VE230 Assignment 1

Issue date: September 29th, 2017

Due date: October 13th, 2017

*While solving the problems you should detail the calculations and write short sentences explaining the process, theorems used etc…

Pb#1 (10%): Find the unit vector $\vec{u}$ perpendicular in the right hand sense to the vectors given by

$\vec{v} = -\vec{\imath} + \vec{\jmath} + \vec{k}$ and $\vec{w} = \vec{\imath} - \vec{\jmath} + \vec{k}$

What is the angle between $\vec{v}$ and $\vec{w}$?

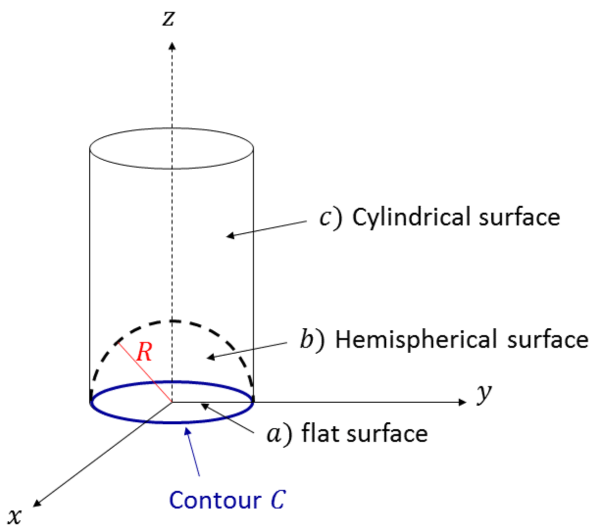
Pb#2 (10%): Find the **gradient** of each of the following functions

a) $f(x, y, z) = ax^2y + by^3z$
b) $g(r, \theta, z) = ar^2 sin\theta + brz sin2\theta$
c) $h(r, \theta, \phi) = \frac{a}{r} + br sin\theta cos\phi$

Pb#3 (30%): Verify **Stokes theorem** with the vector field,

$\vec{A} = -y\vec{\imath} + x\vec{\jmath} - z\vec{k}$

for the circular contour in the xy plane bounding the flat circular surface a, the hemispherical surface b and the cylindrical surface c shown in the figure.
To which of Maxwell's equation can you link this theorem?



Pb#4 (30%): Verify the **divergence theorem** for the vector position

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

Through a rectangular volume having one of its corners at the origin of the Cartesian frame with three of its sides located at distance $\boldsymbol{a}$ along the x −axis, $\boldsymbol{b}$ along the y-axis and $\boldsymbol{c}$ along the z −axis.

Pb#5 (10%): Line integral

Let $\vec{F} = ye^{xy}\vec{i} + xe^{xy}\vec{j} + (cosz)\vec{k}$ be the gradient of a function $f(x, y, z)$. Compute the line integral $\int_L \vec{\mathrm{F}}.d\vec{r}$ along L given by the curve line from $(0,0,\pi)$ to $(1,1,\pi)$, followed by the parabola $z = \pi x^2$ in the plane $y = 1$ to a point $(3,1,9\pi)$.

What would $\vec{F}$ represent regarding its relation to the function $f(x, y, z)$. In other words would these two quantities remind you a parallel in classical physics?

Pb#6 (10%): manipulating scalar and vector fields at once

Let be $f(x, y, z)$ a scalar field and $\vec{A}(x, y, z)$ be a vector field. Give a proof of the relation below,

$\vec{\nabla}.\left(f\vec{A}\right) = f\vec{\nabla}.\vec{A} + \vec{A}.\vec{\nabla}f$