} \vec{E}$$

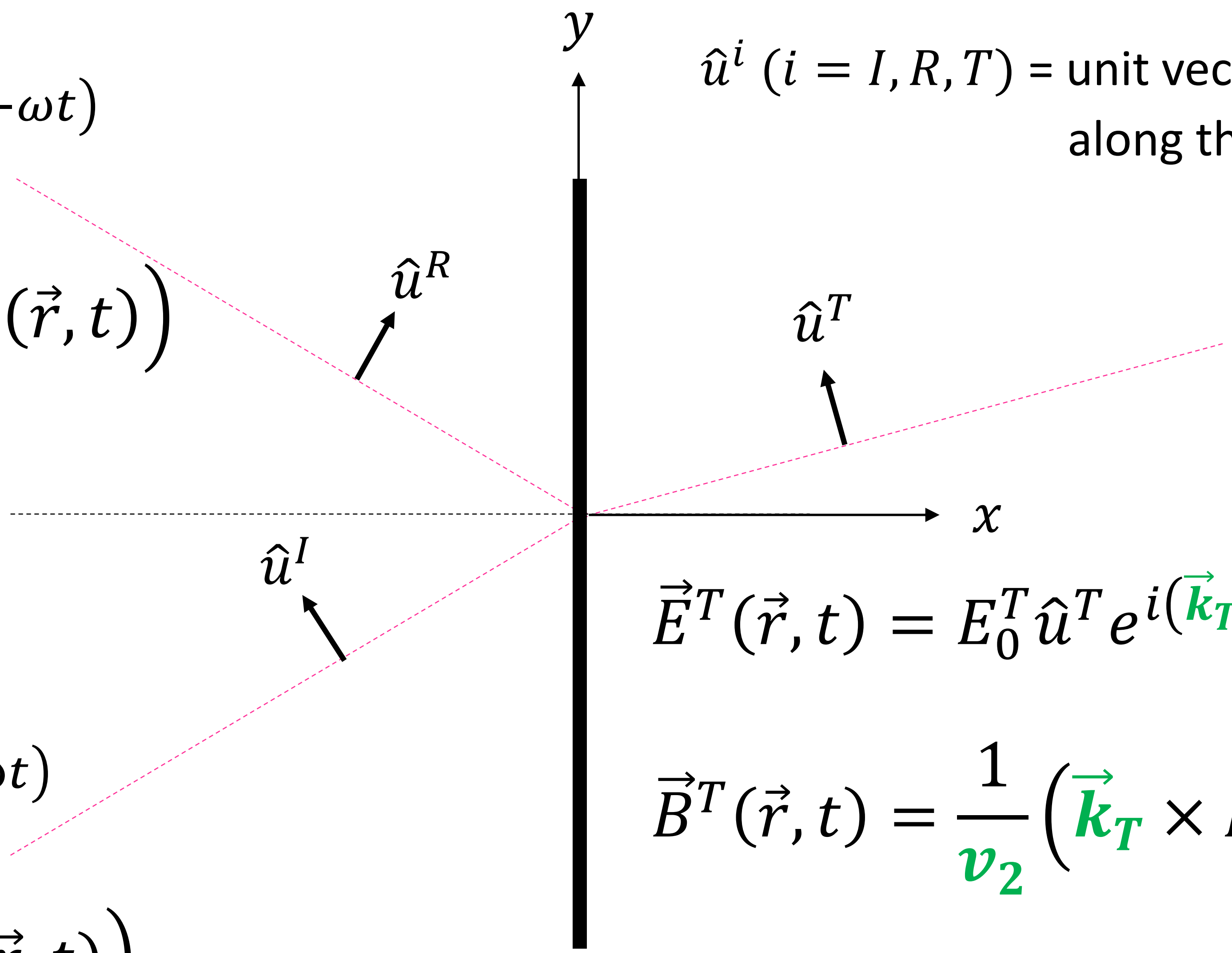
What about the three angles ?

← Medium 1 — — Medium 2 →

$$\vec{E}^R(\vec{r},t) = E_0^R \hat{u}^R e^{i(\vec{k}_R.\vec{r}-\omega t)}$$

$$\vec{B}^R(\vec{r},t) = \frac{1}{v_1}\left(\vec{k}_R \times \vec{E}^R(\vec{r},t)\right)$$

$\hat{u}^i$ $(i = I, R, T)$ = unit vectors along the $\vec{E}$ field



$$\vec{E}^T(\vec{r},t) = E_0^T \hat{u}^T e^{i(\vec{k}_T.\vec{r}-\omega t)}$$

$$\vec{B}^T(\vec{r},t) = \frac{1}{v_2}\left(\vec{k}_T \times \vec{E}^T(\vec{r},t)\right)$$

$$\vec{E}^I(\vec{r},t) = E_0^I \hat{u}^I e^{i(\vec{k}_I.\vec{r}-\omega t)}$$

$$\vec{B}^I(\vec{r},t) = \frac{1}{v_1}\left(\vec{k}_I \times \vec{E}^I(\vec{r},t)\right)$$

Monochromatic wave ➡ $\boldsymbol{\omega = kv}$ is the same for all three waves ➡

$$k_I . v_1 = k_R . v_1 = k_T . v_2$$

➡

$$k_I = k_R = \frac{v_2}{v_1} k_T = \frac{\boldsymbol{n_1}}{\boldsymbol{n_2}}$$

$\boldsymbol{n_i}$ = index of refraction of medium $i = \frac{c}{v_i}$

Boundary conditions at the plane of separation

$$\vec{E}^I(\vec{r},t) + \vec{E}^R(\vec{r},t) \quad \Big| \quad \vec{E}^T(\vec{r},t)$$

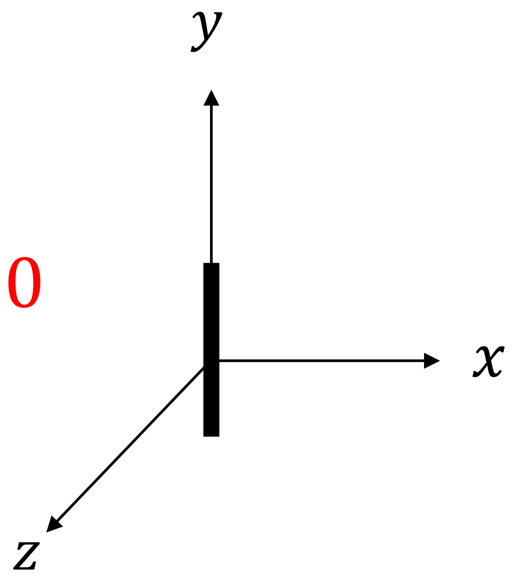
$$\vec{B}^I(\vec{r},t) + \vec{B}^R(\vec{r},t) \quad \Big| \quad \vec{B}^T(\vec{r},t)$$

$$E_0^I e^{i(\vec{k_I}.\vec{r}-\omega t)} + E_0^R e^{i(\vec{k_R}.\vec{r}-\omega t)} = E_0^T e^{i(\vec{k_T}.\vec{r}-\omega t)}$$

Boundary

$$\vec{k_I}.\vec{r} = \vec{k_R}.\vec{r} = \vec{k_T}.\vec{r}$$

At $x = 0$



$$y.k_{Iy} + z.k_{Iz} = y.k_{Ry} + z.k_{Rz} = y.k_{Ty} + z.k_{Tz}$$

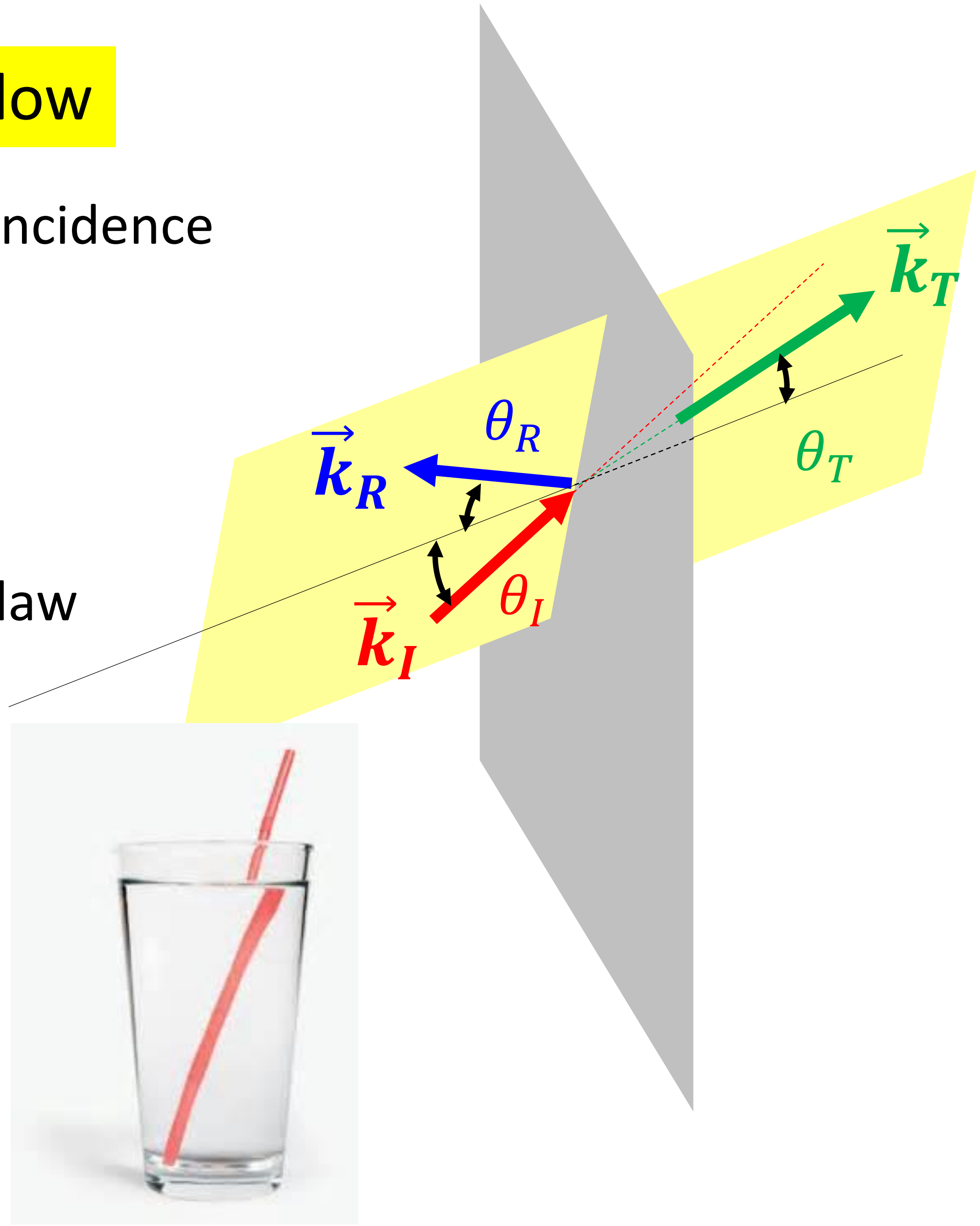
$$\boxed{\begin{array}{c} k_{Iy} = k_{Ry} = k_{Ty} \\ k_{Iz} = k_{Rz} = k_{Tz} \end{array}}$$

Three laws follow

1) $\vec{k}_I, \vec{k}_R$ ***and*** $\vec{k}_T$ form a single plane: plane of incidence

2) $\theta_I = \theta_R$ Law of reflection or Fermat's law

3) $\dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{\boldsymbol{n_1}}{\boldsymbol{n_2}}$ Law of refraction



$$\vec{k}_I.\vec{r} = \vec{k}_R.\vec{r} = \vec{k}_T.\vec{r} \Rightarrow \left.\begin{aligned} \vec{E}^I(\vec{r},t) &= E_0^I e^{i(\vec{k}_I.\vec{r}-\omega t)} \\ \vec{E}^R(\vec{r},t) &= E_0^R e^{i(\vec{k}_R.\vec{r}-\omega t)} \\ \vec{E}^T(\vec{r},t) &= E_0^T e^{i(\vec{k}_T.\vec{r}-\omega t)} \end{aligned}\right\}$$

Exponent factors are all equal

Boundary conditions (slide #79)

$$\left.\begin{aligned} \varepsilon_1(E_0^I + E_0^R)_x &= \varepsilon_2(E_0^T)_x \\ (B_0^I + B_0^R)_x &= (B_0^T)_x \end{aligned}\right\}$$

Normal components at the interface

$$\left.\begin{aligned} (E_0^I + E_0^R)_{y,z} &= (E_0^T)_{y,z} \\ \frac{1}{\mu_1}(B_0^I + B_0^R)_{y,z} &= \frac{1}{\mu_2}(B_0^T)_{y,z} \end{aligned}\right\}$$

Tangential components at the interface

Reflection

$$\vec{E}^R(x,t) = \left(\frac{\alpha-\beta}{\alpha+\beta}\right) E_0^I e^{i(-k_1 x-\omega t)}\hat{u}^R \quad \text{and} \quad \vec{B}^R(x,t) = -\frac{1}{v_1}\left(\frac{1-\beta}{1+\beta}\right) E_0^I e^{i(-k_1 x-\omega t)}\left(\frac{\vec{k}_R}{|\vec{k}_R|}\times\hat{u}^R\right)$$

$$\vec{B}^R(x,t) = -\left(\frac{\alpha-\beta}{\alpha+\beta}\right) B_0^I e^{i(-k_1 x-\omega t)}\left(\frac{\vec{k}_R}{|\vec{k}_R|}\times\hat{u}^R\right)$$

$$\alpha = \frac{cos\theta_T}{cos\theta_I}$$

Normal incidence

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$$

$$\alpha = 1$$

Transmission

$$\vec{E}^T(x,t) = \left(\frac{2}{\alpha+\beta}\right) E_0^I e^{i(k_2 x-\omega t)}\hat{u}^T \quad \text{and} \quad \vec{B}^T(x,t) = B_0^T e^{i(k_2 x-\omega t)}\hat{\boldsymbol{k}} = \frac{E_0^T}{v_2} e^{i(k_2 x-\omega t)}\left(\frac{\vec{k}_T}{|\vec{k}_T|}\times\hat{u}^T\right)$$

$$\vec{B}^T(x,t) = \frac{v_1}{v_2}\left(\frac{2}{\alpha+\beta}\right) B_0^I e^{i(k_2 x-\omega t)}\left(\frac{\vec{k}_T}{|\vec{k}_T|}\times\hat{u}^T\right)$$