

# Magnetostatic

## Concepts

- Electric field
  - Permittivity  $\epsilon_0$
  - Electric force ON a stationary charge
  - Electric field OF a stationary charge
  - Coulomb's law
  - Gauss's law
  - Scalar potential
  - Electric dipole
- >
- Magnetic field
  - Permeability  $\mu_0$
  - Magnetic force ON a moving charge
  - Magnetic field OF moving charge
  - Biot & Savart's law
  - Ampere's law
  - Vector potential
  - Magnetic "dipole"

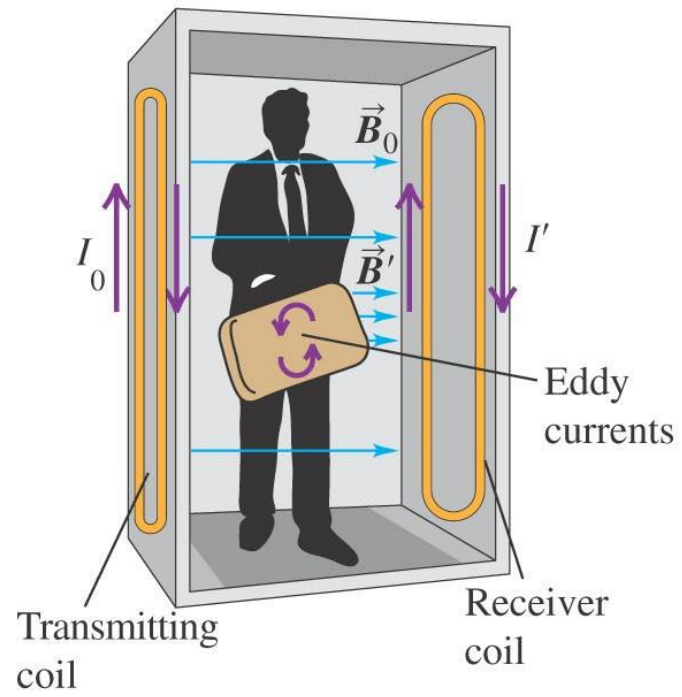
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \times \vec{E} = \vec{0}$$

Gauss's law: Flux through a closed surface

Stokes' theorem: Flux through a surface defined by a closed path

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$



# Magnetostatic versus Electrostatic: A perfect correspondence

## Electric force

The presence of the electric field is taken as an experimental fact

## Coulomb law

How a charge (moving or not) creates an electric field

## Gauss law

Exploits symmetry in relating  $\vec{E}$  to source (charge distribution)

## Magnetic force

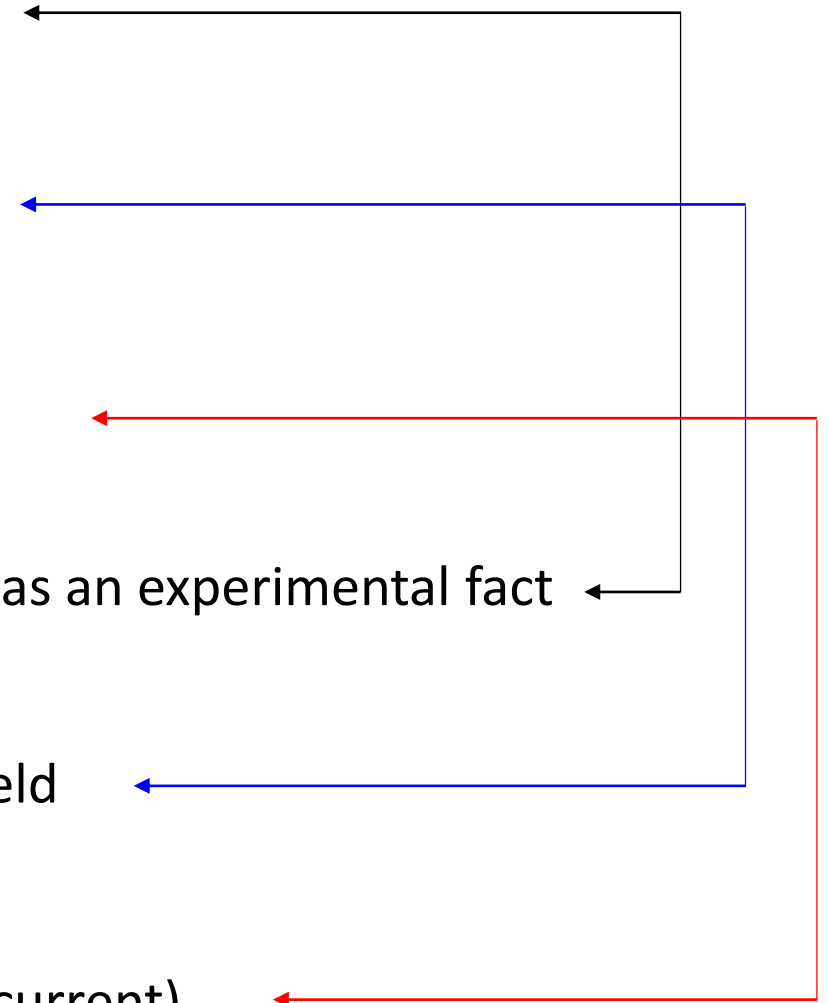
The presence of the magnetic field is taken as an experimental fact

## Biot & Savart law

How a moving charge creates a magnetic field

## Ampere's law

Exploits symmetry in relating  $\vec{B}$  to source (current)



# Lorentz force in the presence of electric and magnetic fields

## Two types of forces apply to a charge

- One due to electric field  $\vec{E}$ :  
The force  $\vec{F}_E(\vec{r})$  depends **ONLY** on where the charge is (whether it is moving or not)
- One due to magnetic field  $\vec{B}$ :  
The force  $\vec{F}_B(\vec{r}, \vec{v})$  depends on where the charge is but also on **HOW FAST** it moves

**Superposition principle:** In case both fields exist and the charge is moving

$$\vec{F} = \vec{F}_E(\vec{r}) + \vec{F}_B(\vec{r}, \vec{v})$$

$$\vec{F}(\vec{r}, \vec{v}) = q[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})]$$

$$\vec{F}(\vec{r}, \vec{v}) = q[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})]$$

Electric force directed parallel  
to the vector position

$$[E] = \frac{N}{C} = \frac{V}{m} = \frac{kg \ m \ s^{-2}}{C}$$

Magnetic force perpendicular to  
the plane defined by the two  
vectors  $\vec{v}$  and  $\vec{B}$

$$[B] = \frac{Ns}{Cm} = \frac{Vs}{m^2} = \frac{Weber}{m^2} \rightarrow Flux/unit \ area$$

$$[B] = \frac{F}{qv} = \frac{kg}{As^2} = Gauss = 10^{-4} \ Tesla$$

$10^{-9} - 10^{-8}$  G – the magnetic field of the human brain

0.25– 0.60 G – the Earth's magnetic field at its surface

50 G – a typical refrigerator magnet

$(6 - 7)10^5$  G – inside an atom and in a medical magnetic resonance  
imaging machine

## Question:

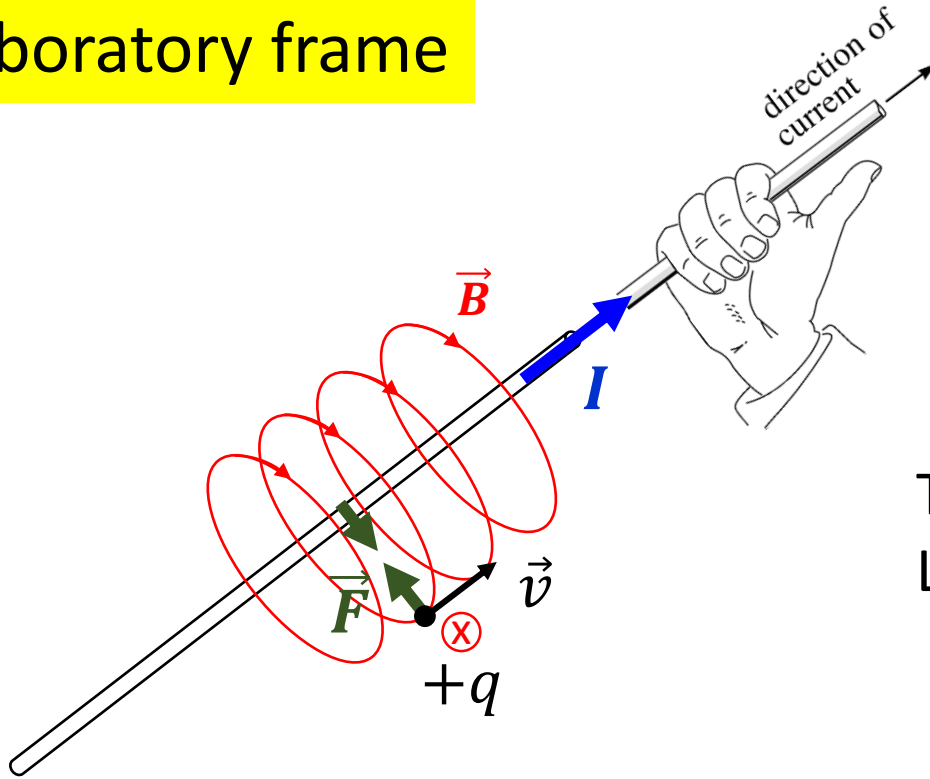
- But what exactly is the Magnetic field?
- Why should there exist some field that acts **ONLY** on moving charges?



**Answer:** Special relativity

*Postulate:* *Physics must be consistent in every “frame of reference”*

## Laboratory frame



The magnetic field of the wire generates a Lorentz **attractive force** on the moving charge

$$F = qvB$$

Charge frame: charge is not moving



*There can be no magnetic force!*



Physics must be consistent in both frames of reference



There must be some attractive force in the charge frame



What is this force ???

Special relativity  Electric  $E = vB$   $F = qE$    $F = qvB$

If a charge moves in  $\vec{E}$  and  $\vec{B}$ , the principle of superposition applies  
and the charge undergoes the Lorentz force

$$\vec{F}(\vec{r}, \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{B} = \vec{0}$$

$$\vec{F}(\vec{r}) = q\vec{E}$$

$$\vec{B} \neq \vec{0}$$

$$\vec{v} = \vec{0}$$

$$\vec{F}(\vec{r}) = q\vec{E}$$

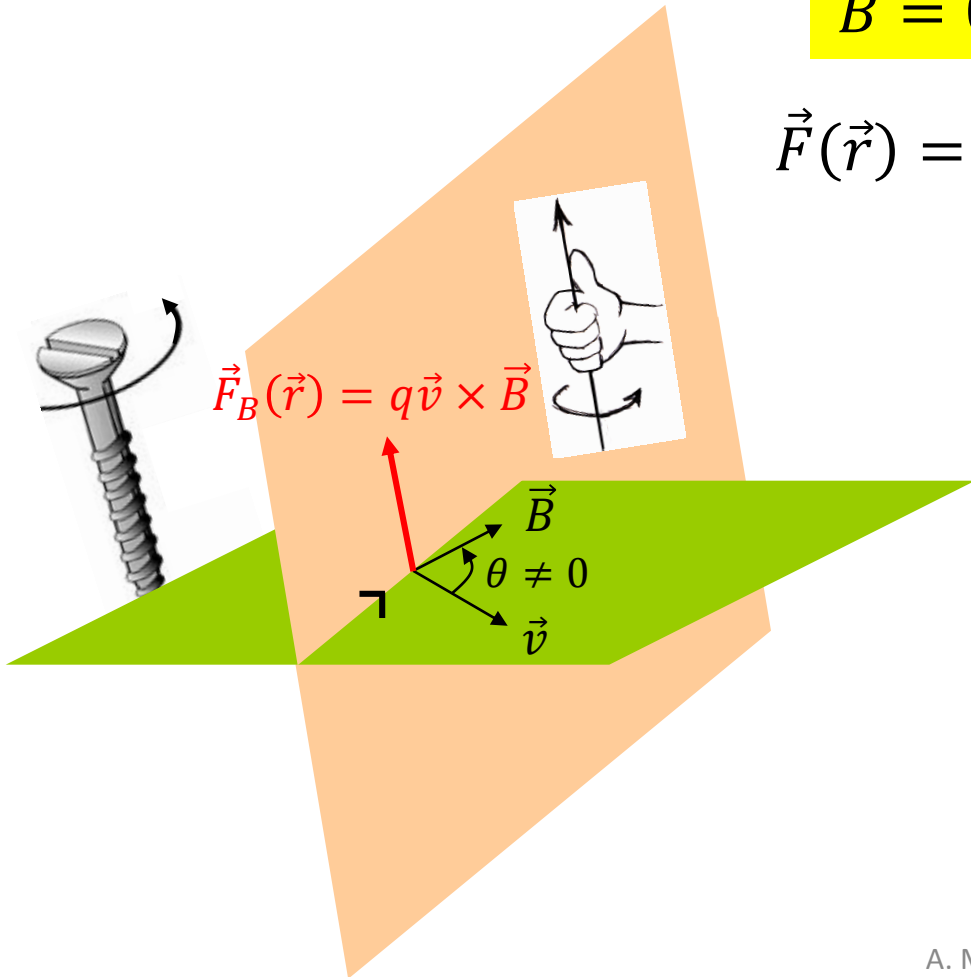
$$\vec{v} \neq \vec{0}$$

$$\vec{v} // \vec{B}$$

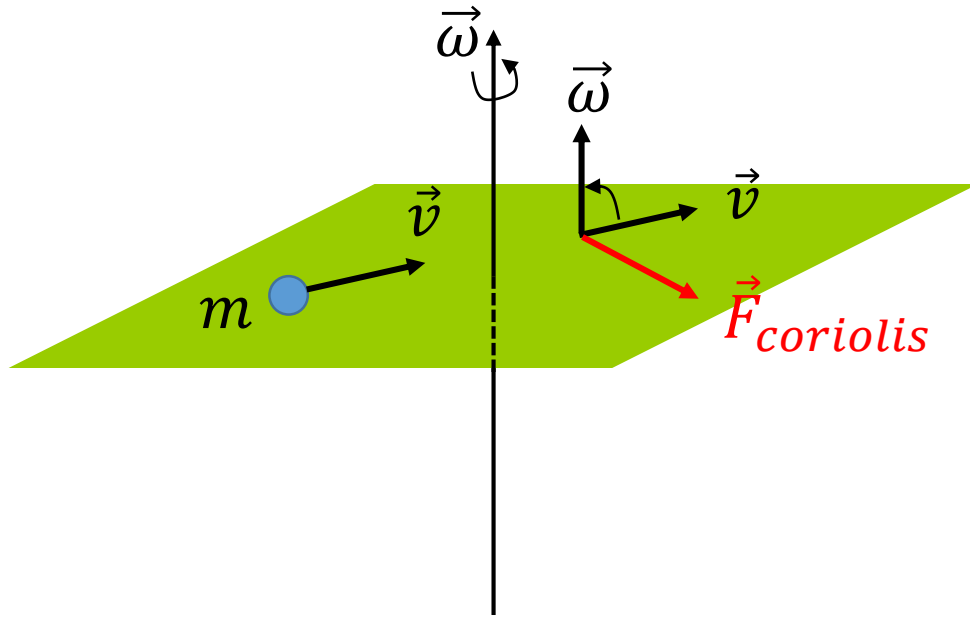
$$\vec{F}(\vec{r}) = q\vec{E}$$

$$\angle \theta \neq 0$$

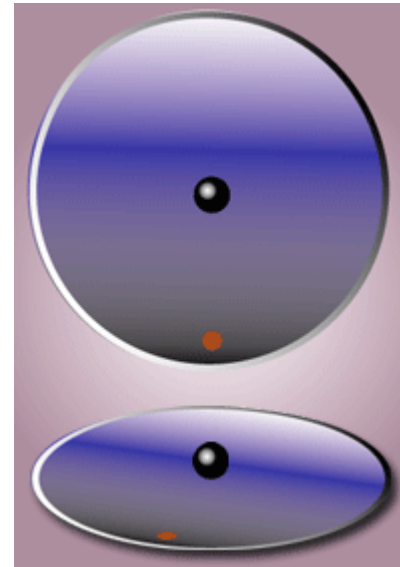
$$\vec{F}(\vec{r}, \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$



# Mechanical equivalence of Lorentz force



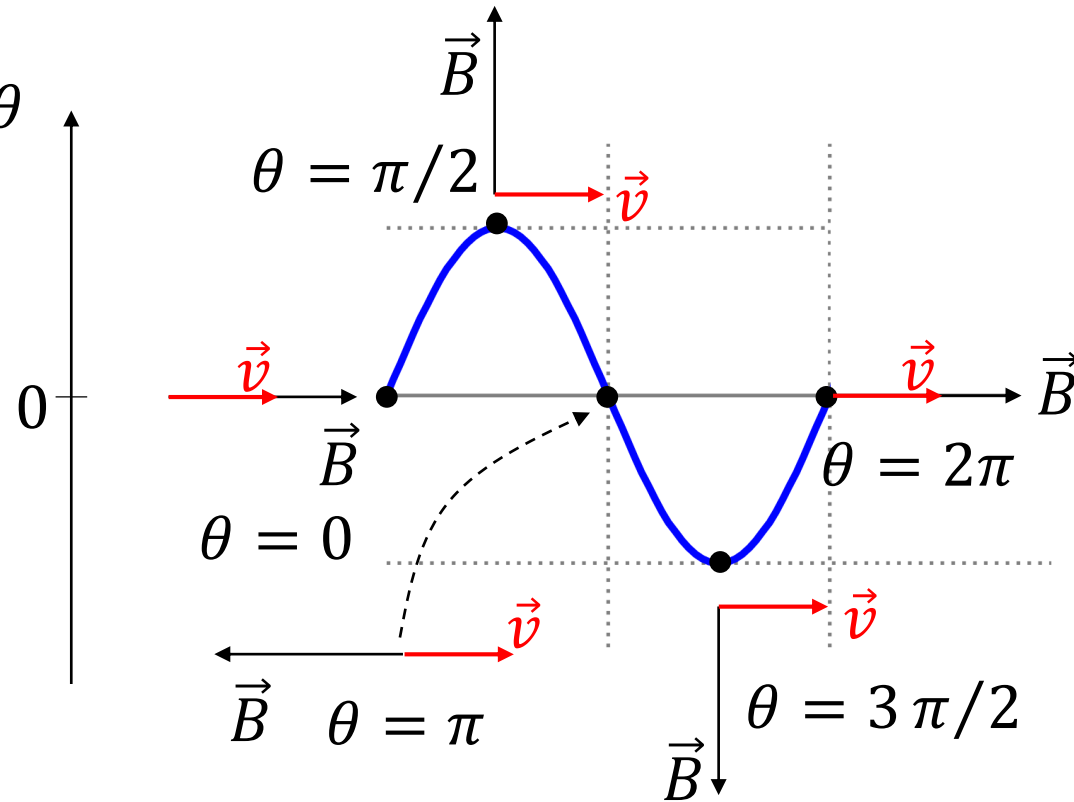
$$\vec{F}_C = 2m\vec{v} \times \vec{\omega}$$



External observer

Observer sitting on the red point

$$F_B = qvB\sin\theta$$



$$[B] \propto [F/qv]$$

Dimension of the magnetic field

$$= \frac{N}{C.ms^{-1}} = N/Am = \text{Tesla}$$

$$1 \text{ Gauss (G)} = 10^{-4}T$$

$$B \text{ on the earth} = 10^{-4}T = 1G$$

$$B \text{ inside atom} = 10T = 10^5G$$

$$B_{atom} = 10^5 B_{earth}$$

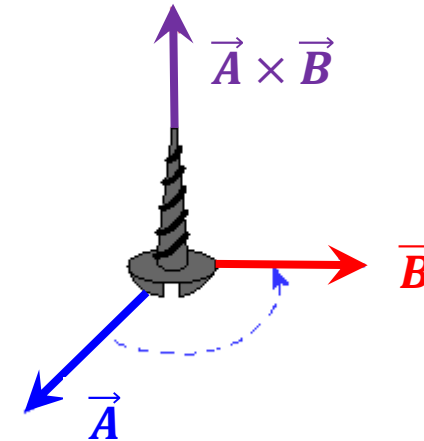
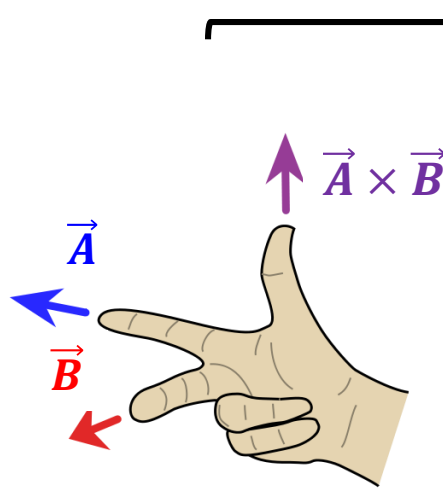
# Main tool to help understanding magnetism

Wire carrying current



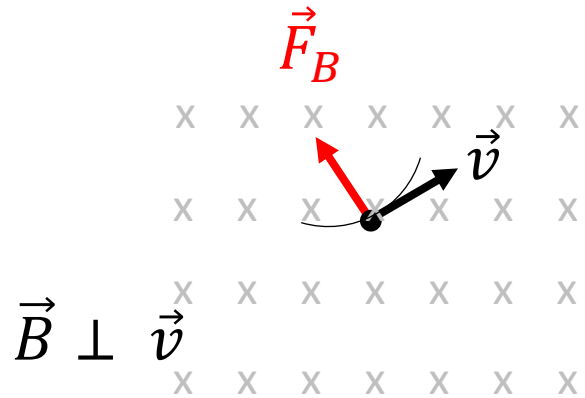
Right hand rule

Lorentz force on moving charge



Screwing and unscrewing rule

# Motion path of a charge in a magnetic field



Why the magnetic force affects the direction of  $\vec{v}$  only **NOT** the speed?

$$dW = \vec{F}_B \cdot d\vec{l} = 0 \quad \vec{F}_{//} = \vec{0}$$

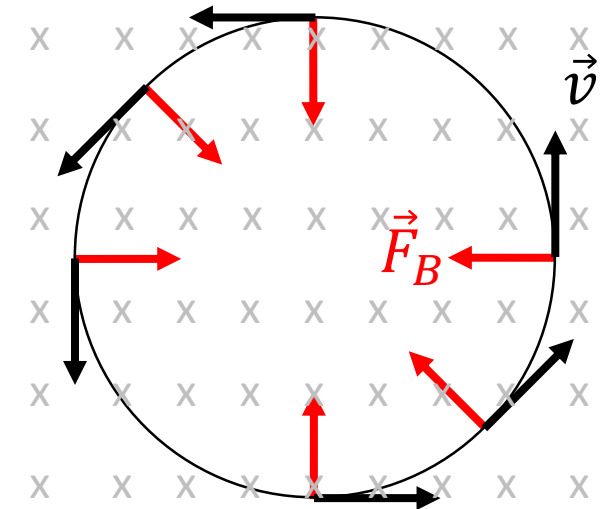
The magnetic force does not do any work on the moving charge

**Work - Energy theorem**  $\Delta K = \Delta W = \vec{F}_B \cdot \Delta \vec{l} = \vec{F}_B \cdot (\vec{v} \Delta t) = 0$

**NO** work  $\Rightarrow$  **NO** change in kinetic energy  $\Rightarrow$  **NO** change of speed

**Consequences:**

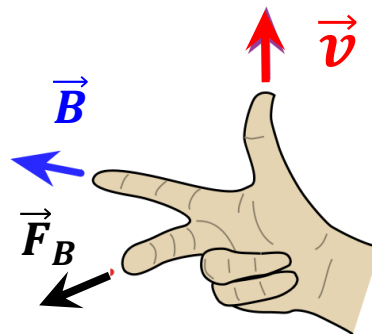
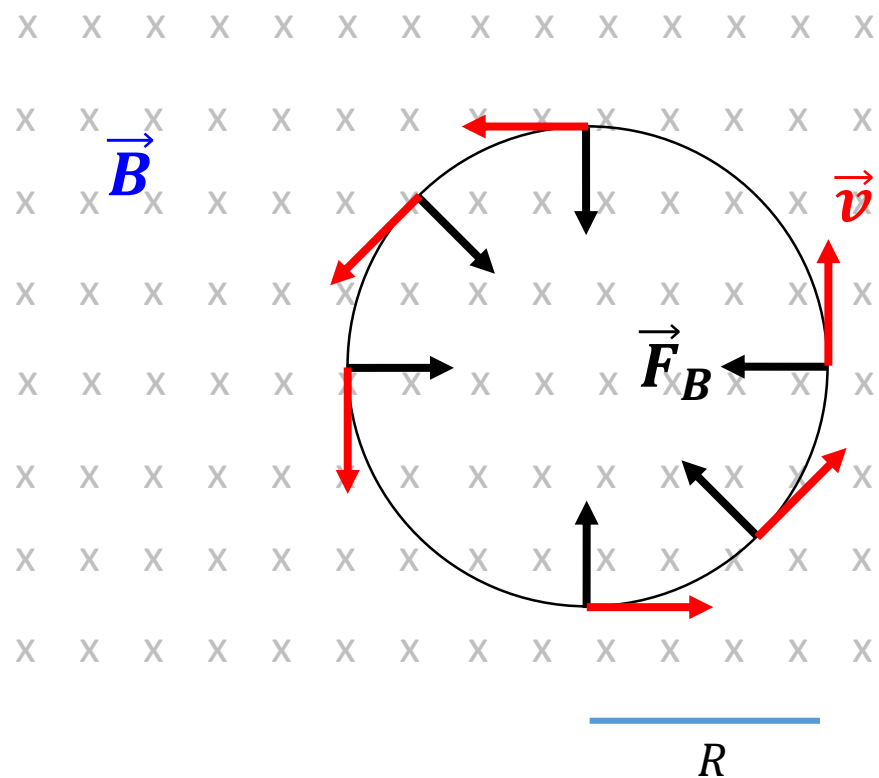
- Only direction changes
- Particle moves in a circle



# Application of the Lorentz force

- Mass spectrometer
- Thomson experiment
- Velocity selector





Two particles A and B with the same charge but not the same mass enter the magnetic field with the same speed.

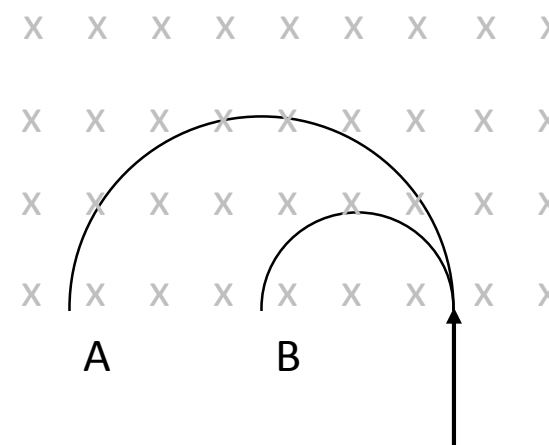
What is the relation between mass and curvature?

Bigger mass  $\Rightarrow$  larger inertia  $\Rightarrow$  less acceleration  
Thus larger radius

## Mass spectrometer

From mechanics

$$\text{Centripetal force} = \frac{mv^2}{R} = qvB$$

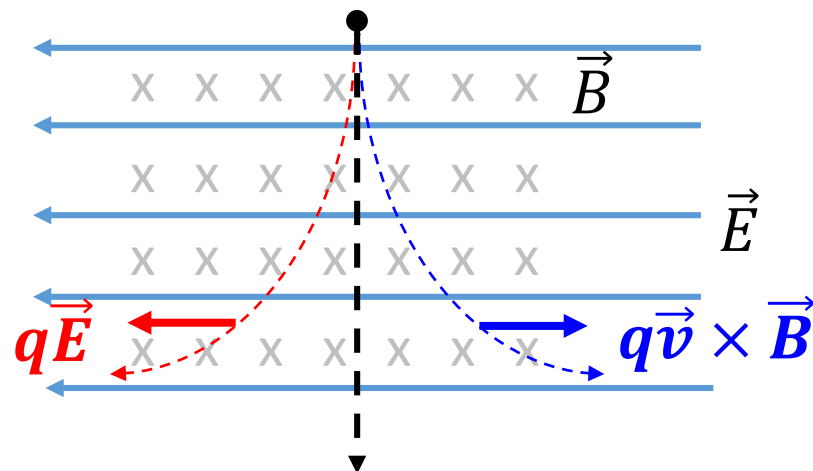


$$R = \frac{mv}{qB}$$

# Thomson experiment: discovery of the electron

**Question:** Given a charge  $q$  moving with a velocity  $\vec{v}$ , it crosses a magnetic field that we want to measure. How can we proceed ?

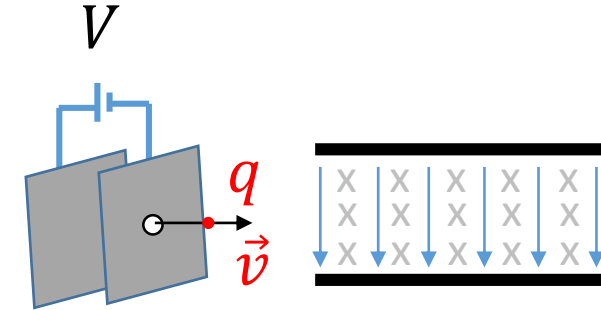
## Thomson experiment



$$qE = qvB$$

$$B = \frac{E}{v}$$

**Does not depend on  $q$  !**



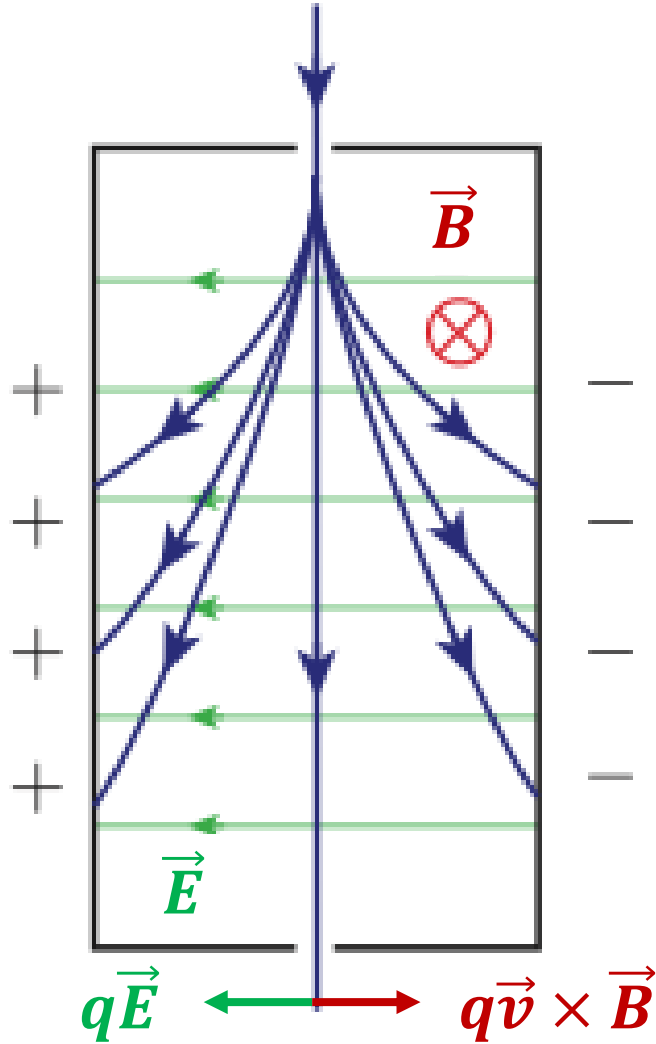
$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \frac{E}{B}$$

$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

Same particles getting in  
with different velocities

## Velocity selector



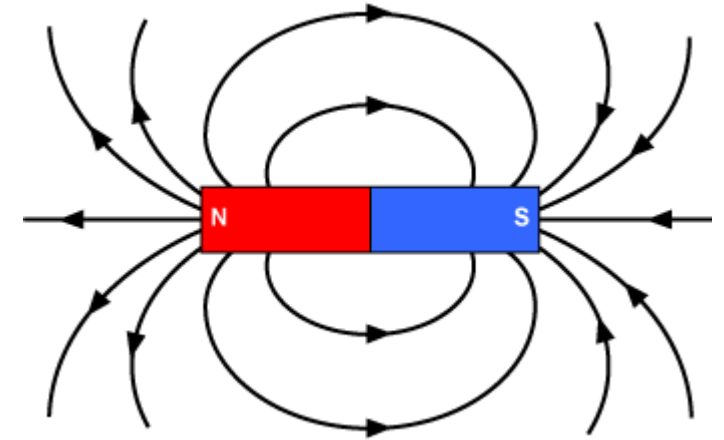
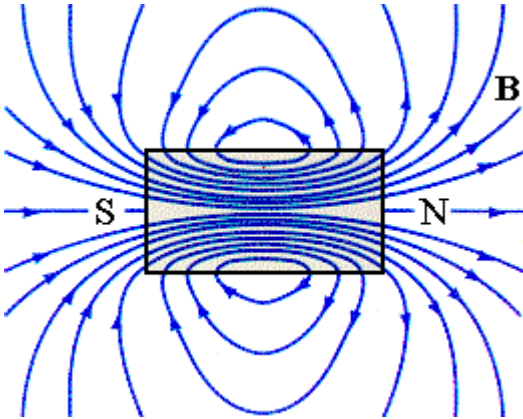
- By adjusting  $E$  and  $B$  appropriately, we can select particles with a particular speed to accelerate for other purposes.
- If all particles have the same mass, then the setup can be used as a source of mono-kinetics particles used for accelerators

## Origin of the magnetic field

- Magnet
- Current carrying wires

# Magnet: Magnetic field lines

Which one of these representations is correct ?



***This is not correct***

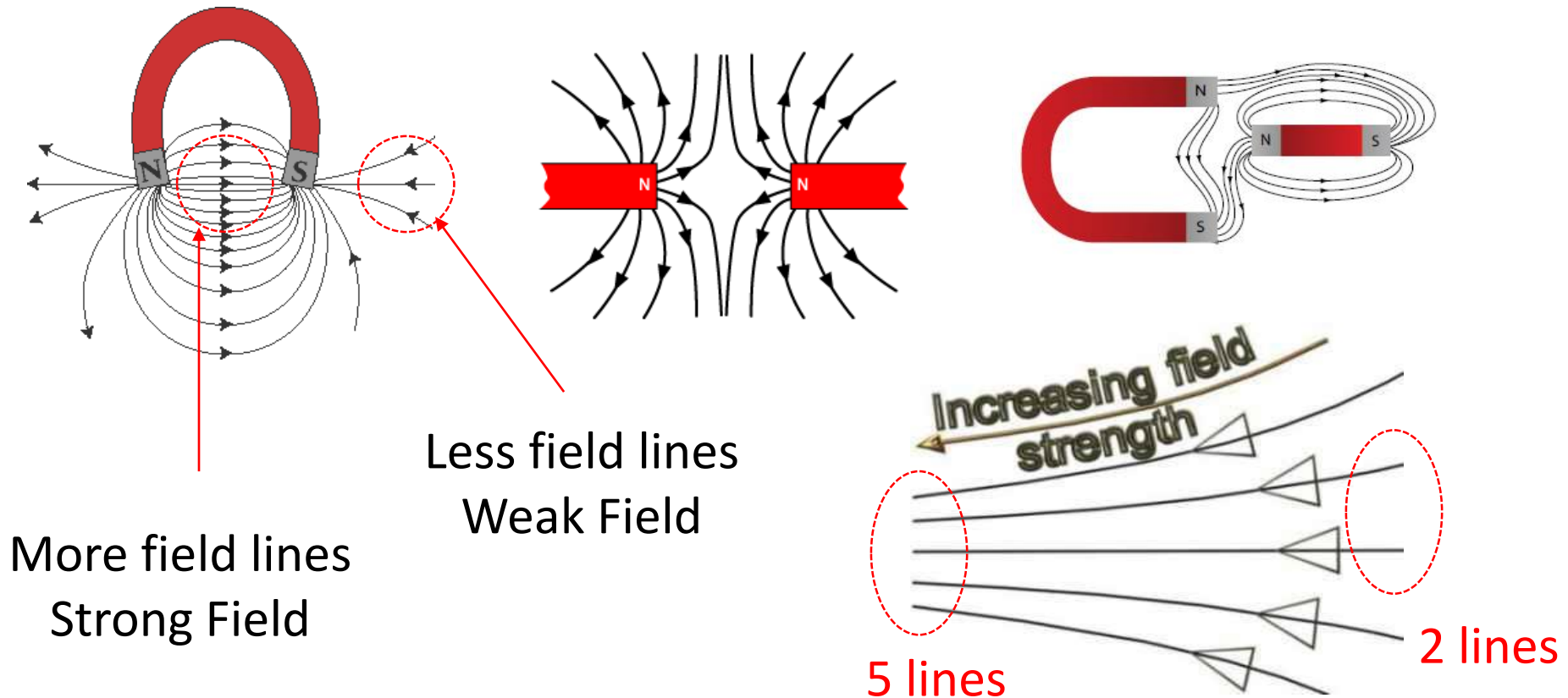
- Outside the magnet lines go from N to S
- Inside the magnet lines go from S to N

Every magnetic field line forms a loop  
There is no start and no end to the field lines

Source and sink for magnetic field lines cannot be separated

As for electrostatic, the magnetic field vectors are tangent to the field lines

As at every point in space the field is unique  $\Rightarrow$  Lines never cross



# Field lines

Electric monopole exists

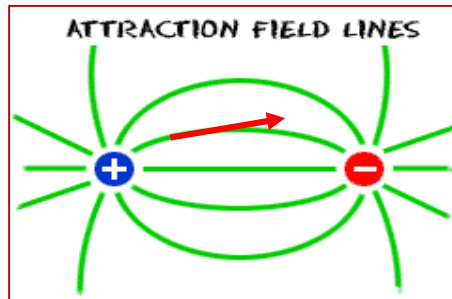
$E$  lines start at **+'s** and end at **-'s**

Magnetic monopole does not exist

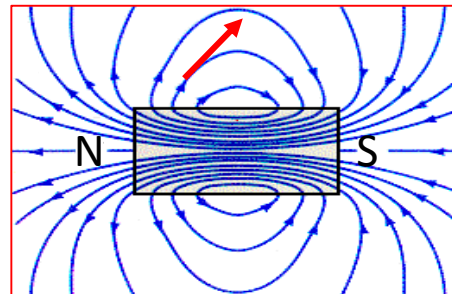
$B$  lines form loops

Comparison makes sense for dipoles...**BUT...**

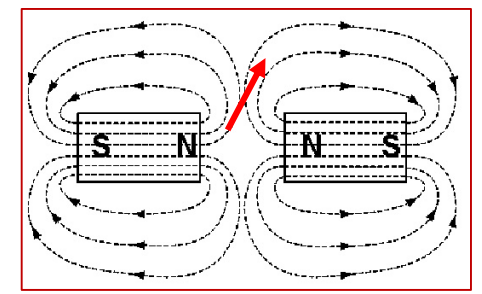
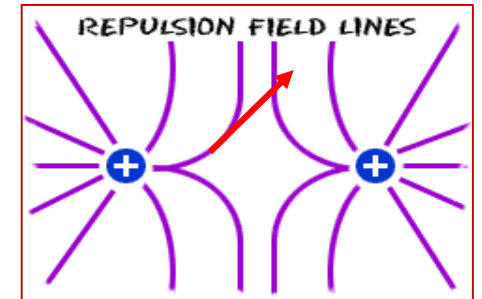
Lines are concave everywhere



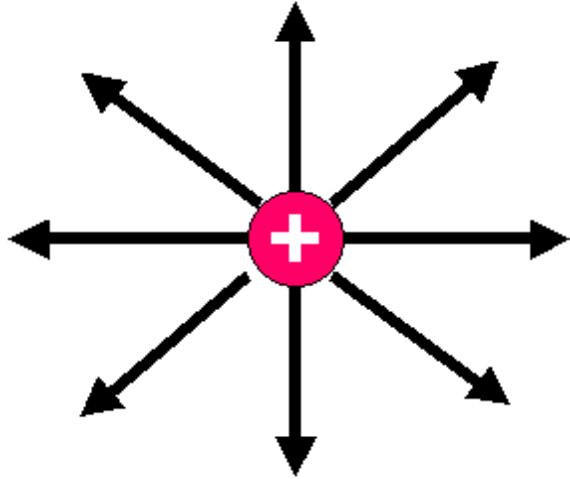
Lines are concave outside magnet and convex inside



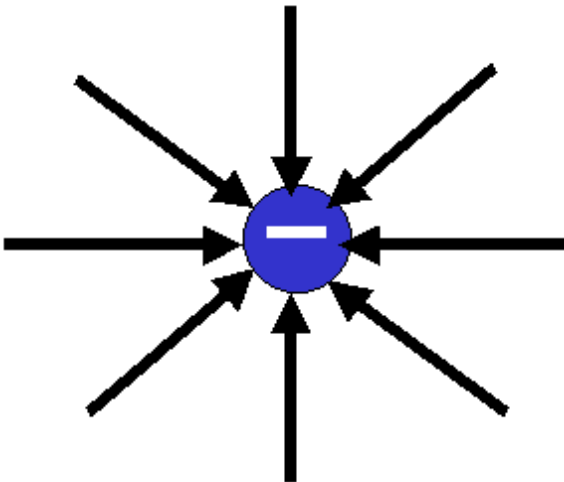
Vector field ( $E$  or  $B$ ) is always Tangent to the field line



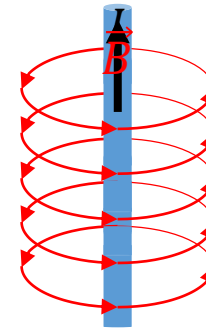
# Magnetic flux and Gauss law for magnetism



- Charges radiate outward (inward) for  $+q$  ( $-q$ )
- $\vec{E}$  lines have a start ( $+q$ ) and end ( $-q$ )



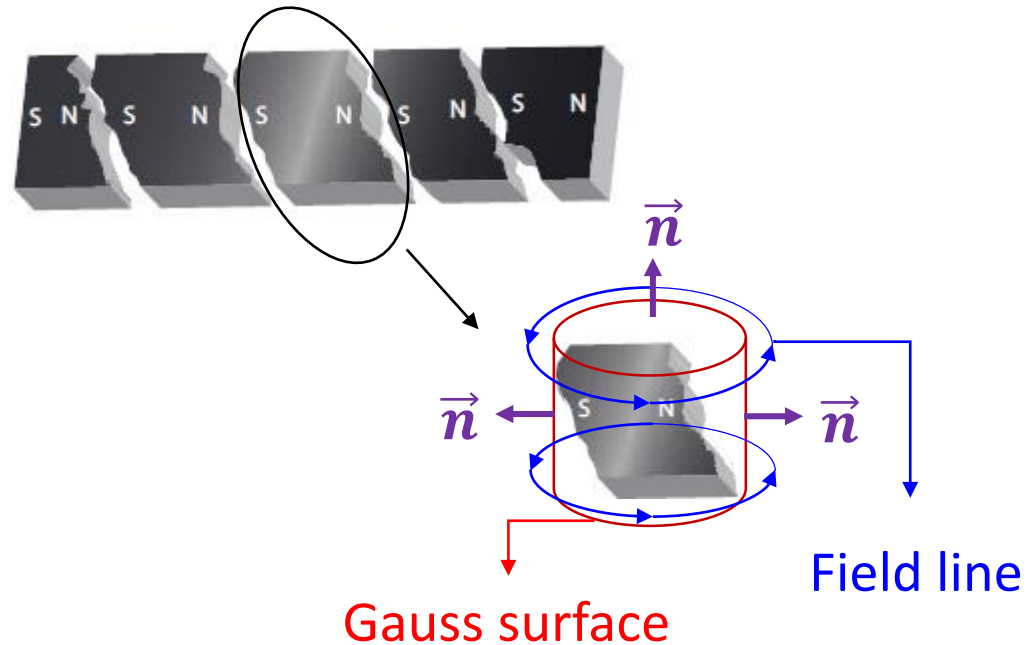
Magnetic monopole  
Does not exist



- $\vec{B}$  lines encircle the current
- $\vec{B}$  lines never end



# Magnetic flux and Gauss law for magnetism

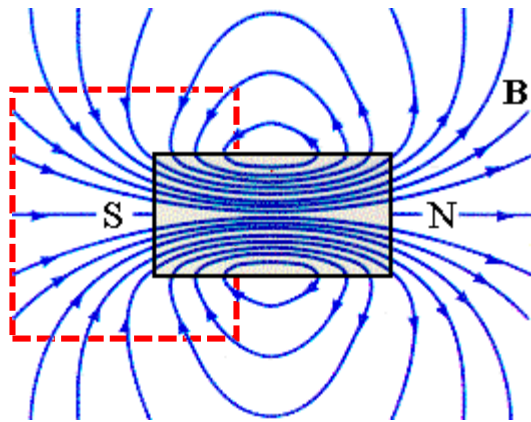


$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Or Gauss theorem

$$\vec{\nabla} \cdot \vec{B} = 0$$

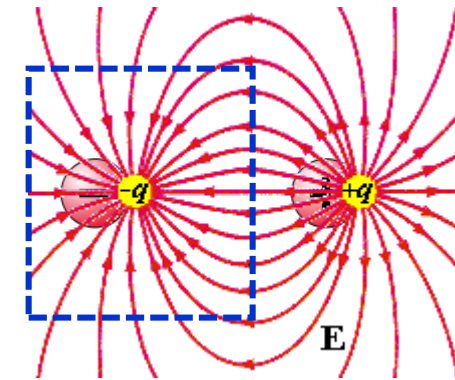
*It does not mean that  $\vec{B} = \vec{0}$  inside the Gaussian surface*



Divergence of the field



Two of Maxwell's equations



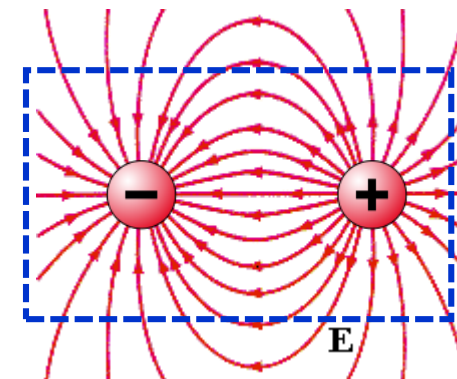
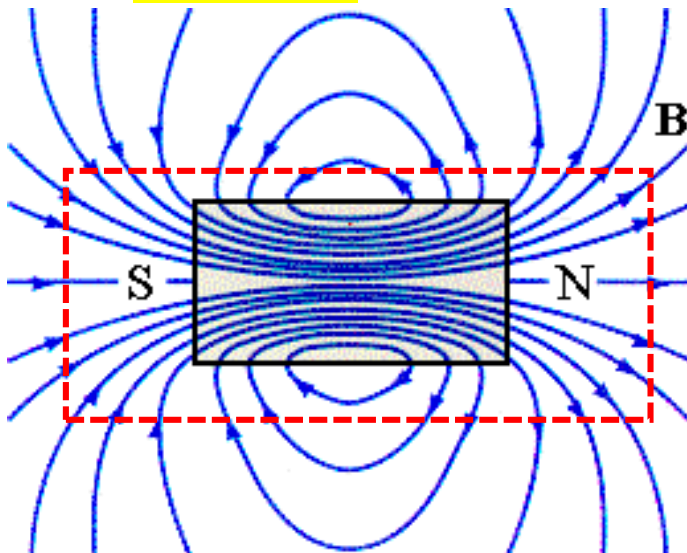
**B** never diverges: Always loops around the pole

**E** diverges: Extends to or from infinity to the pole

$$\vec{\nabla} \cdot \vec{B} = 0$$

Gauss Laws

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

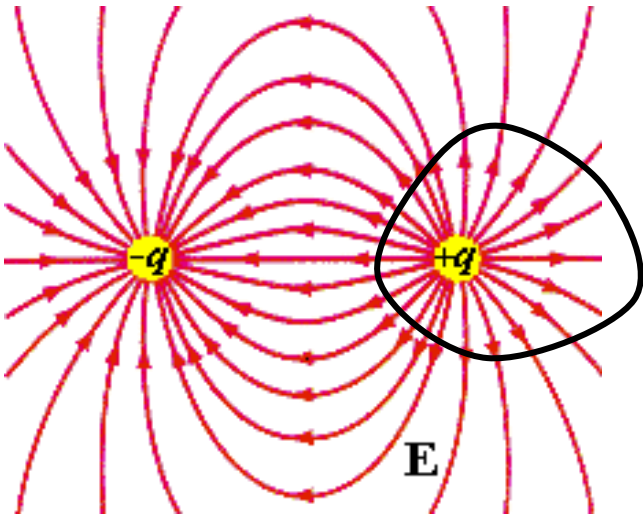


$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$



- $\vec{E}$  diverges out of or toward the charges
- $\vec{E}$  is the response to the charges located somewhere
- $\rho$  is the source of  $\vec{E}$  like a source of fluid
- $\epsilon_0$  is the ability of vacuum to adapt the field lines in response to the charge source



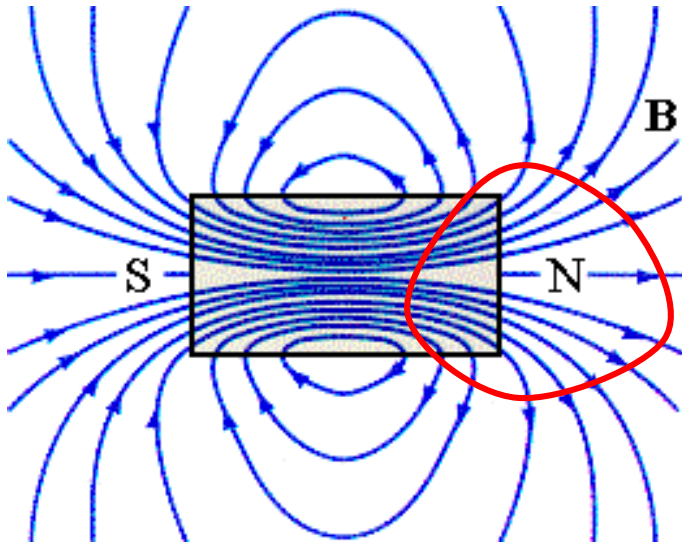
Inside this closed path the lines are all going out of the charge  $+q$  and they are all going toward  $-q$

With a magnet things are fundamentally different.

$$\vec{\nabla} \cdot \vec{B} = 0$$

$\vec{B}$  has no isolated source

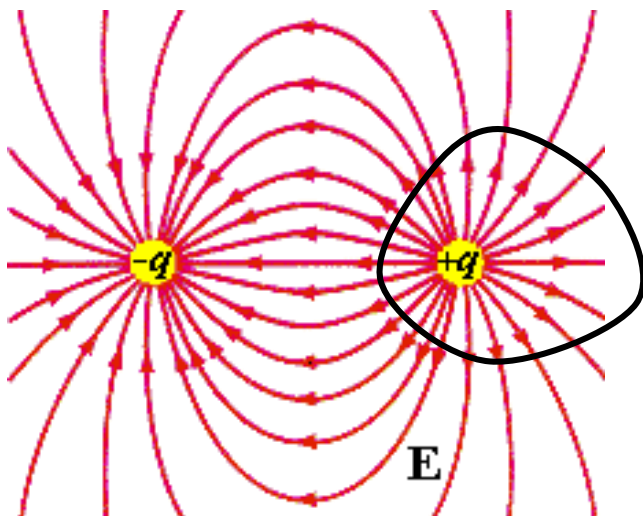
$\vec{B}$  lines have no start and no end



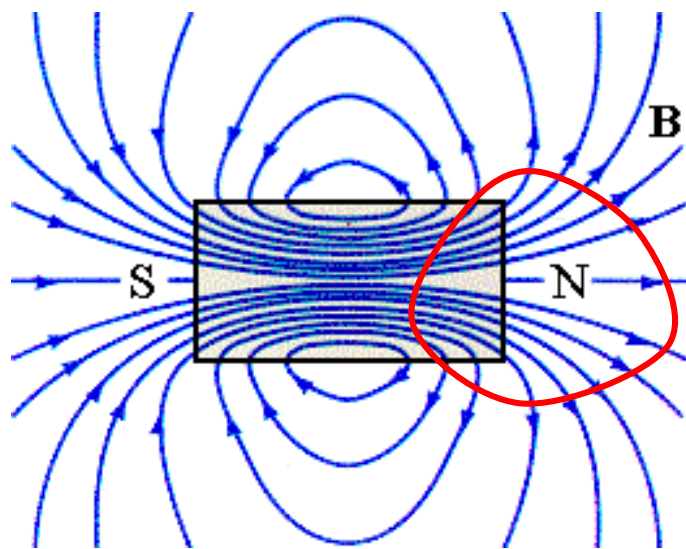
The area inside the closed path is divided into two regions:

- Region I outside the magnet: The lines are all going out of the North pole
- Region II inside the magnet: The lines are back to the North pole coming from the South pole

The density of field lines  $\Leftrightarrow$  Strength of field



Source and sink are independent



Huge difference

Source and sink are **NOT** independent

# Current as a source of magnetic field

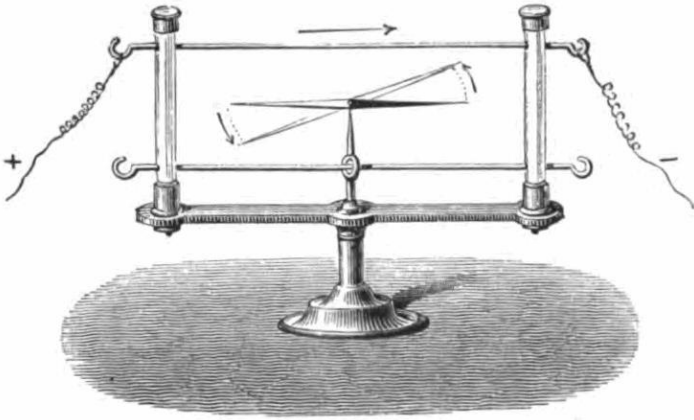
## Biot & Savart's law

## What was known at the time of Biot & Savart? Coulomb's law (1785)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{e}_r$$

- $\vec{E}$  is along the unit vector  $\vec{e}_r / |\vec{r}|$
- $\vec{E}$  is proportional to the charge  $q$
- $\vec{E}$  is inversely proportional to the charge  $r^2$
- $\vec{E}$  is inversely proportional to the permittivity  $\epsilon_0$

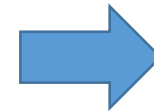
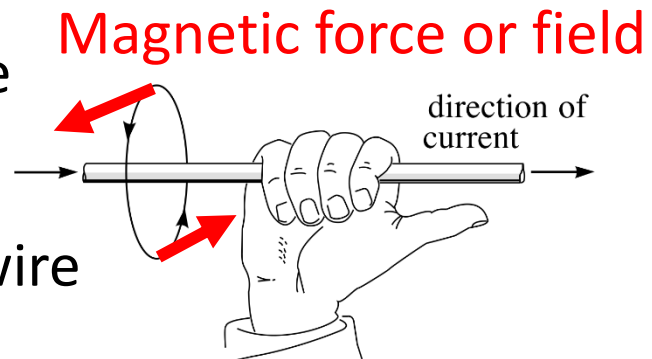
# 1820 Oersted experiment: experimental facts on the magnetic field



Charges set into motion  
generate a magnetic field

Compass above the wire

Compass below the wire



Inverting the direction of motion  
of the charges inverts the direction  
of the magnetic field

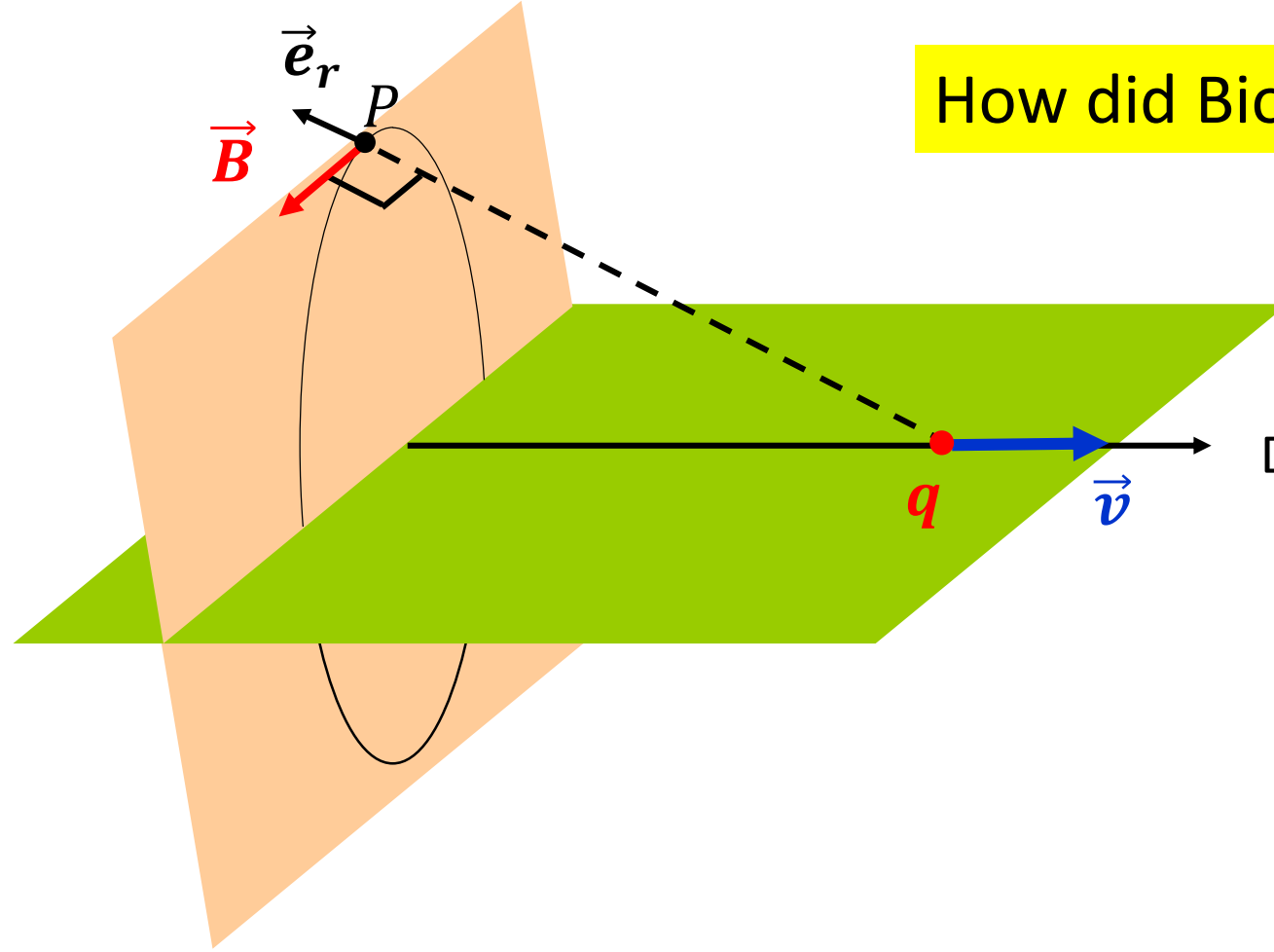


## 1820 Biot & Savart law: linking charge motion to magnetic field

What are the ingredients determining the law resulting from Orsted's observation?

- $\vec{B}$  is proportional to the velocity  $\vec{v}$
- $\vec{B}$  is perpendicular to the plane defined by  $\vec{v}$  and  $\vec{e}_r$   
(The magnetic field is thus proportional to  $\vec{v} \times \vec{e}_r$ )
- $\vec{B}$  is proportional to the charge  $q$
- $\vec{B}$  is inversely proportional to the charge  $r^2$
- $\vec{B}$  is proportional to the permeability  $\mu_0$

## How did Biot & Savart to establish their law?



Magnetic field of a moving charge

Direction of motion of the charge

$\vec{B}$  is perpendicular to the plan  $(\vec{v}, \vec{e}_r)$



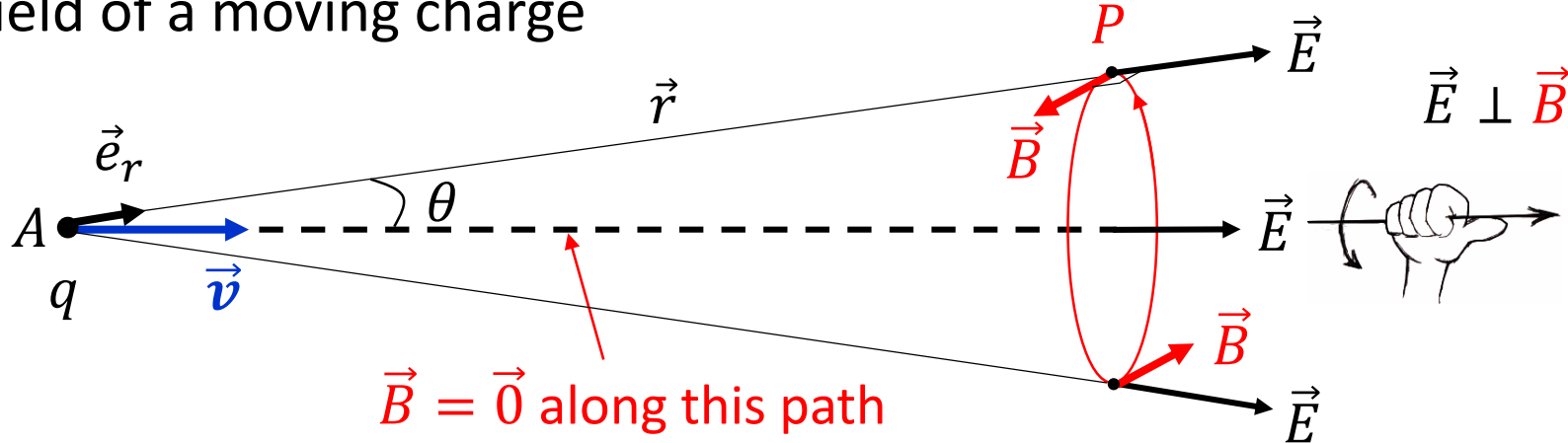
$\vec{B}$  is perpendicular to both  $\vec{v}$  and  $\vec{e}_r$



$\vec{B}$  must result from a cross product of  $\vec{v}$  and  $\vec{e}_r$

# Origin of the magnetic field: some more imagination

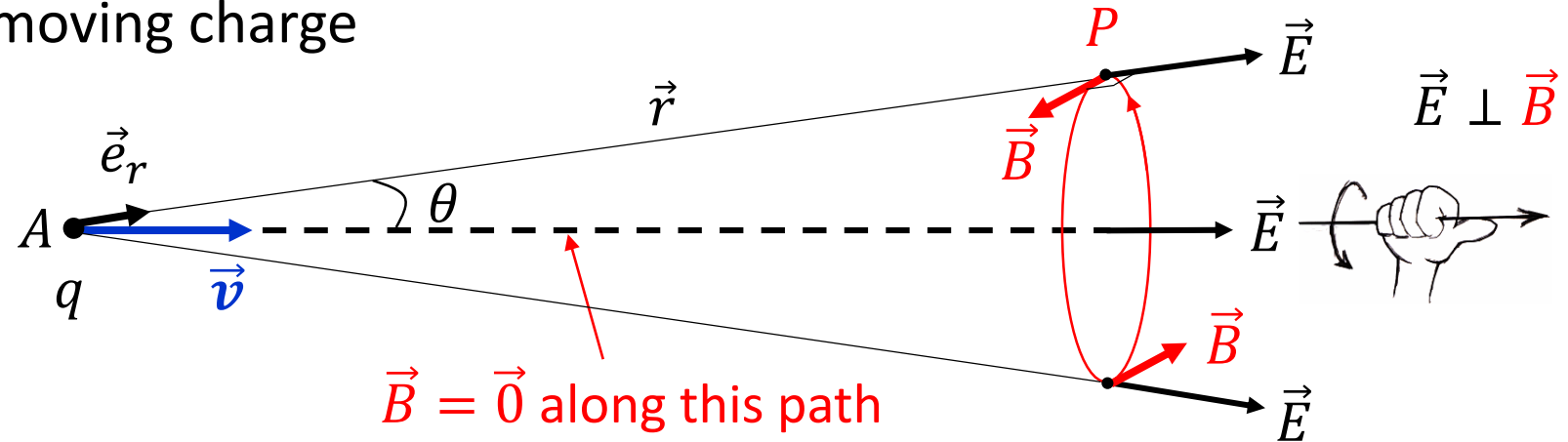
## Magnetic field of a moving charge



- $\vec{B}$  must decrease with increasing  $r$  Why not following  $1/r^2$  as in coulomb law !
- $\vec{B}$  must result from a cross product of  $\vec{v}$  and  $\vec{e}_r$
- As for the electric field, there must be a permeability of  $\vec{B}$  in vacuum:  $\mu_0$

# Origin of the magnetic field

## Magnetic field of a moving charge



### Coulomb's law

Electrostatic

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{e}_r$$

$\vec{E}$  is along the unit vector  $\vec{e}_r$

### Biot & Savart's law

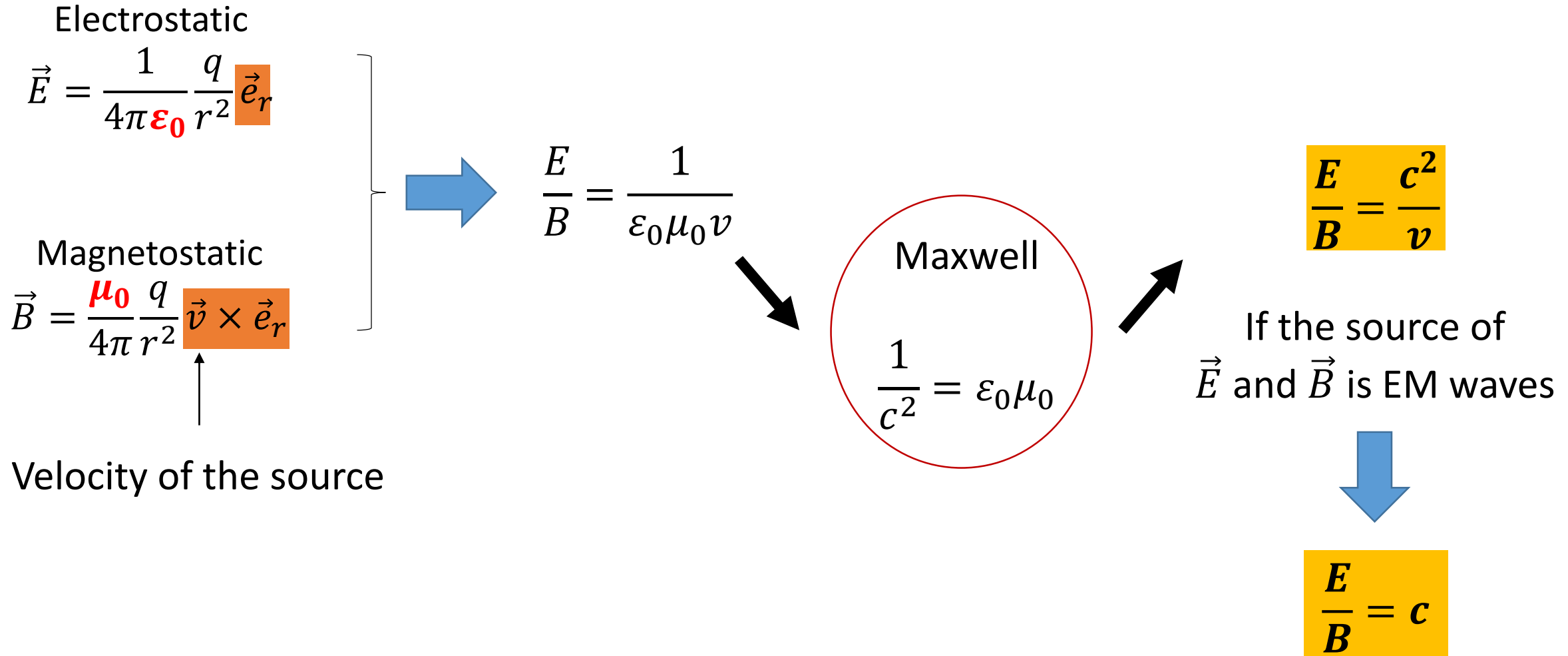
Magnetostatic

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q}{r^2} \vec{v} \times \vec{e}_r$$

$\vec{B}$  is perpendicular to the plan  $(\vec{v}, \vec{e}_r)$

# Origin of the magnetic field

Electric and Magnetic fields produced by a steady moving charge



## Conceptual difficulty with a single moving charge

At a given fixed position  $P[\vec{r}(t)]$   The field is **non-stationary**

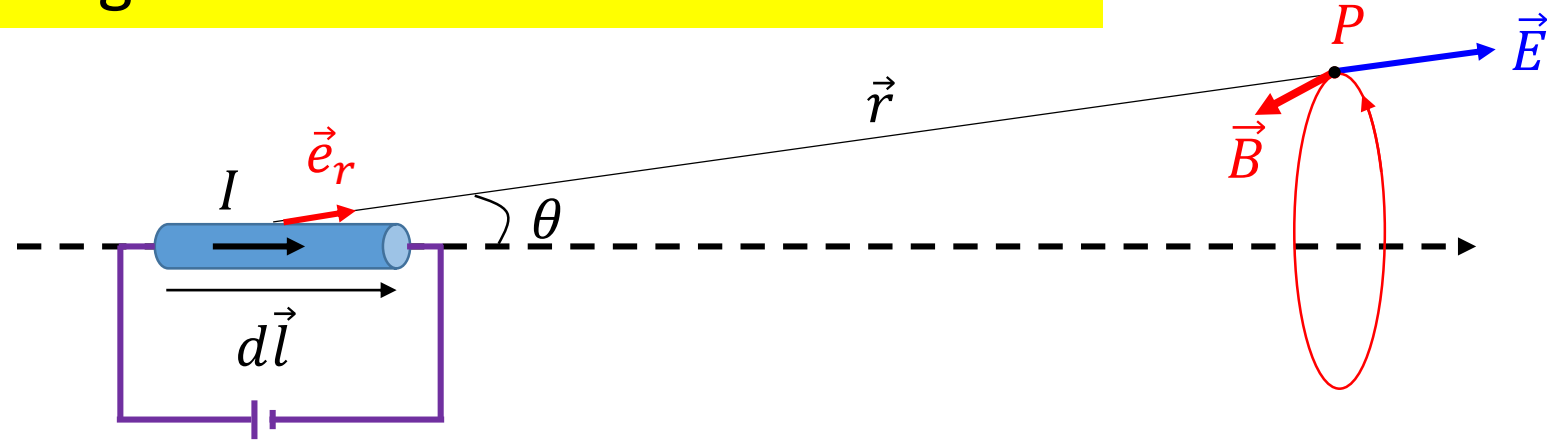
As soon as the particle starts moving from  $A$  at  $t_0$ , at position  $P$  and at a later time  $t_1$  the vector  $\vec{r}_{AP}$  has changed from  $\vec{r}_{AP}(t_0)$  to  $\vec{r}_{AP}(t_1)$

$\vec{B}[\vec{r}(t)] = \frac{\mu_0}{4\pi} \frac{q}{\vec{r}_{AP}(t)^2} \vec{v} \times \vec{e}_r$    $\vec{B}(t)$  at  $P$  is time dependent

The process is no longer magnetostatic

# Magnetic field of a current element

Biot and Savart law: 1820



$$dQ = \rho dV = nqAdl$$

$\vec{v}$  = drift velocity

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{dQ}{r^2} \vec{v} \times \vec{e}_r \quad \Rightarrow \quad d\vec{B} = \frac{1}{4\pi\mu_0} \frac{nqAdl}{r^2} \vec{v} \times \vec{e}_r \quad \Rightarrow \quad d\vec{B} = \frac{1}{4\pi\mu_0} \frac{nqAv}{r^2} d\vec{l} \times \vec{e}_r$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{r^2} d\vec{l} \times \vec{e}_r$$

For a complete circuit

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I}{r^2} d\vec{l} \times \vec{e}_r$$

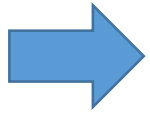
## Flux of magnetic field and Gauss's theorem

Demonstration of  $\vec{\nabla} \cdot \vec{B} = 0$  and  $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$  based on Biot & Savart law



$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{r^2} d\vec{l} \times \vec{e}_r \quad \text{With} \quad \vec{e}_r \rightarrow \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|} \quad \frac{1}{r^2} \rightarrow \frac{1}{(\vec{r} - \vec{r}')^2} \quad d\vec{l} \rightarrow d\vec{l}'$$



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\overset{\text{I}}{\boxed{\phantom{I}}}}{|\vec{r} - \vec{r}'|^3} d\vec{l}' \times (\vec{r} - \vec{r}') = \frac{\mu_0 I}{4\pi} \int d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}) ?$$

$$\text{Steady current} \rightarrow \frac{dI}{dt} = 0$$

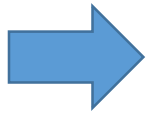


$$\vec{\nabla} \cdot \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \vec{\nabla} \cdot \left( \int d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right) = \frac{\mu_0 I}{4\pi} \int \vec{\nabla} \cdot \left[ d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right] \quad \vec{\nabla} \text{ acts on } \vec{r} \text{ only}$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \vec{\nabla} \cdot \left( \int d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right) = \frac{\mu_0 I}{4\pi} \int \vec{\nabla} \cdot \left[ d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right]$$

$$\vec{\nabla} \cdot (\vec{U} \times \vec{V}) = \vec{V} \cdot (\vec{\nabla} \times \vec{U}) - \vec{U} \cdot (\vec{\nabla} \times \vec{V})$$

$$\vec{U} = d\vec{l}' \quad \vec{V} = \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$



$$\vec{\nabla} \cdot \left( d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right) = \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \cdot \vec{\nabla} \times d\vec{l}' - d\vec{l}' \cdot \left( \vec{\nabla} \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right)$$

$\uparrow$  This acts on  $r$        $\uparrow$  This acts on  $r'$

$$\frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = -\vec{\nabla} \frac{1}{|\vec{r} - \vec{r}'|}$$

$$-d\vec{l}' \cdot \left( \vec{\nabla} \times \vec{\nabla} \frac{1}{|\vec{r} - \vec{r}'|} \right)$$

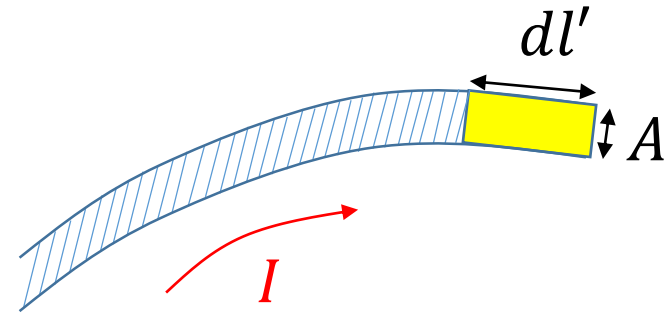


$$\vec{\nabla} \cdot \vec{B} = 0$$

There are **NO** independent sources or sinks for magnetic fields

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \vec{\nabla} \times \left( \int d\vec{l}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right)$$



$$I d\vec{l}' = \vec{J}(\vec{r}') A \cdot d\vec{l}' \\ = \vec{J}(\vec{r}') dV'$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \underbrace{\vec{\nabla} \times \left( \vec{J}(\vec{r}') \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right)}_{\vec{U}} dV'$$

$$\vec{U} = \vec{J}(\vec{r}')$$

$$\vec{V} = \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{\nabla} \times (\vec{U} \times \vec{V}) = (\vec{V} \cdot \vec{\nabla}) \vec{U} - (\vec{U} \cdot \vec{\nabla}) \vec{V} + \vec{U}(\vec{\nabla} \cdot \vec{V}) - \vec{V}(\vec{\nabla} \cdot \vec{U})$$

$$\vec{V} \cdot \vec{\nabla} = V_x \frac{\partial}{\partial x} + V_y \frac{\partial}{\partial y} + V_z \frac{\partial}{\partial z}$$

$$\vec{V} \times (\vec{U} \times \vec{V}) = (\vec{V} \cdot \vec{V})\vec{U} - (\vec{U} \cdot \vec{V})\vec{V} + \vec{U}(\vec{V} \cdot \vec{V}) - \vec{V}(\vec{V} \cdot \vec{U})$$

$$\rightarrow \underbrace{\left( \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \cdot \vec{V} \right)}_{\text{Acting on } r} \vec{J}(\vec{r}') = \vec{0}$$

$$-(\vec{J}(\vec{r}') \cdot \vec{V}) \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\rightarrow \vec{V} \left( \vec{V} \cdot \vec{J}(\vec{r}') \right) = \vec{0}$$

Acting on  $r$

$$\vec{U}(\vec{V} \cdot \vec{V}) \rightarrow \vec{J}(\vec{r}') \vec{V} \cdot \frac{(\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3} = 4\pi \vec{J}(\vec{r}') \delta(\vec{r} - \vec{r}')$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int -(\vec{J}(\vec{r}') \cdot \vec{\nabla}) \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV' + \mu_0 \int \vec{J}(\vec{r}') \delta(\vec{r} - \vec{r}') dV'$$

$$= -\frac{\mu_0}{4\pi} \int \underbrace{(\vec{J}(\vec{r}') \cdot \vec{\nabla}) \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}}_{-(\vec{J}(\vec{r}') \cdot \vec{\nabla}') \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}} dV' + \mu_0 \vec{J}(\vec{r})$$

For steady current

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

**Ampere's law**

## Other supplementary questions

**Q2:** At a given instant, a **positive charge** moves in the positive x direction in a region where there is a magnetic field in the negative z direction. What is the direction of the magnetic force? Does the charge continue to move in the positive x direction?

**Q3:** Two charged particles are projected into a region where there is a magnetic field perpendicular to their velocities. If the charges are deflected in opposite directions, what can you say about them?

**Q4:** If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is zero?

**Q5:** A **negative charge** moving along the positive x axis perpendicular to a magnetic field experiences a magnetic deflection in the negative y direction. What is the direction of the magnetic field?

**Q6:** An electron is projected into a uniform magnetic field  $\mathbf{B} = (1.4 \mathbf{i} + 2.1 \mathbf{j})$  T. Find the vector expression for the force on the electron when its velocity is  $\mathbf{v} = 3.7 \times 10^5 \mathbf{j}$  m/s