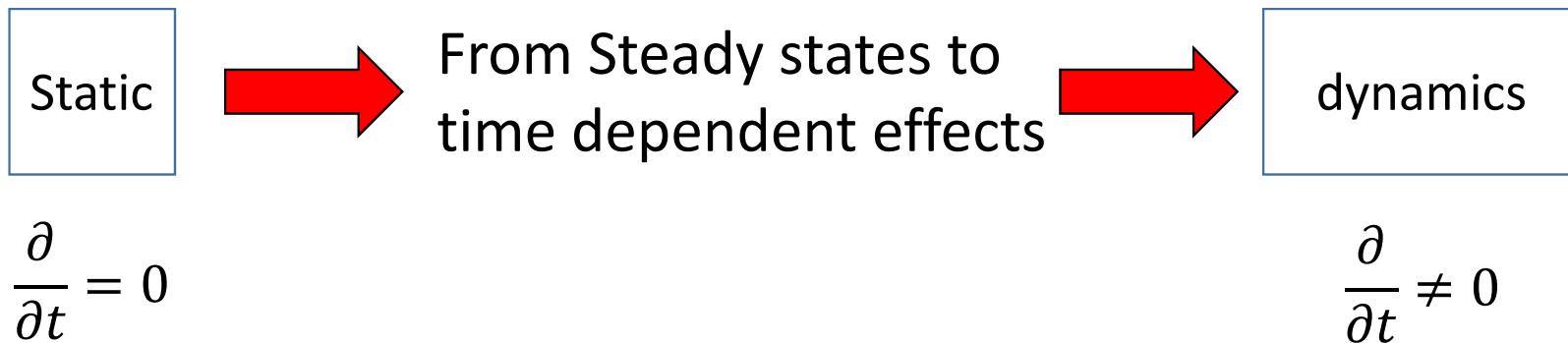


From Static to dynamics



Faraday's discovery and the **birth of electrical engineering (EC)**

Orsted 1820: Discovery of the close connection between electricity and magnetism

- Current in a wire creates a magnetic field (compass deviation)
 - Wire carrying current in a magnetic field manifests action - reaction
- } Involves steady current

and Faraday 1821:

- Changing magnetic field creates a changing electric field
 - Changing electric field creates a changing magnetic field
- } Involves time varying current

Can be used to produce **WORK**



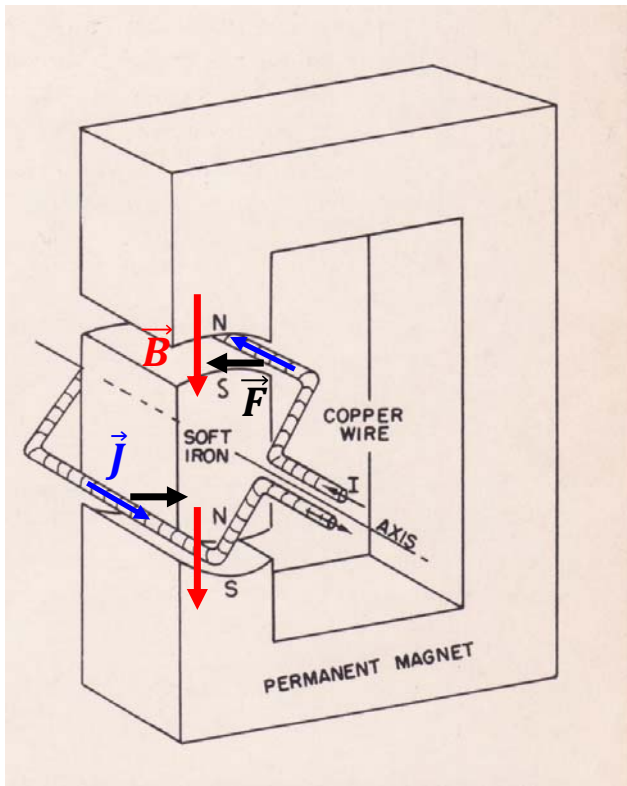
Birth of electrical engineering

FORCE and / or TORQUE



Magnetic field produces mechanical force

Application of Lorentz force

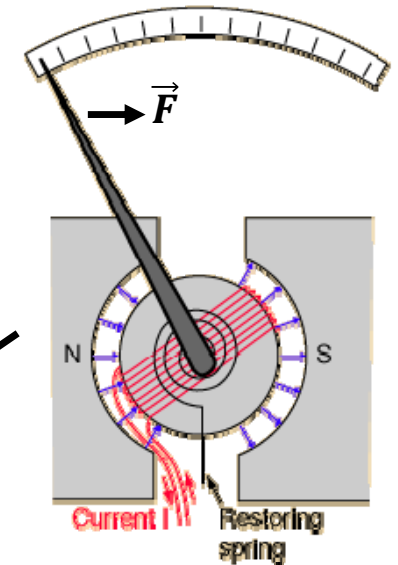


Electromagnetic Motor

Make a motor (current reversed each half-turn)

Torque in action

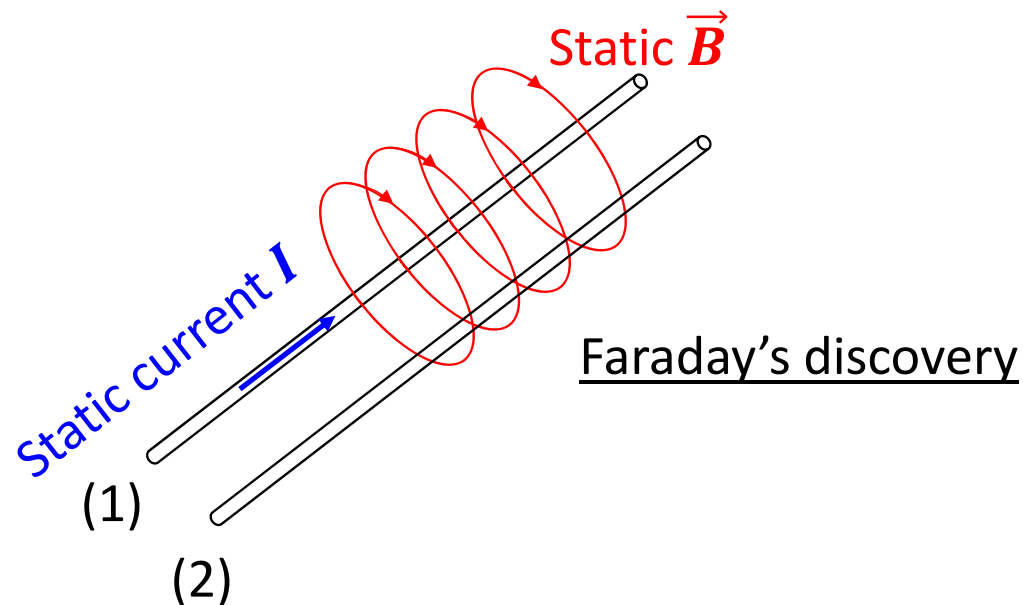
Makes a sensitive instrument for measuring current



Galvanometer

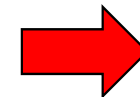
Orsted 1820: Current in a wire creates magnetic field

Faraday 1821: Magnetic field **MAY** create an electric field \Rightarrow Generating thus a current in wire (2)



Nothing appears in wire (2) If current in (1) is steady and wires immobile

Requires that something changes with **TIME**



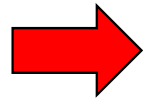
From Static to **dynamics**



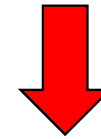
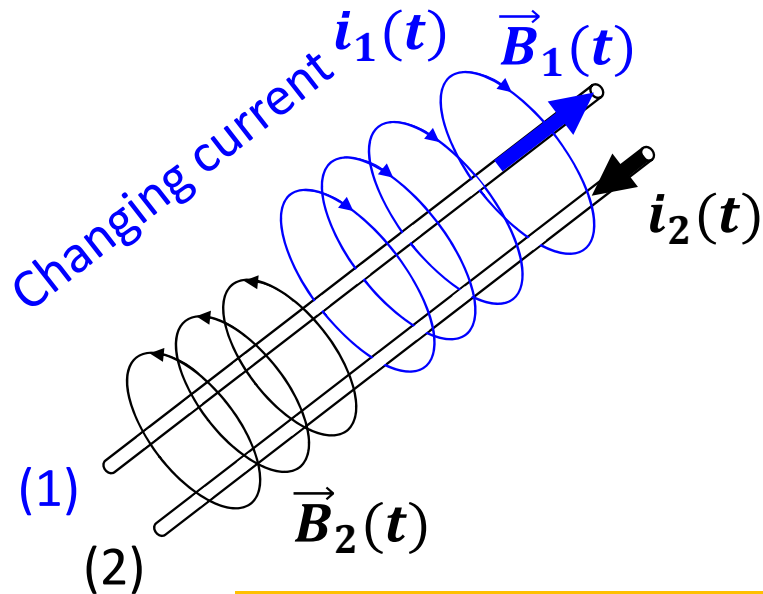
Current induction

No Current without \vec{E}

A varying current in wire (1)



A varying current $i_2(t)$ appears in wire (2)

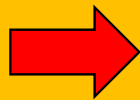


Induction of electric field in wire (2) driving electrons creating thus a current



EMF (ElectroMotive Force)

$i_1(t)$



$i_2(t)$

$\vec{J}_1 = -\vec{J}_2 \longrightarrow$ Opposing magnetic field

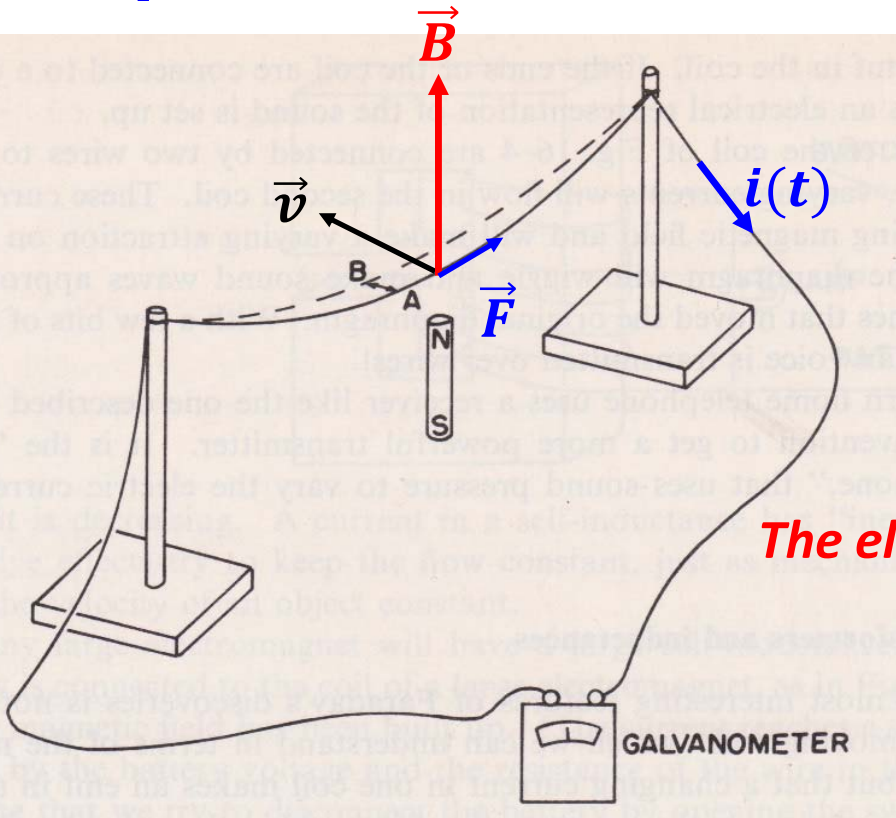
Essential feature discovered by Faraday: **Time changing processes**

Is something missing in this figure?

Yes: A second magnetic field is generated by wire (2)

Current induction: Lorentz force was not known to Faraday

$$\vec{F} = q\vec{v} \times \vec{B}$$



The electrons move by electric repulsion over long distances

\vec{v} = mechanical velocity

Current induced by Lorentz force



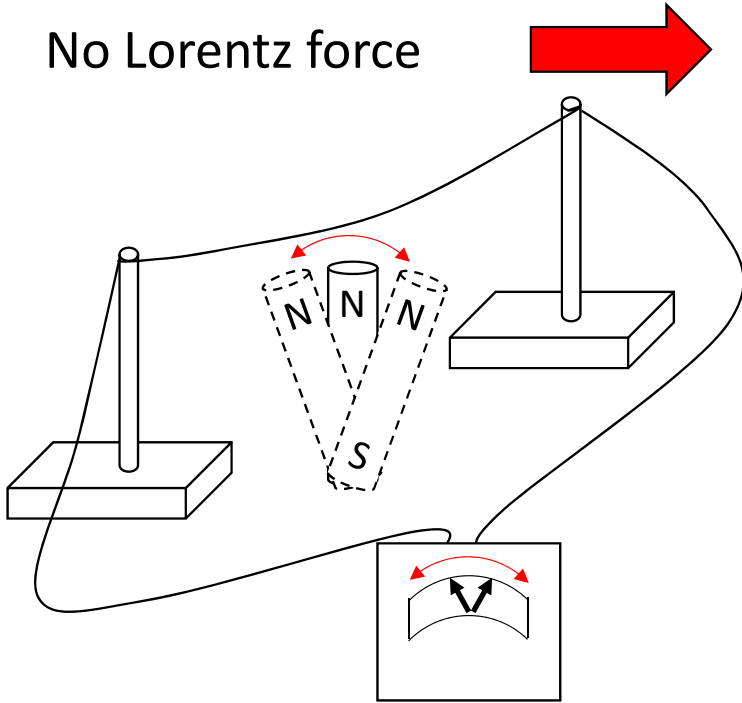
Concept of motional *emf*

Why the galvanometer shows a current while it is far from the Lorentz force ?

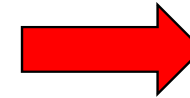
First telegraph discovered by Gauss and Weber

Another way of Current induction discovered by Faraday

No Lorentz force



By shaking the magnet



Relativistic effect

- No current induction if magnet is static
- Current induced by change in magnetic field $\frac{\partial B}{\partial t}$

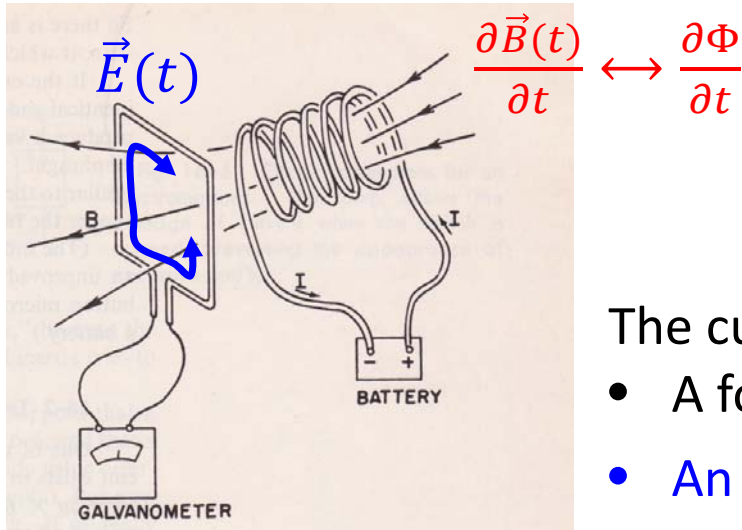


Concept of **nonmotional** *emf*

Again why the galvanometer shows a current while it is far from the Lorentz force ?

There is a net integrated push around the complete circuit. This net push MUST come from an electric field which then leads to the repulsive push over long distance

Faraday's discovery: **emf**



Current induced by **change in magnetic field** $\frac{\partial B}{\partial t}$

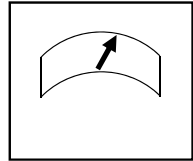
The current noticed in the loop is due to motion of electrons

- A force has pushed the electrons \Rightarrow
- **An electric field** has been generated by $\frac{\partial B}{\partial t}$

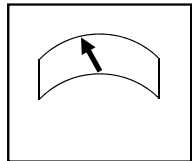
$$\vec{E}(t) \propto \frac{\partial \vec{B}(t)}{\partial t}$$

From Feynman lecture (Volume II)

1) We switch **ON** the battery



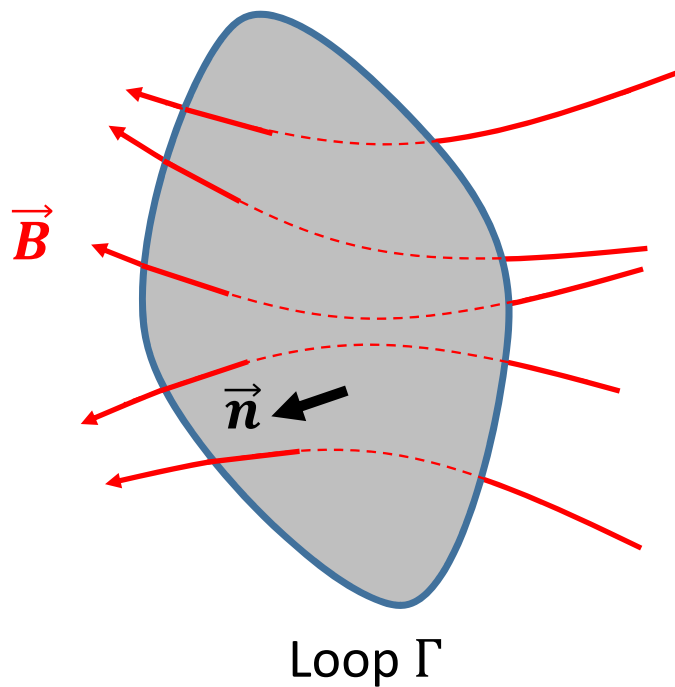
2) We switch **OFF** the battery



emf can be generated in a wire by:

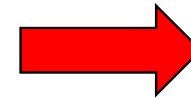
- Moving mechanically the wire near a magnet or vice versa
- Changing current in a nearby wire

Faraday's rule: Is it the changing magnetic field or changing flux that matters?



$$\frac{\partial \vec{B}(t)}{\partial t} \longleftrightarrow \frac{\partial \Phi}{\partial t}$$

Area kept unchanged



$$\Phi = \int \vec{B} \cdot d\vec{A} = \int \mathbf{B}_{\perp} dA$$

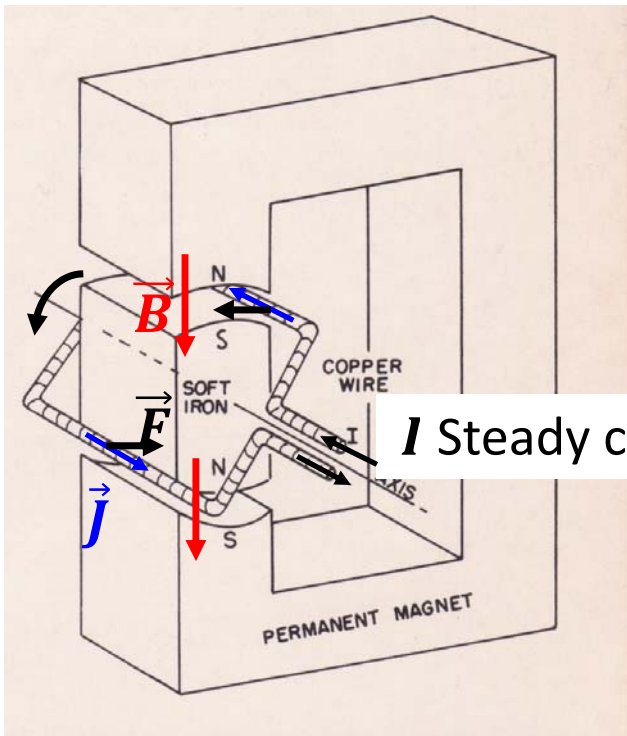
Normal component of \vec{B} integrated over the whole area of the loop

emf: ONLY and ONLY IF the flux is changing with time

- Changing \vec{B}
- Changing \vec{A}
- Changing both \vec{A} and \vec{B}

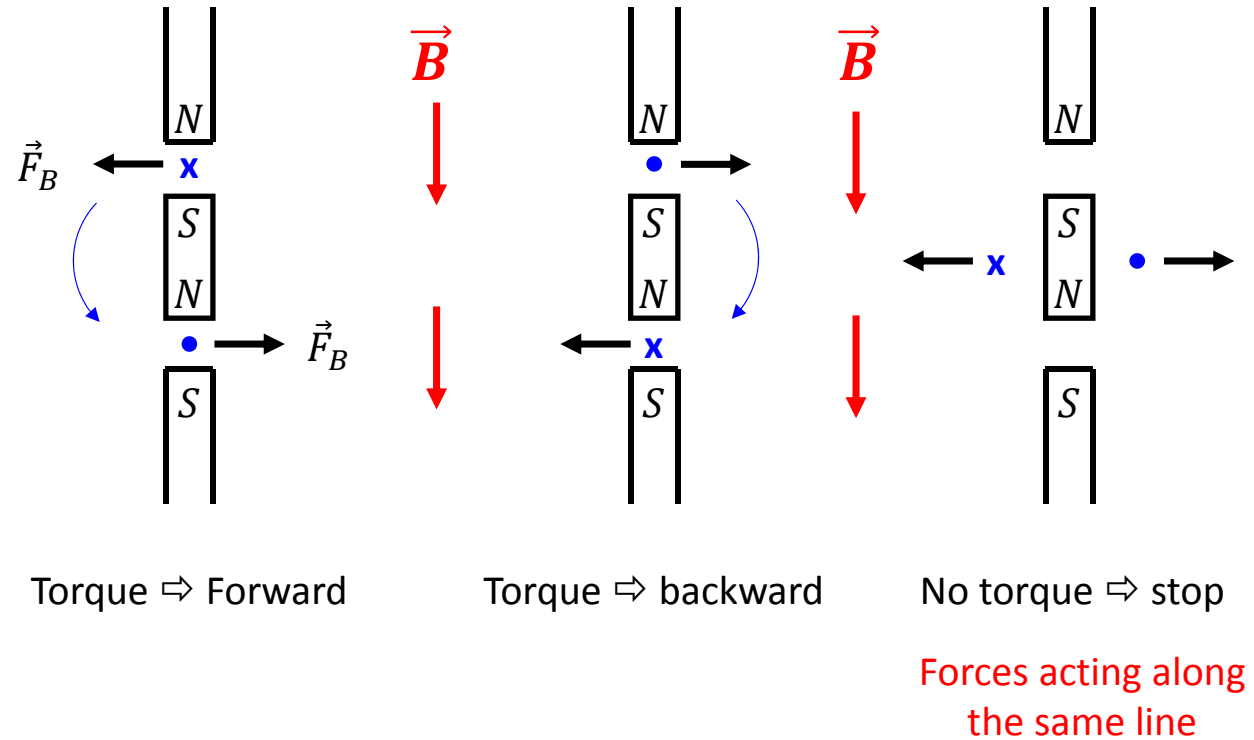
Motor

The loop rotates due to Lorentz force



I Steady current

From Feynman lecture (Volume II)



Forces acting along
the same line

Current **MUST** be reversed each half-turn
otherwise the loop stops rotating

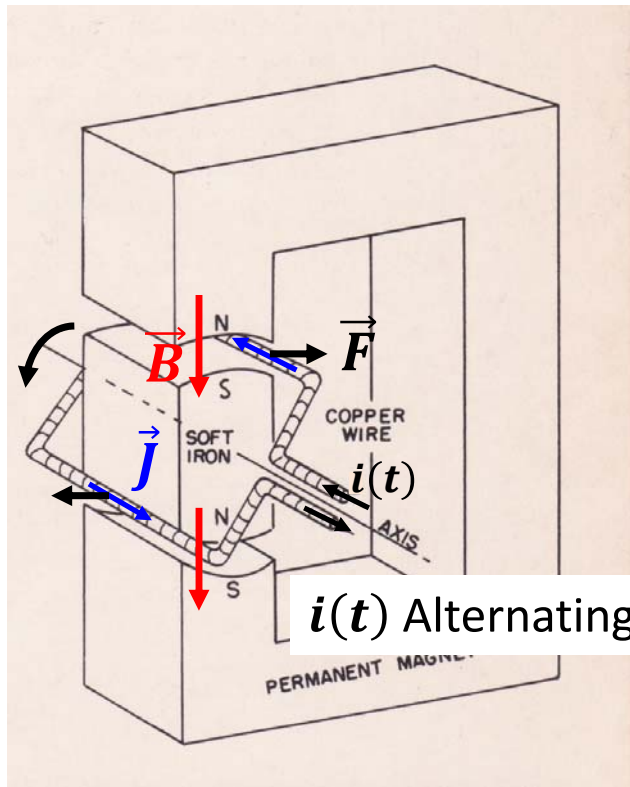
From steady to variable current $I \rightarrow i(t)$

Alternating current generator

The loop rotates mechanically



Induction of an emf: **ElectroMotive Force**



From Feynman lecture (Volume II)

- Area of the loop is fixed
- The normal component of loop's area \vec{n} is changing



Changing flux $\Phi(t)$

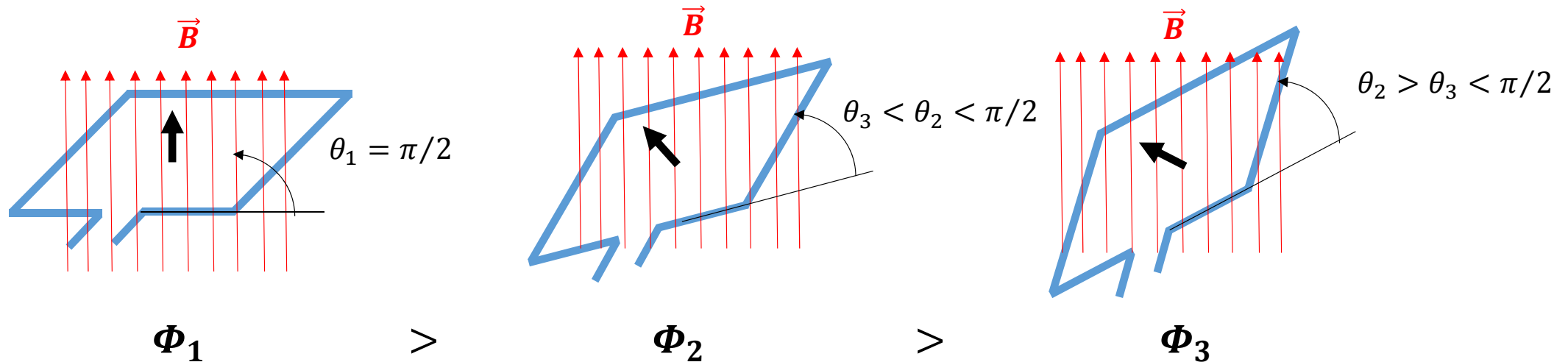


Electric field generated in the wire
Circulation of $\vec{E}(t)$ along the loop



Generation of an alternating current $i(t)$ with
frequency = frequency to the mechanical rotation

Emf = rate of change of magnetic flux through the loop



What matters in the flux is the **normal component of the field** along the unit vector of the area

From Φ_1 to Φ_3

$$\frac{\partial \Phi}{\partial t} \rightarrow E(t) \rightarrow emf \rightarrow i(t)$$

Lenz's rule

- The induced emf creates a current
- This current generates a magnetic field
- This magnetic field tends to oppose the original one



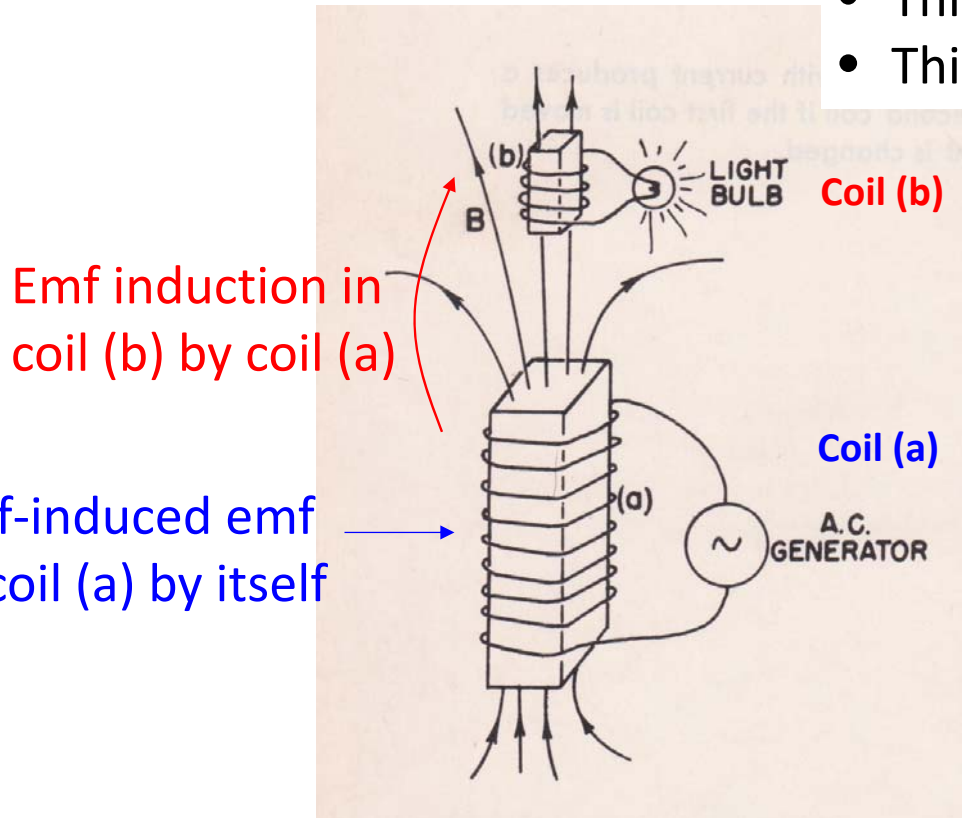
The polarity of the induced emf is **ALWAYS** such as to oppose the change of the flux of the original magnetic field

$$emf = -N \frac{d\Phi}{dt}$$

N = # of turns

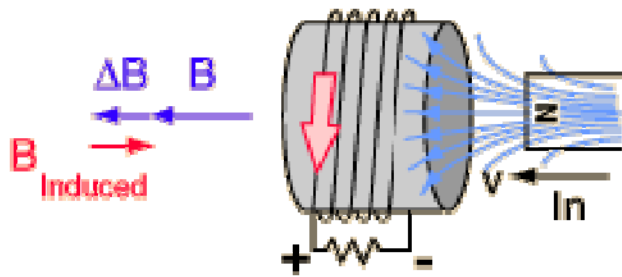
Φ = Flux of B through area A

Lenz's law



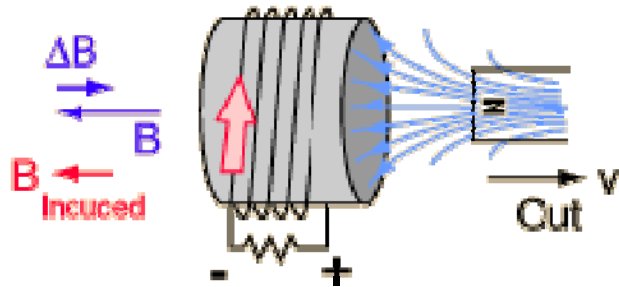
From Feynman lecture (Volume II)

Lenz's law



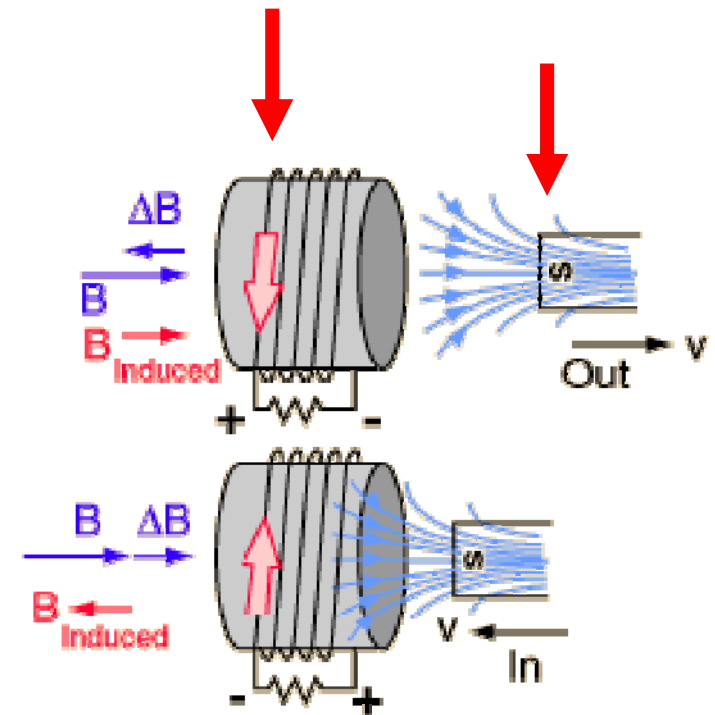
⇒ Flux increases ⇒ B increases by ΔB

The current generated by the emf goes in a direction such as to induce a magnetic field in the opposite direction



⇒ When the flux is decreased the emf is reversed

Caution! Here the arrows are reversed as it is the south pole of the magnet that is moving in and out



Towards time dependent Maxwell's equations

$$\vec{\nabla} \cdot \vec{E}(t) = \frac{\rho(t)}{\epsilon_0}$$

Gauss' law

$$\vec{\nabla} \cdot \vec{B}(t) = 0$$

No magnetic charges

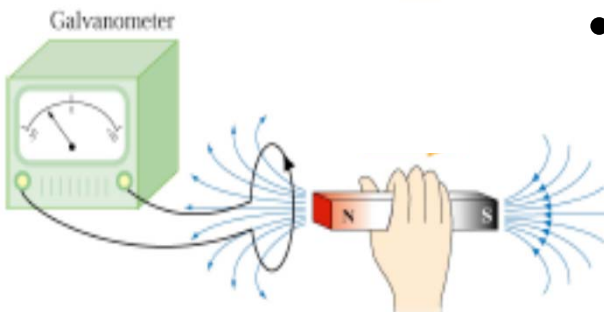
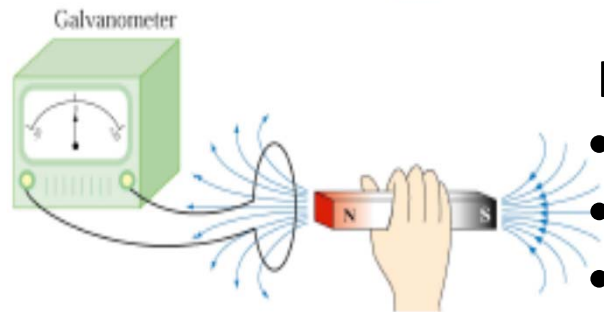
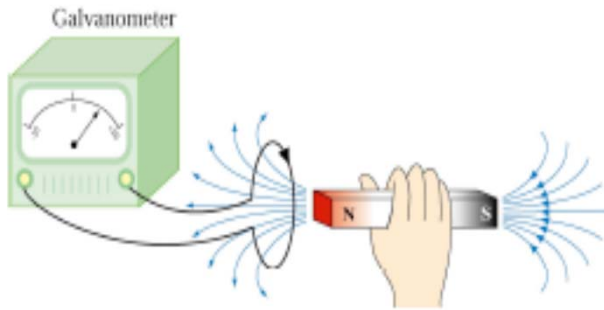
$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law (\vec{E} is no longer conservative)

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell

Faraday's law of induction



If no motion of the loop and magnet

- **NO** change of the flux
- **NO** electric force
- **NO** motion of the free charges
- **NO** current in the loop

Relative motion induces current in the loop



There **MUST** be an induced electric field
Along the loop



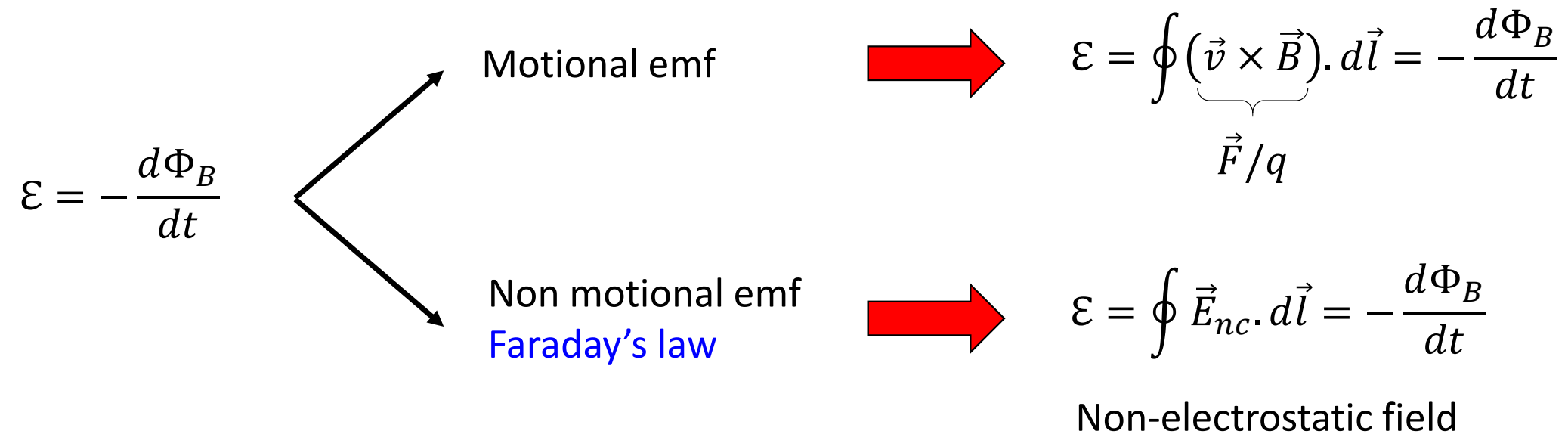
$$\oint \vec{E}_{nc} \cdot d\vec{l} \neq 0$$

$$\rightarrow i(t)$$

Electrostatic \Rightarrow Conservative field

$$\oint \vec{E}_c \cdot d\vec{l} = 0$$

Faraday's law: electromotive force



$\oint \vec{E}_{nc} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	What is the link between these two relationships?	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Integral form	Faraday's law	Differential form

$$\oint \vec{E}_{nc} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} + \oint \vec{E}_c \cdot d\vec{l} = 0$$



$$\oint (\vec{E}_c + \vec{E}_{nc}) \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$\underbrace{\hspace{10em}}$

\vec{E} (superposition principle)

Stoke's theorem



$$\oint \vec{E} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{E}) \cdot d\vec{A}$$

Gauss's theorem

$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$$

$$\text{if } \frac{\partial \vec{B}}{\partial t} = \vec{0} \Rightarrow \oint (\vec{E}_c + \vec{E}_{nc}) \cdot d\vec{l} = 0$$

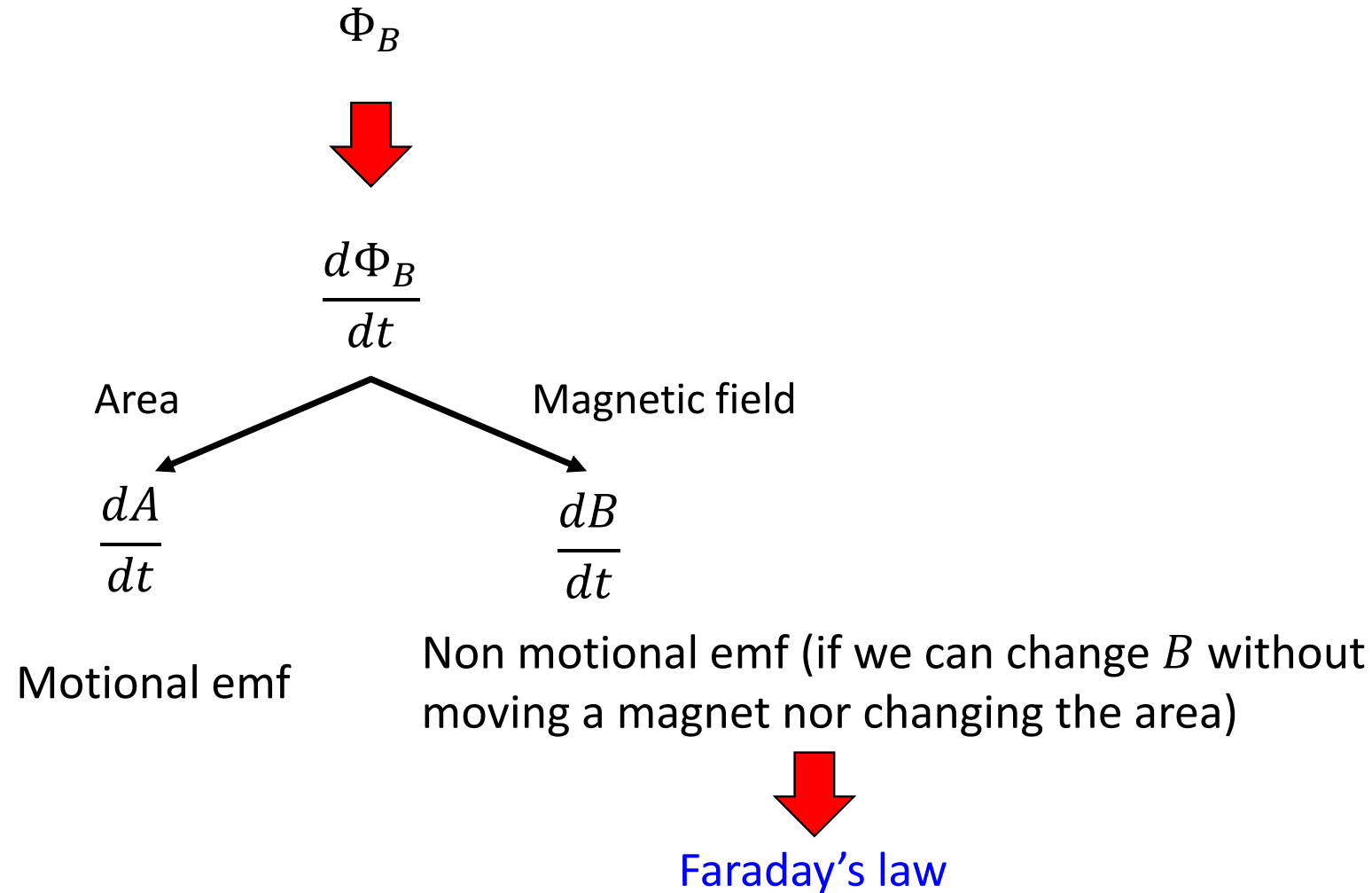


$$\vec{\nabla} \times \vec{E} = \vec{0} \quad \text{Electrostatic}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

True always

The Heart of Faraday's law of induction



$$\frac{d\Phi_B}{dt} = \frac{dBA}{dt}$$

B constant

Part of the circuit is moving $\Rightarrow \vec{v} \times \vec{B} \neq \vec{0}$

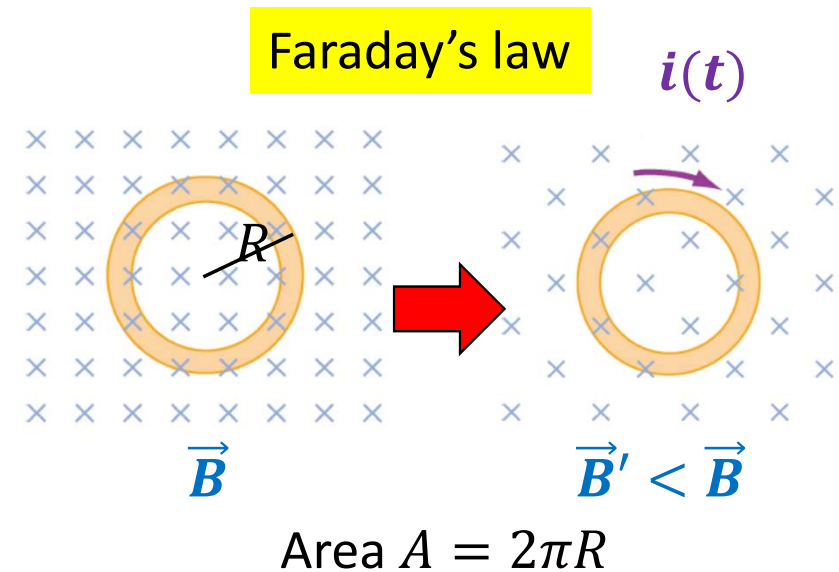
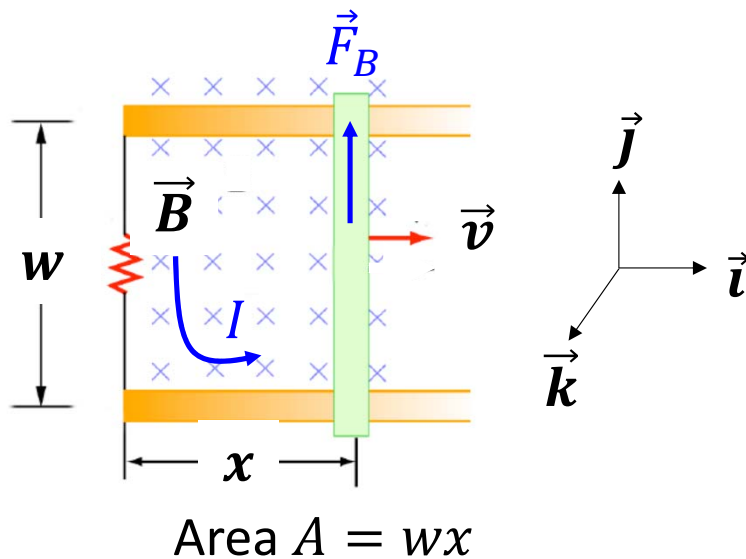
$$A \text{ changes} \Rightarrow \frac{dA}{dt} = w \frac{dx}{dt}$$

A constant

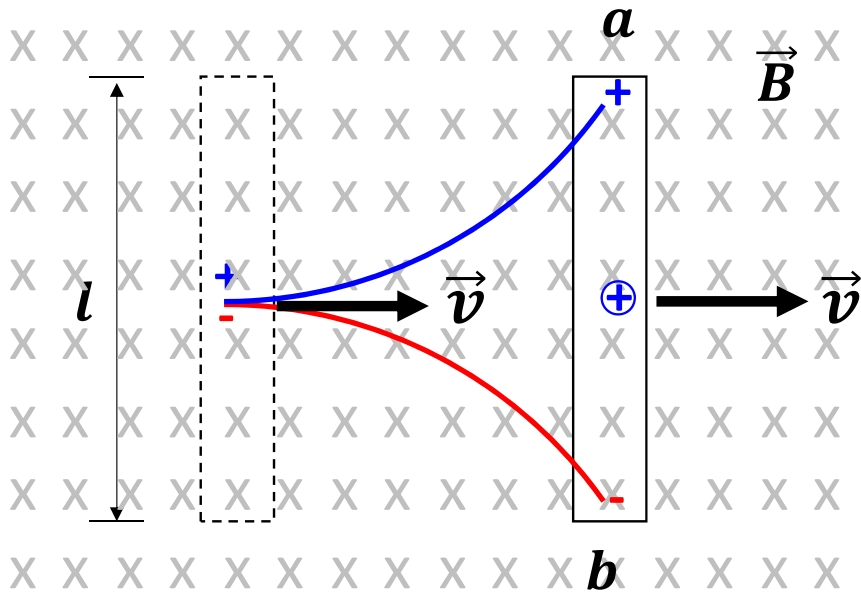
Circuit is immobile $\Rightarrow \vec{v} \times \vec{B} = \vec{0}$

$$B \text{ changes} \Rightarrow \frac{dB}{dt} \rightarrow E(t) \rightarrow \vec{F} = q\vec{E}$$

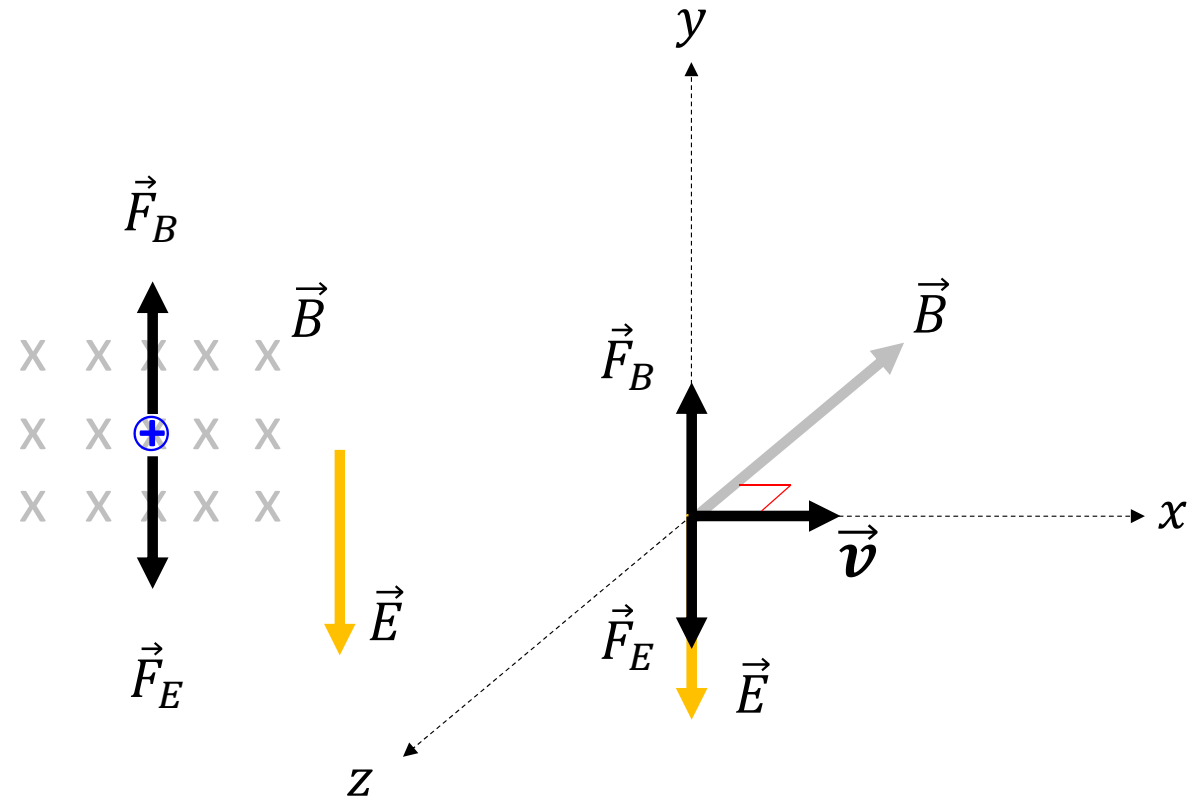
In both cases electrons move due to Coulomb force



Motional emf



An electric field is building up inside the conducting bar !



Equilibrium !

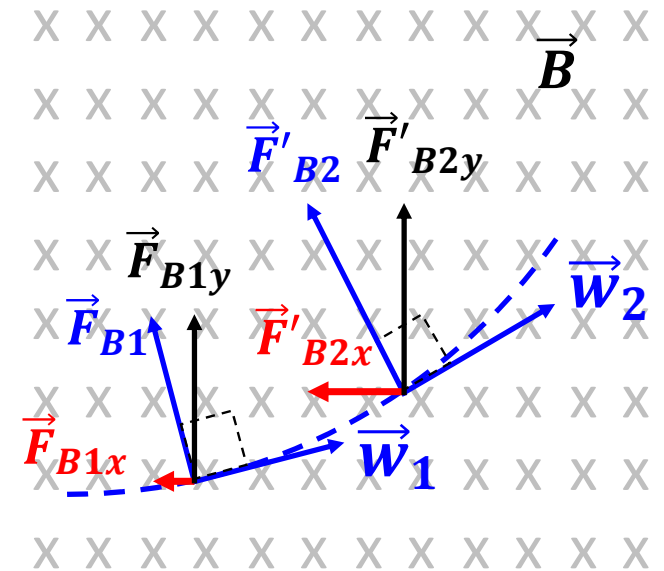
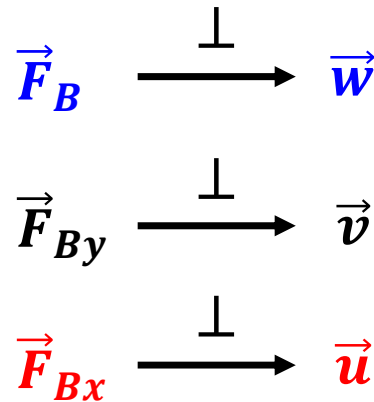
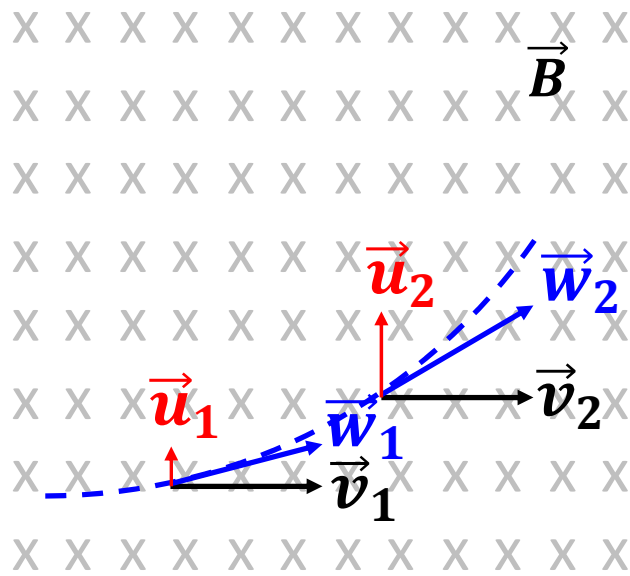
$$\vec{F}_B + \vec{F}_E = \vec{0}$$

Equilibrium $\rightarrow qvB = qE \rightarrow E = vB \rightarrow V_{ab} = V_a - V_b = El$

$$V_{ab} = Blv$$

Motion \rightarrow **Motional** emf = Potential difference $\rightarrow \mathcal{E} = V_{ab} = Blv$

For **Motional** emf both \mathbf{B} and \mathbf{v} are required to get a potential difference



None of these forces do work on the charge

BUT a force \vec{F}_{Bx} opposing the motion of the bar is building-up

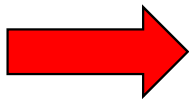
Work **MUST** be done against this force to keep the bar moving at constant velocity \vec{v}

Opposite force $\vec{F}_{Bx} = -quB\vec{l}$

Work done per unit charge against this force must be $\frac{\Delta W}{q} = - \int_x^{x+\Delta x} -uB\vec{l} \cdot d\vec{x}$

$$\frac{\Delta W}{q} = -\Delta V = V_{ab} = - \int_x^{x+\Delta x} -uB dx = uB\Delta x$$

$$\Delta x = v\Delta t$$



$$\Delta x = \frac{v\Delta l}{u}$$



$$\Delta l = u\Delta t$$

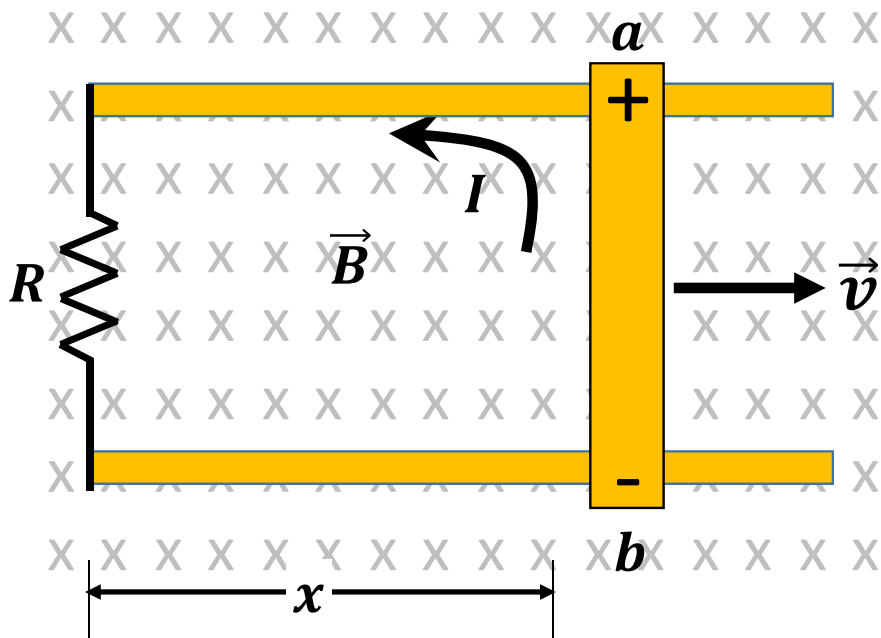
$$\frac{\Delta W}{q} = -\Delta V = V_{ab} = \mathcal{E} = Blv$$

Motional emf found in **slide # 22**

This work is useless because the circuit is **not complete** !

The circuit is complete and the bar moves at constant velocity

The charges will **no longer accumulate** at the ends of the bar and a current is generated
Counterclockwise: The motion of the charges in the circuit is due to coulomb repulsion

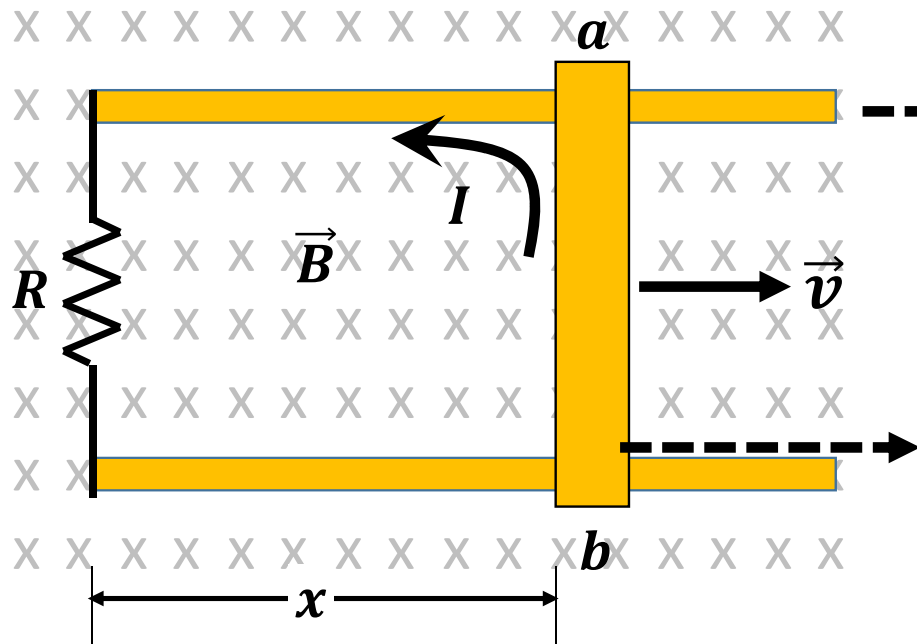


Question: Why putting a resistance R in the circuit ?

Answer: To reduce the current to a minimum in order to avoid generating an important magnetic field which may then disturb the external one

Remember: In electrostatic the test charge must be weak enough to avoid perturbing the source

The circuit is complete and the bar moves at constant velocity



Stationary U-shaped conductor

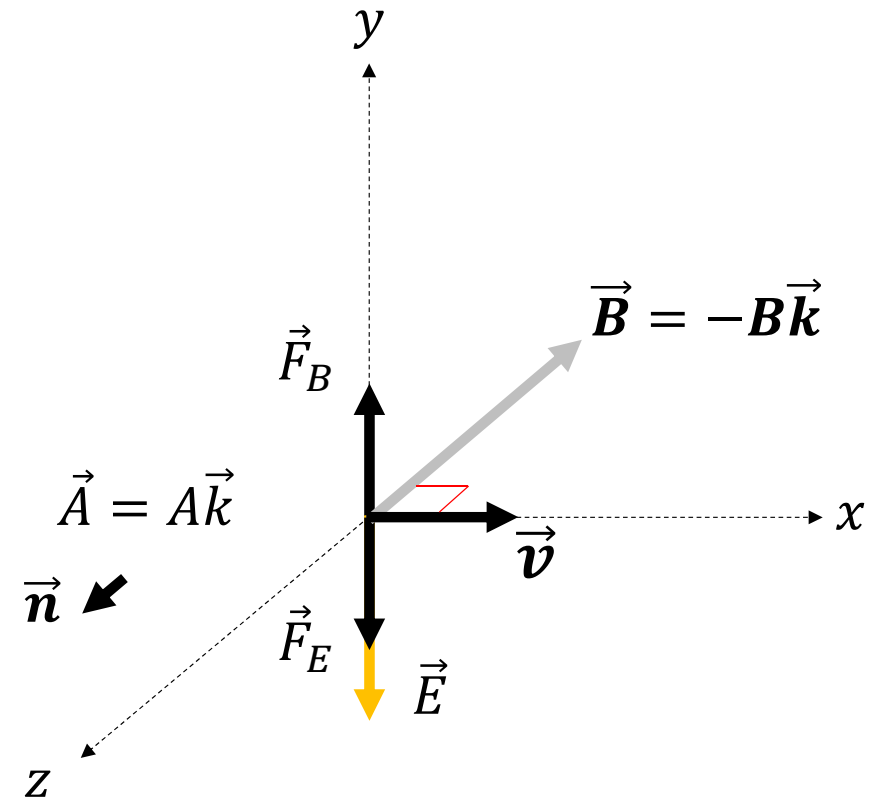
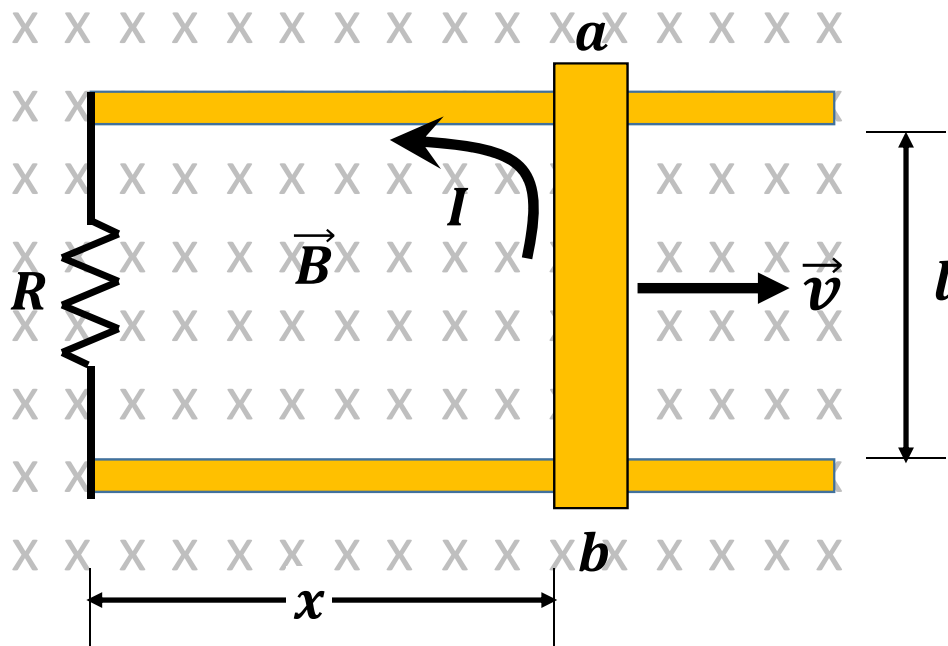
Moving conductor rod

Magnetic force drives charges to the ends of the moving rod from which they redistribute in the stationary U-shaped part

Moving rod = source of electromotive force

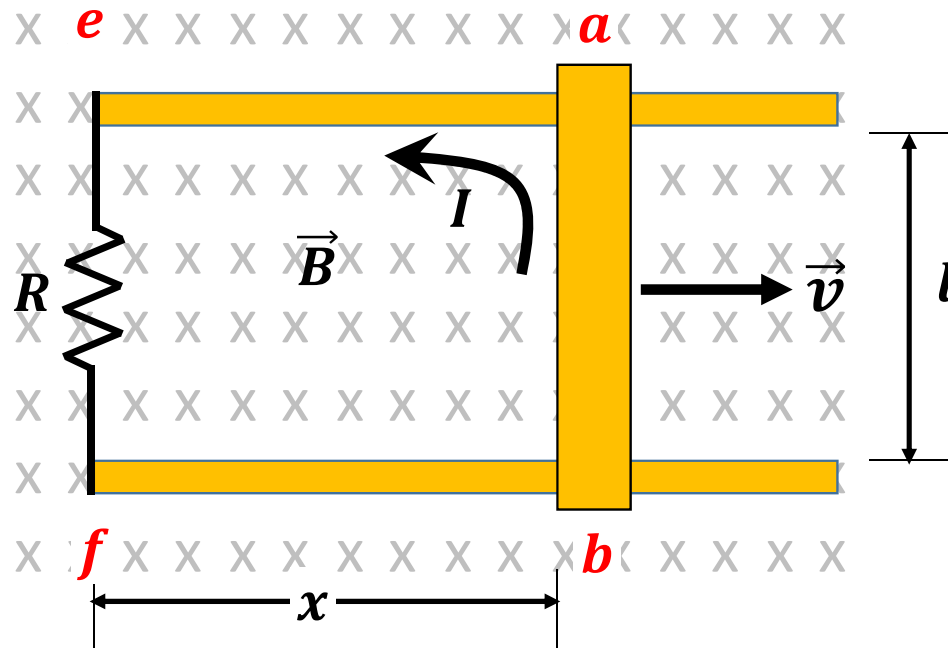
Faraday's law: electromotive force

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$



The magnetic flux changes because the area of the circuit is changing

$$\Phi_B = \vec{B} \cdot \vec{A} = -Blx \quad \Rightarrow \quad \frac{d\Phi_B}{dt} = \frac{d}{dt}(-Blx) = -Bl \frac{dx}{dt} \quad \Rightarrow \quad \frac{d\Phi_B}{dt} = -Blv$$



$$\frac{d\Phi_B}{dt} < 0$$

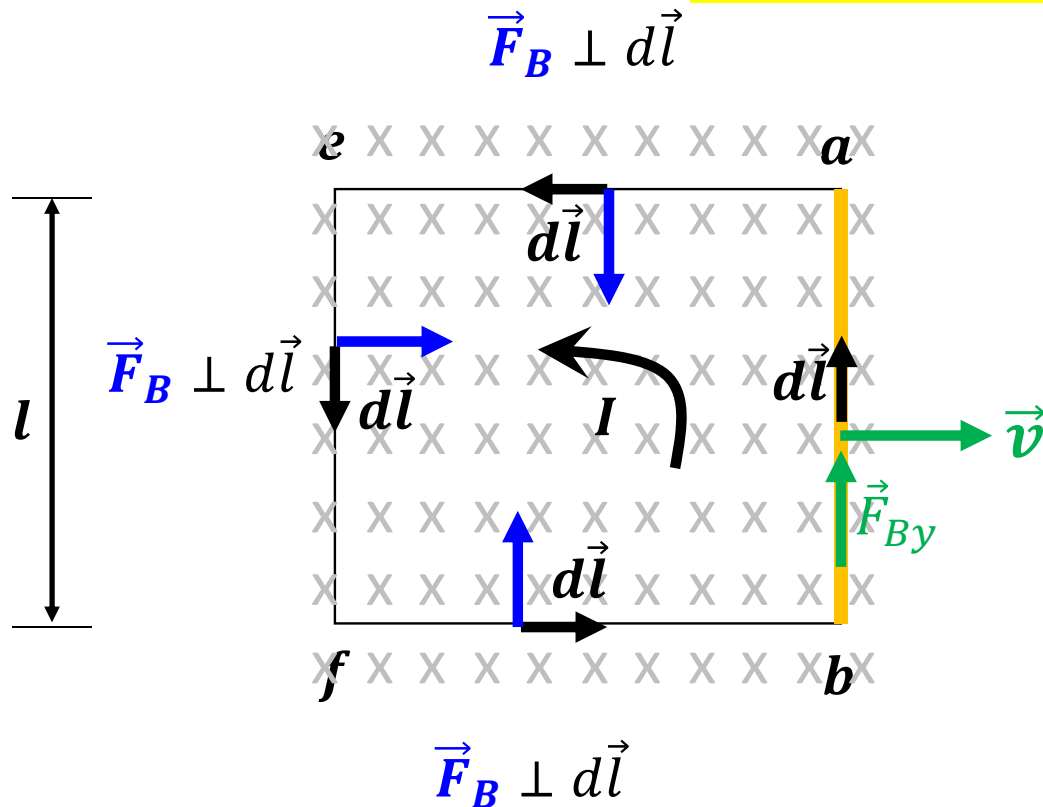
The closed path for the charges is ae**f**b

From slides # 17

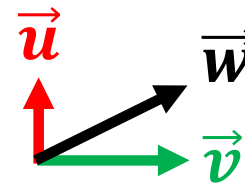
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

The magnetic field generated by the induced current opposes the original

The electromotive force

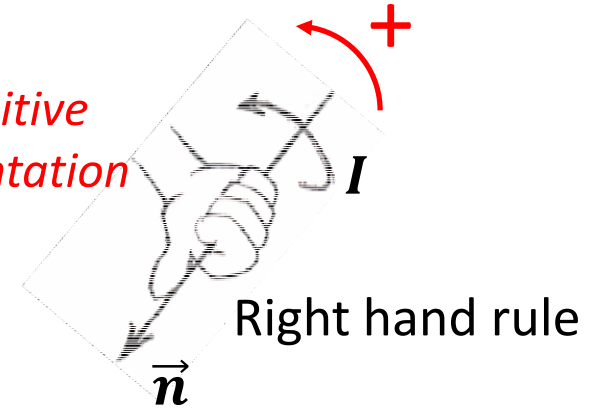


Conventional positive
trigonometric orientation



$$\begin{aligned}\vec{F}_{By} &= q\vec{v} \times \vec{B} = qv\vec{i} \times B(-\vec{k}) \\ &= qvB\vec{j}\end{aligned}$$

$\vec{F}_B \parallel d\vec{l}$ along the moving rod



The only contribution is along the moving bar

Slide #17

$$\mathcal{E} = \oint \frac{\vec{F}_{By}}{q} \cdot d\vec{l} = \oint vB\vec{j} \cdot dy\vec{j}$$

$$\mathcal{E} = Blv$$

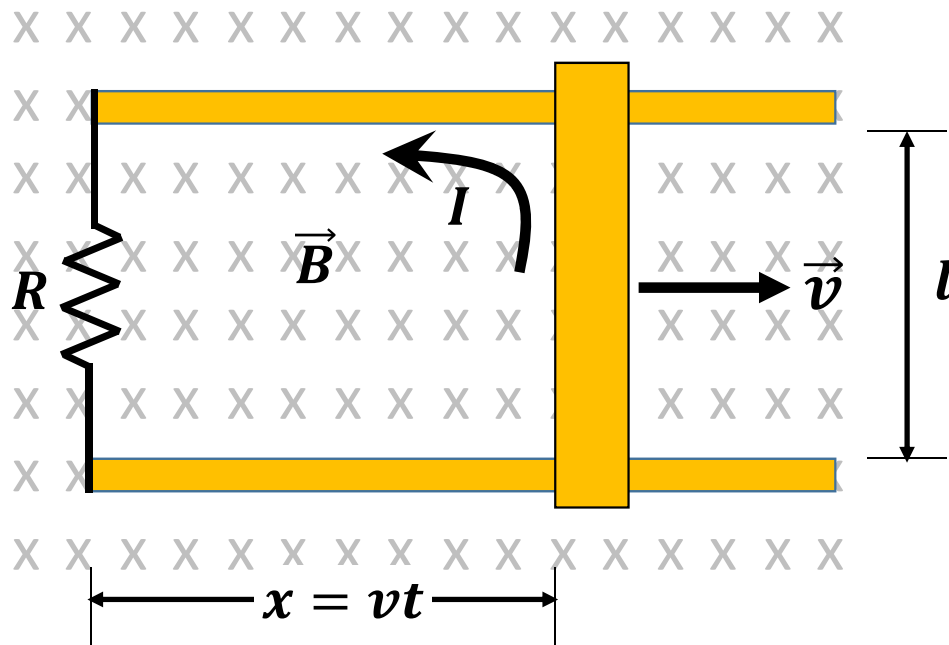
Final exam 2017

A metal bar of mass m slides without friction on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field B , pointing into the page, fills the entire region.

- 1) If the bar moves to the right at speed v , what is the current in the resistor and in what direction does it flow ?
- 2) What is the magnetic force on the bar and what is its direction?
- 3) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t ?

1) If the bar moves to the right at speed v , what is the current in the resistor and in what direction does it flow ?

1) The flux through the loop increases. The area of the loop is lvt , so that the flux and its time derivative through the loop is,



$$\Phi_B = -lvtB \rightarrow \frac{d\Phi_B}{dt} = -Blv$$

By Faradays law, this produces and emf in the loop, whose magnitude is

$$emf = -\frac{d\Phi_B}{dt} = +Blv = IR$$

What does the sign mean?

Circulation according to trigonometric orientation

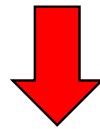
From which we can extract the current $I = +\frac{Blv}{R}$

2) What is the magnetic force on the bar and what is its direction?

$$\text{Laplace } F = Idl \times B$$

Direction of the force?

To the left otherwise we have discovered the perpetual motion or creation of energy from nothing !



The bar must slow down to ultimately stop

$$F = IlB = - \frac{B^2 l^2 v}{R}$$

Force directed to the left

3) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t ?

The second Newton's law

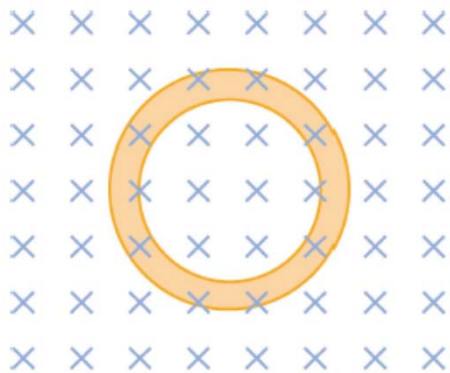
$$m \frac{dv}{dt} = - \frac{B^2 l^2 v}{R}.$$

$$v(t) = v_0 e^{-\frac{B^2 l^2 t}{Rm}}$$

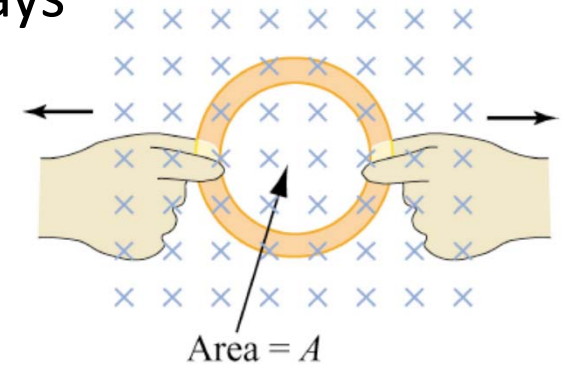
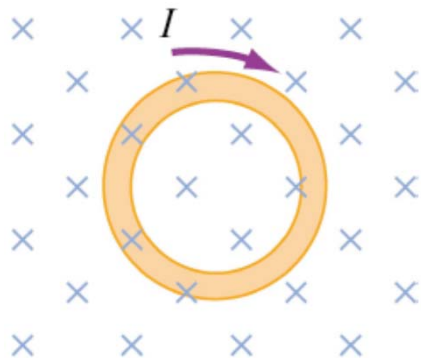
Stationary closed conducting path (coil)

The flux can change following two ways

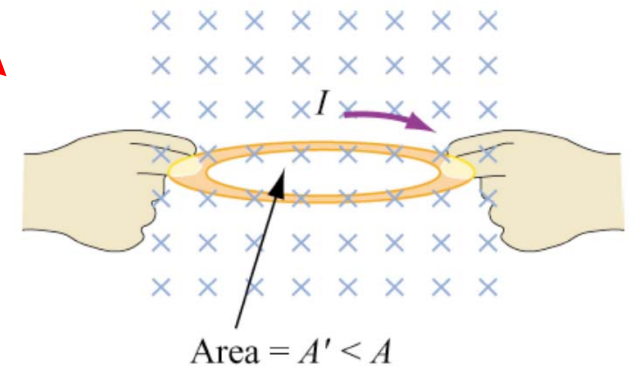
$$\Phi_B = \vec{B} \cdot \vec{A}$$



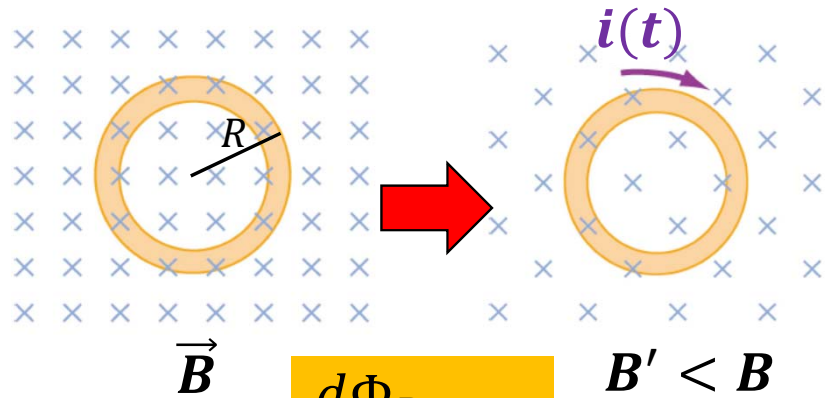
\vec{B} changes



\vec{A} changes



Genius conjecture of Faraday



$$\frac{d\Phi_B}{dt} < 0$$

The circuit is immobile $\Rightarrow \vec{v} = \vec{0} \Rightarrow \vec{F}_B = \vec{0}$

$$\vec{F} = q(\vec{E} + \underbrace{\vec{v} \times \vec{B}}_{\vec{0}})$$

$$\frac{d\Phi_B}{dt}$$

$$\vec{0}$$

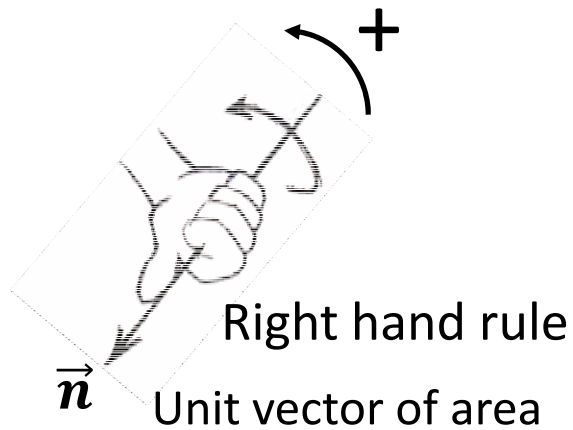
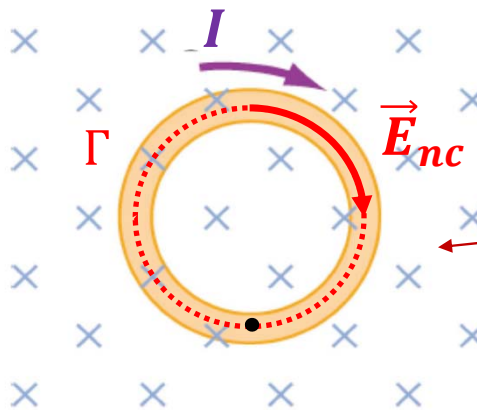
$$\mathcal{E}(\text{emf}) = \oint_{\Gamma} \vec{E}_{nc} \cdot d\vec{l}$$

Slide #17

Faraday's Conjecture

$$\vec{E} \perp \vec{B}$$

\vec{E}_{nc} circulates tangentially to the loop



Electromotive force for both motional and non motional case

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

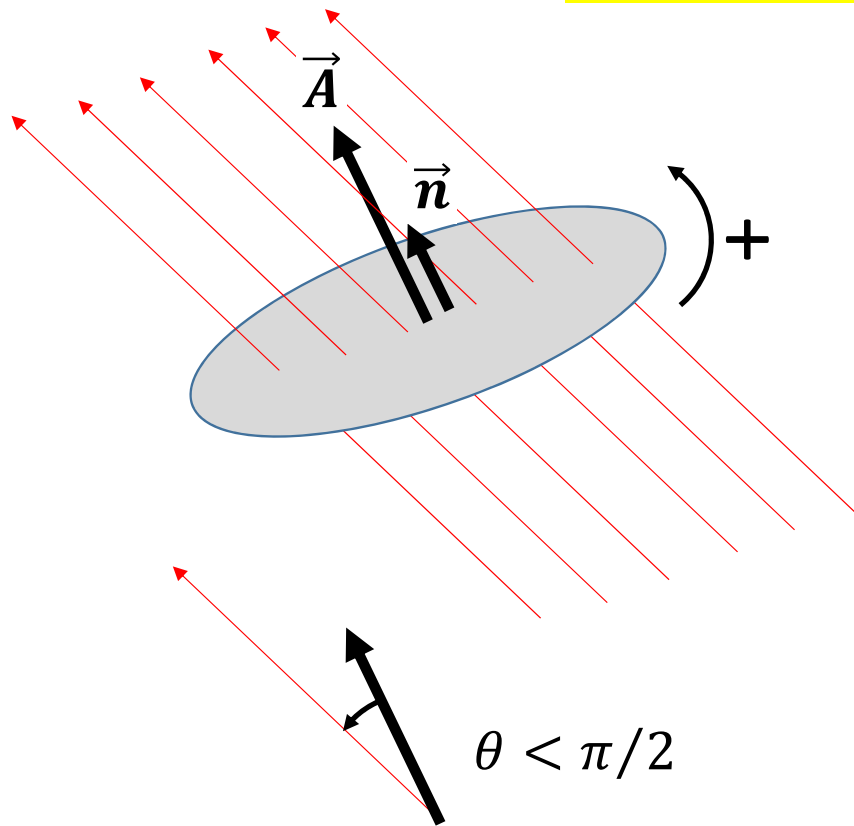
For motional electromotive force

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = \oint_{\Gamma} \vec{E}_{nc} \cdot d\vec{l}$$

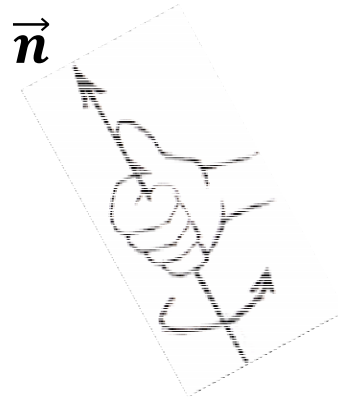
For non-motional electromotive force:
Faraday's law

Lenz's law

Lenz's law and sign convention for emf

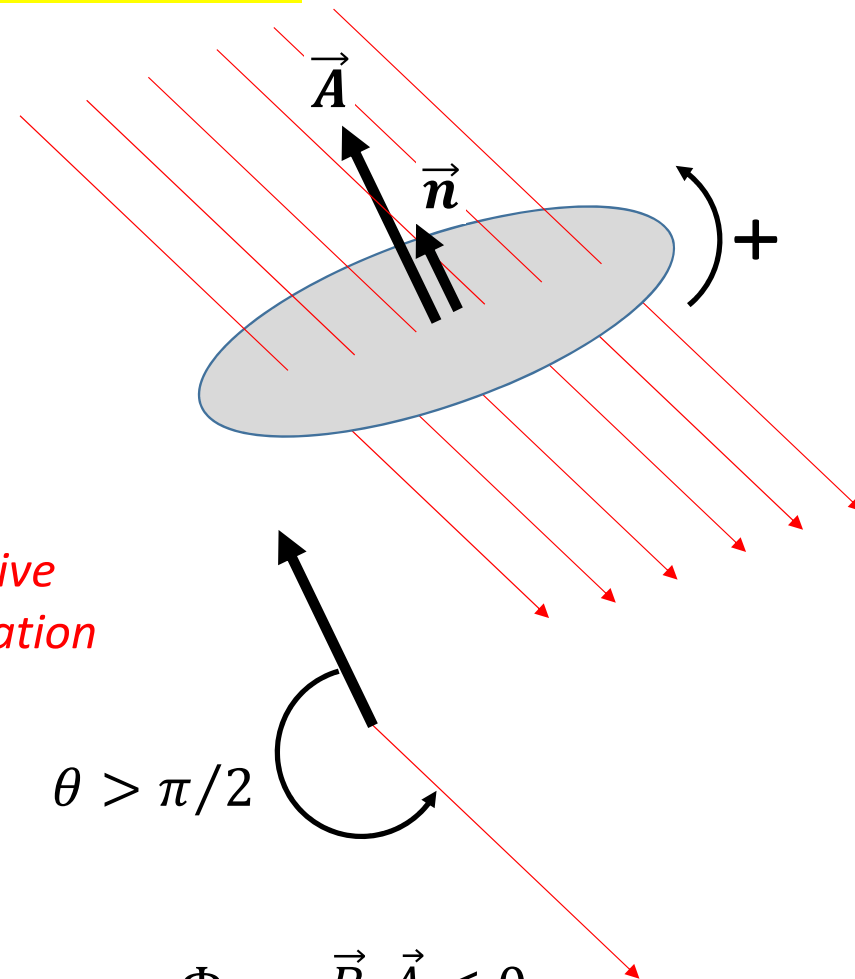


$$\Phi_B = \vec{B} \cdot \vec{A} > 0$$

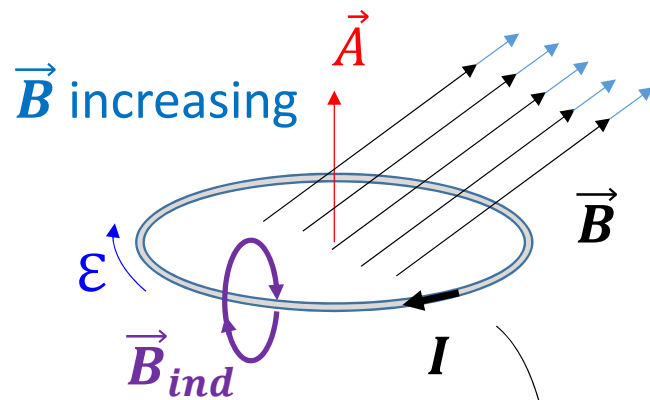


$\vec{A} \equiv \text{thumb}$

*Conventional positive
trigonometric orientation*



$$\Phi_B = \vec{B} \cdot \vec{A} < 0$$

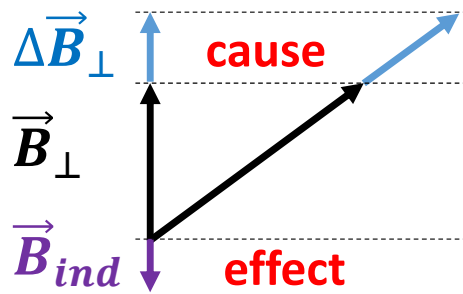
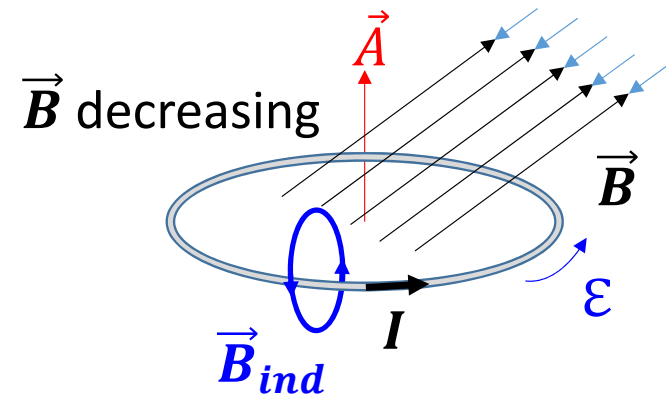


$$\cos\theta > 0$$

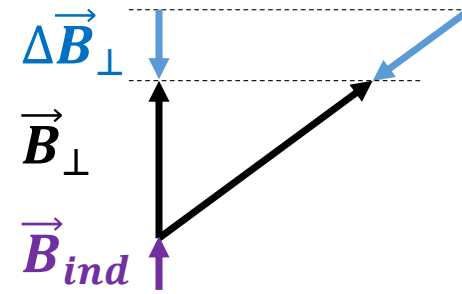
$$\Phi_B > 0$$



Trigonometric orientation



$$\frac{d\Phi_B}{dt} > 0$$



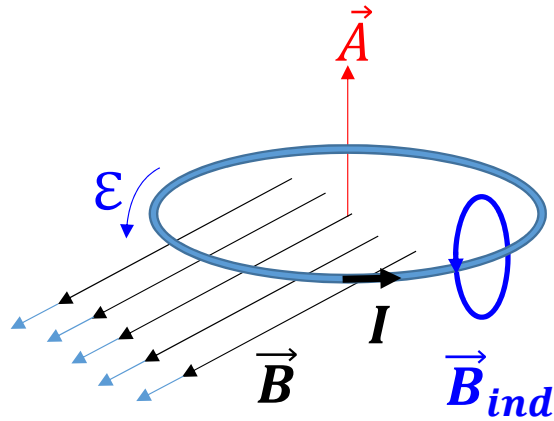
$$\frac{d\Phi_B}{dt} < 0$$

Induced emf \Rightarrow induces current \Rightarrow induces \vec{B}_{ind} \Rightarrow opposing the **cause of the effect**

$$\cos\theta < 0$$

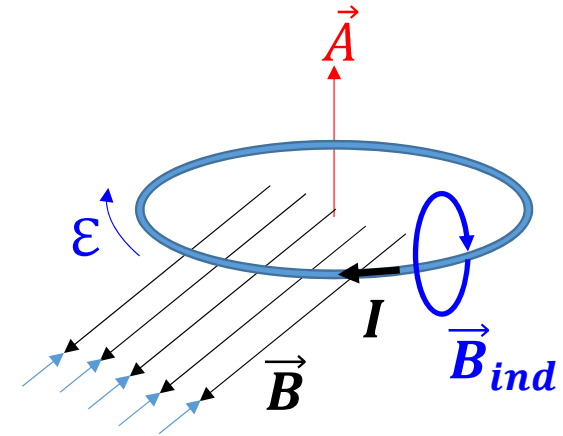
$$\Phi_B < 0$$

$\left. \vphantom{\int} \right) +$
Trigonometric orientation



\vec{B} increasing

$$\frac{d\Phi_B}{dt} < 0$$



\vec{B} decreasing

$$\frac{d\Phi_B}{dt} > 0$$

\vec{B}_\perp parallel \vec{A}

$$\Phi_B = \vec{B} \cdot \vec{A} > 0$$

$$\theta < \pi/2$$

$$\frac{d\Phi_B}{dt} > 0$$

$$\frac{d\Phi_B}{dt} < 0$$

\vec{B}_\perp anti-parallel \vec{A}

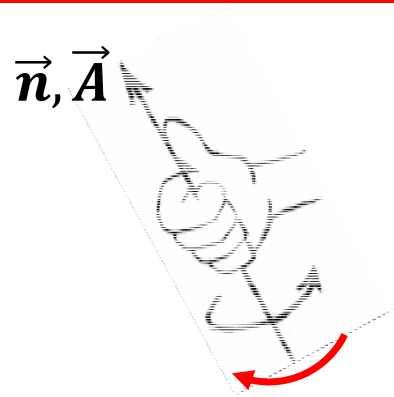
$$\Phi_B = \vec{B} \cdot \vec{A} < 0$$

$$\theta > \pi/2$$

$$\frac{d\Phi_B}{dt} > 0$$

$$\frac{d\Phi_B}{dt} < 0$$

\vec{n}, \vec{A}

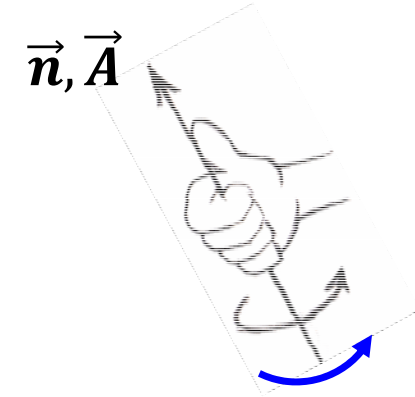


$$\varepsilon < 0$$

$\vec{A} \equiv \text{thumb}$

$$\varepsilon > 0$$

\vec{n}, \vec{A}



$\vec{A} \equiv \text{thumb}$

Summarizing Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



$$\oint \vec{E}_{nc} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$



$$\oint \underbrace{(\vec{E}_c + \vec{E}_{nc})}_{\vec{E}} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$



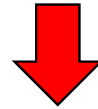
Stoke's theorem

+

Gauss's theorem

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday



Varying **magnetic field** gives rise to an induced **electric field**



Maxwell

Remarkable symmetry of nature

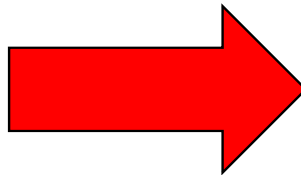


Varying **electric field** gives rise to an induced **magnetic field**

Varying **electric field** gives rise to an induced **magnetic field**

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J}$$

Ampere's law



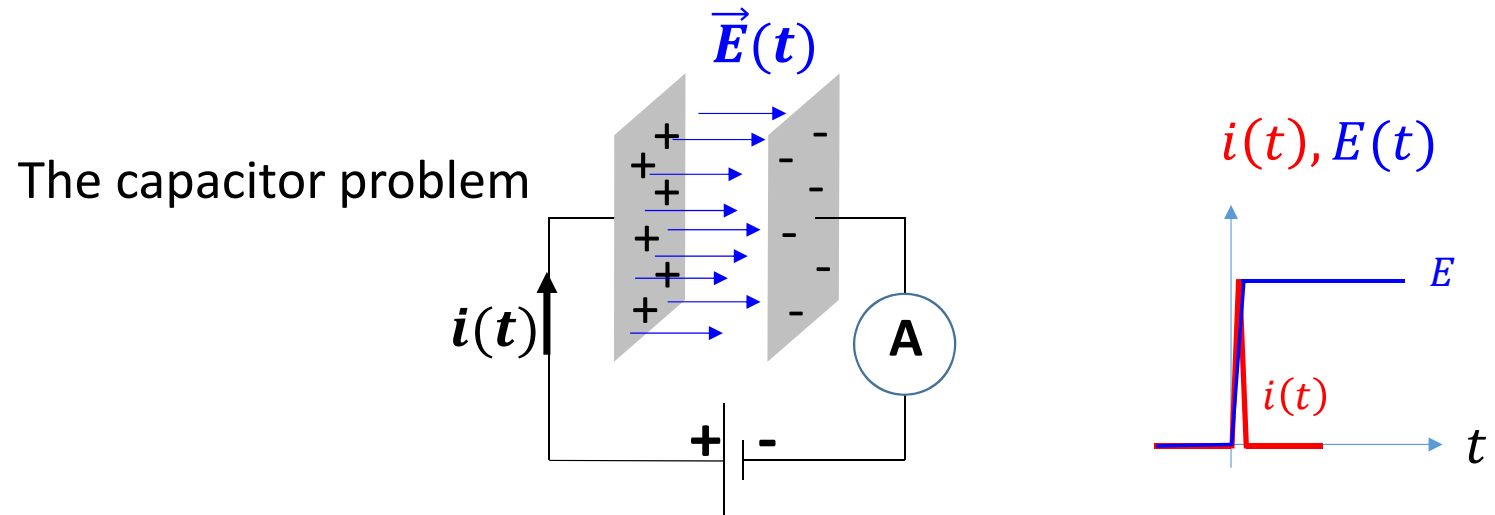
$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

Maxwell's law

The term that brings symmetry

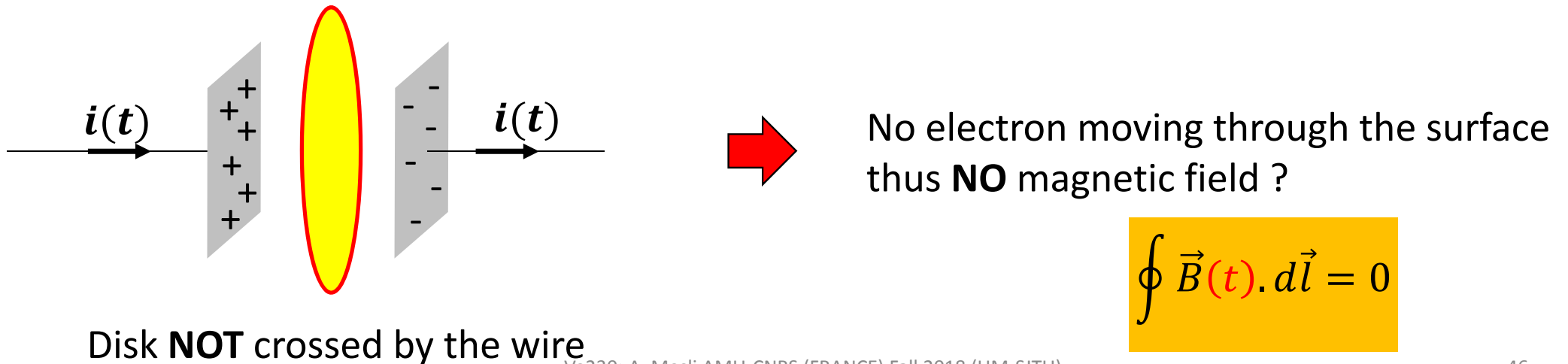
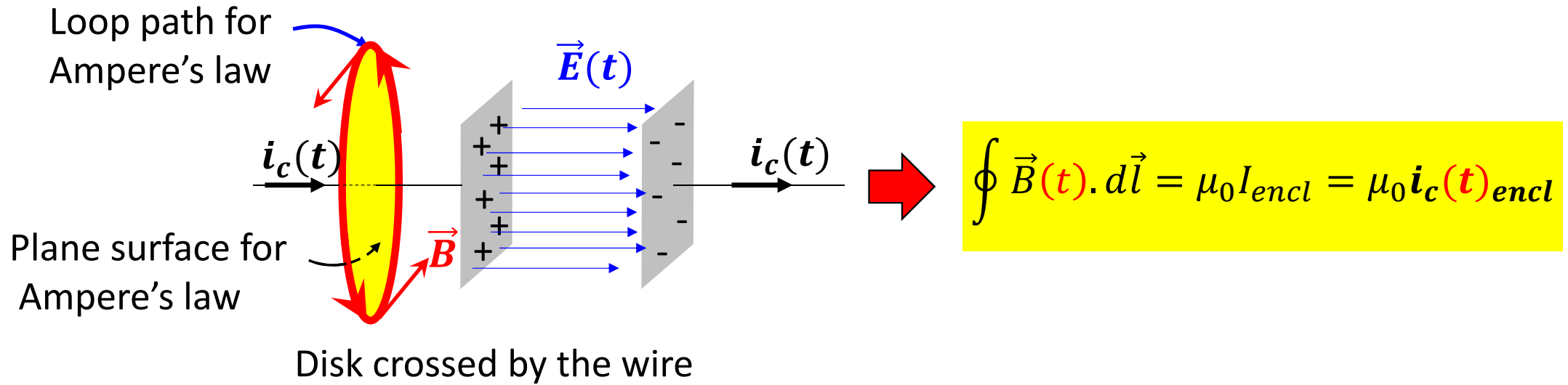
Displacement current

Displacement current or how Maxwell saved Ampere's law

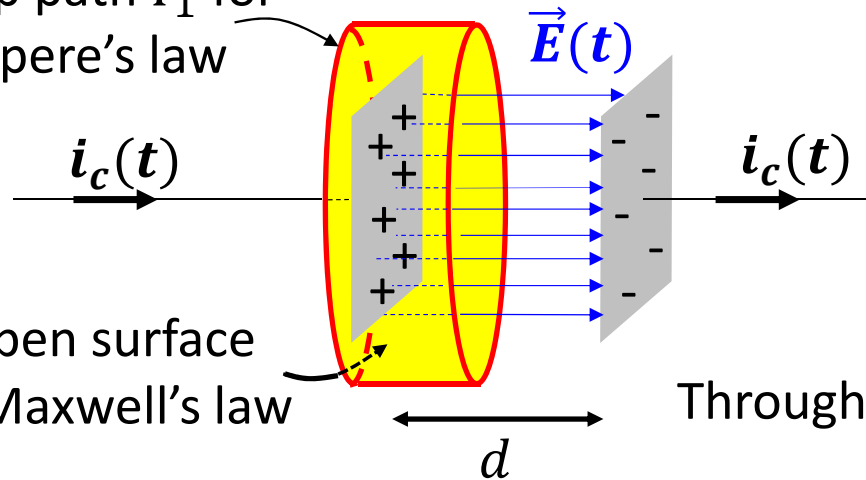


This is not a closed loop **BUT** a current $i(t)$ flows in the circuit
Clearly no electron is jumping from one plate to the other !

Displacement current or how Maxwell saved Ampere's paradoxical law



Loop path Γ_1 for
Ampere's law



Cylinder open on the left hand side =
Maxwell's surface.

The wire does not cross Maxwell's surface

3D open surface
for Maxwell's law

Through Maxwell's surface a flux $\Phi_E(t)$ is building-up

$$Q(t) = CV(t) = \frac{\epsilon_0 A}{d} E(t) \cancel{d} = \epsilon_0 \boxed{AE(t)} = \epsilon_0 \boxed{\Phi_E(t)}$$

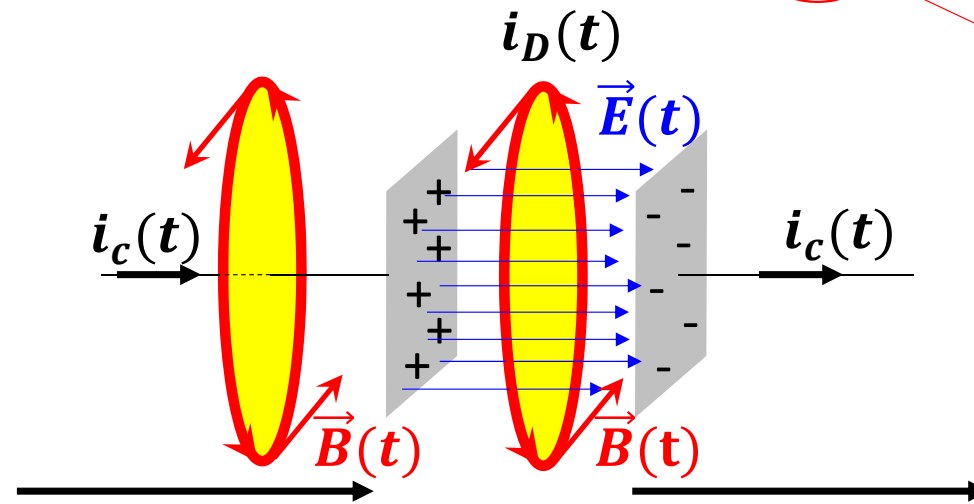
$$i(t) = \frac{dQ(t)}{dt} = \epsilon_0 \frac{d\Phi_E(t)}{dt}$$

$i_D(t)$

Maxwell's genius idea ➡ $i_D(t) = \epsilon_0 \frac{d\Phi_E(t)}{dt}$

Final step: Fourth Maxwell's equation

$$\oint \vec{B}(t) \cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 [i_c(t) + i_D(t)]_{encl}$$



$$\mu_0 i_D(t) = \mu_0 \epsilon_0 \frac{d\Phi_E(t)}{dt}$$

\uparrow
 c^2


Continuity ➔

$i_c(t) \neq 0$	$i_c(t) = 0$	$i_c(t) \neq 0$
$i_D(t) = 0$	$i_D(t) \neq 0$	$i_D(t) = 0$
$\vec{E}(t) = 0$	$\vec{E}(t) \neq 0$	$\vec{E}(t) = 0$

What is the major outcome of Maxwell's contribution?

Time-varying field of either kind induces a field of the other kind in neighboring region

Towards the prediction of electromagnetic **disturbances** consisting of \vec{E} and \vec{B} and propagating in free space without requiring any medium



Waves

How did Maxwell's correct Ampere?

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law ...

... completed by Maxwell

$$\oint \vec{B}(t) \cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 [i_c(t) + i_D(t)]_{encl} \quad \rightarrow \quad \oint \vec{B}(t) \cdot d\vec{l} - \mu_0 i_D(t)_{encl} = \mu_0 i_c(t)_{encl}$$

$$i_D(t) = \varepsilon_0 \frac{d\Phi_E(t)}{dt} \quad \rightarrow \quad \oint \vec{B}(t) \cdot d\vec{l} - \mu_0 \varepsilon_0 \frac{d\Phi_E(t)}{dt} = \mu_0 i_c(t)_{encl}$$

Stoke's theorem

+

Gauss's theorem

$$\oint \vec{B} \cdot d\vec{l} = \iint (\vec{\nabla} \times \vec{B}) \cdot d\vec{A}$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{A} = \mu_0 \varepsilon_0 \iint \frac{d\vec{E}}{dt} \cdot d\vec{A}$$

$$\vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$$

This is all about the story with Maxwell's equations

$$\vec{\nabla} \cdot \vec{E}(t) = \frac{\rho(t)}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B}(t) = 0 \quad \text{No magnetic charges or monopoles}$$

Gauss' law

$$\vec{\nabla} \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

Faraday's law

$$\vec{\nabla} \times \vec{B}(t) = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

Ampere's law completed by Maxwell

In electric field, the stored energy is:

$$U = \frac{\epsilon_0}{2} \int E^2 dV$$

All space

$$\frac{\epsilon_0}{2} E^2$$

In a magnetic field, the stored energy stored:

$$U = \frac{1}{2\mu_0} \int B^2 dV$$

All space

$$\frac{1}{2\mu_0} B^2$$