An alternative proof of the question on page 57 of *G_Lecture 12&13_Dielectrics*

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Case of a dielectric sphere inserted in a uniform electric field: Uniformly polarized dielectric sphere

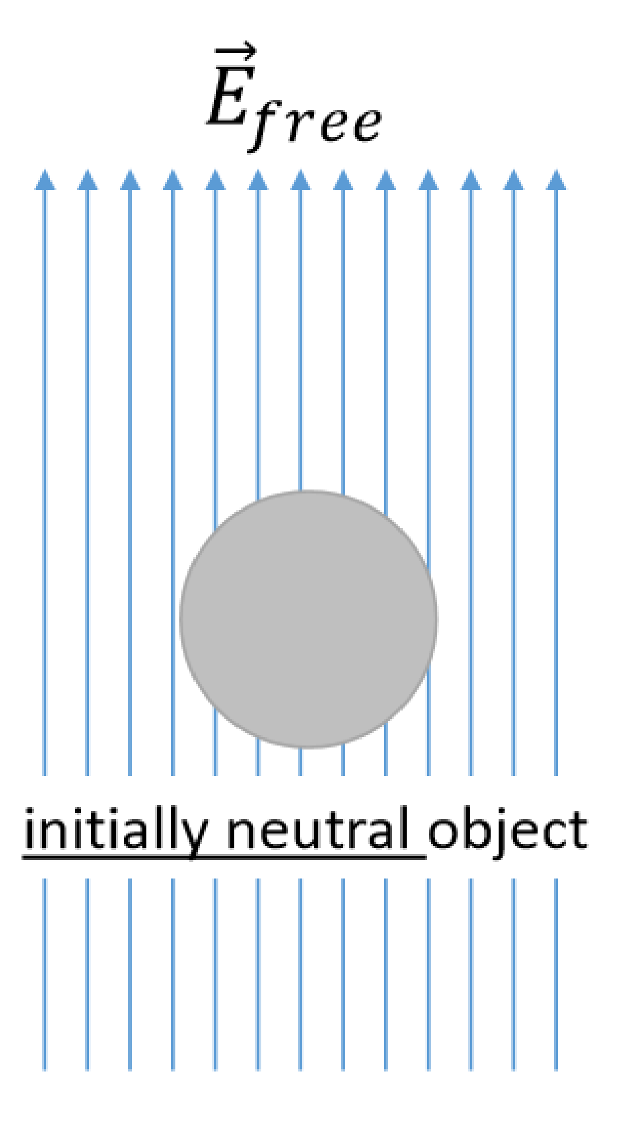
- Question: (On G_Lecture Page 57)

1. Once in the external field $\vec{E}_{\mathrm{free}}$, bound surface and / or

bulk charges are induced in the initially neutral object

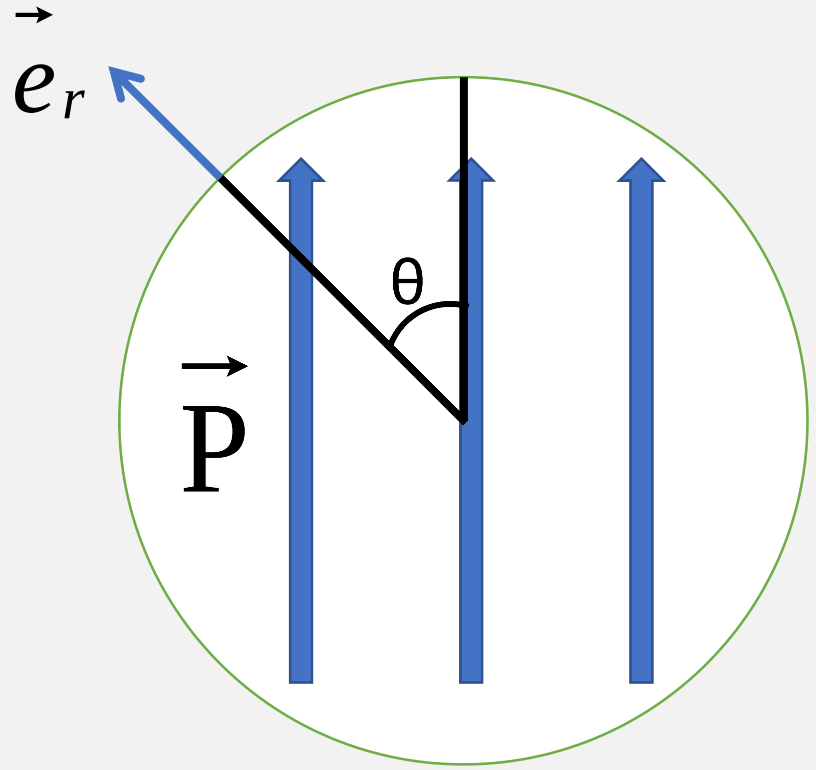
2. A new field $\boldsymbol{E}$ is now generated by the object itself (dipoles):

Resulting field $\vec{E}_{free} + \vec{E}_{diel}$



The surface charge density

$$\sigma_b = \vec{P}.\vec{e}_r = P\cos\theta$$



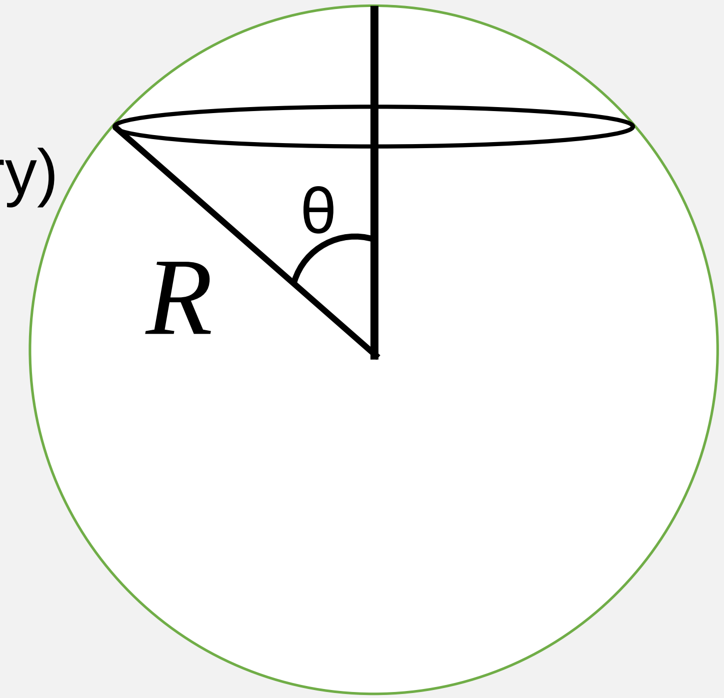
Step to find Electric field due to polarized sphere

- Choose a unit ring with width Rdθ
- Charge density on the ring is the same.(symmetry)

$$\sigma_b = P\cos\theta$$

$$dA = 2\pi R\sin\theta \cdot Rd\theta$$

$$dq = 2\pi R^2 \sin\theta d\theta \cdot \sigma_b$$



- So the electric field on the center due to the ring:

$$dE = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{x}{R^3} = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{\cos\theta}{R^2} = \frac{2\pi R^2\sin\theta d\theta\cdot\sigma_b}{4\pi\varepsilon_0}\cdot\frac{\cos\theta}{R^2}$$

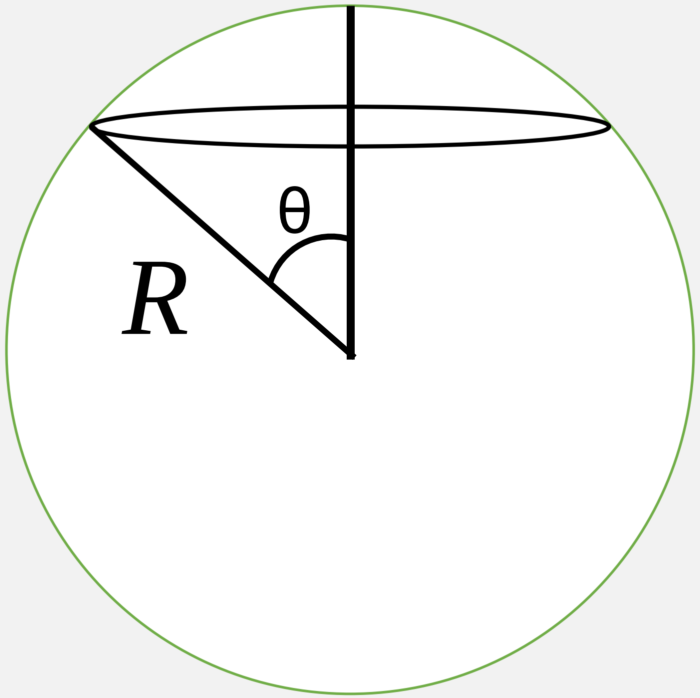
Integral to find the electric field due to the sphere

$$E_{diel} = \int_{sphere} dE$$

$$= \int_0^{\pi} \frac{2\pi R^2 \sin\theta \cdot \sigma_b}{4\pi\varepsilon_0} \cdot \frac{\cos\theta}{R^2} d\theta$$

$$= \frac{2P}{4\varepsilon_0} \int_0^{\pi} \sin\theta \cos^2\theta d\theta$$

$$= \frac{P}{3\varepsilon_0}$$



Total Electric field

$$E$$
$$= \left| \vec{E}_{free} + \vec{E}_{diel} \right|$$
$$= E_{free} - \frac{P}{3\varepsilon_0}$$
$$= E_{free} - \frac{\chi\varepsilon_0 E}{3\varepsilon_0}$$

Therefore

$$\vec{E} = \frac{3}{3+\chi}\vec{E}_{free} = \frac{3}{2+\varepsilon_r}\vec{E}_{free}$$