

# Ve 280

## Programming and Introductory Data Structures

### **Recursion; Function Pointers; Function Call Mechanism**

#### **Learning Objectives:**

Understand recursion and know how to write recursive functions

Understand how to write more general code with function pointers

Understand function call mechanism

# Outline

- Recursion
- Function Pointers
- Function Call Mechanism

# Recursion

- Recursion is a nice way to solve problems
  - “Recursive” just means “refers to itself”.
  - There is (at least) one “trivial” base or “stopping” case.
  - All other cases can be solved by first solving one smaller case, and then combining the solution with a simple step.
- Example: calculate factorial  $n!$

$$n! = \begin{cases} 1 & n = 0 \\ n \cdot (n-1)! & n > 0 \end{cases}$$

```
int factorial (int n) {  
    // REQUIRES: n >= 0  
    // EFFECTS:  computes n!  
    if (n == 0) return 1; // base case  
    else return n*factorial(n-1); // recursive step  
}
```

# Recursive Helper Function

- Sometimes it is easier to find a recursive solution to a problem if you change the original problem slightly, and then solve that problem using a **recursive helper function**.

```
soln()  
{  
    ...  
    soln_helper();  
    ...  
}
```

```
soln_helper()  
{  
    ...  
    soln_helper();  
    ...  
}
```

# Recursive Helper Function

## Example

- A palindrome is a string that is equal to itself when you reverse all characters.
  - For example: rotor, racecar

- Write a function to test if a string is a palindrome.

```
bool is_palindrome(string s);  
// EFFECTS: return true if s is  
// a palindrome.
```

# Palindrome Example

- If a string is empty, it is a palindrome.
- If a string is of length one, it is a palindrome.
- Given a string of length more than one, it is a palindrome, if
  - its first character equals its last one, **and**
  - the substring without the first and the last characters is a palindrome.
- In order to test whether a substring is a palindrome, we define a **helper** function

```
bool is_palindrome_helper(string s,  
    int begin, int end);  
// EFFECTS: return true if the substring  
// of s starting at begin and ending at  
// end is a palindrome.
```

# Palindrome Example

```
bool is_palindrome_helper(string s,  
    int begin, int end)  
// EFFECTS: return true if the substring  
// of s starting at begin and ending at  
// end is a palindrome.  
{  
    if(begin >= end) return true;  
    if(s[begin] == s[end])  
        return is_palindrome_helper(s,  
            begin+1, end-1);  
    else return false;  
}
```

# Palindrome Example

- With the helper function, `is_palindrome()` can be realized as

```
bool is_palindrome(string s)
// EFFECTS: return true if s is
// a palindrome.
{
    return is_palindrome_helper(s, 0,
                                s.length()-1);
}
```



# Outline

- Recursion
- **Function Pointers**
- Function Call Mechanism

# Function Pointers

## Motivation

- If you were asked to write a function to add all the elements in a list, and another to multiply all the elements in a list, your functions would be almost exactly **the same**.
- Writing almost the exact same function twice is a bad idea!  
Why?
  1. It's wasteful of your time!!
  2. If you find a better way to implement some common parts, you have to change **many different** places; this is prone to error.

# Our Example: list\_t type

- A list can hold a sequence of zero or more integers.
- There is a recursive definition for the values that a list can take:
  - A valid list is:  
either    an empty list  
or        an integer followed by another valid list

# Function Pointers

## Background on lists

- Here are some examples of valid lists:

```
( 1 2 3 4 ) // a list of four elements
( 2 5 2 )   // a list of three elements
( )         // an empty list
```

- There are also several operations that can be applied to lists. We will use the following three:
  - `list_first( )` takes a list, and returns the first element (an integer) from the list. **REQUIRES: non-empty list!**
  - `list_rest( )` takes a list and returns the list comprising all but the first element. **REQUIRES: non-empty list!**
  - `list_isEmpty( )` takes a list and returns the Boolean “true” if the argument is an empty list, and “false” otherwise.

# Function Pointers

## Using lists

- Suppose we want to write a **recursive** function to find the smallest element in a list.
  - The function requires the input list to be non-empty.

Question: how do you do it **recursively**?

- **Answer:**

`smallest(list) = the element (if list has only a single element)`  
`or the minimum of the first element and the smallest element from the rest of the list`

# Function Pointers

Using recursion to find the smallest element in a list

```
int smallest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS:  returns smallest element
    // in the list
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = smallest(rest);
    if(first <= cand) return first;
    return cand;
}
```

# Function Pointers

## Using lists

- Now suppose we want to write a recursive function to find the largest element in a list.
  - The function also requires the input list to be non-empty.

- Recursive definition:

`largest(list)` = the element (if list has only a single element)  
or the maximum of the first element and the largest element from the rest of the list

# Function Pointers

Using recursion to find the largest element in a list

```
int largest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS:  returns largest element
    // in the list
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = largest(rest);
    if(first >= cand) return first;
    return cand;
}
```



# Function Pointers

## More Motivation

- `largest` is almost identical to the definition of `smallest`.
- Unsurprisingly, the solution is almost identical, too.
- In fact, the **only** differences between `smallest` and `largest` are:
  1. The names of the function
  2. The comment in the EFFECTS list
  3. The polarity of the comparison: `<=` vs. `>=`
- It is silly to write almost the same function twice!

**Function pointers to rescue!**

# Function Pointers

## A first look

- So far, we've only defined functions as entities that can be called. However, functions can also be referred to by **variables**, and passed as **arguments** to functions.
- Suppose there are two functions we want to pick between: `min()` and `max()`. They are defined as follows:

```
int min(int a, int b);  
    // EFFECTS: returns the smaller of a and b.  
int max(int a, int b);  
    // EFFECTS: returns the larger of a and b.
```

# Function Pointers

## A first look

```
int min(int a, int b);  
    // EFFECTS: returns the smaller of a and b.  
int max(int a, int b);  
    // EFFECTS: returns the larger of a and b.
```

- These two functions have precisely the same type signature:
  - They both take two integers, and return an integer.
- Of course, they do completely different things:
  - One returns a min and one returns a max.
  - **However, from a syntactic point of view, you call either of them the same way.**

# Function Pointers

## The basic format

- How do you define a **variable** that points to a function that takes two integers, and returns an integer?

- Here's how:

```
int    (*foo) (int, int) ;
```

- You read this from "inside out". In other words:

<code>foo</code>	“foo”
<code>(*foo)</code>	“is a pointer”
<code>(*foo) (</code>	“to a function”
<code>(*foo) (int, int) ;</code>	“that takes two integers”
<code>int    (*foo) (int, int) ;</code>	“and returns an integer”

# Function Pointers

The basic format

```
int    (*foo) (int, int);
```

- Once we've declared foo, we can **assign** any function to it:

```
foo = min;
```

- Furthermore, after assigning min to foo, we can just call it as follows:

```
foo(3, 5)
```

- ...and we'll get back 3!

# Function Pointers v.s. Variable Pointers

- For function pointers, the compiler allows us to **ignore** the “**address-of**” and “**dereference**” operators.

```
int (*foo)(int, int);  
foo = min; // min() is predefined  
foo(5, 3);
```

We don't write:

```
foo = &min;  
(*foo)(5, 3);
```

- In contrast, for variable pointers:

```
int foo;  
int *bar;  
bar = &foo;  
*bar = 2;
```

# Function Pointers

Re-write `smallest` in terms of function pointers

```
int compare_help(list_t list, int (*fn)(int, int))
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = compare_help(rest, fn);
    return fn(first, cand);
}

int smallest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS: returns smallest element in list
{
    return compare_help(list, min);
}
```

```
int min(int a, int b);
    // EFFECTS: returns the
    // smaller of a and b.
```

# Function Pointers

Re-write `largest` in terms of function pointers

```
int compare_help(list_t list, int (*fn)(int, int))
{
    int first = list_first(list);
    list_t rest = list_rest(list);
    if(list_isEmpty(rest)) return first;
    int cand = compare_help(rest, fn);
    return fn(first, cand);
}

int largest(list_t list)
    // REQUIRES: list is not empty
    // EFFECTS: returns largest element in list
{
    return compare_help(list, max);
}
```

```
int max(int a, int b);
    // EFFECTS: returns the
    // larger of a and b.
```



# Outline

- Recursion
- Function Pointers
- Function Call Mechanism

# Call Stacks

How a function call really works

- When we call a function, the program does following steps:
  1. Evaluate the actual arguments to the function (order is not guaranteed).  

Example: `y = add(4-1, 5);`
  2. Create an “**activation record**” (sometimes called a “**stack frame**”) to hold the function's **formal parameters** and **local variables**.
    - When call function `int add(int a, int b)`, system creates an activation record:  

`a, b (formal), result (local)`
  3. Copy the actuals' values to the formals' storage space.

`a=3`  
`b=5`
  4. Evaluate the function in its local scope.
  5. Replace the function call with the result.

`y=8`
  6. Destroy the activation record.

# Call Stacks

How a function call really works

- It is typical to have multiple function calls. How the activation records are maintained?
  - Answer: stored as a **stack**.
- Stack: a set of objects which is modified as **last in first out**.  
Example: a stack of plates in a cafeteria
  - Each time you clean a plate, you add it to the top of the stack
  - Each time a new plate is needed, the one at the top is taken **first**



# Call Stacks

How a function call really works

- When a function  $f()$  is called, its **activation record** is added to the “top” of the stack.
- When the function  $f()$  returns, its **activation record** is removed from the “top” of the stack.
- In the meantime,  $f()$  may have called **other functions**.
  - **These functions** create corresponding activation records.
  - **These functions** must return (and destroy their corresponding activation records) before  $f()$  can return.

# Call Stacks

## Example

- When a function is called, its **activation record** is added to the “top” of the stack.
- When that function returns, its **activation record** is removed from the “top” of the stack.



double add(double a, double b): a = 1, b = 0, result = 0

double sin(double x): x = 1, result = 0

int main(): x = 1, sinResult = 0

- Note: “top” is placed in quotes, because in reality, stack of activation records grows **down** rather than **up**.

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Main starts out with an activation record with room only for the local “result”:

main:

result: 0
-----------

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Then, main calls plus\_two,  
passing the literal value "0":

main:

result: 0
-----------

plus\_two:

x: 0
------



# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

Which in turn calls plus\_one:

main:

result: 0

plus\_two:

x: 0

plus\_one:

x: 0

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus\_one adds one to x,  
returning the value 1:

main:

result: 0

plus\_two:

x: 0

plus\_one:

x: 0



# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus\_one's activation record  
is destroyed:

main:

result: 0
-----------

plus\_two:

x: 0
------



~~plus\_one:~~

<del>x: 0</del>
-----------------

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus\_two adds one to the result,  
and returns the value 2:

main:

result: 2

plus\_two:

x: 0



# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

plus\_two's activation record  
is destroyed:

main:

result: 2
-----------

2

plus\_two:

x: 0
------

# Call Stacks

## Example

```
int plus_one(int x) {  
    return (x+1);  
}
```

```
int plus_two(int x) {  
    return (1 + plus_one(x));  
}
```

```
int main() {  
    int result = 0;  
  
    result = plus_two(0);  
    cout << result;  
    return 0;  
}
```

main then prints the result:

**2**

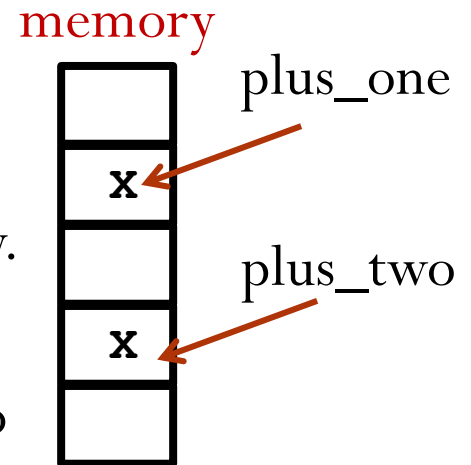
main:

result: 2
-----------

# Call Stacks

Example: Some things to note

- Even though `plus_one` and `plus_two` both have formal parameters called “x”, there is no problem.
  - These two x’s are at different locations in memory.
  - `plus_one` cannot see `plus_two`'s x.
  - Instead, the **value** of `plus_two`'s x is passed to `plus_one`, and stored in `plus_one`'s x.



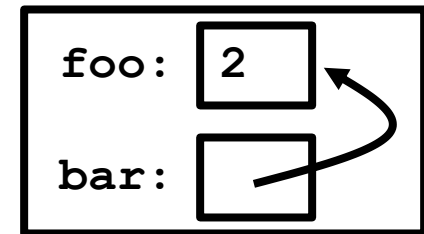
# Call Stack

Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Activation record of main:





# Call Stack

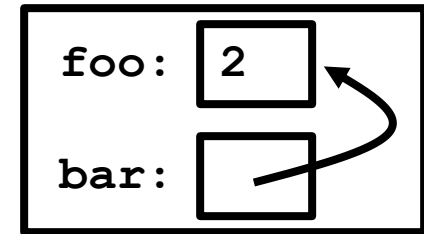
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

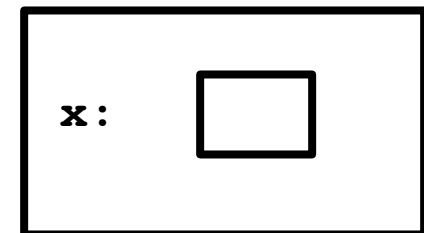
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Main calls `add_one`,  
creating an activation  
record for `add_one`

main:



add\_one:



# Call Stack

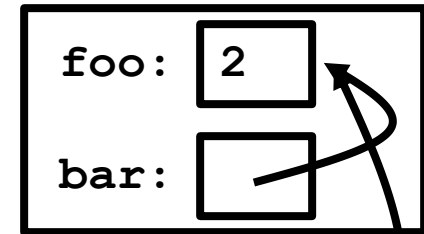
Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

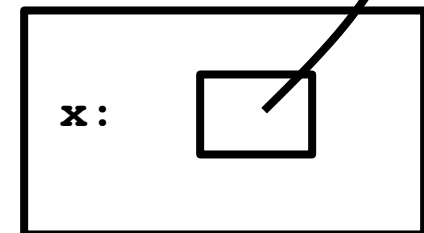
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

Copy the value of bar to add\_one's formal parameter x.

main:



add\_one:



Both x and bar point to foo.

# Call Stack

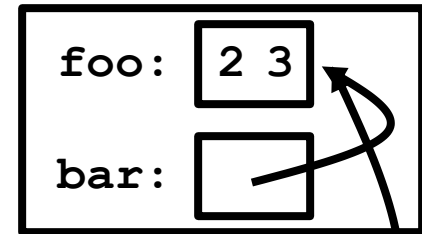
## Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

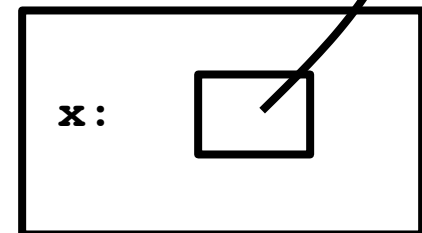
```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

add\_one adds 1 to the object pointed to by x.

main:



add\_one:



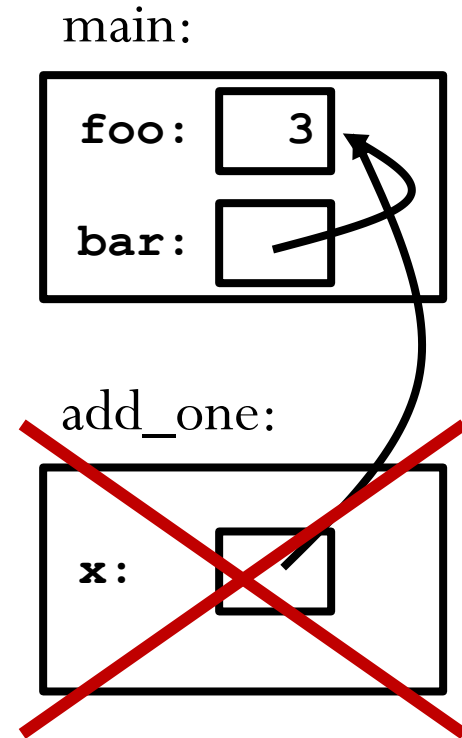
# Call Stack

## Example: Using Pointers

```
void add_one(int *x) {  
    *x = *x + 1;  
}
```

```
int main() {  
    int foo = 2;  
    int *bar = &foo;  
    add_one(bar);  
    return 0;  
}
```

add\_one's activation record is destroyed.



# Call Stack

Example: Recursion

main

x:

- Suppose we call our function as follows:

```
int main()
```

1. {
2. int x;
3. x = factorial(3);
4. }

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

# Call Stack

Example: Recursion

- `main()` calls `factorial` with an argument 3.
- We evaluate the actual argument, create an activation record, and copy the actual value to the formal.

`main`

`x:`

`factorial`

`n:`

`RA: main line #3`

**RA = "Return Address"**

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

# Call Stack

## Example: Recursion

- Now we evaluate the body of factorial:
  - n is not zero, so we evaluate the **else** arm of the if statement:  
return 3 \* factorial(2)
  - So, factorial must call factorial. We will create a **new** activation record for a **new** instance of factorial.

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

# Call Stack

Example: Recursion

- Again,  $n$  is not zero, so we evaluate the **else** arm again:

`return 2 * factorial(1)`

- This creates a new activation record for factorial

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2



# Call Stack

Example: Recursion

- And again, we evaluate the **else** arm:

return 1\*factorial(0)

- This creates a new activation record for factorial

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

# Call Stack

Example: Recursion

- In evaluating factorial(0), n is zero, so we evaluate the **if** arm rather than **else** arm.
- Return the value “1”
- Popping the most recent activation record off the stack.

```
int factorial (int n) {  
1. if (n == 0) return 1;  
2. else return n*factorial(n-1);  
}
```

main

x:

factorial

n:

RA: main line #3

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

factorial

n:

RA: factorial line #2

1



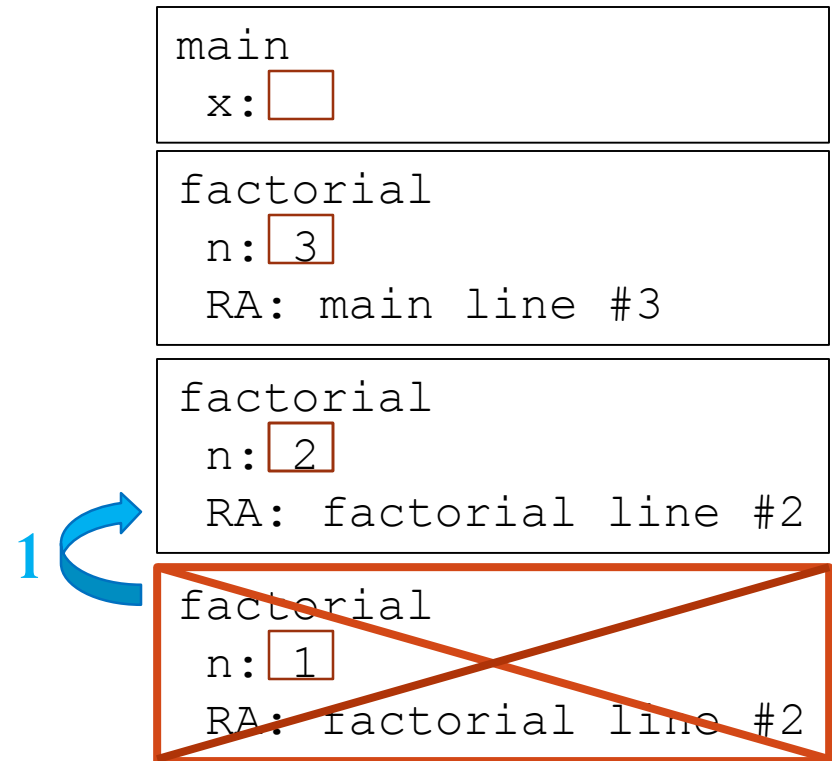
# Call Stack

Example: Recursion

- In **factorial(1)**, we called factorial(0) as follows:  
    return 1 \* factorial(0)
- Now we know the value of factorial(0), so we complete factorial(1):

    return 1 \* 1   => return 1;  
from factorial(1)

- This pops another activation record off the stack



# Call Stack

Example: Recursion

- Now it allows us to complete evaluating **factorial(2)**:

return  $2 * \text{factorial}(1)$   $\Rightarrow$

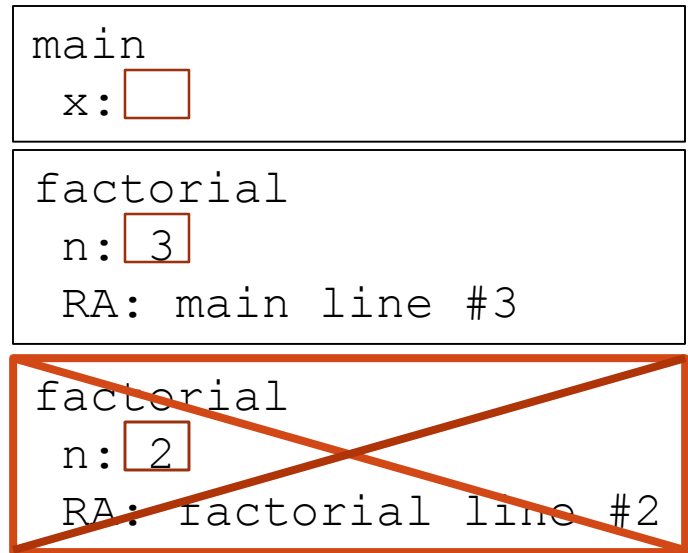
return  $2 * 1$   $\Rightarrow$

return 2

from factorial(2)

- Now pop off another activation record.

2 



# Call Stack

Example: Recursion

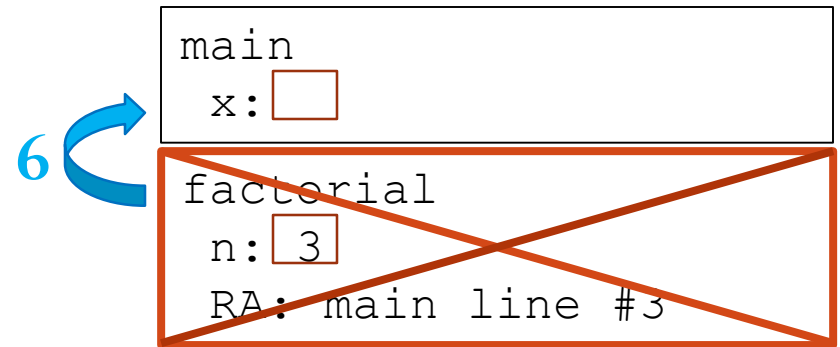
- Now we can complete evaluating **factorial(3)**:

return 3 \* factorial(2) =>

return 3 \* 2 =>

return 6

- That is the correct answer.
- Don't forget that last pop!





# Which Statements Are True?

Select all the correct answers.

- **A.** The number of recursive calls of factorial can be as high as we want.
- **B.** The number of calls of factorial could be just 1.
- **C.** We can change the function factorial so that the number of calls of factorial could be **reduced by 1**.
- **D.** None of the above.

```
int factorial (int n) {  
    if (n == 0) return 1;  
    else return n*factorial(n-1);  
}
```



# Reference

- Recursion
  - Problem Solving with C++, 8<sup>th</sup> Edition, Chapter 14
- Function pointers
  - C++ Primer (4<sup>th</sup> Edition), Chapter 7.9