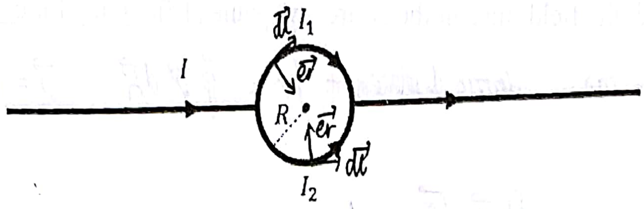
10

Pb#4 (20 pts): A single infinitely long straight piece of wire carrying a current I is split and bent so that it includes two half circular loops of radius R, as shown in the Figure. If current I_1 goes through the top half loop (and I_2 through the lower half loop) and if the magnetic field at the center of the loop is $\vec{B} = \pi kI/(2R)\vec{k}$ (constant) where the $z-axis$ is out of the paper, what are the currents I_1 and I_2?



For I_1, $dB_1 = \frac{\mu_0}{4\pi}\cdot\frac{I_1}{R^2}\vec{dl}\times\vec{e_r} = \frac{\mu_0}{4\pi}\frac{I_1}{R^2}\cdot dl$

$\therefore B_1 = \int \frac{\mu_0 I_1}{4\pi R^2} dl = \pi R\cdot\frac{\mu_0 I_1}{4\pi R^2} = \frac{\mu_0 I_1}{4R}$, pointing into the paper.

For I_2, $dB_2 = \frac{\mu_0}{4\pi}\cdot\frac{I_2}{R^2}\vec{dl}\times\vec{e_r} = \frac{\mu_0}{4\pi}\cdot\frac{I_2}{R^2}\cdot dl$

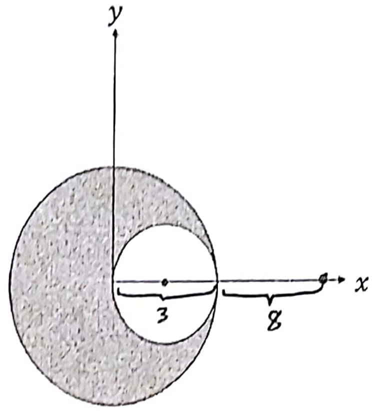
$\therefore B_2 = \int \frac{\mu_0 I_2}{4\pi R^2} dl = \frac{\mu_0 I_2}{4R}$, pointing out of paper.

$\therefore B = B_2 - B_1 \Rightarrow \begin{cases} \frac{\mu_0}{4R}(I_2 - I_1) = \frac{\pi k I}{2R} \\ I_1 + I_2 = I \end{cases} \Rightarrow \begin{cases} I_1 = \left(\frac{1}{2} - \frac{\pi k}{\mu_0}\right) I \\ \cancel{I_1 = \left(\frac{1}{2} - \frac{2\pi}{\mu_0}\right)} \\ I_2 = \left(\frac{\pi k}{\mu_0} + \frac{1}{2}\right) I \end{cases}$

Pb#1 (30 pts): A total current of 52 mA flows through an infinitely long cylindrical conductor of radius $r = 3\ cm$ which has an infinitely long cylindrical hole through it of diameter r centered at $r/2$ along the x-axis as shown.

$R = 11\ cm$

(a) What is the magnitude of the magnetic field at a distance of 11 cm along the positive x-axis? The permeability of free space is $4\pi \times 10^{-7}\ T \cdot m/A$. Assume the current density is constant throughout the conductor.



We can assume the cylindrical hole is full of conductor with the same current density ~~52 mA~~, and another conductor with the same current density ~~52 mA~~ in the opposite direction is placed.

$\therefore$ Using superposition principle,

$$\oint \vec{B_1} \cdot d\vec{l} = 2\pi \cdot R \cdot B_1 = \mu_0 I_1 \Rightarrow B_1 = \frac{\mu_0 I_1}{2\pi R} = \cancel{\frac{4\pi\times10^{-7}\times52\times10^{-3}}{2\pi\times11\times10^{-2}} = 9.455\times10^{-8}\,(T)}$$

$$\oint \vec{B_2} \cdot d\vec{l} = +B_2 \cdot 2\pi \cdot (R - \tfrac{r}{2}) = \mu_0 I_2 \Rightarrow B_2 = \frac{\mu_0 I_2}{2\pi(R-\frac{r}{2})} = \cancel{\frac{4\pi\times10^{-7}\times52\times10^{-3}}{2\pi\times(11-1.5)\times10^{-2}} = 1.095\times10^{-7}\,(T)}$$

$$J = \frac{I}{\pi r^2 - \pi \cdot (\frac{r}{2})^2} = \frac{4I}{3\pi r^2} = \frac{4\times52\times10^{-3}}{3\pi\times(3\times10^{-2})^2} = 24.52\ (A/m^2)$$

$$\therefore I_1 = J \cdot \pi r^2 = 69.33\ mA. \qquad I_2 = J \cdot \pi \cdot (\tfrac{r}{2})^2 = 17.33\ mA.$$

$$\therefore B_1 = \frac{\mu_0 I_1}{2\pi R} = \frac{4\pi\times10^{-7}\times 69.33\times10^{-3}}{2\pi\times11\times10^{-2}} = 1.26\times10^{-7}\ (T).$$

$$B_2 = \frac{\mu_0 I_2}{2\pi(R-\frac{r}{2})} = \frac{4\pi\times10^{-7}\times17.33\times10^{-3}}{2\pi\times9.5\times10^{-2}} = 3.648\times10^{-8}\ (T).$$

$$\therefore B = B_1 - B_2 = 8.952\times10^{-8}\ (T). \qquad \therefore \text{The magnitude is } 8.952\times10^{-8}\ (T).$$

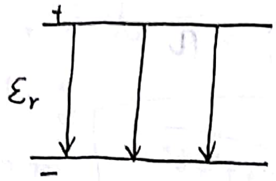
9p

Pb#3 (30 pts, 10, 20): given as Pb#1 in HW #4

Two plates capacitor whose space is filled with a dielectric of relative permittivity ε_r.

a) Determine the $\boxed{\text{energy density}}$ stored in the capacitor. A clear demonstration is needed. Giving the formula will not be accepted

b) How can we express the universality character of this energy density?



(a) energy density $\Delta U = \frac{u}{V}$, $V = A \cdot d$.

$\bar{D} = \varepsilon \bar{E}$ where $\varepsilon = \varepsilon_0 \varepsilon_r$ ✓

this is not a demonstration

and $w = \frac{1}{2} \bar{D} \bar{E}$, so $w = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$

which means the energy density is $\underline{\frac{1}{2} \varepsilon_0 \varepsilon_r E^2}$

(b) universality character of energy density:

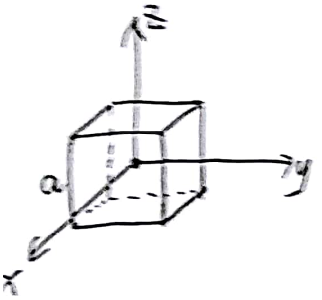
✓ $w = \frac{1}{2} \bar{D} \cdot \bar{E}$

$w = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$

energy density increase as E increase, and will never be negative.

20

Pb#2 (20 pts): A dielectric cube of side a, centered at the origin, carries a "frozen" polarization $\vec{P} = \alpha\vec{r}$. Where α is constant. Find all bound charges density and check your results.



① For the right surface, $\vec{n}_1 = \vec{j} = (0,1,0)$ $\vec{P} = \alpha\vec{r} = \alpha(x,y,z)$

$\therefore \sigma_1 = \vec{P}\cdot\vec{n}_1 = \alpha(x,y,z)\cdot(0,1,0) = \alpha y = \alpha\cdot\frac{a}{2}$

② For the left surface, $\vec{n}_2 = -\vec{j} = (0,-1,0)$ $\vec{P} = \alpha(x,y,z)$

$\therefore \sigma_2 = \vec{P}\cdot\vec{n}_2 = -\alpha(-\frac{a}{2}) = \alpha\cdot\frac{a}{2}$

For other surfaces, we can also get that

$\sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = \alpha\cdot\frac{a}{2}$.

$\therefore$ All the bound charges density is $\alpha\cdot\frac{a}{2}$.

To check,

$\rho_b = -\nabla\cdot\vec{P} = -(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})\cdot\alpha(x,y,z) = -\alpha(1+1+1) = -3\alpha$.

$\therefore \int \rho_b\, dV = -3\alpha\cdot a^3$

$\int \sigma\cdot dA = 6a^2\cdot\alpha\cdot\frac{a}{2} = 3\alpha a^3$

$\therefore \int \sigma\, dA + \int \rho_b\, dV = 0$

$\therefore$ It is right.