Homework 2: Vector calculus

1. Demonstrate that the projection of a vector $\vec{F}$ on a vector $\vec{w}$ is given by Proj($\vec{F}$) $= \frac{\vec{F}.\vec{W}}{|\vec{W}|^2}\vec{W}$

2. Find the flux of $\vec{F} = (x^2 + y^2)\vec{k}$ through the disk of radius 3 centered at the origin in the xy −plane and oriented upwards

3. For constants a, b, c, m, consider the vector field,
$\vec{F} = (ax + by + 5z)\vec{\imath} + (x + cz)\vec{\jmath} + (3y + mx)\vec{k}$

(a) Suppose the flux of $\vec{F}$ through any closed surface is 0. What does this tell you about the value of the constants a, b, c, m?

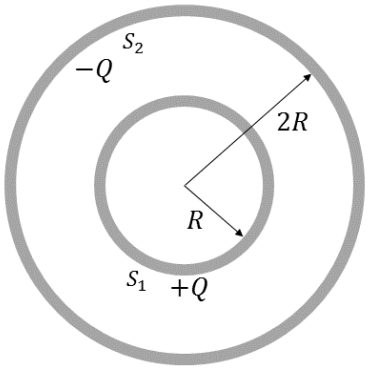
(b) Suppose instead that the line integral of $\vec{F}$ around any closed curve is 0. What does this tell you about the value of the constants a, b, c, m?

Gauss law: Application

4. Consider a spherical conducting shell $S1$ of radius R on which charge $+Q$ is placed. Without touching or disturbing it, this shell is now surrounded concentrically by a similar shell $S2$ of radius $2R$ on which charge $-Q$ is placed (see Figure).

(a) What is the magnitude of the electric field in the region between the two shells ($R < r < 2R$)?

(b) What is the electric field inside shell S1 (r < R)?



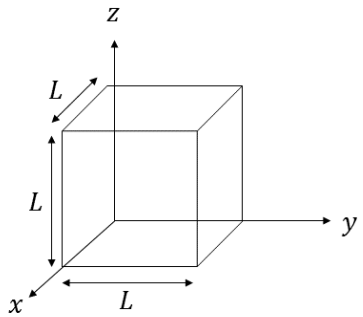
5. A solid insulating sphere of radius R has charge distributed uniformly throughout its volume. What fraction of the sphere's total charge is located within the region $r < R/2$?

6. A solid insulating sphere of radius R has a *non-uniform* volume charge distribution given by $\rho(r) = ar$, where a is a constant. What is the total charge Q of the insulating sphere?

7. In a certain region of space within a distribution of charge the electric field is given by $\vec{E}(r) = ar\hat{r}$. It points radially away from the origin and has a magnitude $E(r) = ar$, where $a = 150N/(Cm)$. How much electric charge (in nC) is located inside a shell with an inner radius of $0.5\ m$ and an outer radius of $1.0\ m$?

8. In a certain region of space the electric field is given by $\vec{E}(r) = (a\ /\ r)\hat{r}$. It points radially away from the origin and has a magnitude $E(r) = a/r$, where $a = 90Nm/C$. How much electric charge (in nC) is located inside a sphere with radius $R = 0.5\ m$?

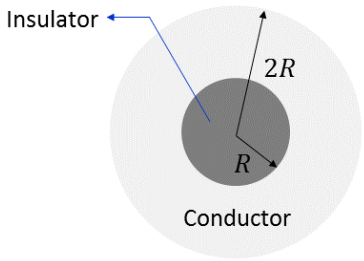
9. Consider a cube of sides $\mathrm{L} = 2\ \mathrm{m}$, as shown in the figure. Suppose that a non-uniform electric field is present and is given by $\vec{\mathrm{E}}(\mathrm{x}) = (\mathrm{a} + \mathrm{bx})\hat{\mathrm{x}}$, where $\mathrm{a} = 1\ \mathrm{N/C}$ and $\mathrm{b} = 0.5\ \mathrm{N/(Cm)}$. What is the total net charge within the cube (in pC)?



10. A solid insulating sphere of radius R has a charge $+Q$ distributed uniformly throughout its volume (the volume charge density ρ is constant). The insulating sphere is surrounded by a solid spherical conductor with inner radius R and outer radius $2R$ as shown in the Figure. The conductor is in static equilibrium and has a net charge $+Q$.

(a) What is the magnitude of the electric field at the point $r = R/2$ inside the insulating sphere?

(b) What is the magnitude of the electric field at the point $r = 3R/2$ inside the conductor?



11. A point charge $+Q$ is located at the center of a solid spherical conducting shell with inner radius of R and outer radius of $2R$ as shown in the Figure. In addition, the conducting shell has a total *net* charge of $+Q$.

(a) How much charges are located on the inner ($r = R$) and outer surfaces ($r = 2R$) of the conducting shell?

(b) What is the magnitude of the electric field in the region $r < R$ inside the hole in the conducting shell?