Establishing Maxwell's equations without a single mathematical manipulation: Getting a flavor of the main concepts

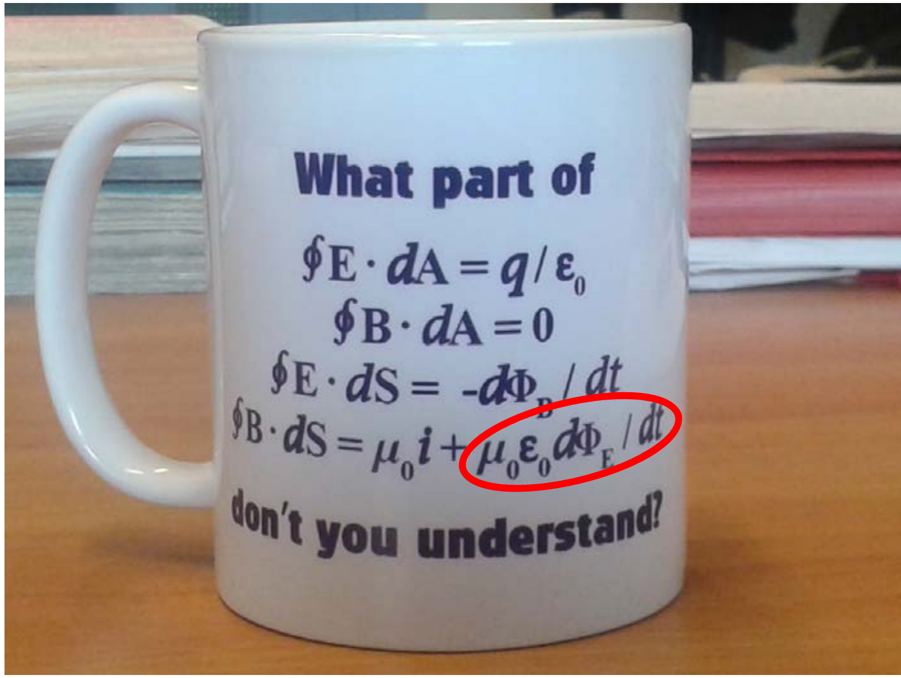
<u>A physical understanding is a</u>:

- Completely unmathematical
- Imprecise
- Inexact

… theory…

… but absolutely necessary for physicist

… R. Feynman



Electricity is all around

- Phone
- Microphone
- Electric clock
- Calculator
- TV
- Video
- Computer
- Light (EM wave)
- Stars
- Car/train/plane
- Nerves
- Cells
- Vision
- Heart beats owing to electricity
- Thinking requires electricity

Since the Greeks (600 BC) we know that amber (electrons) attract dry leaves

Benjamin Franklin (1750): concept of electrical fluid

Two types of electricity

A = Glass
B = Amber

A – A repel
B – B repel
A – B attract

All substances are penetrated by electric fluid

**Question #4 What is the working principle of a touch screen in smartphones?

Answer to **Question #4 Faraday's discovery: see Lecture on capacitor

Concepts reviewed in the next two lectures

- Charge
- Charge conservation
- Force
- Superposition principle
- Charge induction
- Polarization
- Field vector
- Filed line
- Work and Energy
- Flux of field vector
- Circulation of field vector

Concept #1: Charge

From A/B to $+/-$

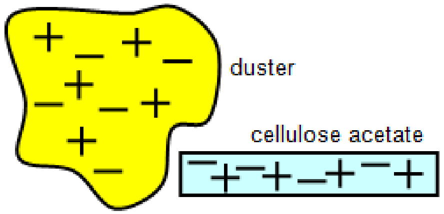
J.J. Thomson

- Discovery of electron 1897
- Determination of its charge 1899

$$|q| = |e| = 1.6 \times 10^{-19} C$$

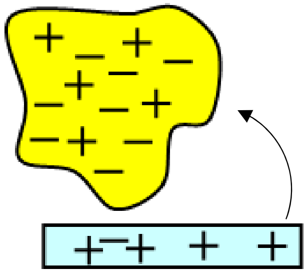
- Charge is quantized

Before rubbing



Both are neutral

After rubbing



Transfer of negative
Charge after friction

Both are charged

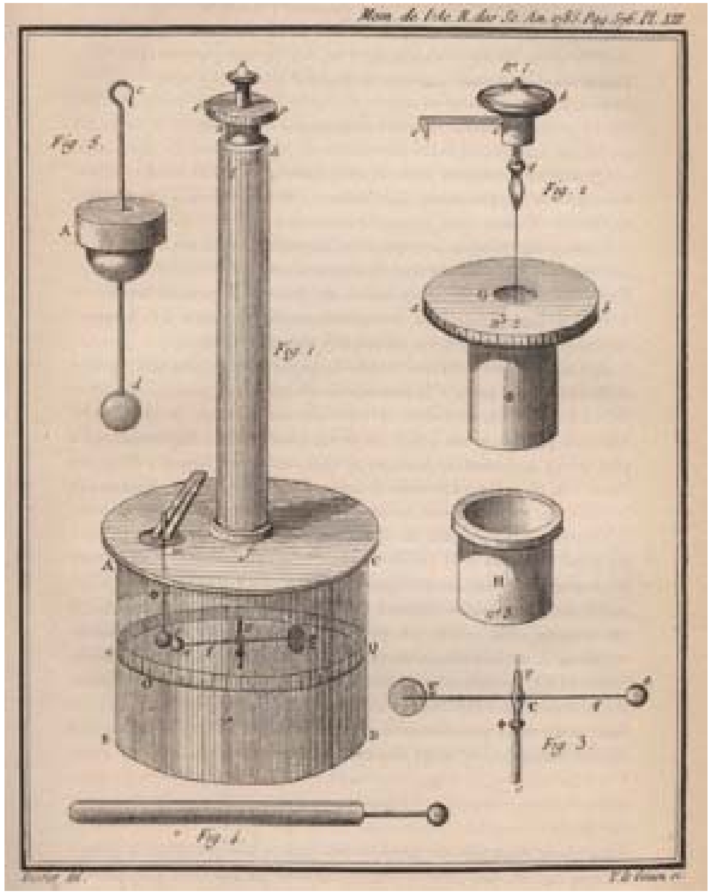
Concept #2: Charge conservation

Charge is neither created nor destroyed:
it is transferred

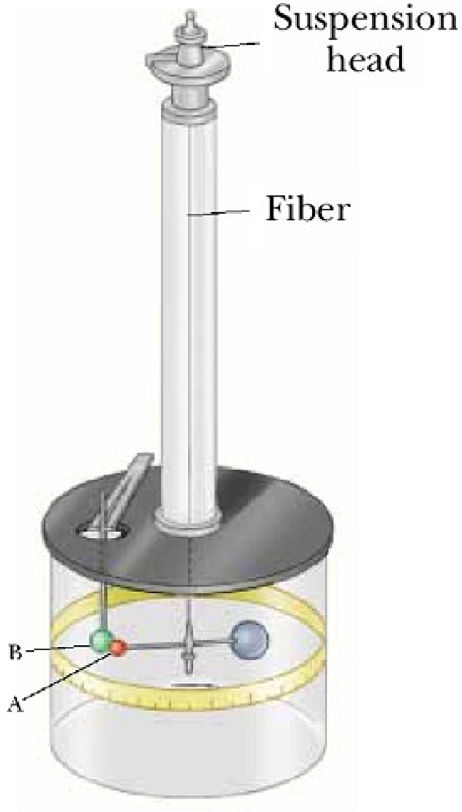
$$\sum_{univers} charges = 0$$

Setup by which Coulomb established his law in 1785

Original version



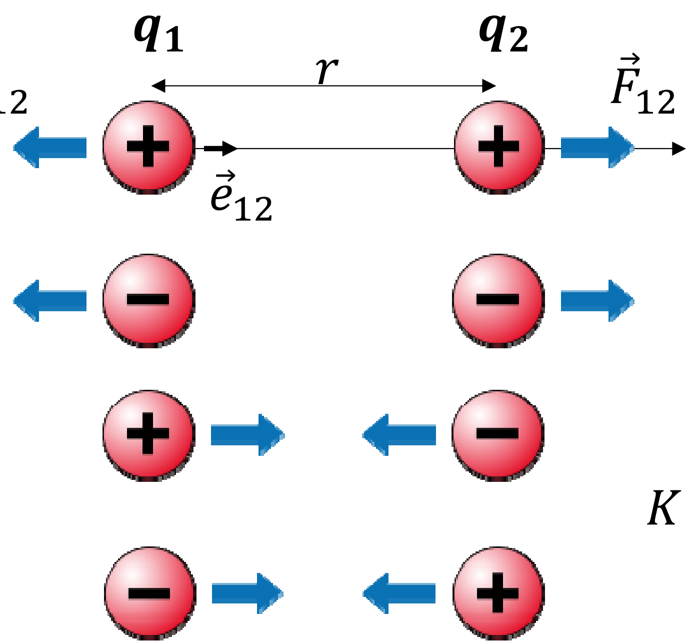
Modern version



Concept #3: Electric force

Coulomb law

$\vec{F}_{21} = -\vec{F}_{12}$



Newtonian's 3rd law

Action – reaction ⇨ $\vec{F}_{21} = -\vec{F}_{12}$

$$\boxed{\vec{F}_{12} = K\frac{q_1 q_2}{r^2}\vec{e}_{12}}$$

Miracle of the nature

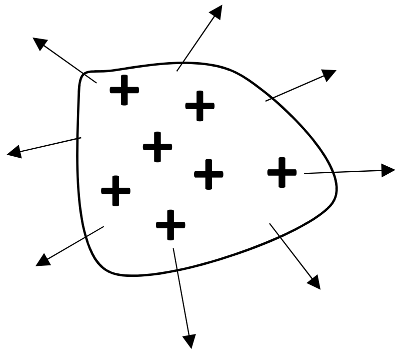
$$K = \frac{1}{4\pi\varepsilon_0} = 9.0\times 10^9 Nm^2/C^2$$

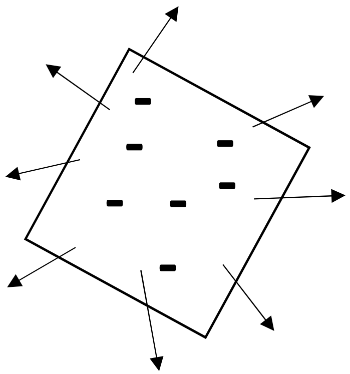
Gravitational **attraction** $\boxed{\vec{F}_{12} = G\frac{Mm}{r^2}\vec{e}_{12}}$ $G = 6.67\times 10^{-11} kg^{-1}m^3s^{-2}$

Does a charge act on itself?

If so, in normal classical electromagnetism this would lead to an infinite self-force

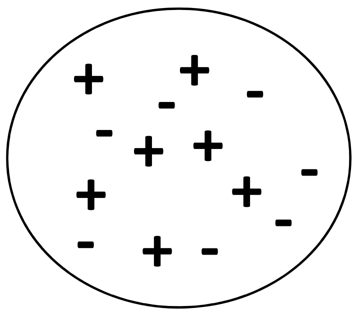
Charges – Electrical and Magnetic forces $\Longrightarrow$ **Motion**





$$\vec{F}_E = 10^{36}\vec{F}_G$$

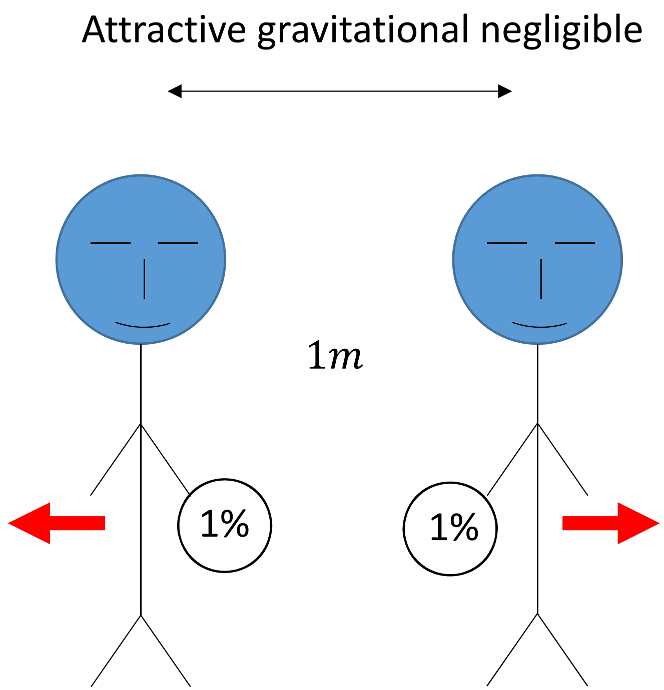
Like charges will fly a part with a terrific force !



$\equiv$

Atom (forces balance)

Stable matter



$n_e - n_p = 1\%$

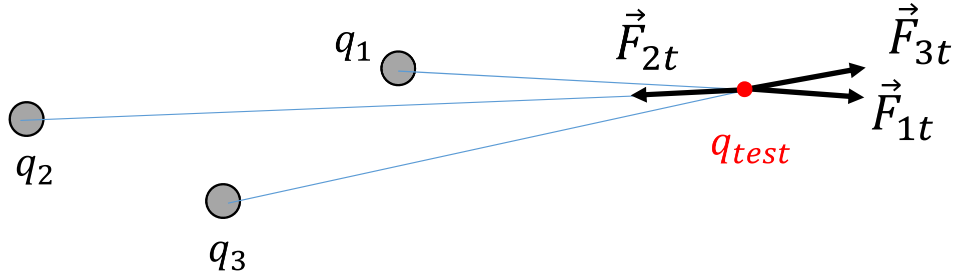
**Question #5

How big would be the repelling force ?

Answer to **Question #5

Coulomb force = Enough to lift the entire earth

Concept #4: Principle of superposition



$$\vec{F}_{tot} = \sum_i \vec{F}_{it}$$

Is the superposition principle intuitive?	Not at all
Is it pertinent?	Yes it is
Is it out of doubt?	It remains consistent with all experiments carried to date

Superposition principle applies to all types of forces: Gravitational – Electric – Magnetic - Nuclear

And beyond….

For linear differential equations the same principle applies

$$Equation\ 1 \rightarrow \vec{F}_1$$
$$Equation\ 2 \rightarrow \vec{F}_2$$
$$Equation\ 3 \rightarrow \vec{F}_3$$

$$\boxed{Equation \rightarrow \vec{F} = \sum individual\ equations}$$

"Tuning" a radio station would not be possible without the principle of superposition

About the notion of charge test

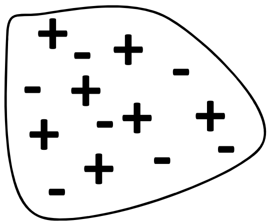
**Question #6

What property should have a test charge to be able to make good predictions?

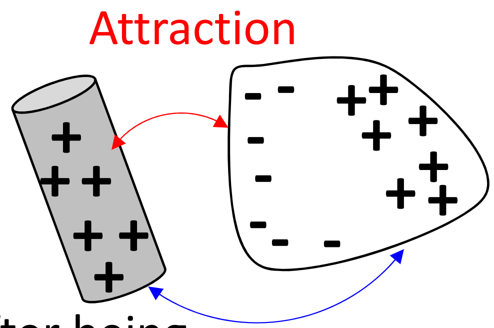
Answer to **Question #6

Have the weakest possible charge in order not to disturb the fields
created by the source to be probed

Concept #5_1: Charge induction: Non contact action or action at distance



Neutral conductor
Charges free to move



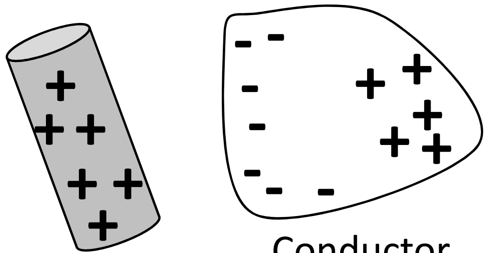
Glass rod after being rubbed with silk

Repulsion

Still Neutral conductor
Charge free to move

Process of induction in conductors ⇔ Induces <u>long</u> distance separation of charges

**Question #7



Positively charged rod

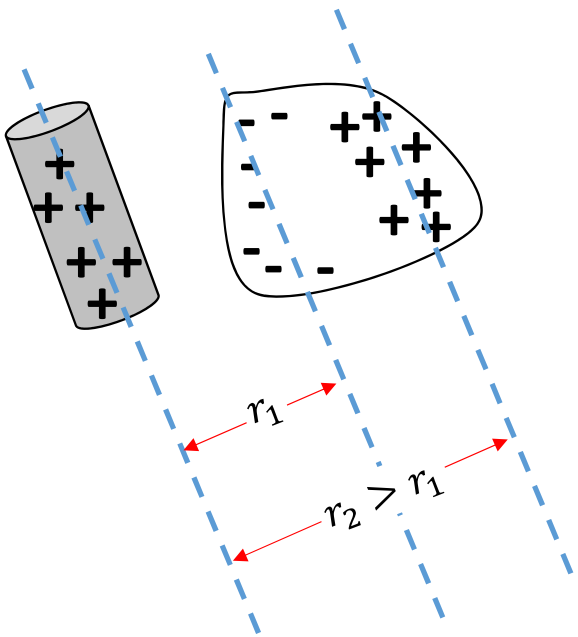
Conductor free to move

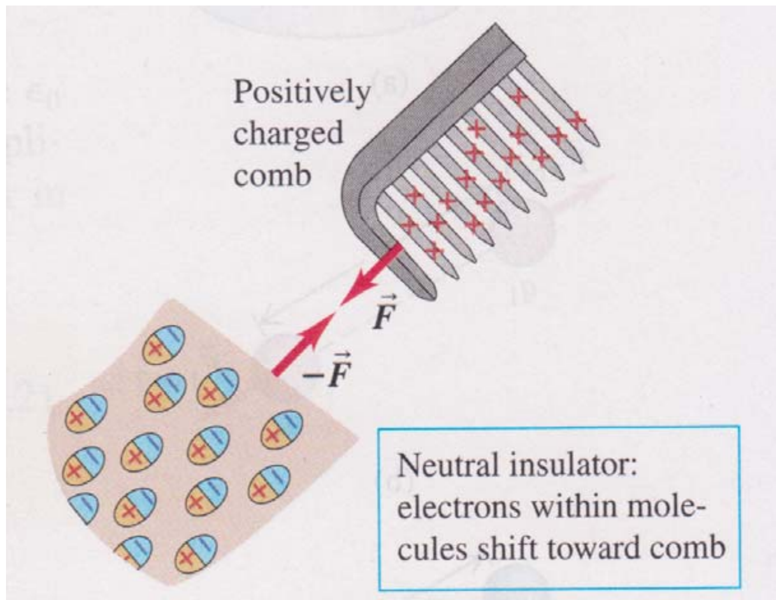
If the whole conductor is free to move
Does it move? why? and in which direction?

Answer to **Question #7:

- Yes because of (induction) polarization
- Net attraction > Net repulsion
- Towards the positively charged rod

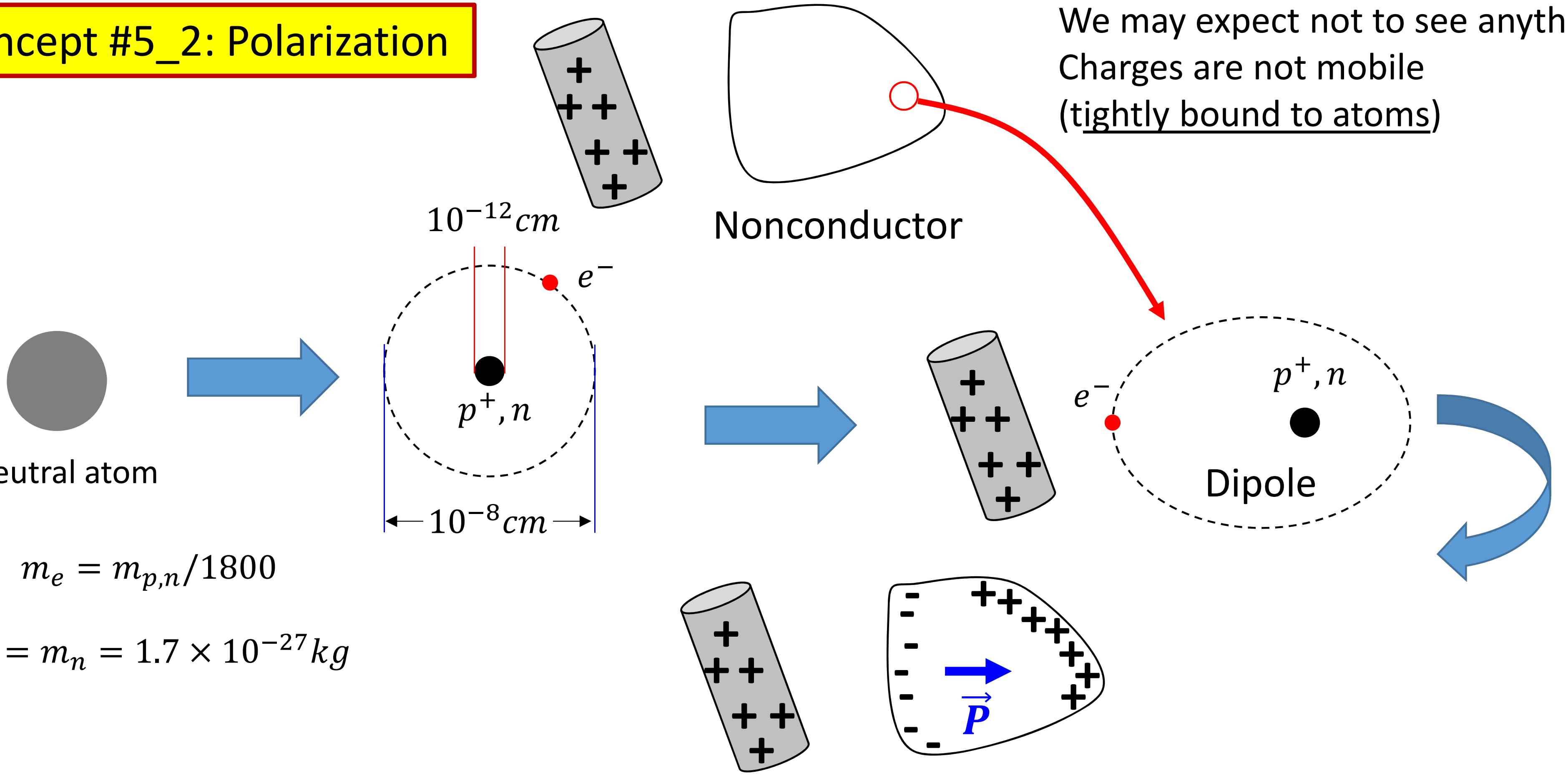
$$F_{attr} \propto {}^{1}/_{r^2}$$





Concept #5_2: Polarization

We may expect not to see anything.
Charges are not mobile
(tightly bound to atoms)

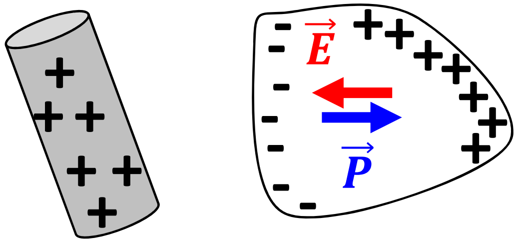


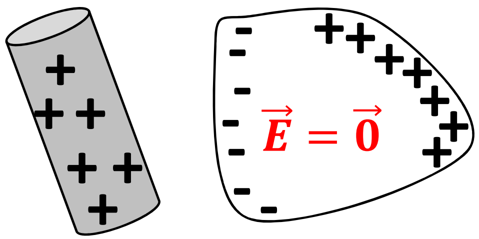
$m_e = m_{p,n}/1800$

$m_p = m_n = 1.7 \times 10^{-27} kg$

Process of polarization of dielectrics ⇔ Induces **short** distance separation of charges

BUT looks like a conductor !





Dielectric
Charge not free
Polarization

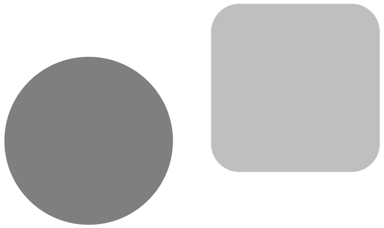
Conductor
Charge free to move
Induction

What is the major difference between these two situations?

- Inside the dielectric there is a field which tends to oppose the external applied field

- Inside the conductor there is no field: The conductor cancels completely the external applied field **(in statics !)**

***Question #8



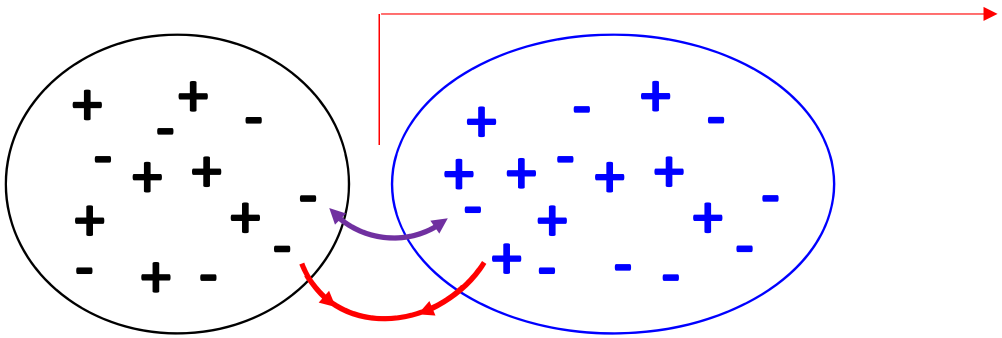
Two perfectly neutral bodies

$$\sum_{1}^{N} q_i^{+} = \sum_{1}^{N} q_j^{-}$$

- Could these two neutral bodies experience a net force if brought close to each other?
- If so how and why?

Answer to ***Question #8:

We did not say anything about spatial distribution ! A net force can arise if negative charge of one body is closer to the positive than to the negative charge of the other body



Neutral piece 1 Neutral piece 2

These forces keep the two NEUTRAL pieces close to each other

Because $\vec{F}_E \propto \frac{1}{r^2}$

How can we proceed to make an interaction between two neutral pieces happen?

By first polarizing them (to make the distribution of charge non-uniform inside each piece)

No transfer of charge without contact ⇨

By friction or electric contact in a circuit

Induction and polarization are **NOT TRANSFER** of charges ! ⟷ Charge separation possible without contact

Coulomb's law for electricity/Newton's law for gravity

Attractive or repulsive

Attractive only

$$F_C \propto \frac{1}{r^2}$$

$$F_G \propto \frac{1}{r^2}$$

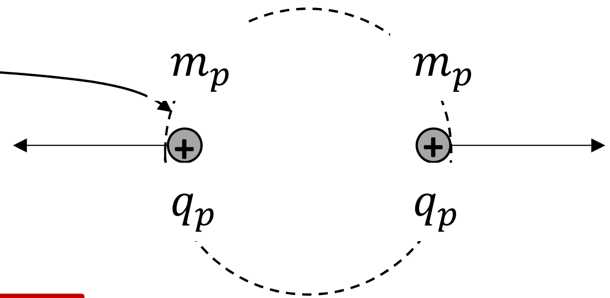
$$K = 9.0 \times 10^9 Nm^2/C^2$$

$$F_C = K\frac{q_p q_p}{r^2}$$

$$F_G = G\frac{m_p m_p}{r^2}$$

$$G = 6.7 \times 10^{-11} Nm^2/kg^2$$

Size of the nucleus



$$\boxed{\frac{F_C}{F_G} = 10^{36}}$$

Acceleration ratio $F = ma$

$$d = 10^{-12} cm$$

$$\boxed{\frac{a_C}{a_G} = 10^{26}}$$

To the earth

$$\frac{F_C}{F_G} = 10^{36}$$

Contradiction that at large distance F_G is playing a much important role !

Reason: Planets have very little charges BUT large masses

Earth / Mars $\quad Q = 4 \times 10^5 C \quad F_G = 10^{17} F_C$

What makes the electron stable? Why all its negative "parts" do not fly apart according to Coulomb?

This question has never been answered !

The coulomb force is terrific !

**Question #9

Why don't protons and electrons end up on the top of each other and the nucleus does not fly apart if the electric forces are so terrific?

Answer to **Question #9

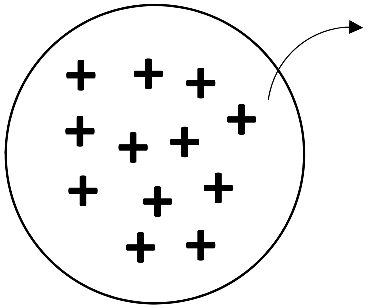
For electrons: → Heisenberg uncertainty principle $\Delta x . \Delta p \approx \hbar$ $\quad$ $\Delta x \rightarrow 0, \Delta p \rightarrow \infty$

→ Repulsion

For protons: → Nuclear force $\propto {}^{1}/_{r^n}$ $\quad n > 2$

↓

Combination of electrical forces and quantum physics



Nucleus

Nuclear attractive force acting between each p^+ and its first neighbors $\propto {}^{1}/_{r^n}$

Electric repulsive force between $p^+ \propto {}^{1}/_{r^2}$

At short distances, Nuclear attractive force >> Coulomb's Electric repulsive force

More protons in a nucleus $\Rightarrow$ Stronger Coulomb's Electric repulsive force $\Rightarrow$ Breaking of nucleus

$\Rightarrow$ Radiactivity for heavy atoms ${}^{92}U$ $\Rightarrow$ Nuclear energy (Atomic bomb)

What is nuclear energy? $\Rightarrow$ Electric energy resulting from Coulomb's force

Two types of Forces involving charges

Immobile charges

$F_E = F_C \propto 1/r^2$

!

Mobile charges

$F_E \neq 1/r^2$ Much more complicated than Coulomb's form !

F_B = Magnetic force

$\vec{F}_B = q\vec{v} \times \vec{B}$

Lorentz force

$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{F}_L(\vec{r}, t)$ because mobile charges change position with time

Concept #6: Field (Faraday 1845)

Scalar field

Vector field

Quantities which depend upon position in space

Scalar field: To each point in space is associated a number which may vary in time:

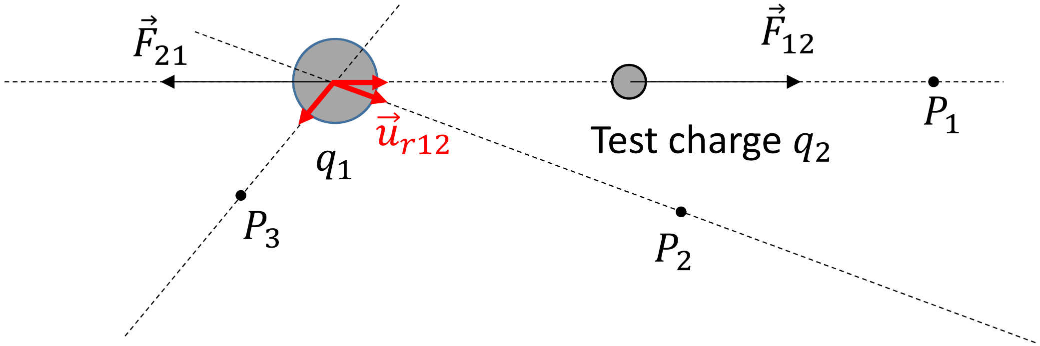
$$\boldsymbol{T(x,y,z,t)}$$
$$\boldsymbol{P(x,y,z,t)}$$

Vector field: To each point in space is associated a number which may vary in time **and direction,**

$$\vec{\boldsymbol{h}}\boldsymbol{(x,y,z,t)}$$
$$\vec{\boldsymbol{v}}\boldsymbol{(x,y,z,t)}$$
$$\vec{\boldsymbol{E}}\boldsymbol{(x,y,z,t)}$$

Vector Field

Electric force requires at least two charges



Action – reaction ⇨ $\vec{F}_{21} = -\vec{F}_{12}$

Is force field meaningful?

No: The force appears only at q_2 when it is there

If there is only one charge q_1 how can we evaluate the effect at P_i when there is no test charge

When there is only one charge q_1 **SOMETHING** builds up all around in the rest of the universe

Concept of Field

Speed of light is **NOT** infinite

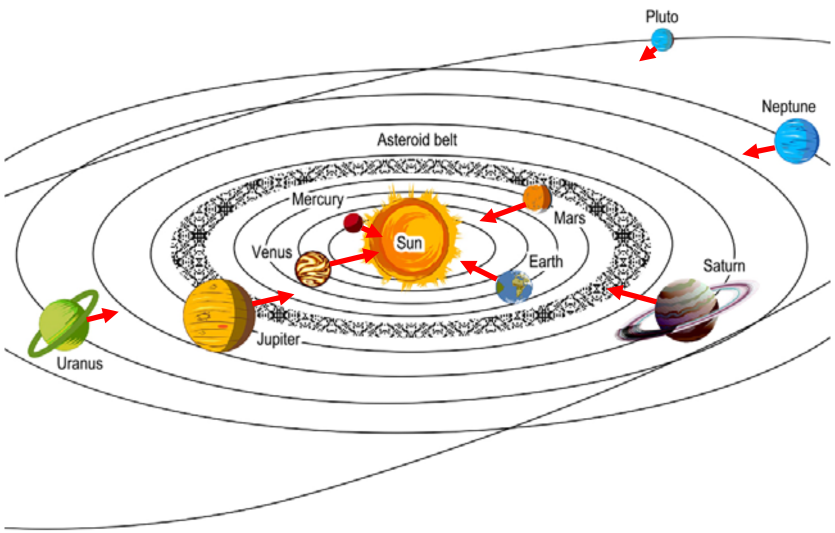
Concept of retarded time

Action at distance versus action at contact

The effect is felt at every point is space

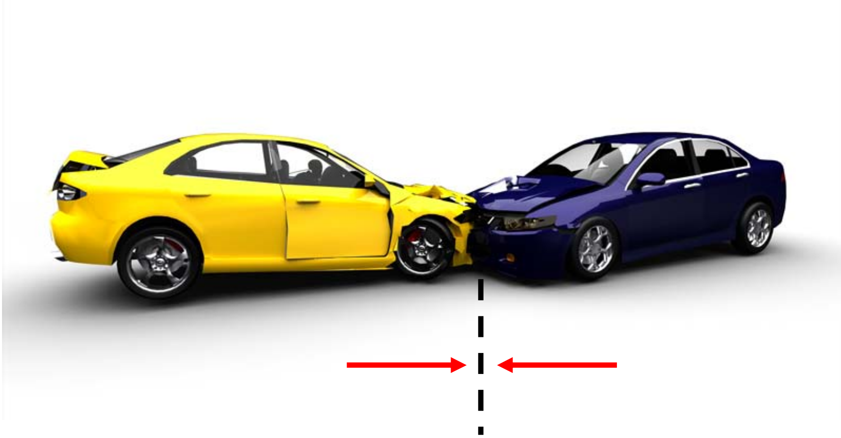
If a planet is removed the attraction effect is still there

A force is acting with nothing to transmit to !



Action at distance: Gravitation field

The effect is felt only at the point of contact



Action at contact

What does that mean ?

Contact force is an illusion

There is something in the universe around a single charge q_1 even when **NO OTHER** charges are present

If we bring a charge q_2, a force acts on it

q_1 is immobile ⇨ $\vec{E}(x, y, z)$

q_1 is mobile ⇨ $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$

Does $\vec{B}(x, y, z, t)$ act on q_2?

- Requires q_2 to be mobile
- Must consider the direction of motion of q_1 relative to q_2

A moving charge settles in the whole universe electric and magnetic vector fields

...Other vector fields

Heat flow $\vec{h}(x, y, z, t)$
Current density flow $\vec{j}(x, y, z, t)$
Velocity flow $\vec{v}(x, y, z, t)$
...

...There are also scalar fields

Temperature $T(x, y, z, t)$
Electric potential $\varphi(x, y, z, t)$
...

$\vec{E}(x, y, z, t)$ $\qquad$ $\vec{E}(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t)$

$\varphi(x, y, z, t)$ $\qquad$ $\varphi(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t)$

$\Rightarrow$ $\dfrac{\partial E}{\partial x_i}, \dfrac{\partial \varphi}{\partial x_i}$ etc ... Differential equation to describing the fields

$\Downarrow$

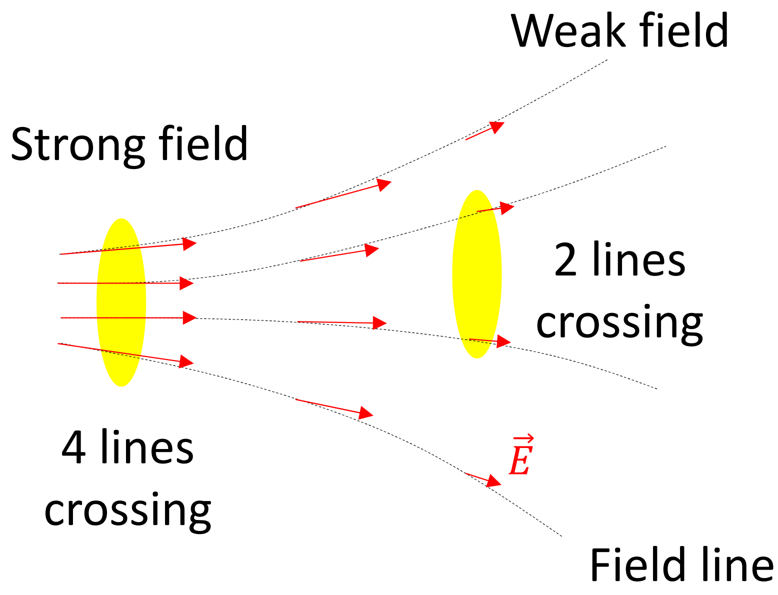
Laws of electrodynamics

Concept #7: Field lines

Action at distance

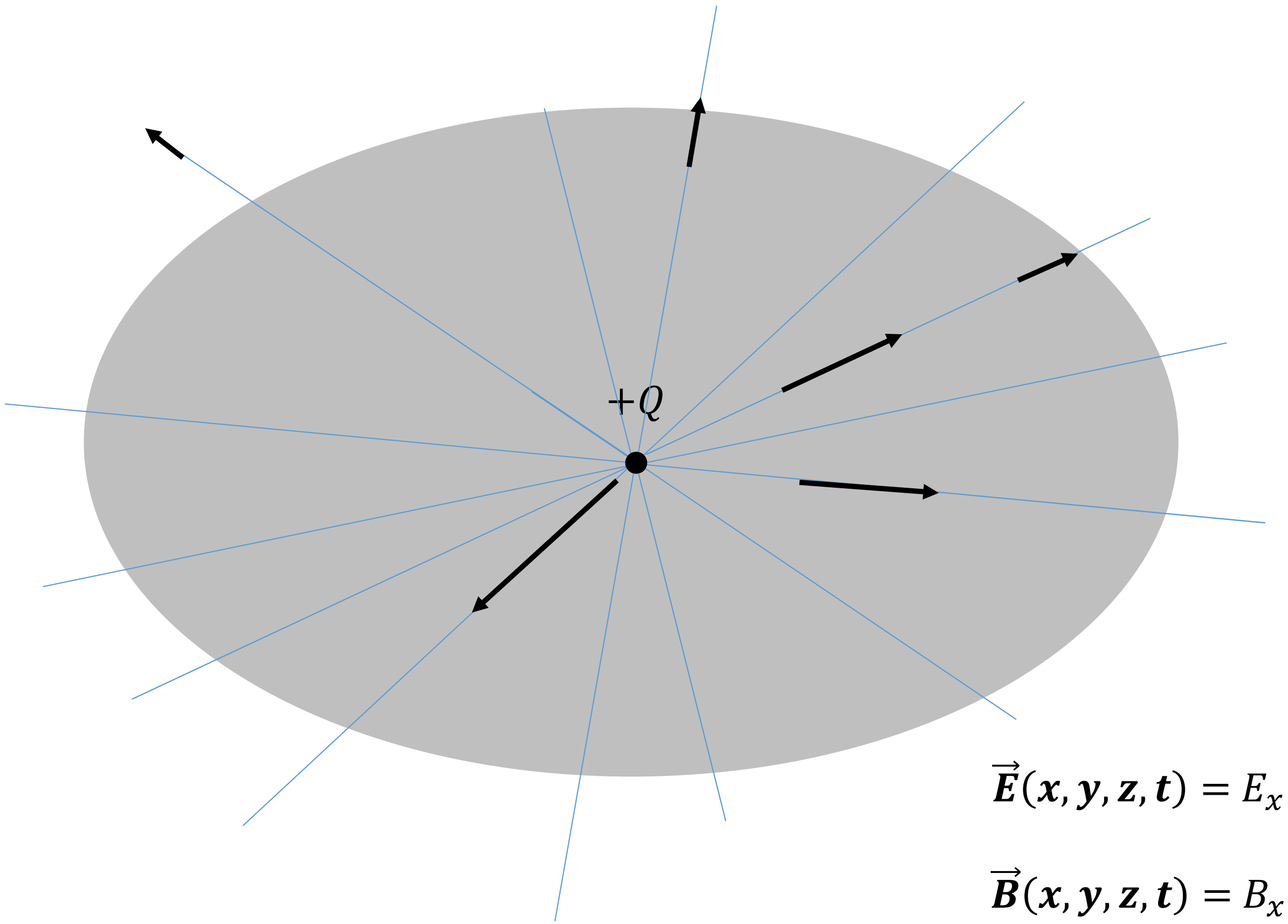
Concept of field filling the whole space like:

- Gravitation field
- Electric field
- Magnetic field



- $\vec{E}$ is **ALWAYS** tangent to the field line
- Local density of field lines /unit area at right angle = strength of the field at each point
- Field lines **NEVER** cross

The concept of **electric field** and **field lines**



$$\vec{F} = q_{test}\vec{E}$$

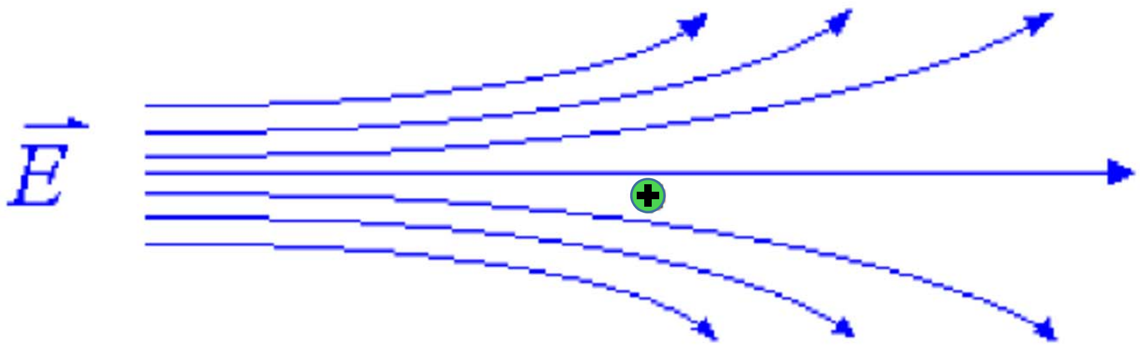
NOT a Field　　　Field

$$q_{test} \ll Q, q_{i's}$$

P

$$\vec{\boldsymbol{E}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}) = E_x(x,y,z,t)\vec{\boldsymbol{\imath}} + E_y(x,y,z,t)\vec{\boldsymbol{\jmath}} + E_z(x,y,z,t)\vec{\boldsymbol{k}}$$

$$\vec{\boldsymbol{B}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{t}) = B_x(x,y,z,t)\vec{\boldsymbol{\imath}} + B_y(x,y,z,t)\vec{\boldsymbol{\jmath}} + B_z(x,y,z,t)\vec{\boldsymbol{k}}$$



A positive charge is placed in a region of electric field as shown. Which way does it move

a) Up

b) Down

c) Left

d) Right

e) It does not move

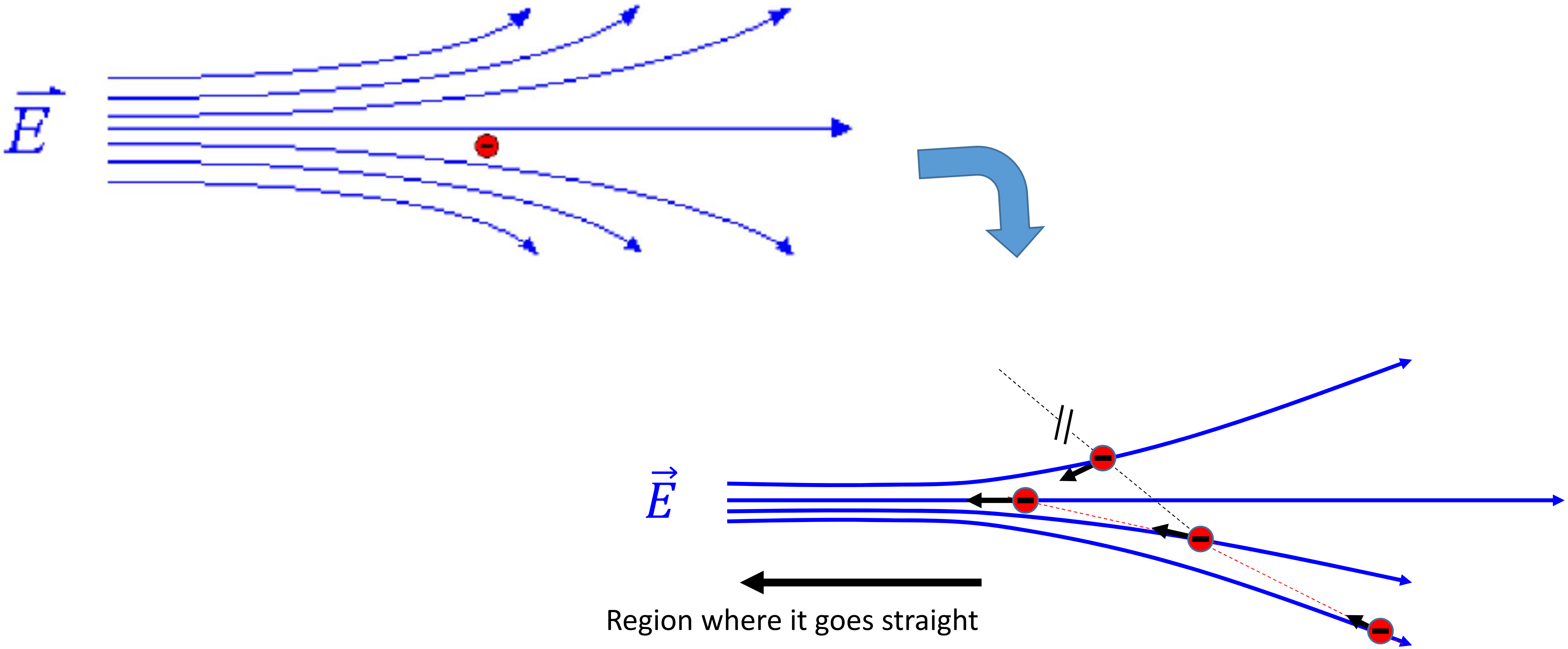
**Question #10

Does it move straight to the right ?

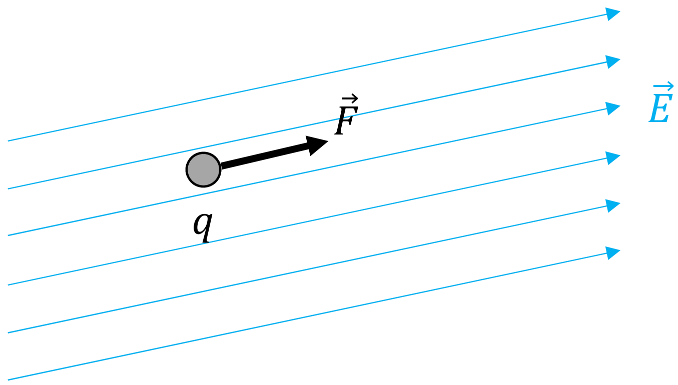
Answer to **question #10

No ! It will follow a very complicated trajectory

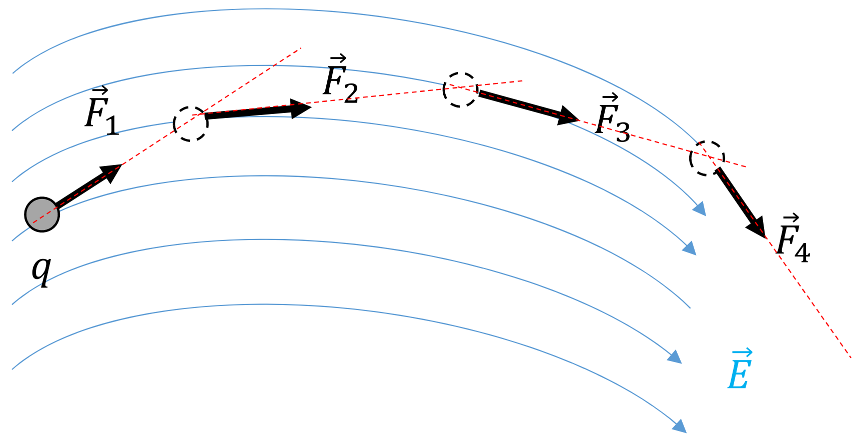
What about a negative charge?


$\vec{E}$
$\vec{E}$
//
Region where it goes straight

Uniform versus no uniform field



The charge goes straight like in a gravitational field
(close to the earth !)



The charge takes a very complicated trajectory