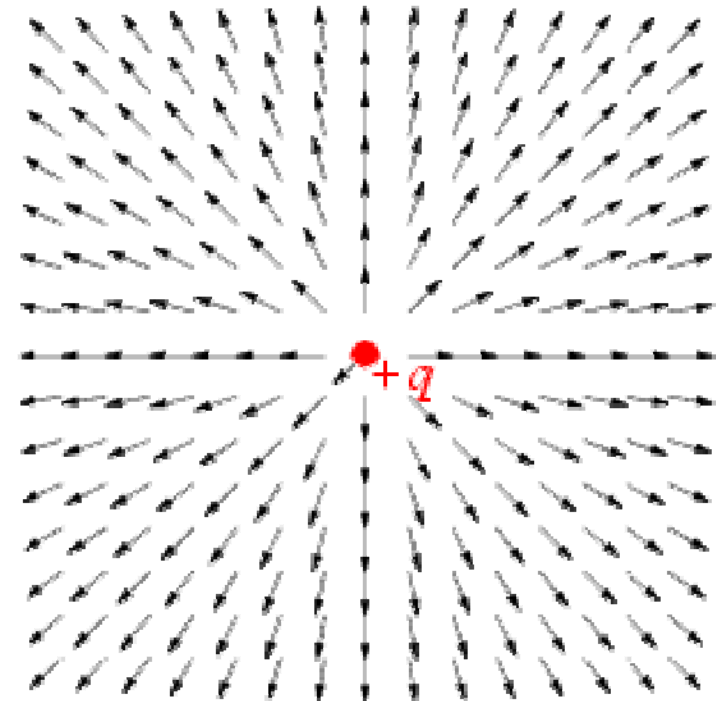
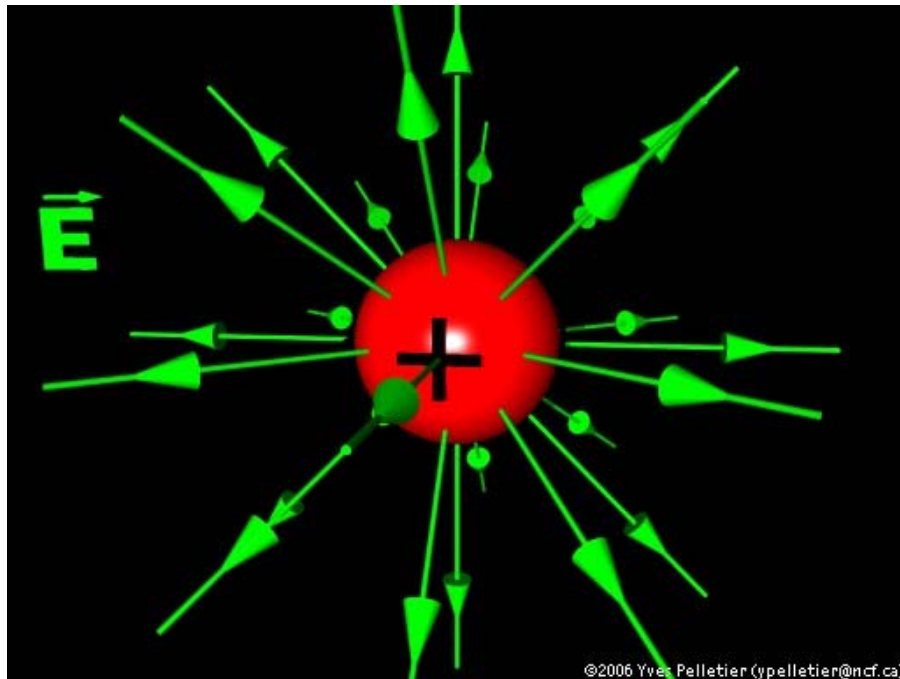
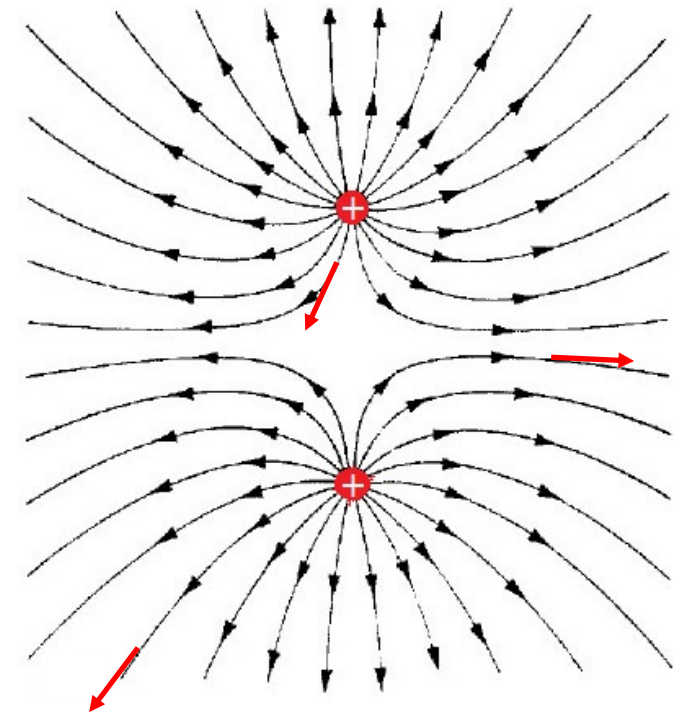
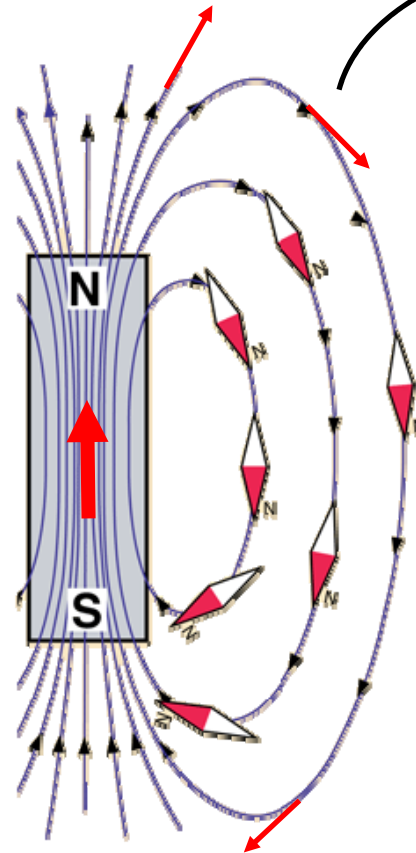
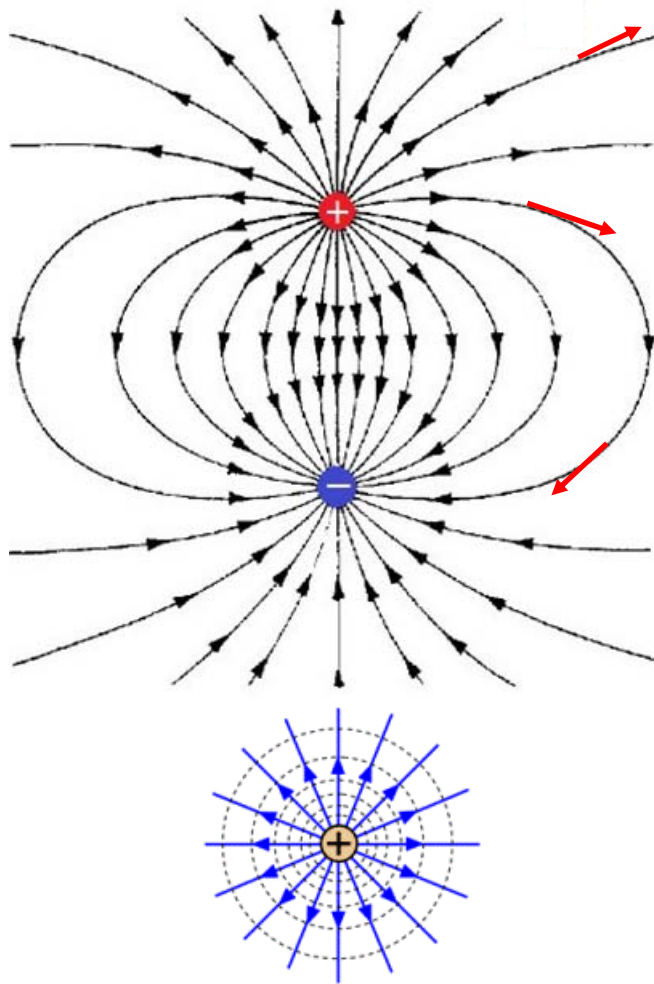


Electric field of a single charge



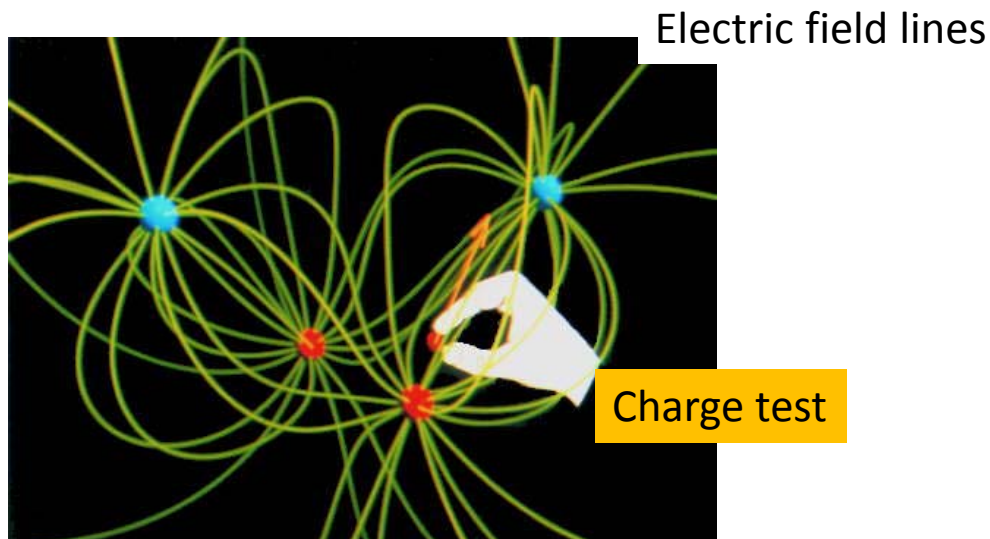
Field lines keep track of the direction of the field

Lines are going from N to S outside the magnet
BUT from S to N inside the magnet !



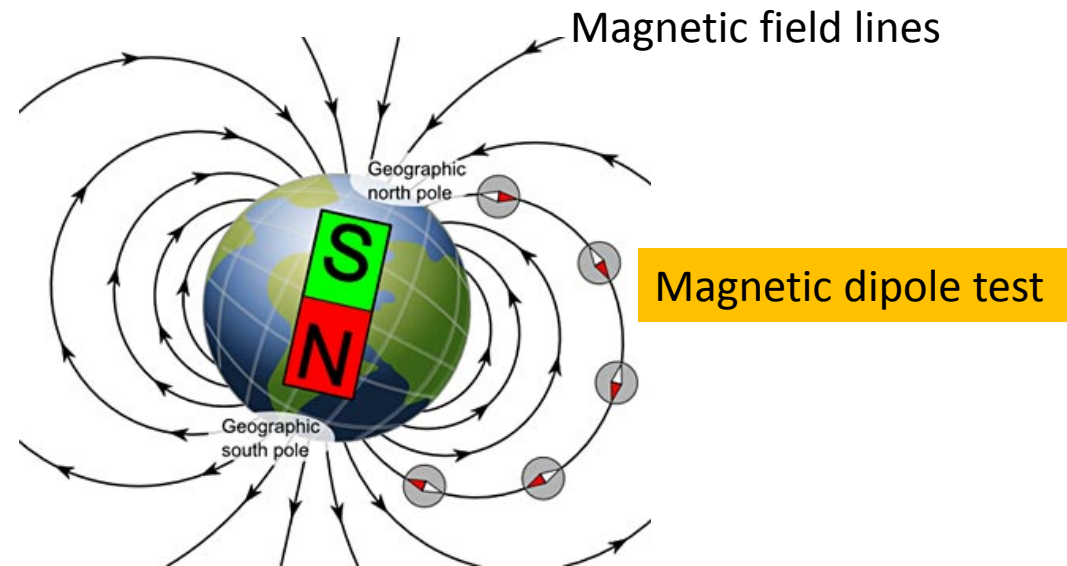
Field vectors are always tangent to the field lines

Action at distance: Electric field of multiple charges and Magnetic field of a dipole magnet



We can imagine how complicated will be the trajectory of a test charge left in this field !

A non negligible test charge may perturb these field lines !



Major difficulty with field line concept

They do not fit with superposition principle



We add vectors

$$\vec{E}_{tot} = \sum_i \vec{E}_i$$

How can we add field lines?

Major difficulty with field line concept

Two charges moving in space, both at the same speed and parallel to each other

For a static observer: There is a magnetic field thus magnetic field lines



For an observer moving with the same speed: Do the field lines move with the charges?

Impossible to say as the field disappears

Best: Restrict to the abstract idea of field vector and scalar

$$\vec{E}(x, y, z, t), \vec{B}(x, y, z, t), \varphi(x, y, z, t), T(x, y, z, t) \text{ etc...}$$

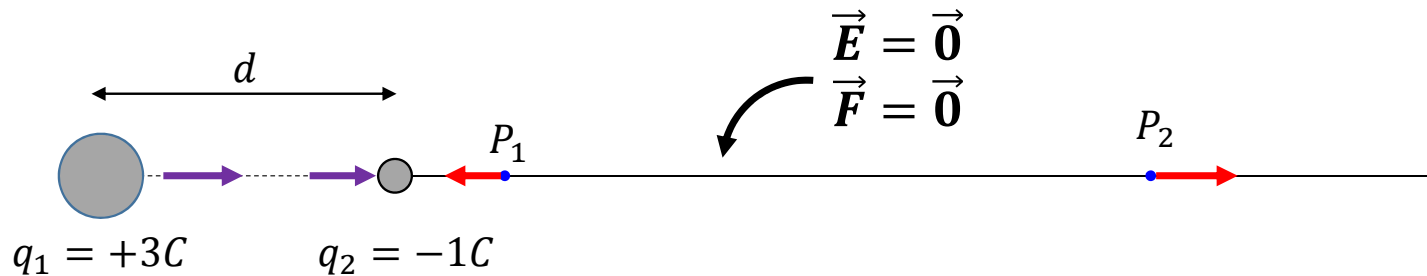
According to Faraday and Maxwell

Fields  $\vec{E}(x, y, z, t), \vec{B}(x, y, z, t)$  'sphere of influence'

- Something which propagates in space
- Something which stores energy and momentum,
“Substance” that mediates interactions between bodies

 To be taken with great care

Two charges system: Towards dipoles



What does a charge test $+q$ feel at P_1 and P_2 ?

Consequence of Coulomb's law

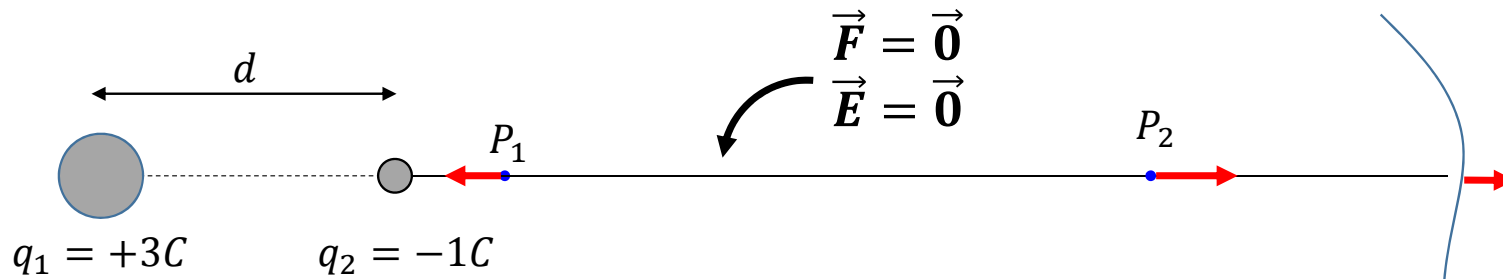
- At P_1 the test charge feels mostly the **attractive** force from q_2 (because of $1/r^2$)
- At P_2 ($r \gg d$) the test charge feels mostly the **repelling** force from $(q_1 - q_2) = +2C$

Can we guess what happens between P_1 and P_2 ?

- Somewhere in between P_1 and P_2 the field must be zero thus the force: Why?

Because the fields are in opposite directions at P_1 and P_2

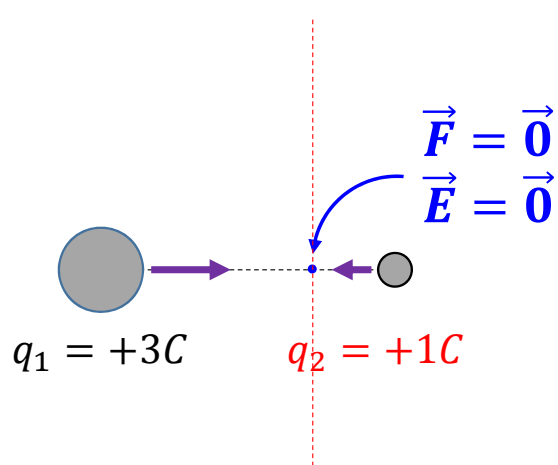
Two charges system: Towards dipoles



In which direction does the field point far away from the two charges ?

Very far away it points outward

$$1/r^2$$



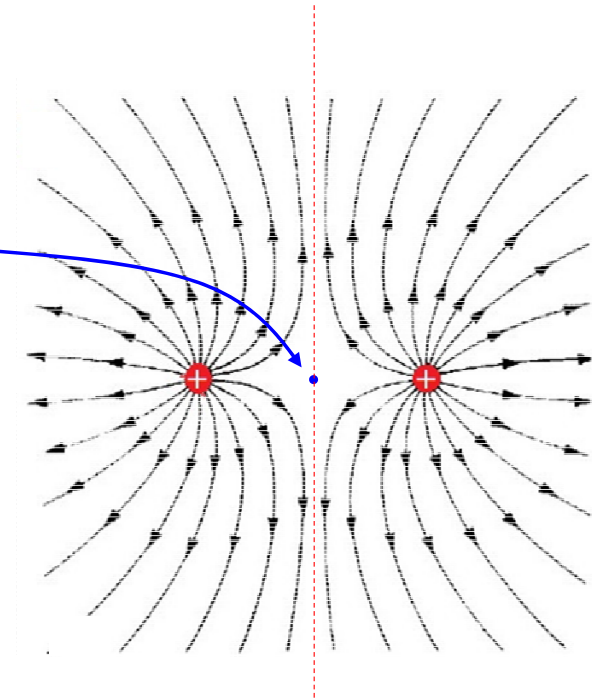
Are these locations considered as **points of equilibrium**?

A point near the moon in the line joining the moon and the earth the gravitation is zero



Notice that here the charges **repel** each other
While the moon and the earth **attract** each other

$\vec{F} = \vec{0}$
 $\vec{E} = \vec{0}$



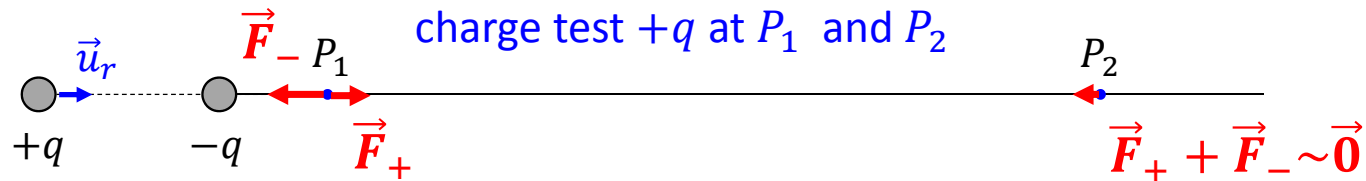
Is the field zero all along these lines?

No !

Consequence for a dipole

For a single charge $q \Rightarrow \vec{E} \propto \frac{1}{r^2} \vec{u}_r$

What about if we consider a dipole ?



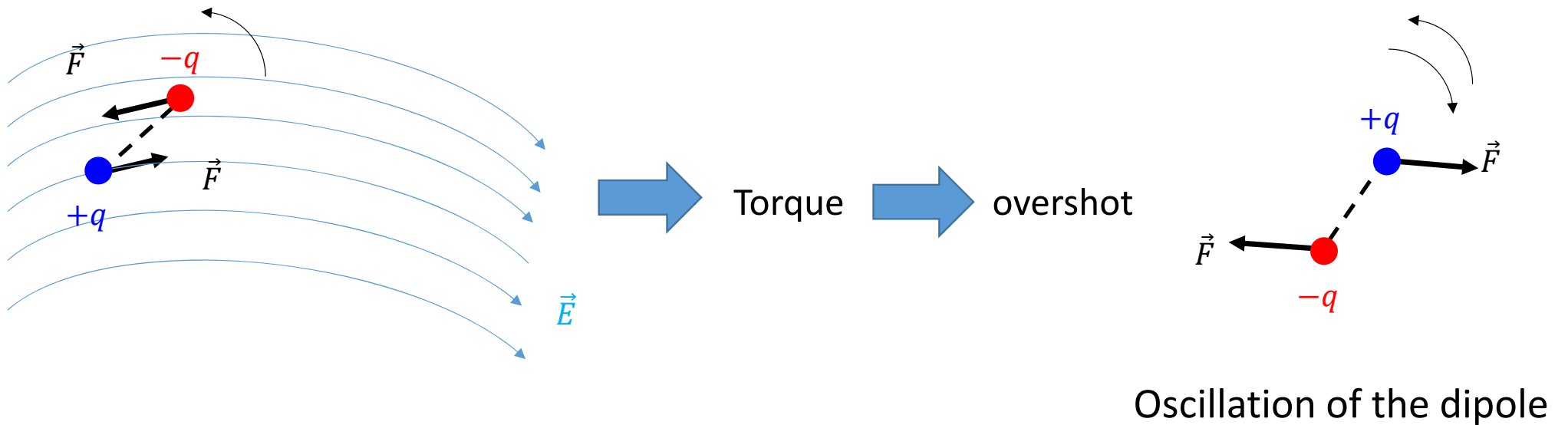
$\Rightarrow \vec{E} \propto \frac{1}{r^n} \vec{u}_r$ **$n > 2$** The field strength falls off much more rapidly

Is there a point in space where the field is zero?

There is **NOT** a single point in space where the field is zero

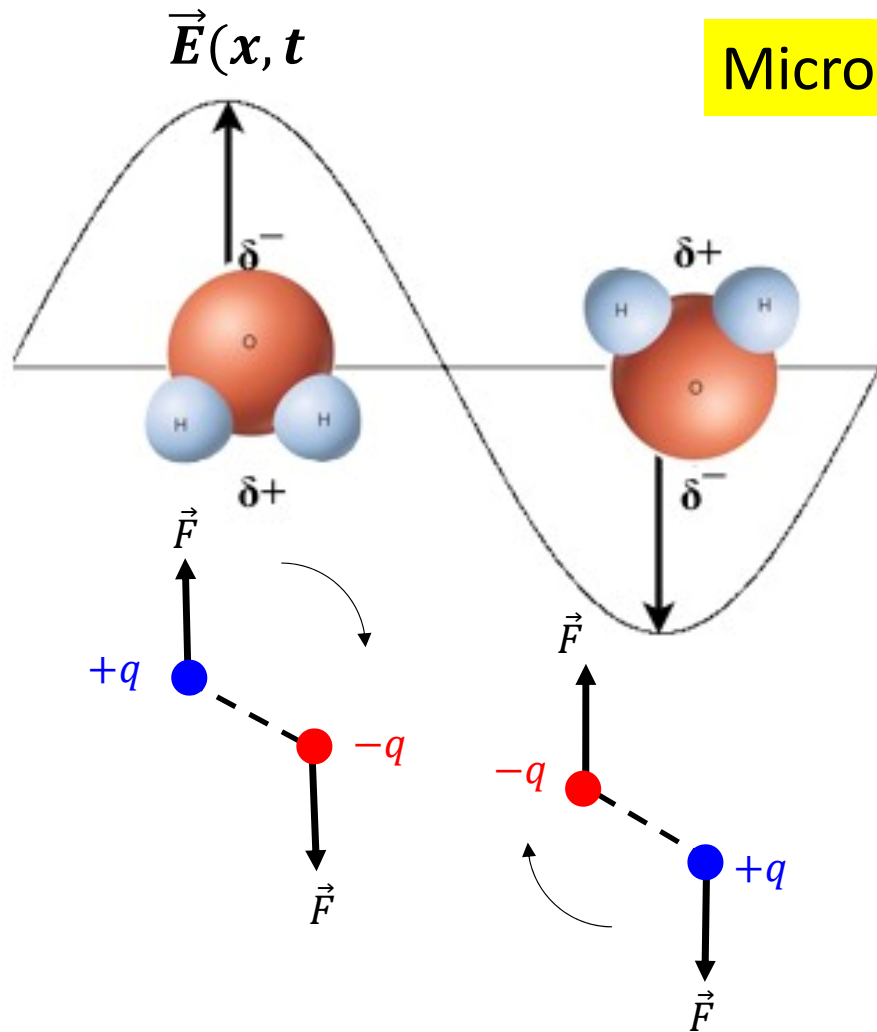
The dipole is a very interesting and useful system

Any symmetric molecule will convert into a dipole once place in an electric field



Can we get something useful from this mechanism?

Microwave oven: 2.45 GHz



Oscillation of the Electric field



Continuous rotation of the molecule



Gain of energy



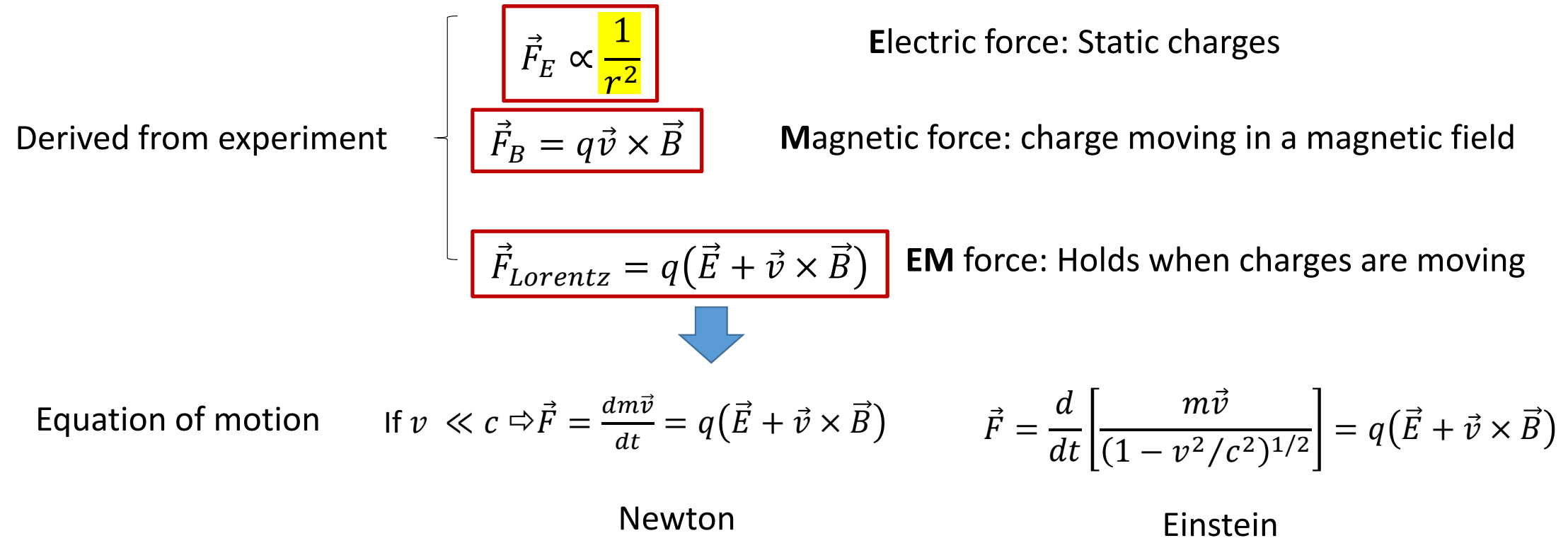
Inelastic collisions with neighboring molecules



Energy transfer

Newtonian mechanism and Maxwell's electromagnetism at once!

Summary of all forces acting on charges



Knowing the Electric and magnetic forces \Rightarrow knowing the motion

How are these forces produced ? \Rightarrow VE230

Force acting on an electron

Mechanical (Newtonian) force

$$\vec{F} = m\vec{a}$$

Pulling a conducting wire



This guy is setting mechanically electrons into motion

The force acting on the electron is in the same direction and parallel to \vec{v}

What happens if a magnet is nearby?

- Charges (electrons) can be set into motion by moving **mechanically** the conducting wire in which the electrons are free to move

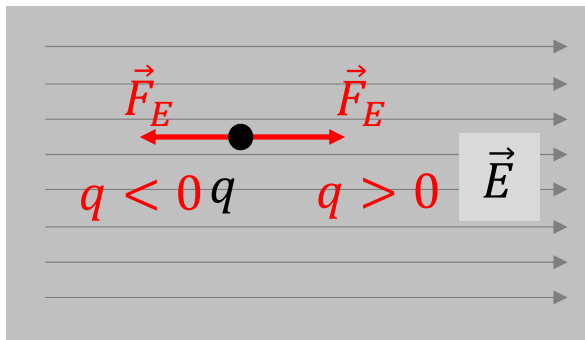


- Reversibly by setting **electrically** into motion free electrons inside the conducting wire, the whole wire can be set into motion **if a magnet is nearby**

Lorentz force acting on a charge

Lorentz force

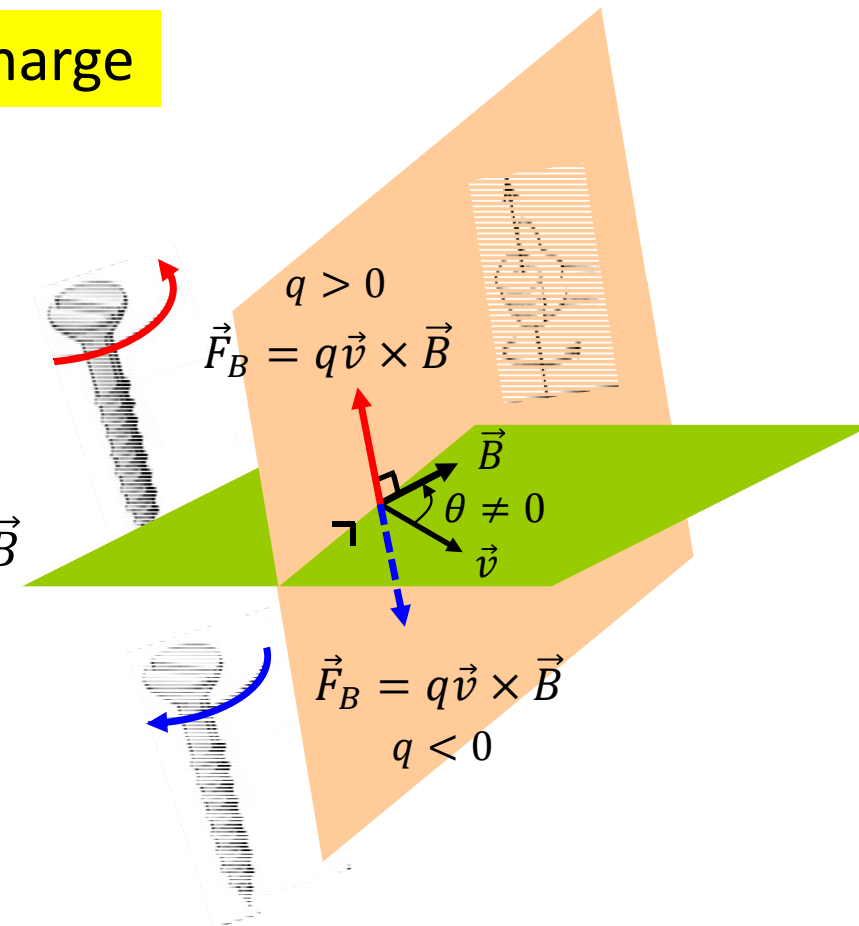
$$\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$$



The electric force acting on the charge is parallel to \vec{E}

$$\vec{F} = q\vec{E}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$



The magnetic force acting on the charge is perpendicular to \vec{v} and \vec{B}

\vec{E} acts on a **static** and **moving** charge
 \vec{B} acts on a moving charge **ONLY**

Concept #8: Work and Energy

As for the gravitational force, electrostatic force is **conservative**: It derives from the potential energy

$$F_E = -\frac{d\varphi}{dx} \quad \longrightarrow \quad d\varphi = -F_E dx \quad \longrightarrow \quad \varphi(b) - \varphi(a) = -\int_a^b F_E dx$$

From Newton

$$F = m \frac{dv}{dt} \quad \longrightarrow \quad F dx = m \frac{dv}{dt} dx = m \frac{dx}{dt} dv = m v dv \quad \longrightarrow \quad K(b) - K(a) = \int_a^b F dx$$

Miracle

If the only acting force is the conservative one $\longrightarrow F = F_E$

$$K(b) - K(a) = -[\varphi(b) - \varphi(a)] \quad \longrightarrow \quad K(b) + \varphi(b) = K(a) + \varphi(a) \quad \text{Conservation of energy}$$

A conservative force ALWAYS acts to “push” the system toward lower energy

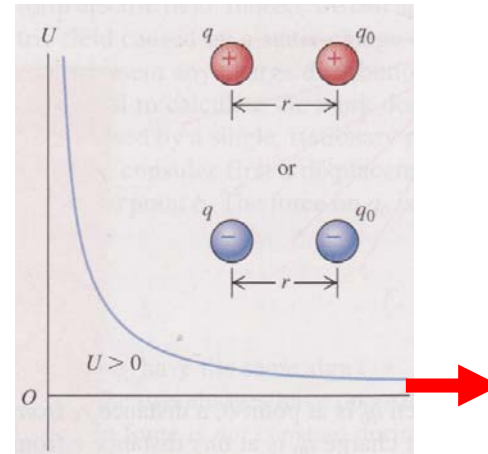


Energy is lowered when the charges are at **infinite** distance

Nature favors situation with minimum energy



Energy is lowered when the charges are as **close** as possible

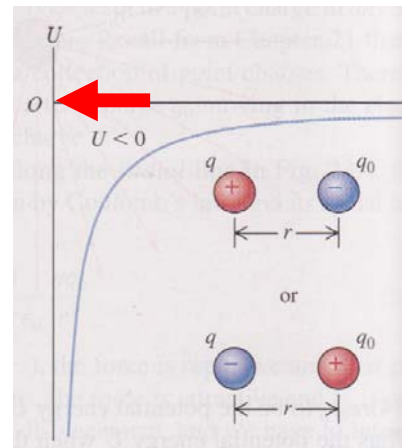


From University Physics, 11th edition

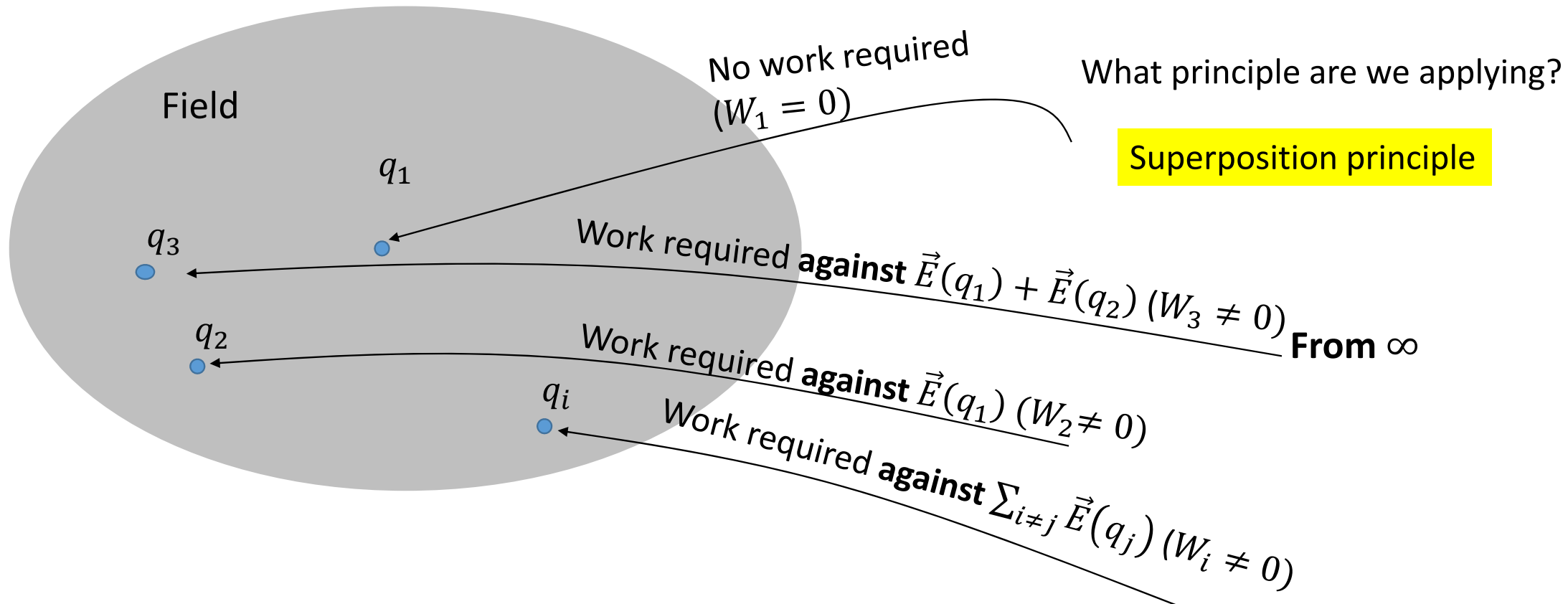
Nucleus predicted unstable !

Classical mechanics

Atom predicted unstable !

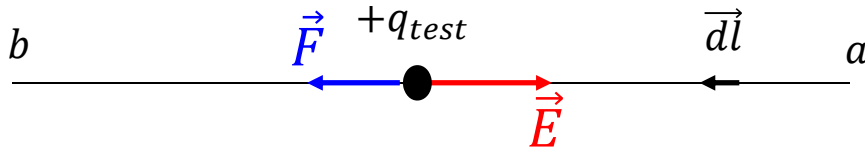


From empty space to space with imported charges



Why does an external force do a work **AGAINST** the electric field?

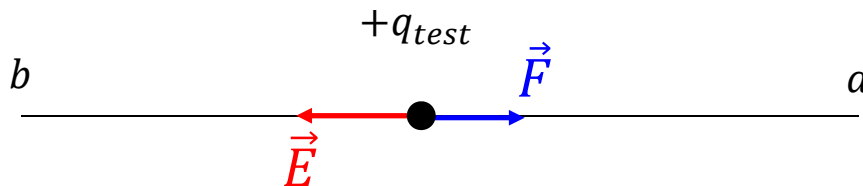
Work required to assemble charges



We push harder and harder **Against** the repulsion

$$\frac{W_{a \rightarrow b}}{q_{test}} = - \int_a^b \vec{E} \cdot d\vec{l}$$

any path



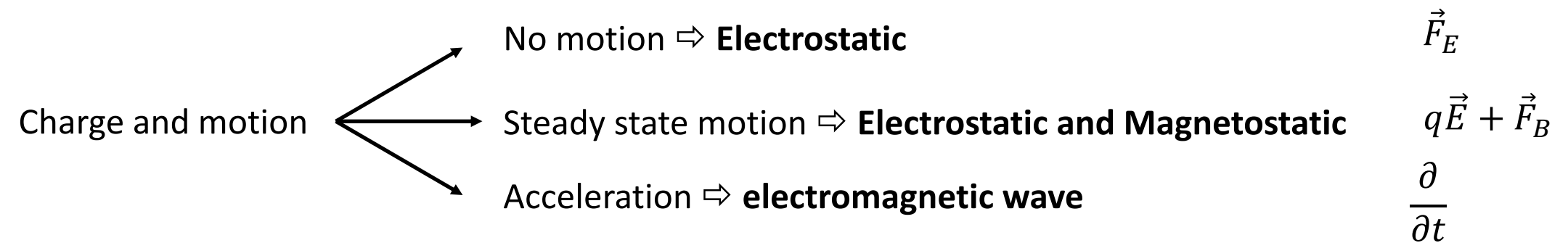
We retain harder and harder **Against** the attraction

$$\vec{E} = \frac{\vec{F}}{q_{test}} \text{ is conservative field}$$



$$\frac{W_{a \rightarrow b}}{q_{test}} = \varphi(b) - \varphi(a) = - \int_a^b \vec{E} \cdot d\vec{l}$$

The electric potential φ



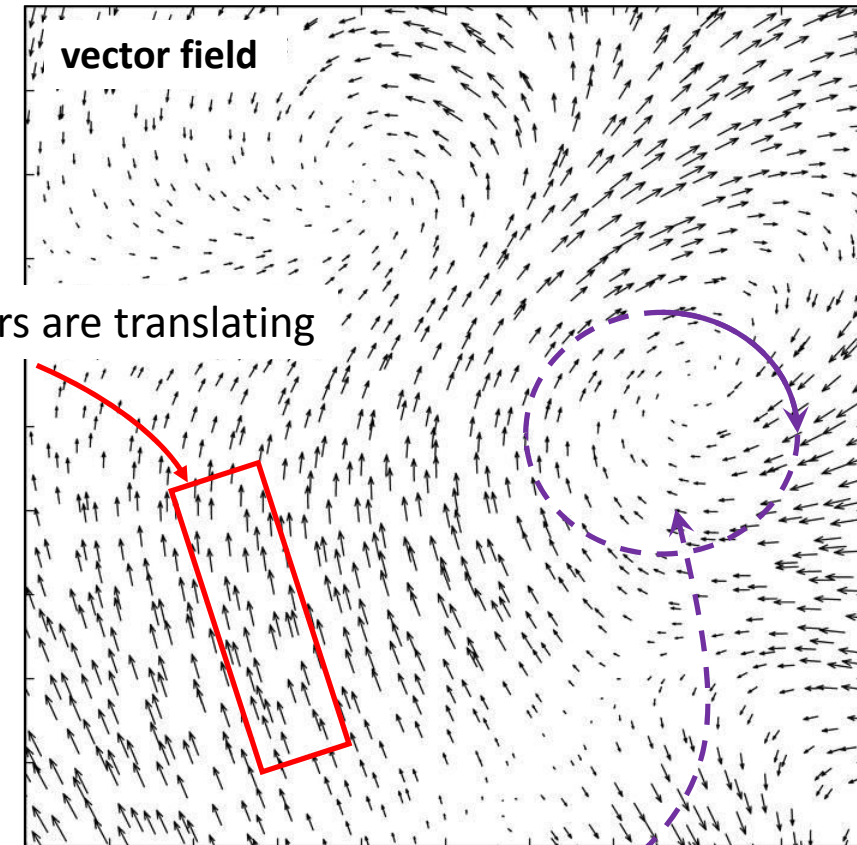
Concepts of flux and circulation of the vector fields: two fundamental properties

Vector field may undergo

- Translation (vibration is a kind of translation)
- Rotation (circulation along a curved path)

- 1) Flux through a surface (closed **OR** not)
- 2) Circulation along a curved path (closed **OR** not)

- Flux
 - Circulation
- ➡ Describe the laws of electromagnetism



Vectors are translating

Vectors are also **rotating**

Concept #9: Flux of the vector fields

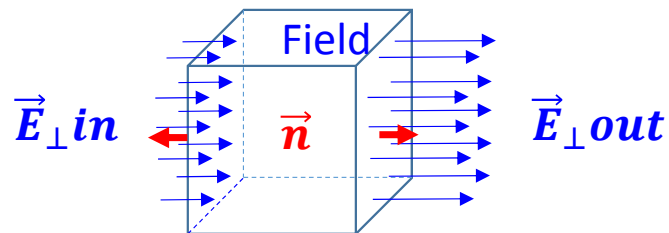
Vectors are **flowing** in and out of the closed surface

Flux = net outward flow

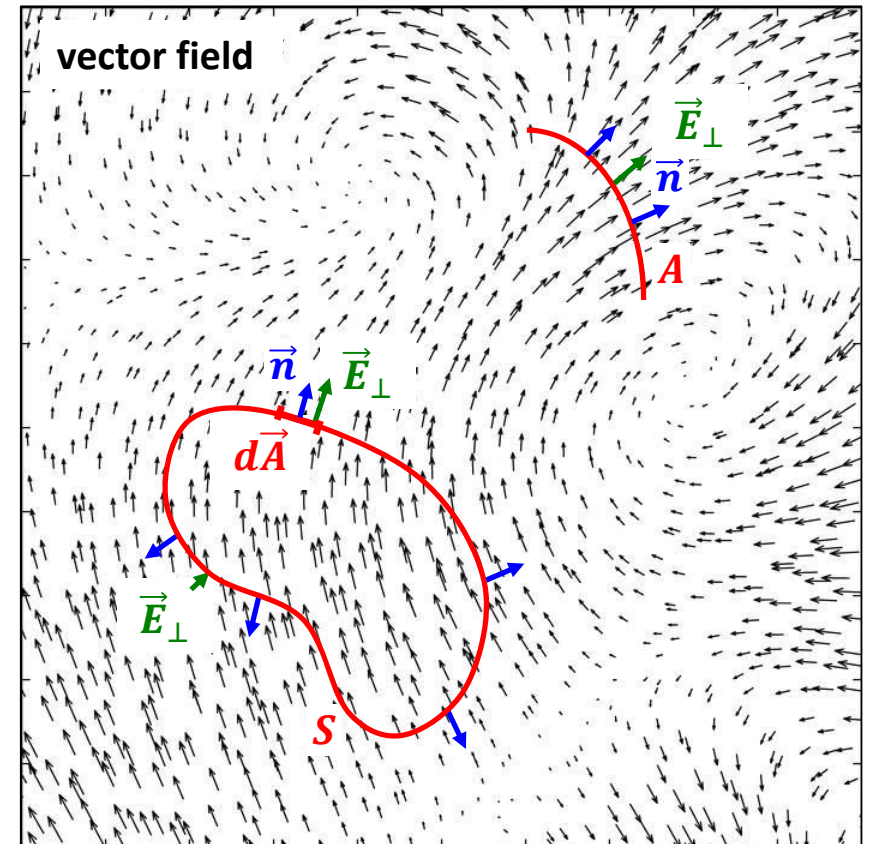
Flux = (average normal component) . (surface area)

$$\int \vec{E}_{\perp} \cdot d\vec{A}$$

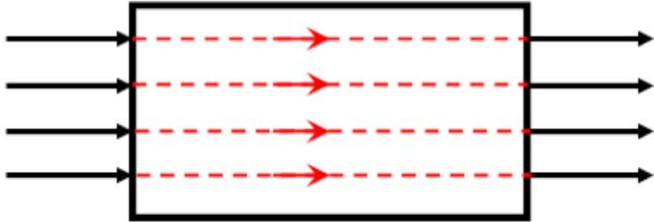
Intensity matters !



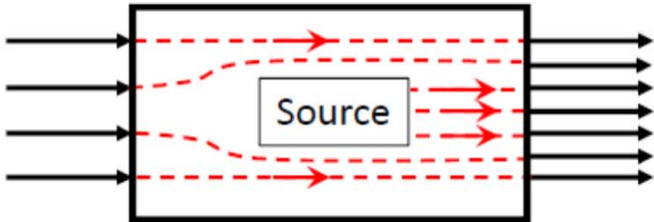
$$E_{\perp in} < E_{\perp out} \Rightarrow Flux > 0$$



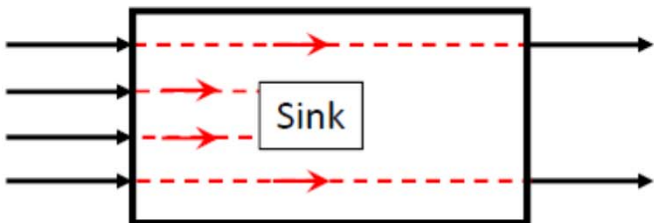
Flux in = Flux out



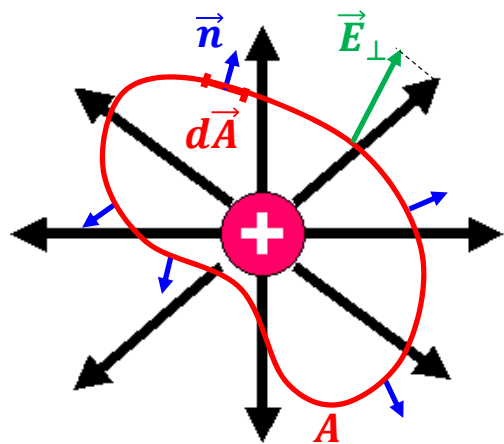
Flux in < Flux out



Flux in > Flux out

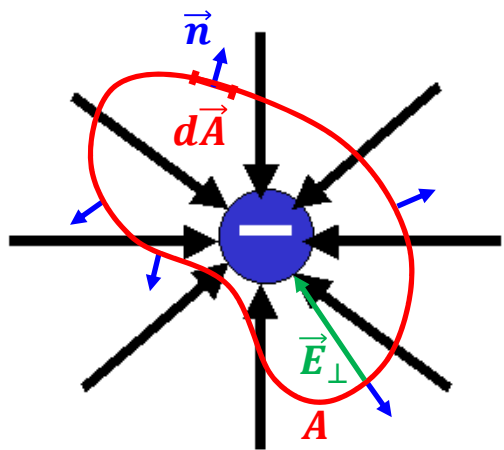


The net flux through a closed surface indicates whether there is a source or sink within the enclosed volume



Source

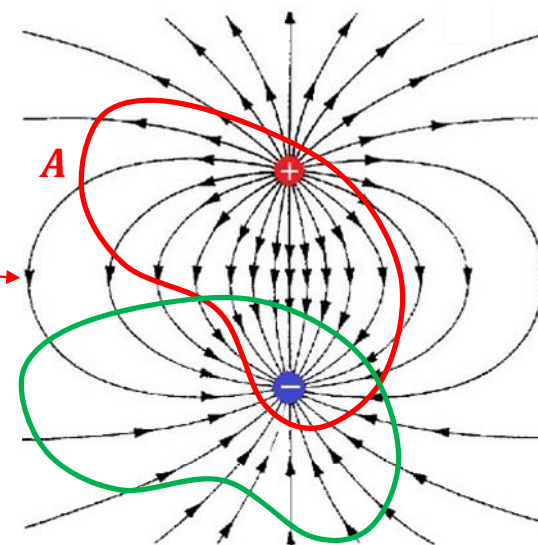
Flux $> 0 \Leftrightarrow$ DIVERGENCE



Sink

Flux $< 0 \Leftrightarrow$ CONVERGENCE
= -DIVERGENCE

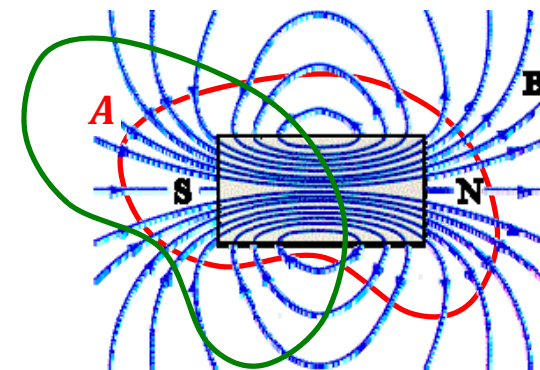
Closed 3D surfaces



Flux $= 0$

Flux < 0

Electric monopoles exist



Flux $= 0$

Flux $= 0$

Inside the magnet lines go from S to N

Magnetic monopoles DO NOT exist

The first Maxwell's equation (in vacuum)

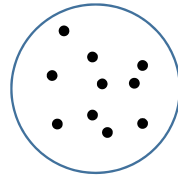


Gauss law

1) The flux of \vec{E} through any closed surface = $\frac{\text{net charge inside}}{\epsilon_0}$

Electric monopoles exist

*Question #11



Why we may have charges inside but **NET flux = 0** ?

Answer to *Question #11

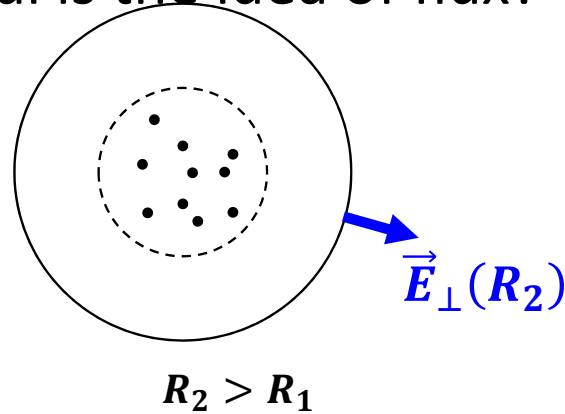
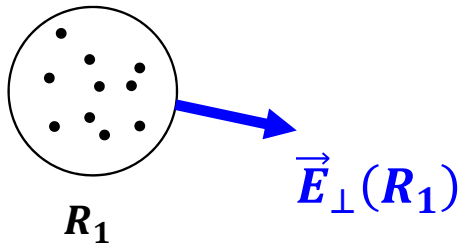
$$\sum_i q_i(> 0) = \sum_j q_j(< 0)$$

If **Net** charge $> 0 \Rightarrow$ Divergence \Rightarrow Source

If **Net** charge $< 0 \Rightarrow$ convergence \Rightarrow Sink

How powerful is the idea of flux?

Net charge inside > 0



Electric field at the surface

$$\text{Flux} = E_\perp \times 4\pi R^2 = Cte$$

...is conserved through closed surface...



If the surface increases, the field must decrease

Charge $+q$ and the idea that the field of a point charge is spherically symmetric \Rightarrow Coulomb's law

Coulomb's law derives from Gauss's law

$$E_\perp \times 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

Slide #31 A_Lecture 0_General

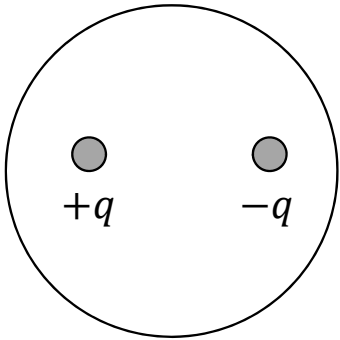
*Question #12

Would Gauss's law exist if the Coulomb law were $\propto 1/r^n$ with $n \neq 2$?

Answer to *Question #12

NO! wait for lecture on Gauss's law

Gauss's law applied to a dipole



Flux = (average normal component) . (surface area)

$$\int \vec{E}_{\perp} \cdot d\vec{A} = 0$$

What can we say about the Electric field outside the surface?

Not much ! Gauss's law is not of great help in this case

The second Maxwell's equation

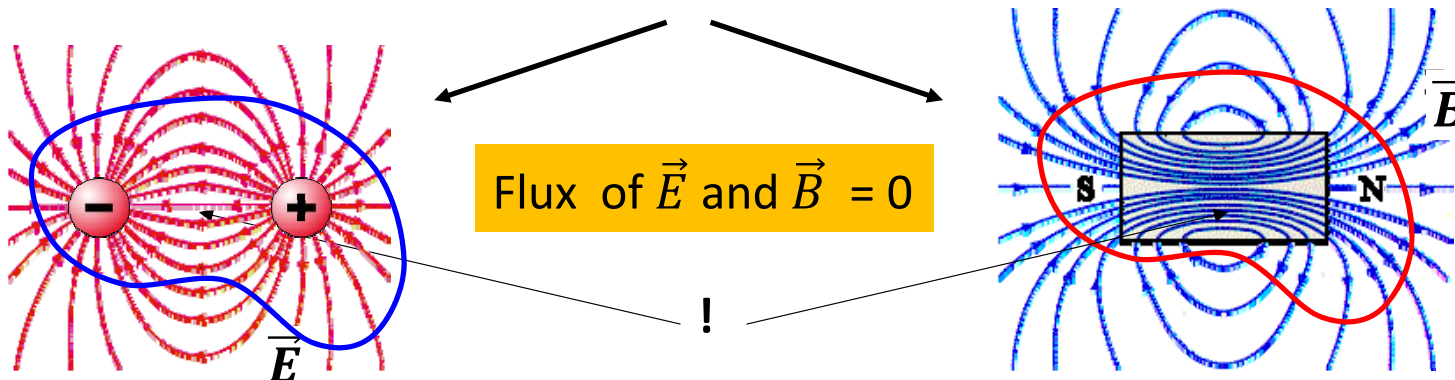


Gauss law

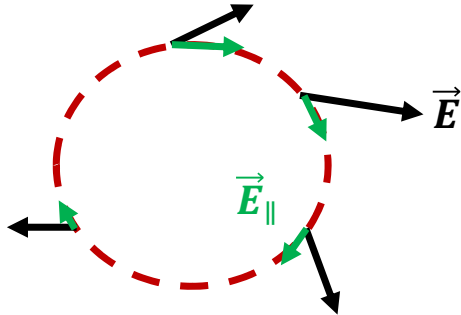
2) The flux of \vec{B} through any closed surface = 0

Magnetic monopoles **DO NOT** exist
Field lines close on themselves

Perfect symmetry of these two laws if we compare an electric dipole with a magnet



Concept #10: Circulation of the vector fields

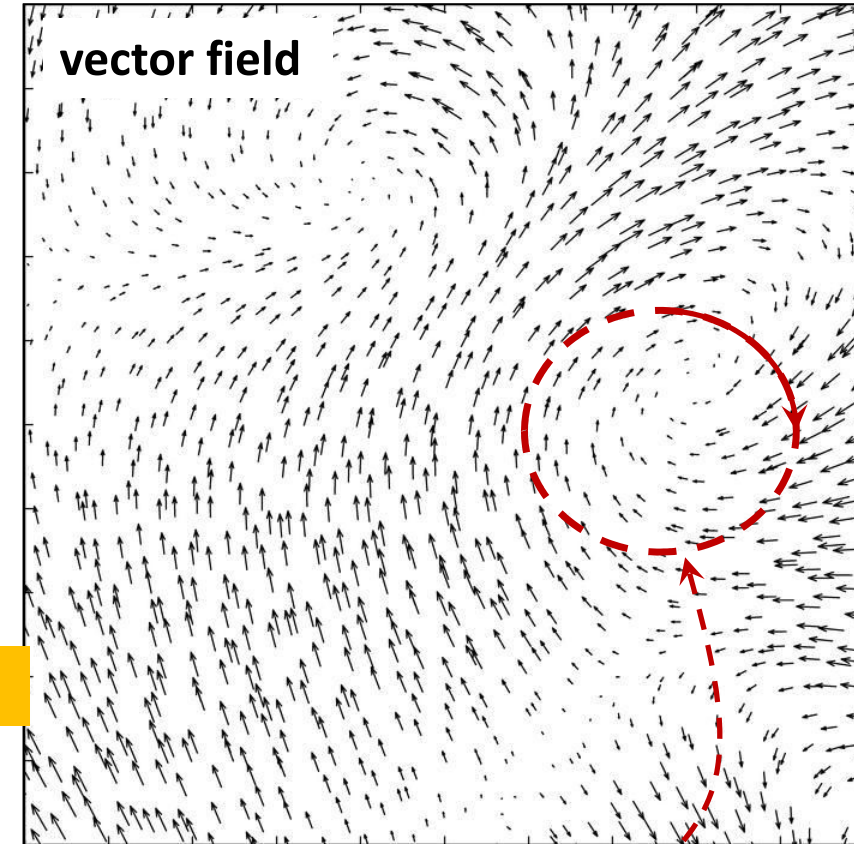


Circulation = net rotational motion

Circulation = (average tangential component) . (distance around)

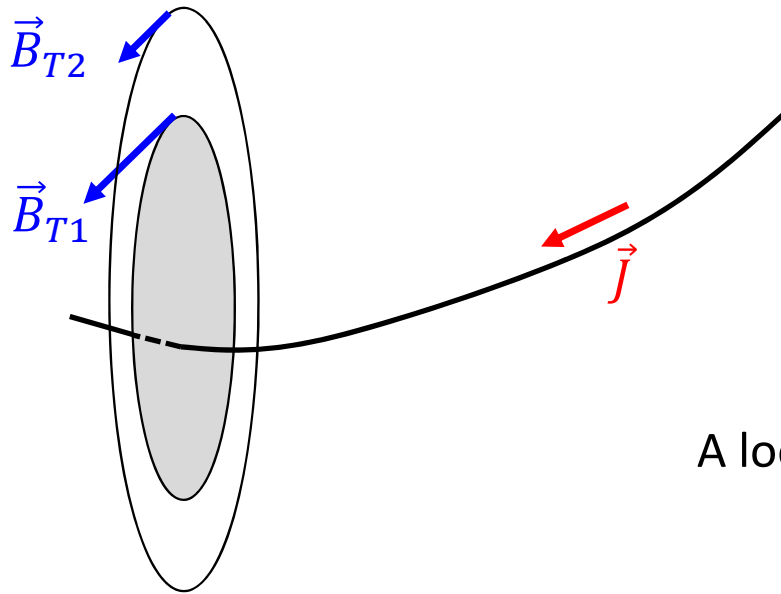
$$\int \vec{E}_{\parallel} \cdot d\vec{l}$$

Intensity matters !



Vectors are **rotating**

How powerful is the idea of circulation?



If \vec{j} is fixed, the circulation must be constant



A loop with radius R_2 is larger than a loop with radius R_1

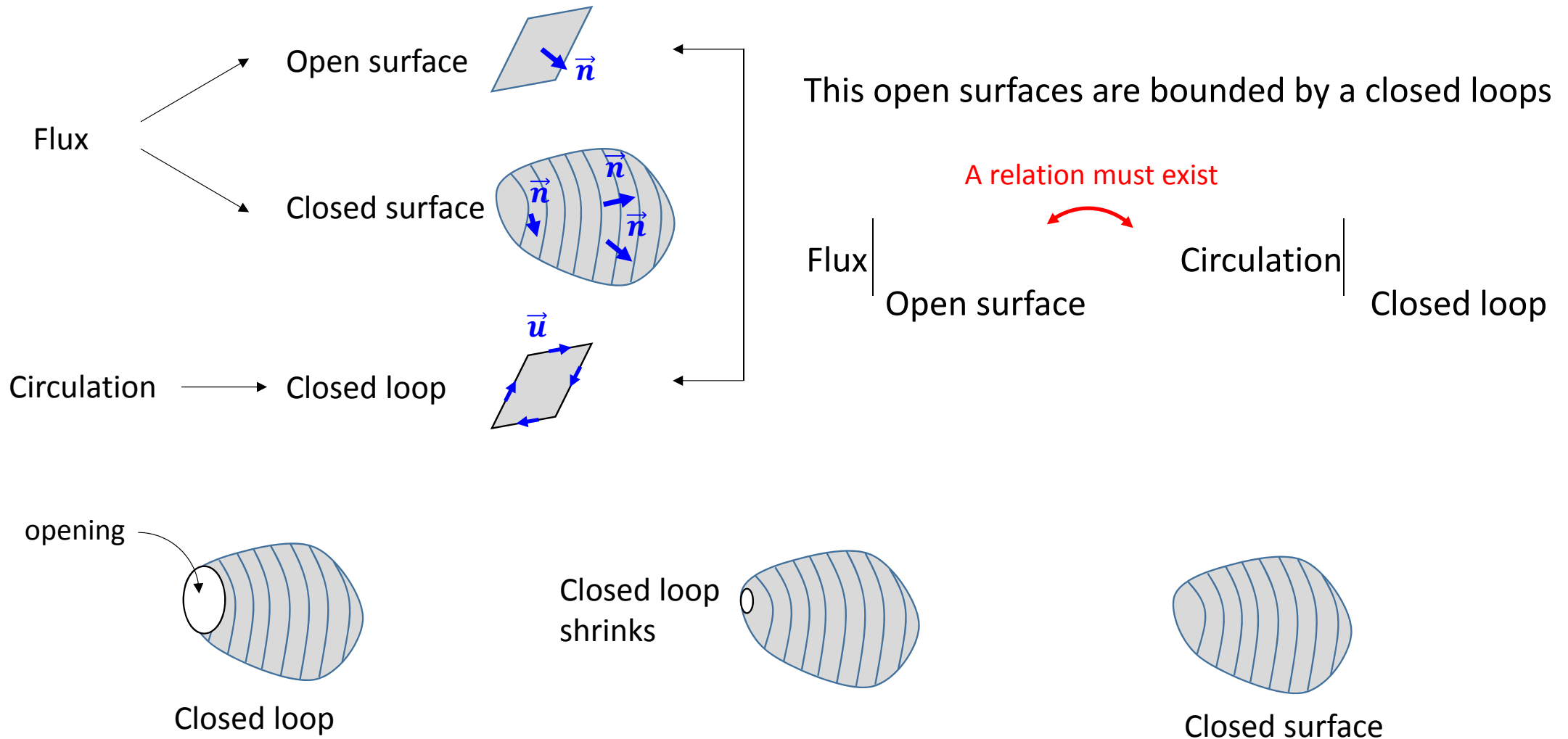


Circulation = (average tangential component) . (distance around)



$$\vec{B}_{T2} < \vec{B}_{T1}$$

Connection of the two concepts: Flux and circulation



Laws of electromagnetism

$$\vec{E}(x, y, z, t)$$

Flux through
closed surface

Circulation along
closed loop

$$\vec{B}(x, y, z, t)$$

Flux through
closed surface

Circulation along
closed loop

open surface bound by closed loop

Another important theorem

- Divergence or Gauss's theorem

Circulation
Line integral



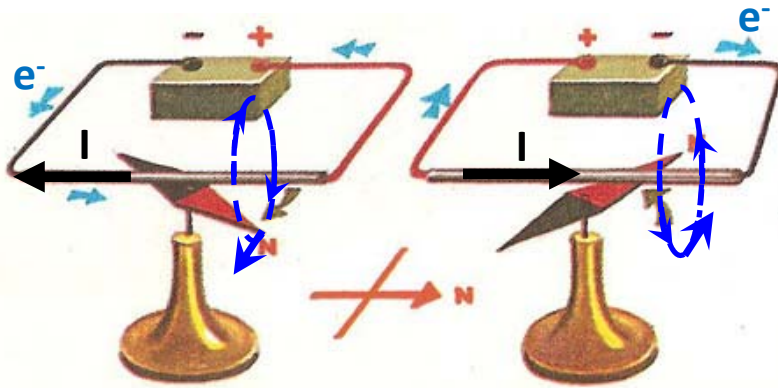
Flux
Surface integral

Surface integral ↔ Volume integral

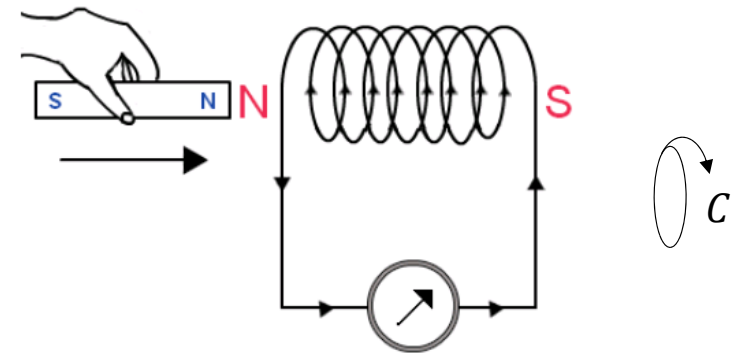
Stokes Theorem

Field induction: Ørsted vs Faraday

Ørsted's experiment (1820)

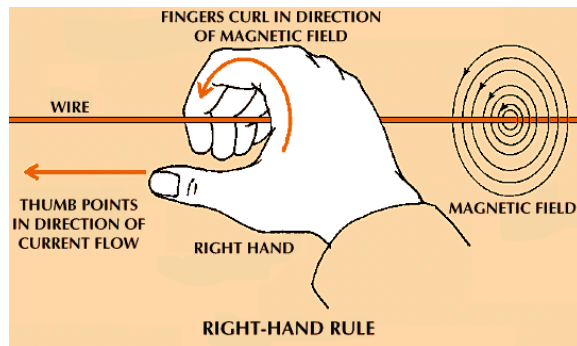


Faraday's experiment (1821)

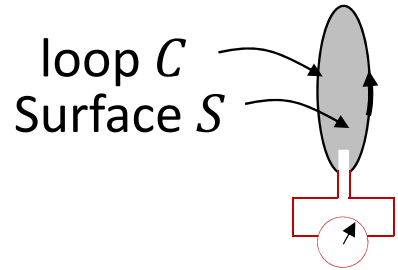


Steady or varying Current flow generates $\vec{B} \Rightarrow$ Lorentz force

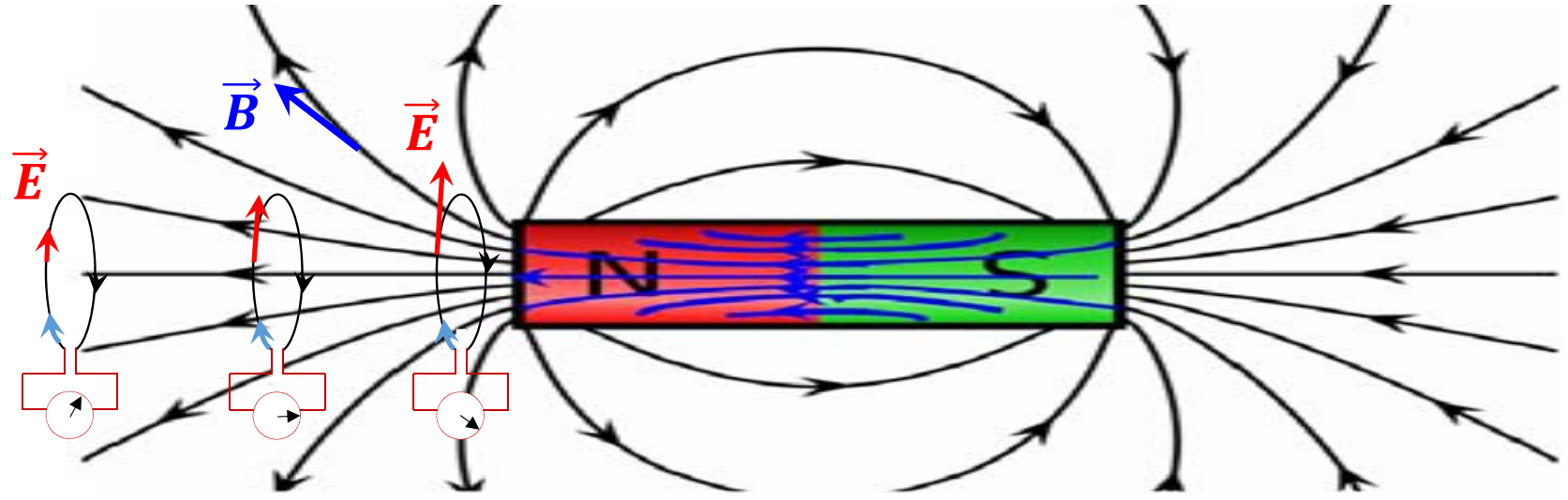
Only Varying \vec{B} generates $\vec{E} \Rightarrow$ Lorentz force responsible for the measured current



The third Maxwell's equation: Faraday's experiment



Circulation of \vec{E} around loop C is responsible for the measured current



Configurations 1

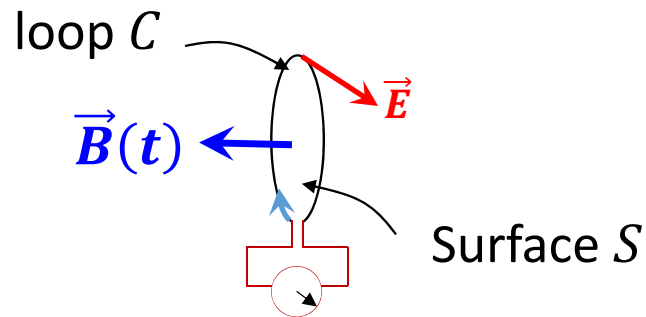
2

3

Flux 1 < Flux 2 < Flux 3

A relation must exist between the **circulation of \vec{E}** around C and the **flux of \vec{B}** through surface S

The third Maxwell's equation



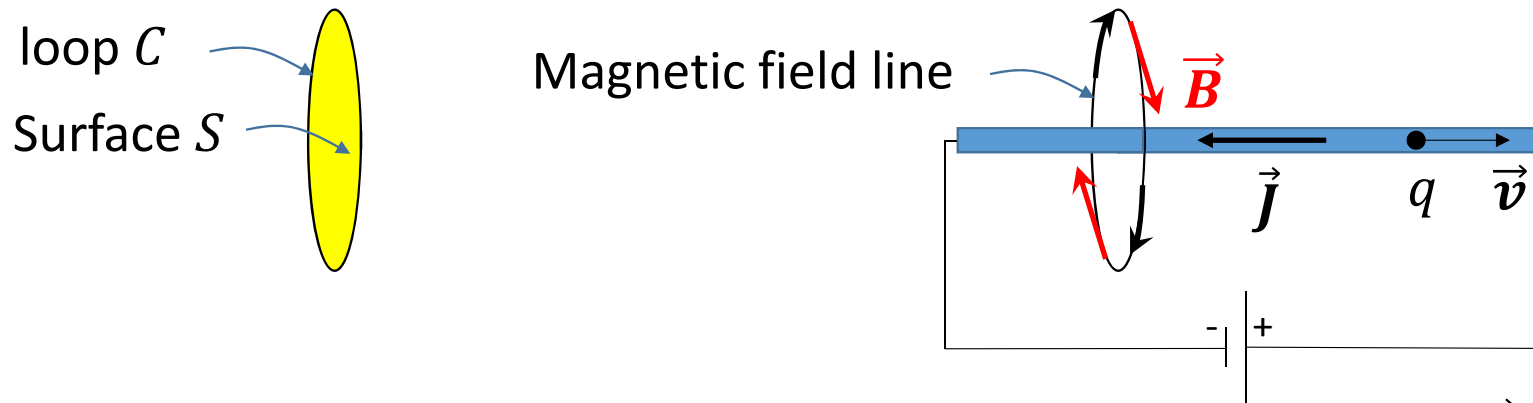
3) Circulation of \vec{E} around C (bordering a surface S) = $\frac{d}{dt}$ (flux of \vec{B} through S)

Sign and circulation directions will be discussed later

We can foresee Stokes theorem behind this statement

The fourth Maxwell's equation: Ørsted's experiment, Biot & Savart's and Ampere's law

Ørsted's experiment: Biot & Savart's and Ampere's law

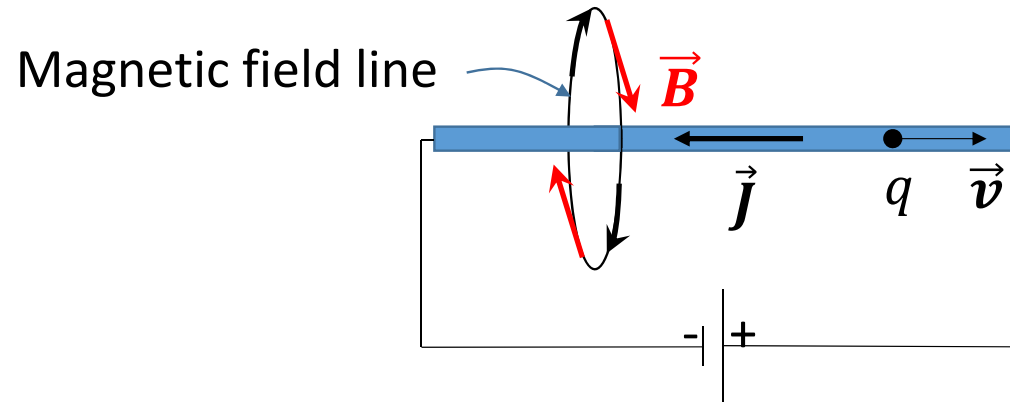
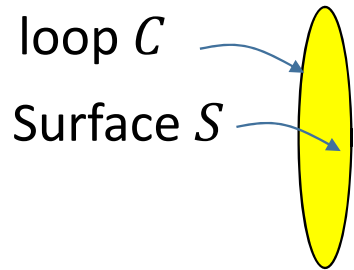


A relation must exist between the **current flow \vec{J}** through **surface S** and the **circulation of \vec{B} around C**

Biot & Savart's and Ampere's law

$$4) \quad c^2 [\text{Circulation of } \vec{B} \text{ around } C \text{ (bordering a surface } S)] = \frac{\text{flux of electric current through } S}{\epsilon_0}$$

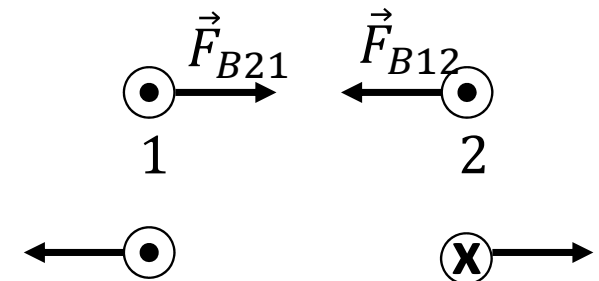
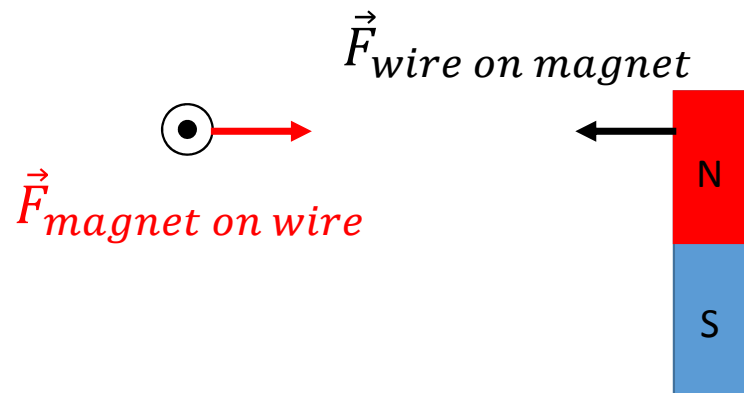
Oersted's experiment

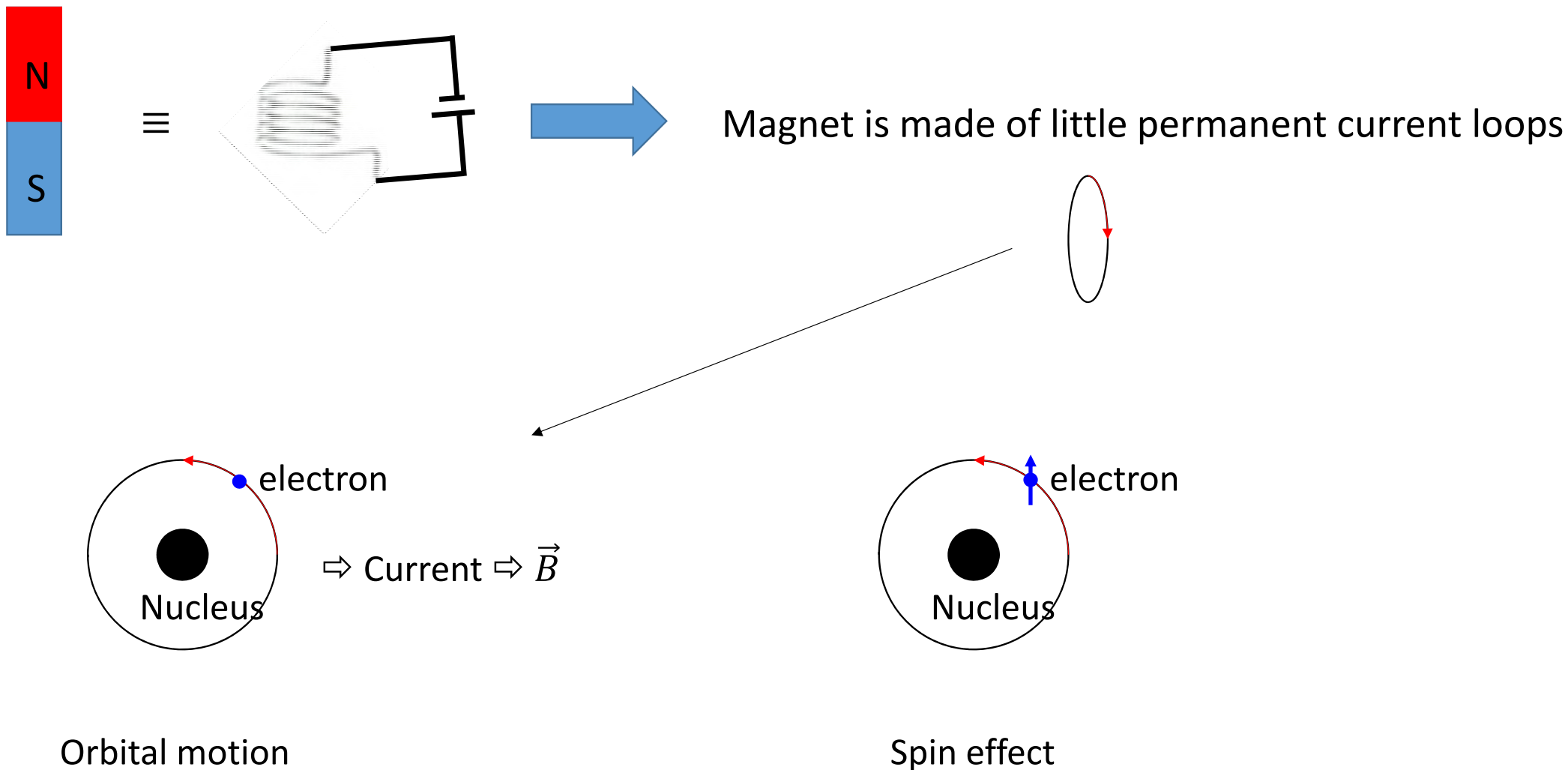


Action - reaction

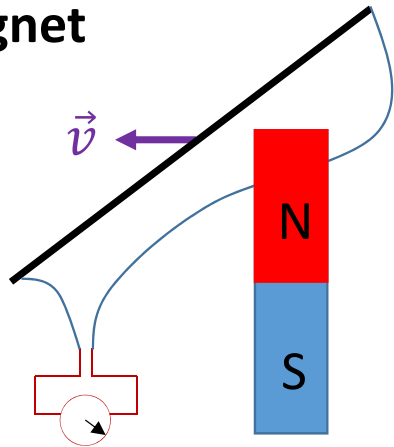
$$\vec{F} = q\vec{v} \times \vec{B}$$

We may replace the magnet by another current carrying wire





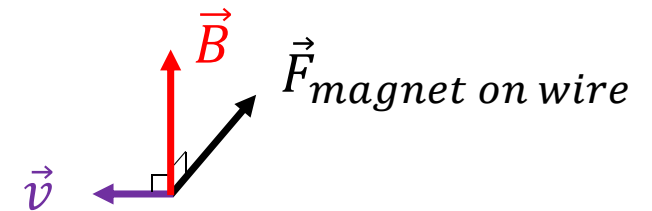
Pulling a conducting wire nearby a magnet



Mechanical motion of electron
in the presence of a magnet



Lorentz force



*****Question #13**

Is that all what happens? Electrons set into motion in the wire ?

Answer to*question #13

Principle of action-reaction requires that the magnet also moves. In which direction?

Close look at the fourth Ampere's law: Genius idea of Maxwell

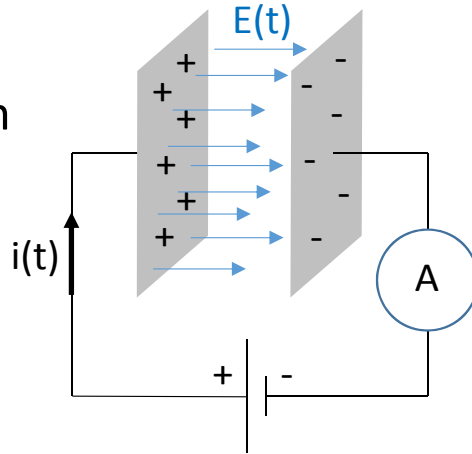
$$4) \quad c^2 [\text{Circulation of } \vec{B} \text{ around } C \text{ (bordering a surface } S)] = \frac{\text{flux of electric current through } S}{\epsilon_0}$$

Ampere and Biot & Savart's law based on Ørsted's experiment

Something is missing in this relation

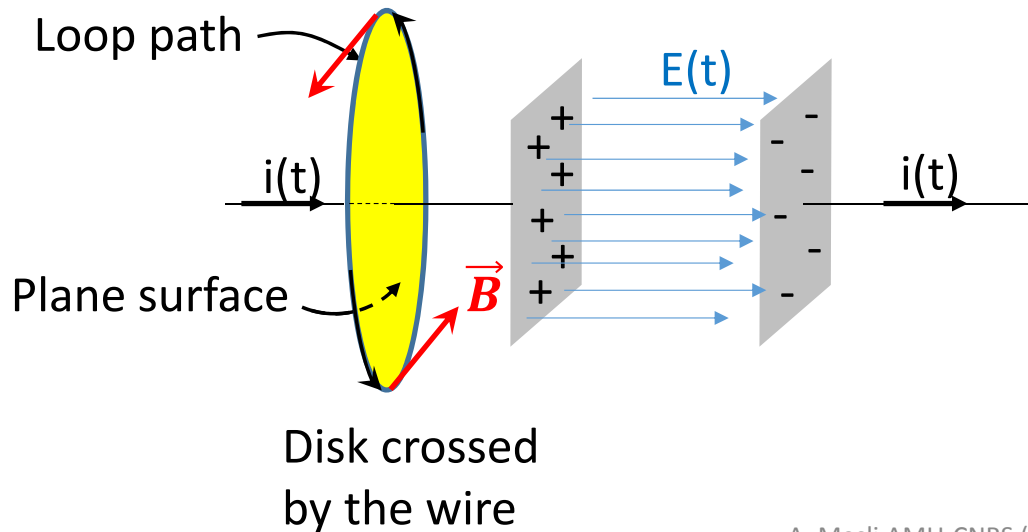
Maxwell's discovery of the displacement current

The capacitor problem

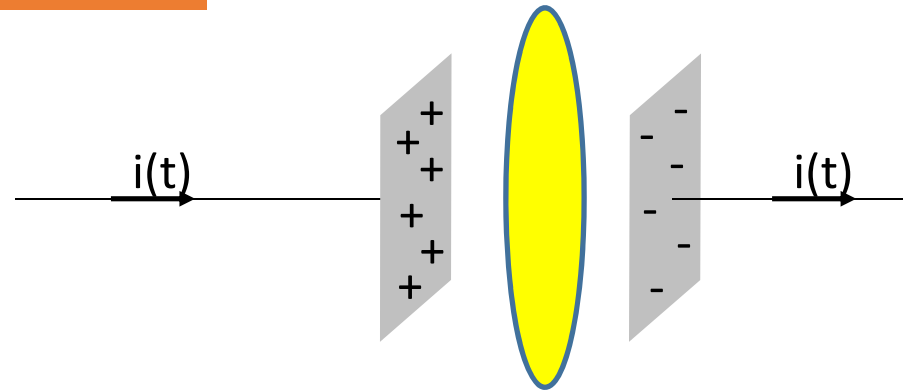


This is not a closed loop **BUT** a current $i(t)$ flows in the circuit
Clearly no electron is jumping from one plate to the other !

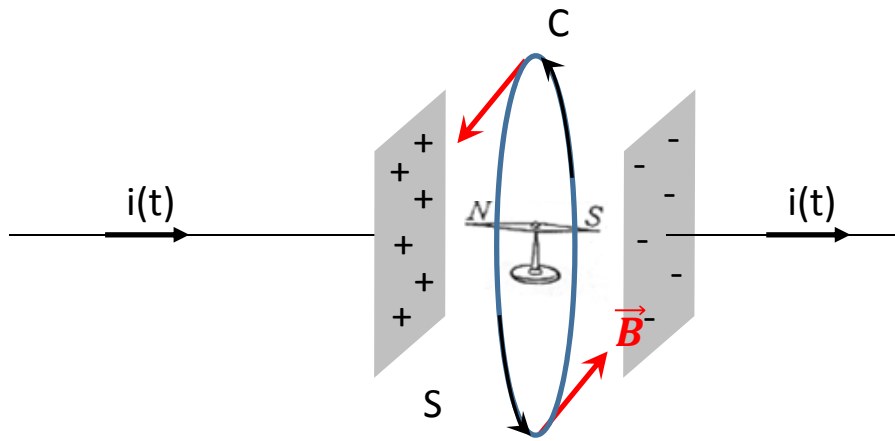
Ampere's law



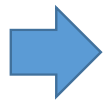
The wire does **NOT**
 Cross the wire



No electron moves through the surface thus **NO** magnetic field ?

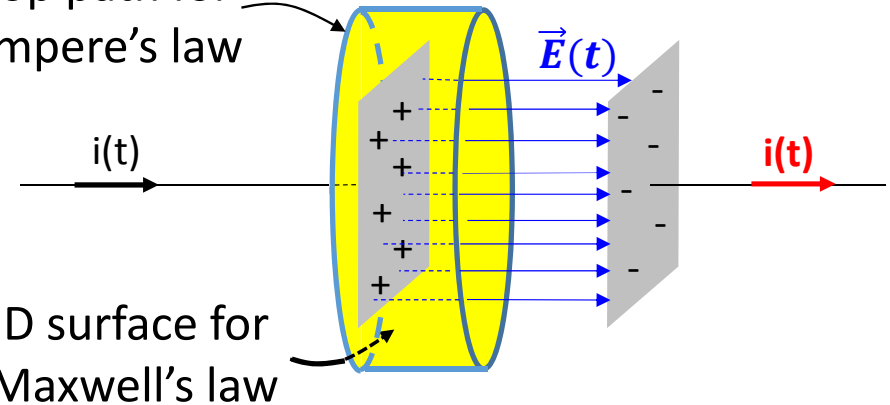


compass ROTATES



There is a magnetic field
Where does it come from?

Loop path for
Ampere's law



3D surface for
Maxwell's law

Cylinder open on the left hand side = Maxwell's surface
The wire does not cross it

$$4) \quad c^2 [\text{Circulation of } \vec{B} \text{ around } C \text{ (bordering a surface } S)] = \frac{\text{flux of electric current through } S}{\epsilon_0} +$$

$$\frac{d}{dt} (\text{flux of } \vec{E} \text{ through } S)$$

Maxwell's discovery

Summary of the complete laws of electrodynamics

$$1) \text{ The flux of } \vec{E} \text{ through any **closed** surface} = \frac{\text{net charge inside}}{\epsilon_0}$$

Poisson - Gauss

*Behavior of \vec{E} and \vec{B} perfectly
Symmetric if **NET** charge inside
the **closed** surface = 0*

$$2) \text{ The flux of } \vec{B} \text{ through any **closed** surface} = 0$$

Gauss

Electric dipole \Leftrightarrow magnet

$$3) \text{ Circulation of } \vec{E} \text{ around a curve (bordering a surface } S) = \frac{d}{dt} (\text{flux of } \vec{B} \text{ through } S)$$

Faraday

$$4) \text{ } c^2 [\text{Circulation of } \vec{B} \text{ around } C \text{ (bordering a surface } S)] = \underbrace{\frac{\text{flux of electric current through } S}{\epsilon_0}}_{\text{Ørsted, Biot \& Savart}} + \underbrace{\frac{d}{dt} (\text{flux of } \vec{E} \text{ through } S)}_{\text{Maxwell}}$$

Ørsted, Biot & Savart
Ampere

Maxwell

1) The flux of \vec{E} through any closed surface = $\frac{\text{net charge inside}}{\epsilon_0}$

2) Circulation of \vec{E} around C (bordering a surface S) = $\frac{d}{dt}$ (flux of \vec{B} through S)

3) c^2 [Circulation of \vec{B} around C (bordering a surface S)]
= $\frac{d}{dt}$ (flux of \vec{E} through S) + $\frac{\text{flux of electric current through S}}{\epsilon_0}$

4) The flux of \vec{B} through any closed surface = 0

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stokes Theorem

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Gauss's theorem

Maxwell's discovery that speed of light emerges from static phenomena !

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

Comments on the so-called universal constants

G : gravitational constant	}	All appear as proportionality constants
ε_0 : Electric permittivity		
μ_0 : Magnetic permeability		

The gravitational force does not depend on the intervening medium



Hence G is a universal constant

ε and μ depend on the medium

- Electric permittivity ε describes how an electric field affects and is affected by a medium. It is determined by the ability of a material to polarize in response to an applied field, and thereby to cancel, partially, the field inside the material
- Similarly, magnetic permeability μ is the ability of a substance to acquire magnetization in magnetic fields. It is a measure of the extent to which magnetic field can penetrate matter



They have different values for different media.

strictly speaking ε and μ are not universal constants

$\varepsilon\mu$ gives the speed v of electromagnetic radiation in the medium through $\varepsilon\mu = 1/v^2$

$$\text{Index of refraction } n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

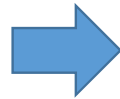
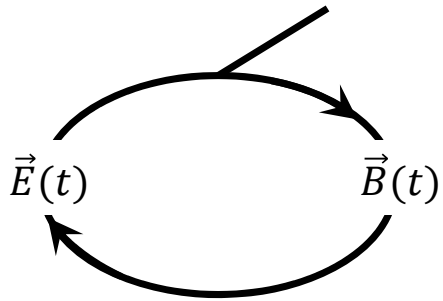
Different media =
different index of refraction

Electromagnetic wave production and radiation

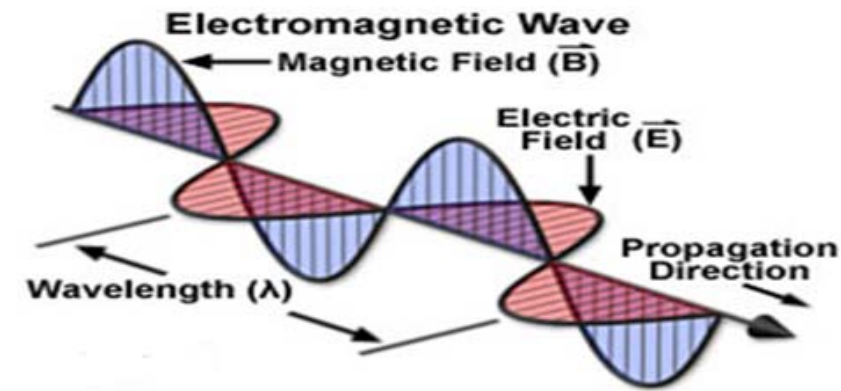
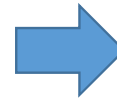
Setting $\vec{J}(t)$ in a circuit



Sets electrons into accelerating motion



Far from the source



Emission of EM waves

$\vec{E}(t)$ and $\vec{B}(t)$ are self sustaining

Some interesting questions

***Question #14

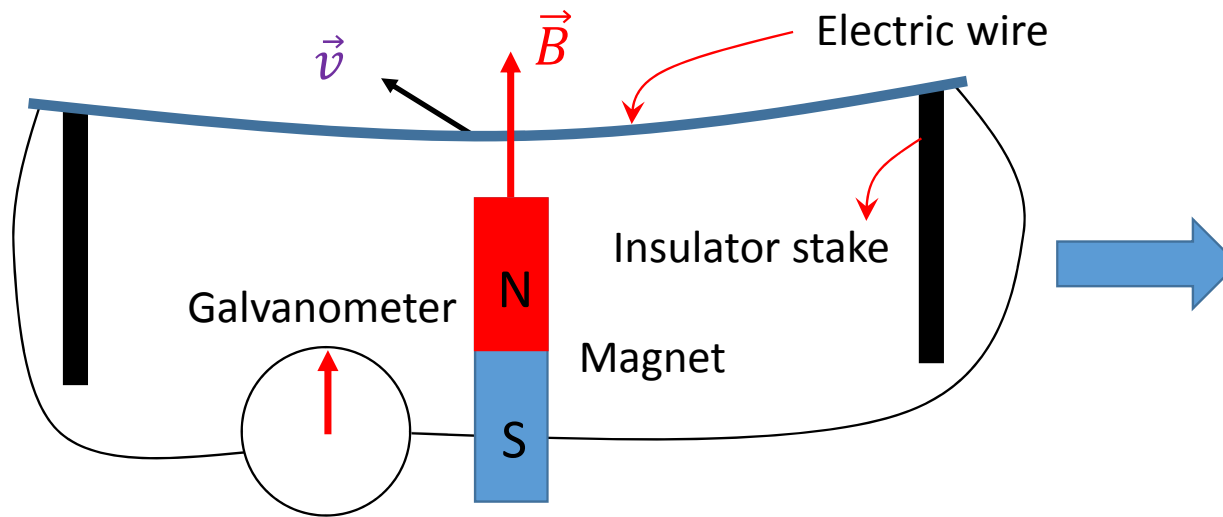
Why c^2 appears in the fourth Maxwell's equation?

$$4) \ c^2 [\text{Circulation of } \vec{B} \text{ around } C \text{ (bordering a surface } S)] = \frac{\text{flux of electric current through } S}{\epsilon_0} + \frac{d}{dt} (\text{flux of } \vec{E} \text{ through } S)$$

Answer to *Question #14

Because B is a relativistic effect in electricity: Wait for last lecture

Illustration: The electrons in the electric wire are at rest. A magnet nearby has no effect on immobile electrons



- Could we initiate current in the wire ?
- If yes how ?

Move the wire sideways mechanically



Electrons in the wire move: Lorentz force



Current induced in the wire

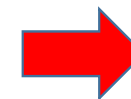
If the wire is immobile and we move the magnet



Relativity principle requires that we observe the same effect



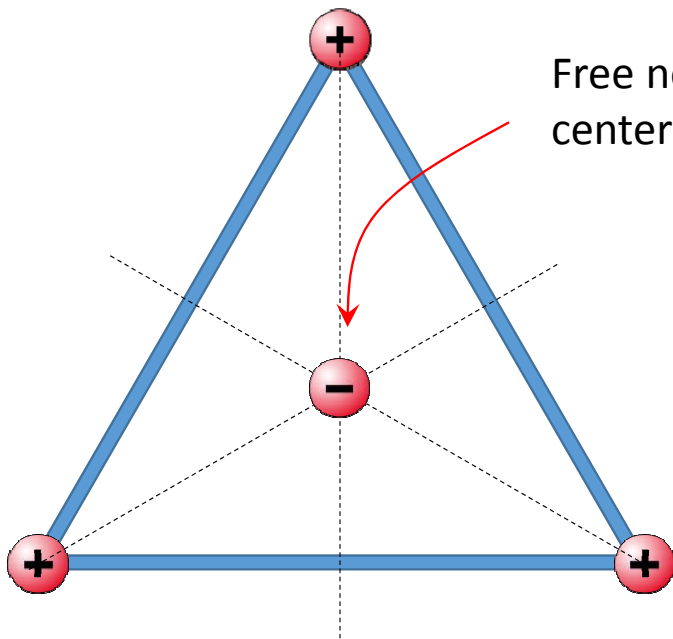
Moving B creates a changing $E \Rightarrow$ Electric force \Rightarrow Current in the wire



REMARQUABLE CONSEQUENCE
of relativity before its discovery

***Question #15

Three positive charges rigidly bound together



Free negative charge put at the center of the equilateral triangle

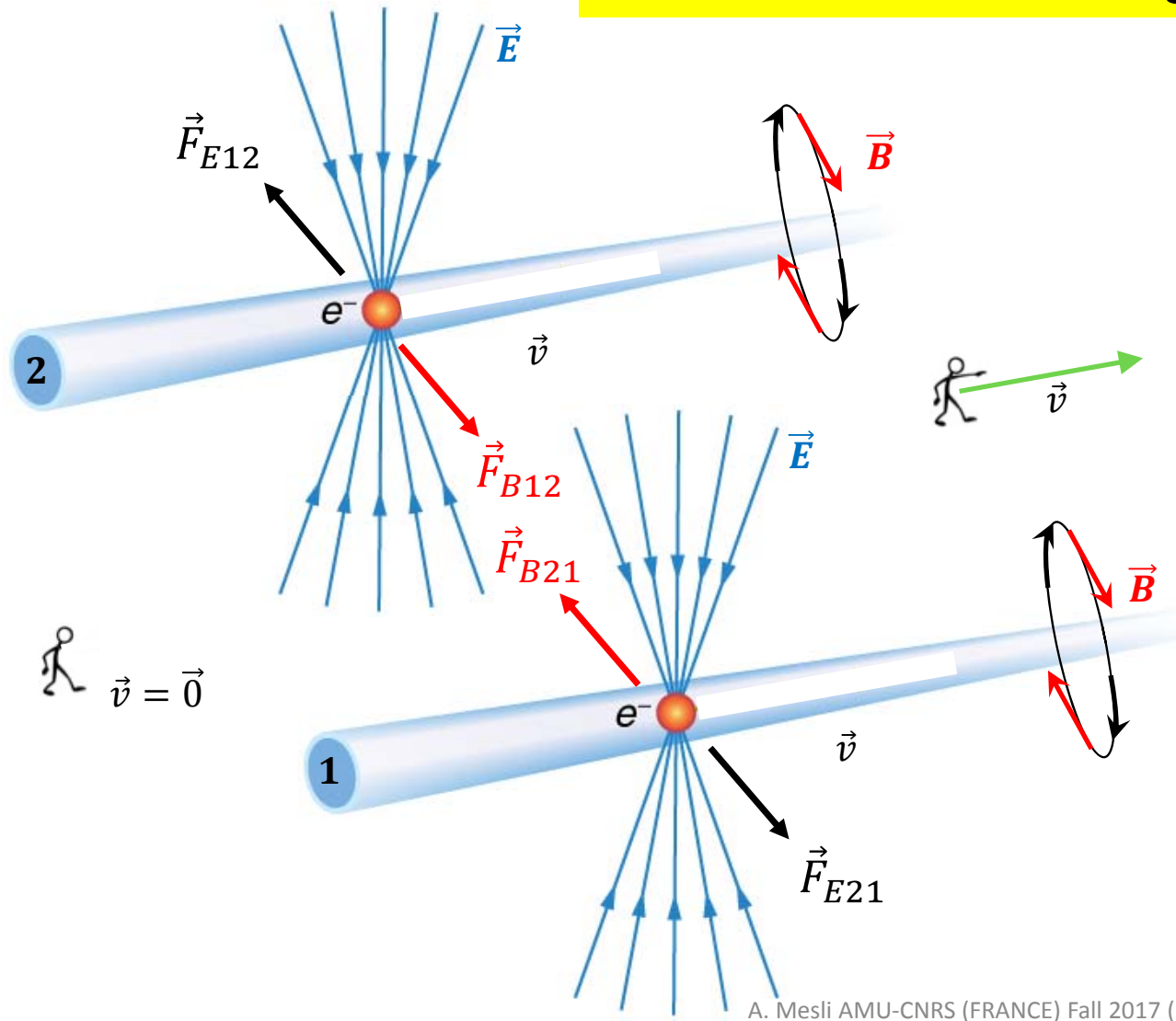
- Is the negative charge
- In a stationary state ?
 - In equilibrium ?

Answer to*question #15

In a stationary state **NOT** in equilibrium

Are the electric and magnetic fields virtual

Two electrons are moving with the same speed \vec{v} and along parallel paths



For a stationary observer

- Should the electrons repel as predicted from the \vec{E} field or attract as predicted from the \vec{B} field ?

For the same observer riding with the same speed as the two electrons?

Does he still observe:

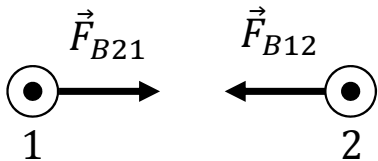
- The electric field ?
- The magnetic field ?

Special relativity:

Whether the observer is immobile or moving with the charges,
All physical laws remain unchanged in any inertial frame

***Question #16

Two wires carrying the same current in the same direction (out of the page)



According to Lorentz force they attract each other via the magnetic field force

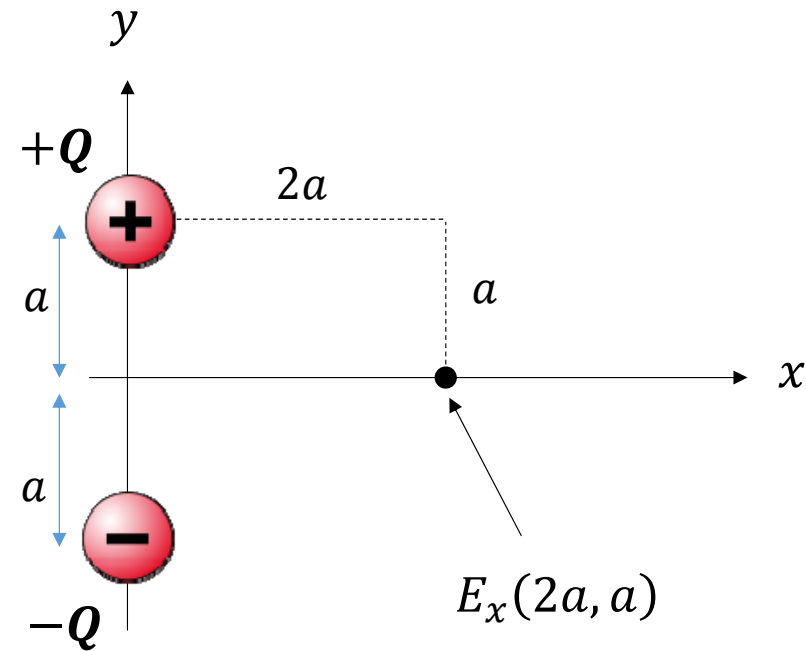
BUT what about the Electric forces du to charges inside the wires? Do they play any role ?

Answer to **question #16

- The net charges everywhere in the two wires $Q = 0 \Rightarrow$ The net electrical forces between the wires are zero
- The relativistic magnetic forces and fields will be of the same sort as in the case of two beams of charges of a single sign. This is true even in the frame of reference of what we think as the moving charges, that is, the electrons
- In the frame of reference moving at the drift velocity of these current-carrying electrons, it is the protons or positively charged ions that are moving in the opposite direction
- Consequently in any frame of reference for current-carrying wires in parallel, the net electrical force will be zero, and there will be a net attractive magnetic force.

Principle of superposition

Consider the following dipole



Which of the following statements about $E_x(2a, a)$ is true?

a) $E_x(2a, a) < 0$

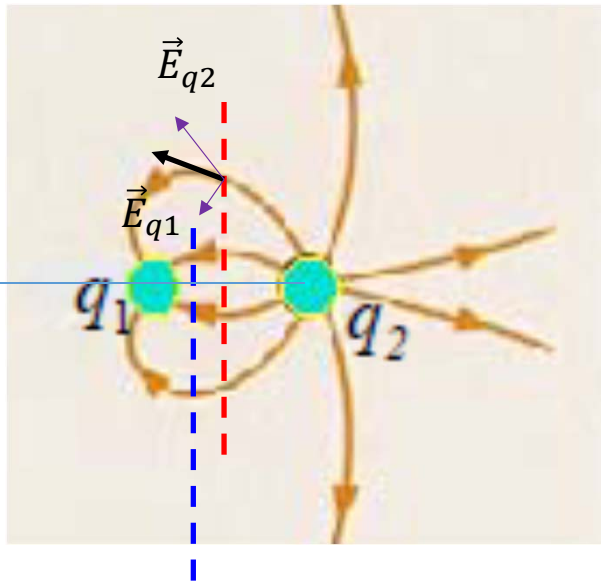
b) $E_x(2a, a) = 0$

c) $E_x(2a, a) > 0$

BUT $E_y(2a, a) < 0$

There is **NOT** a single point in space where the field is zero

Question #17



While examining the electric field lines produced by the two charges in the figure, say which statement is true

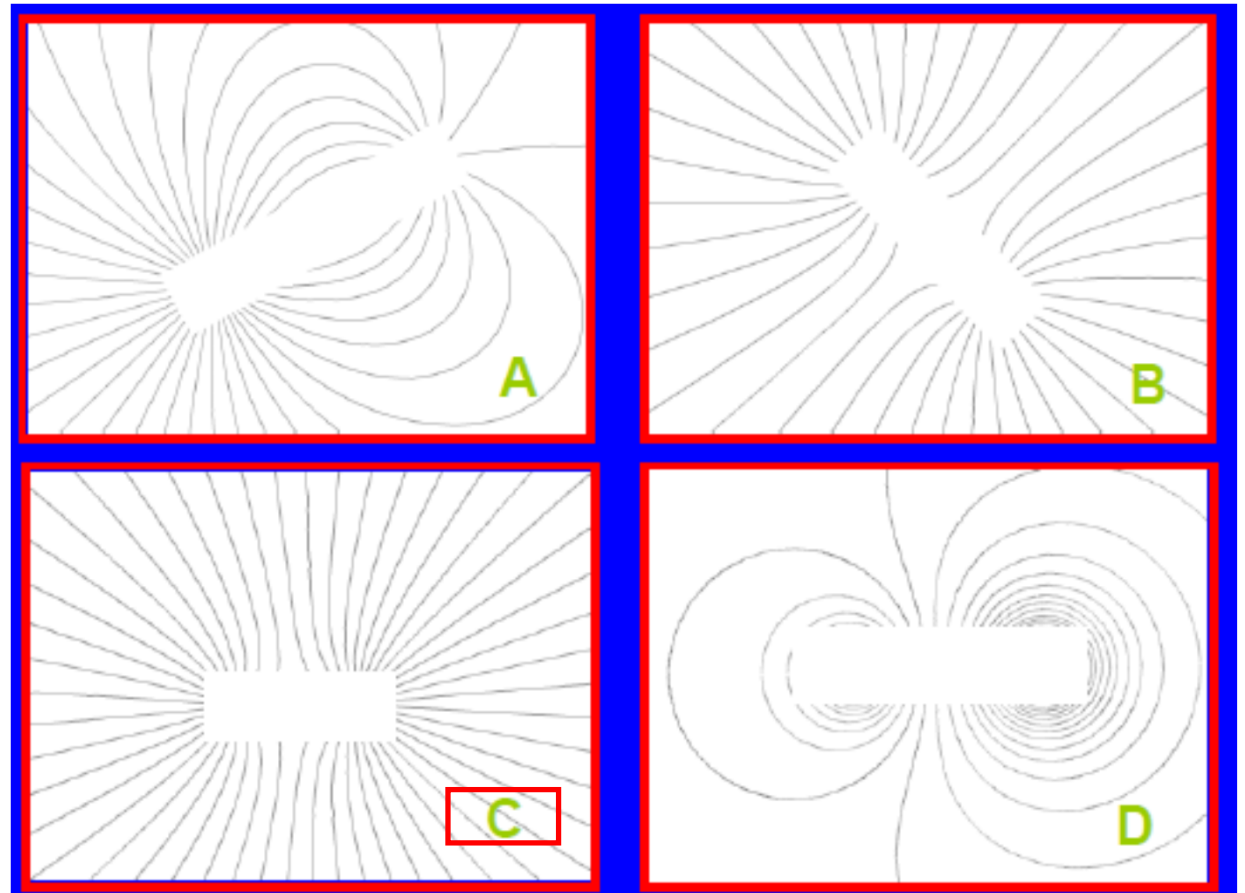
- * 1) q_1 and q_2 have the same sign
- *** 2) q_1 and q_2 have the opposite signs and $|q_1| > |q_2|$
- 3) q_1 and q_2 have the opposite signs and $|q_1| < |q_2|$

Answers to Question #17

- 1) **No !** Field lines go from q_2 to q_1 $\Rightarrow q_2 > 0$ and $q_1 < 0$
- 2) Along a line of symmetry between the two charges the E-field still has a positive y – component. If the charges were equal, the y -component would be zero therefore $|q_2| \neq |q_1|$
- 3) The inflection point (the point where the y – component cancels) is close to q_1 $\Rightarrow |q_1| < |q_2|$

*Question #18

Answers to *Question #18



Which of the following field pictures best represent the electric field from the two charges that have the same sign **BUT** different magnitude ?

Electrostatics I: Summary of basic considerations

- Electromagnetism involves particles that have a property called : **Charge**

Charge is as fundamental as **Mass**

- Mass** is linearly accelerated by gravitational force
 - Charge** is linearly accelerated by electric force
- Work is done by ➡ Gravitational force
Electric force

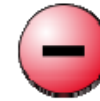
*Question #19

Does magnetic force do work?

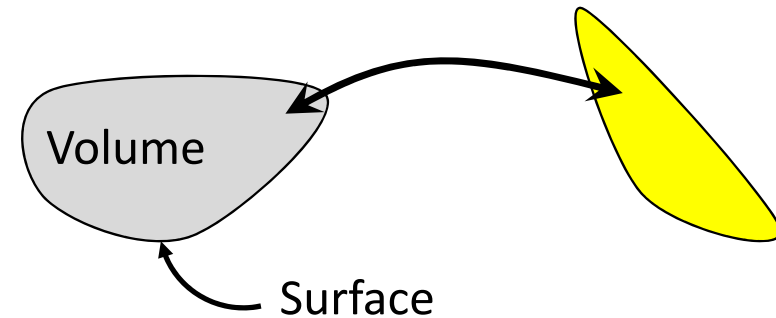
Answer to*question #19

NO because it is always perpendicular to the velocity of the moving charge

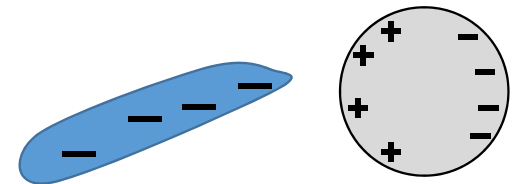
- Electric charge: two types



- **Transfer** of charges by contact or friction:
Charging – discharging process

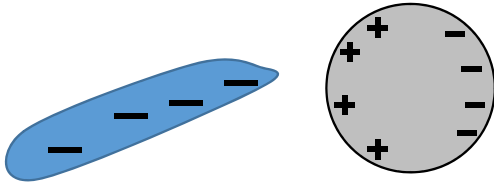


- **Induction** at distance of charges in a body (neutral or initially charged)



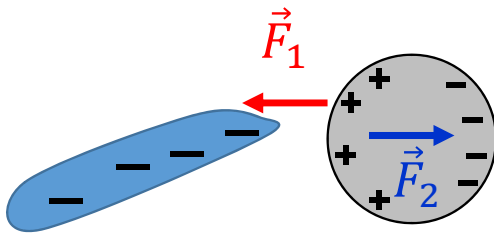
- Some bodies allow charges to move in the volume and the surface: **Conductors**
- Some bodies **DO NOT** allow charges to move in the volume and the surface: **Dielectrics**

**Question #20



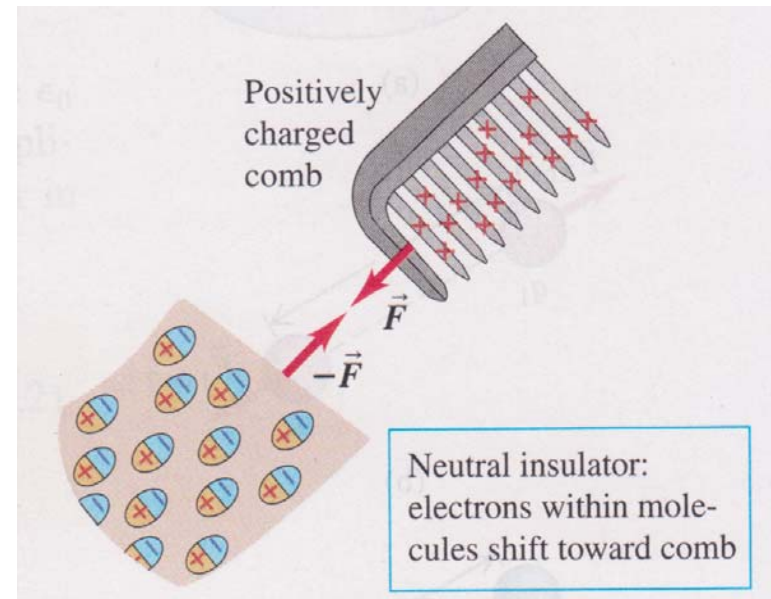
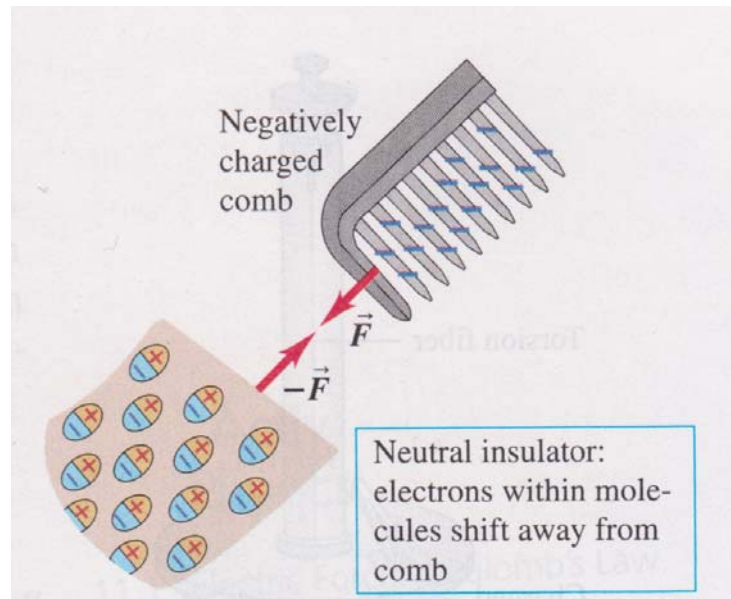
Why can't we accumulate more positive charges by **induction** on the left of the sphere?

Answer to*Question #20



Equilibrium is reached when $\sum_i \vec{F}_i = \vec{0}$

- Dielectrics interact via induced dipoles: **Polarization**



From University Physics, 11th edition



A charged object of either sign (+ or -) attracts **ALWAYS** an uncharged dielectric

Summary for Electrostatic

- Charge (+ and -)
- Electric force
- Coulomb's law \Rightarrow Gauss's law
- Electric field $\vec{E}(\mathbf{r})$
- Field lines
- Electric potential $\varphi(\mathbf{r})$
- Conservative field
- Electric potential energy
- Potential lines

From an empty space ...

...to the presence of objects of different nature:

Impact on the external field

- Induction
- Polarization
- Conductors
- Dielectrics



Boundary considerations

Next Lectures

- **Electrostatics I: Basic concepts**
- Position of the problem: Coordinate systems
- Scalar versus Vector fields: Operators
- Electrostatics II: Gauss law
- Dipoles
- Conductors
- Dielectrics
- Electrostatic energy
- Magnetostatic
- The magnetic field and its applications
- The vector potential
- Induced currents
- Law of induction
- Maxwell equations
- Wave polarization and propagation
- Relation to special theory of relativity

Statics



If $\frac{\partial}{\partial t} = 0$ (no time dependence) in all four Maxwell's equations

Electrostatics

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

Magnetostatics

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Two completely different domains

Electrostatics

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0 \quad \Leftrightarrow \text{Electrostatic field is conservative}$$

$$\vec{E} = -\vec{\nabla} \varphi$$

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot (-\vec{\nabla} \varphi) = -\nabla^2 \varphi = \frac{\rho}{\epsilon_0} \quad \Rightarrow \quad \text{Poisson equation } \nabla^2 \varphi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$