Week 12
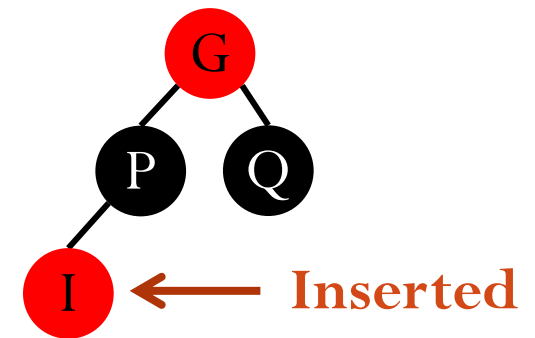# Review Class

VE281 TA Group

Violation at Leaf
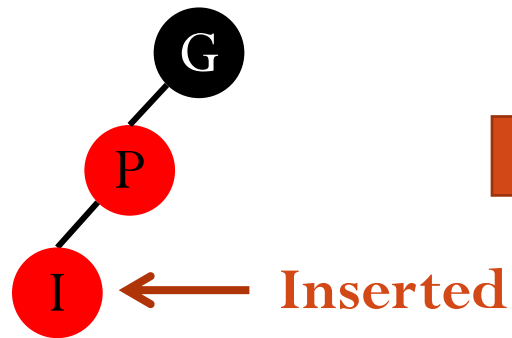
- Case 1: Q is a **red leaf**.
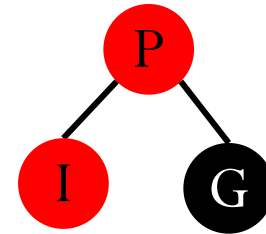

Recoloring

Inserted

Inserted
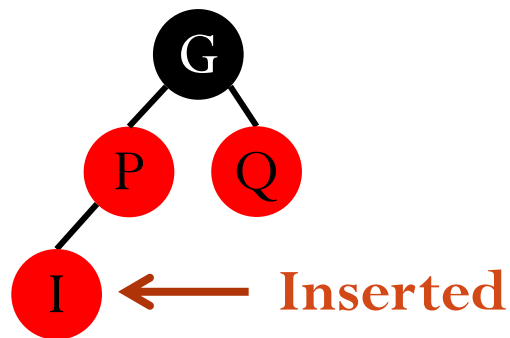
May **recurse**, since G's parent may be red.
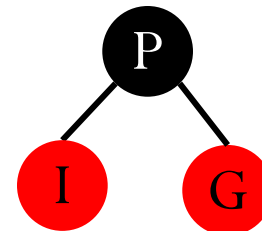
Violation at Leaf

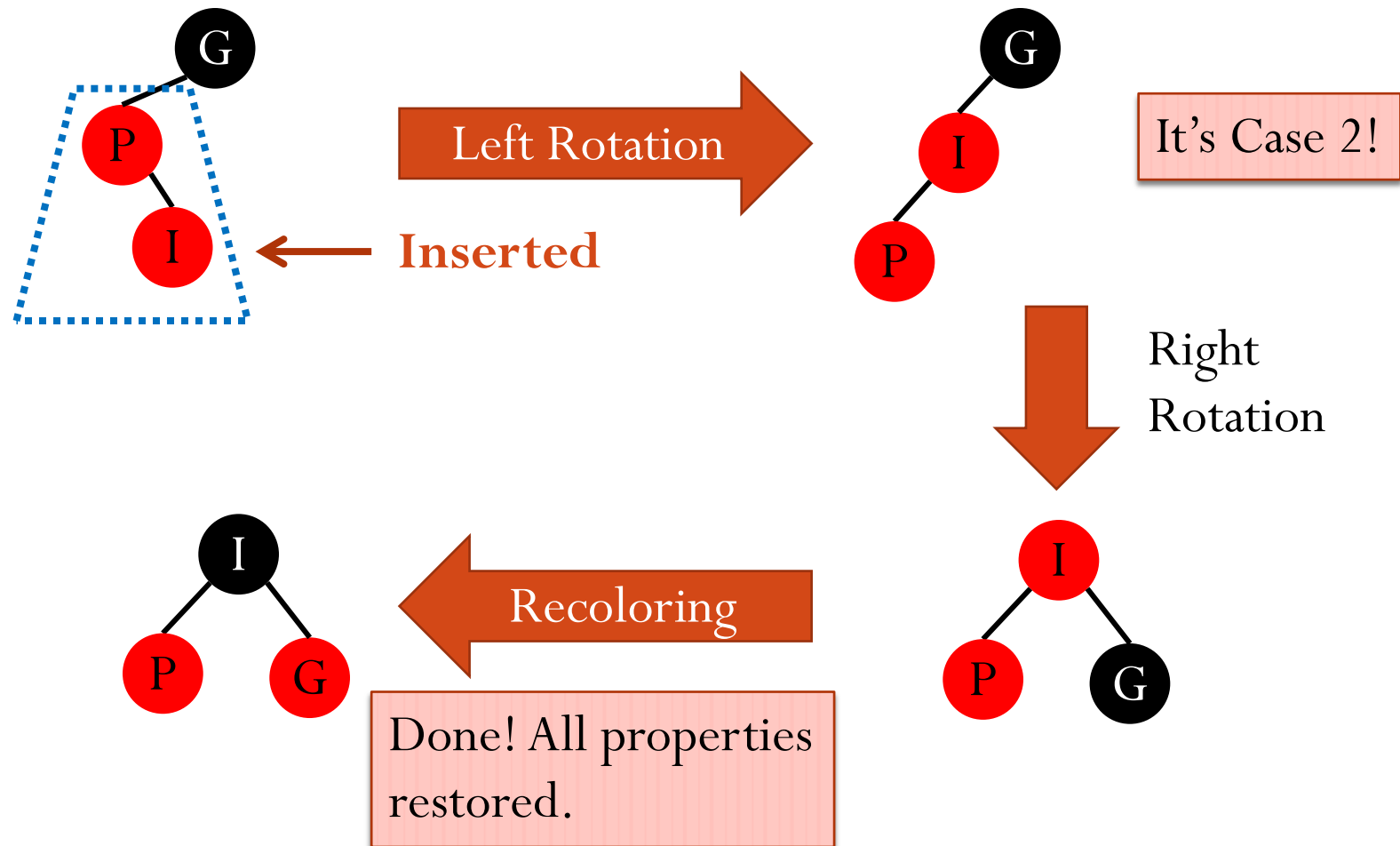- Case 2: Q is empty; I is P's **left** child.

Right Rotation

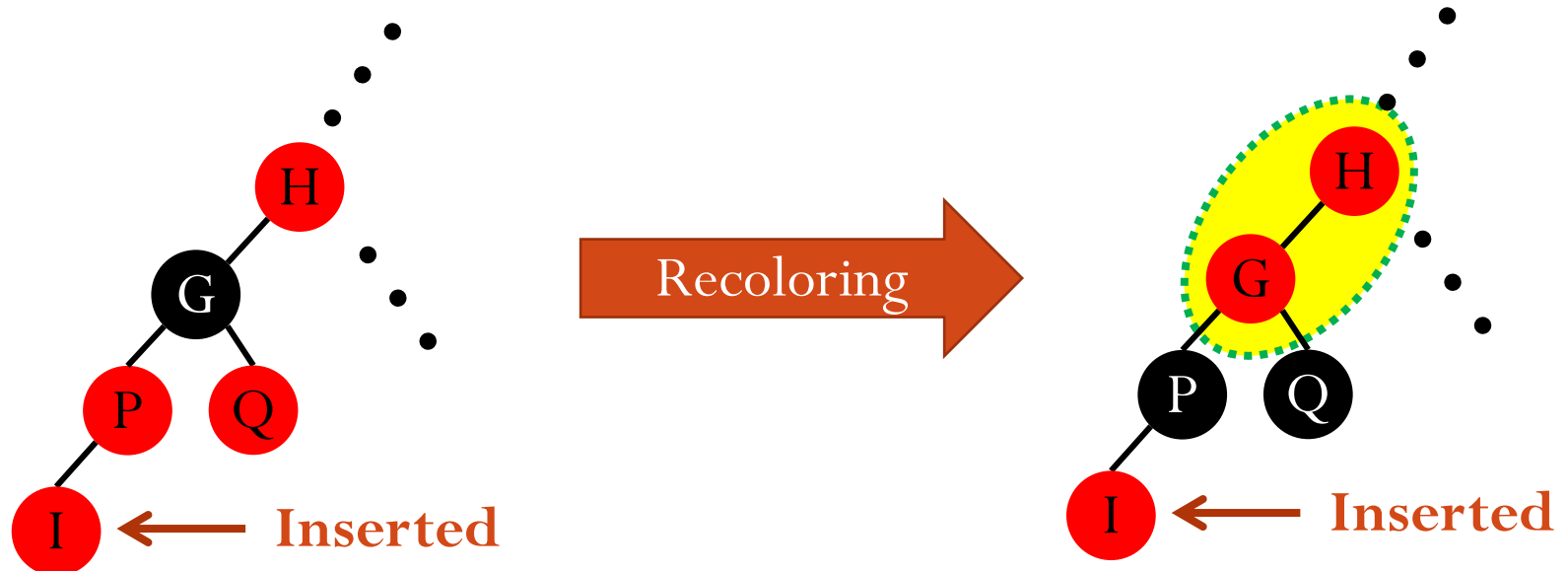Recoloring

Done! All properties restored. (Why?)

Violation at Leaf

- Case 3: Q is empty; I is P's **right** child.

Left Rotation

Inserted

It's Case 2!

Right Rotation

Recoloring

Done! All properties restored.

Violation at Leaf: Summary

- For Case 2 (Q is empty; I is P's **left** child) and Case 3 (Q is empty; I is P's **right** child), **we're done**.

- For Case 1 (Q is a **red leaf**), we may recurse.
  - Violation of **red rule**.
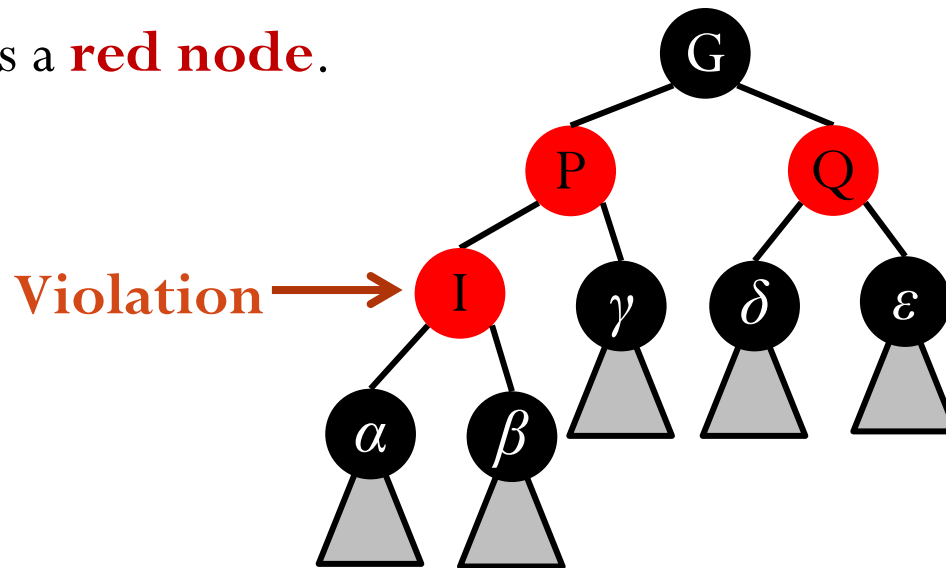
Violation at Internal Nodes

- Caused by **moving the violation up** the tree.

- When violating, its **parent** is **red** and its **grandparent** is **black**.

- **Assume**: the parent "P" is the **left child** of the grandparent "G". (The "right child" case is **symmetric**.)

- **Denote**: the right child of the grandparent to be Q.
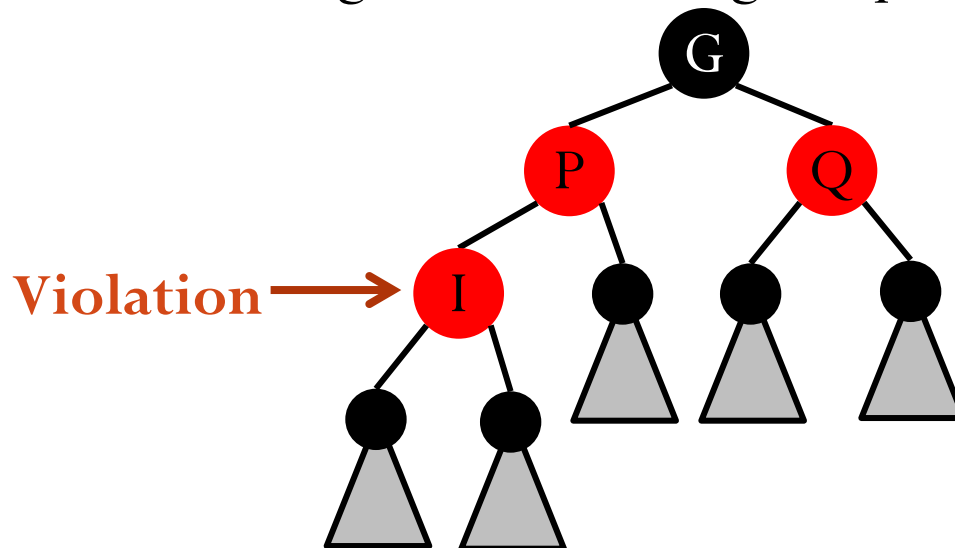
Violation at Internal Nodes

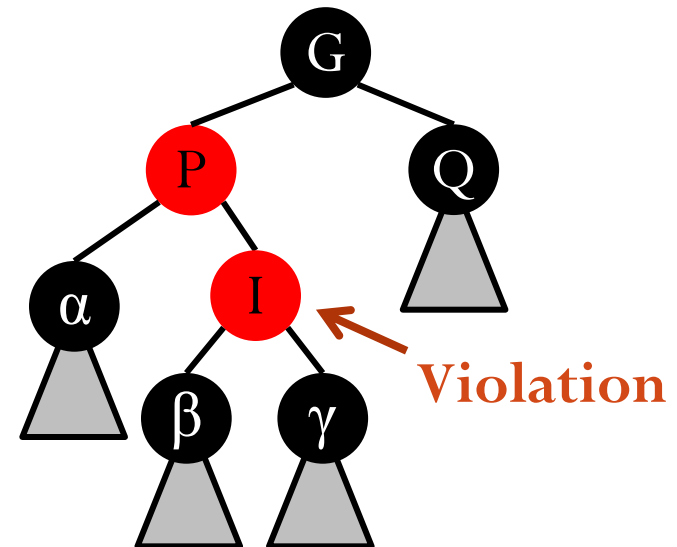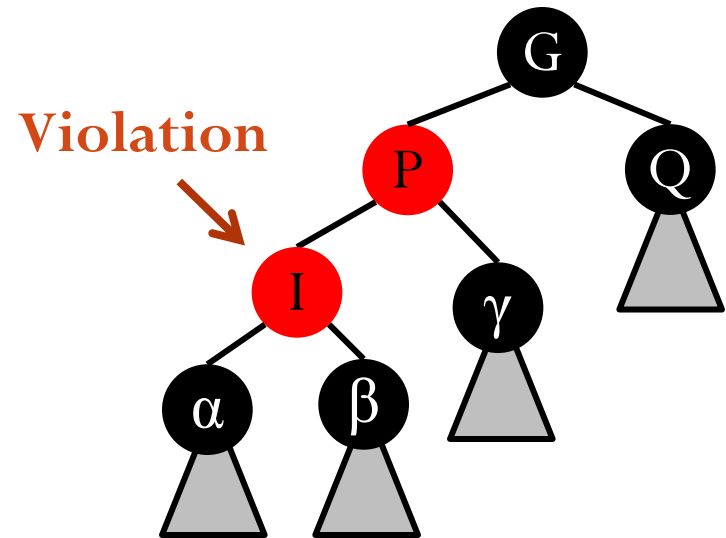- Three Cases:
  1. Q is a **red node**.

- **Claim**:
  - α, β, γ, δ, ε are trees with **black root**.
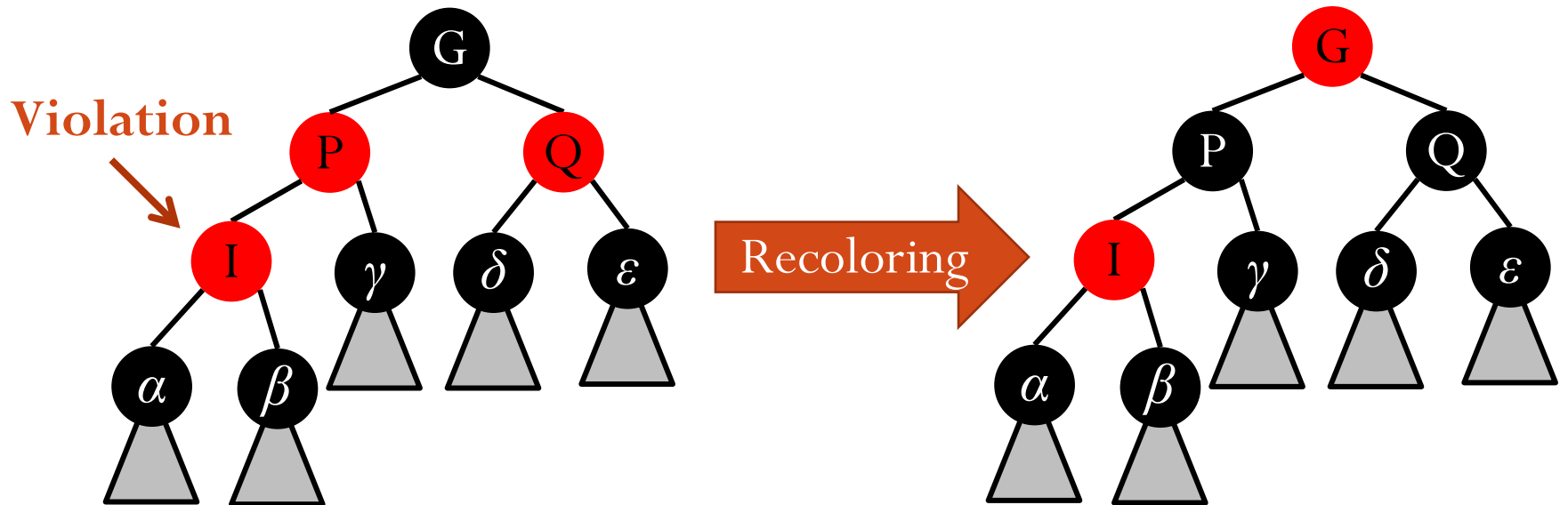 - α, β, γ, δ, ε have the **same** **black height**.

Violation at Internal Nodes

- Three Cases:

  2. Q is a **black node**; I is P's **left** child.

  3. Q is a **black node**; I is P's **right** child.

- **Claim** for Case 2 and 3:
  - α, β, γ, Q are trees with **black root**.
  - α, β, γ, Q have the **same black height**.

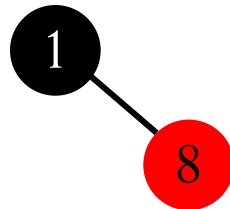# Violation at Internal Nodes

- Case 1: Q is a **red node**.

Violation

Recoloring

May **recurse**, since G's parent may be red.

Example

- Insert 1

- Insert 8

Example (cont.)

- Insert 2

Case 3 at leaf

Right Rotation

Left Rotation

Recoloring

Example (cont.)

- Insert 7

Case 1 at leaf

Recoloring

Recoloring Root

Example (cont.)

- Insert 3

Case 2 at leaf

Right Rotation

Recoloring

Example (cont.)

- Insert 6

Case 1 at leaf
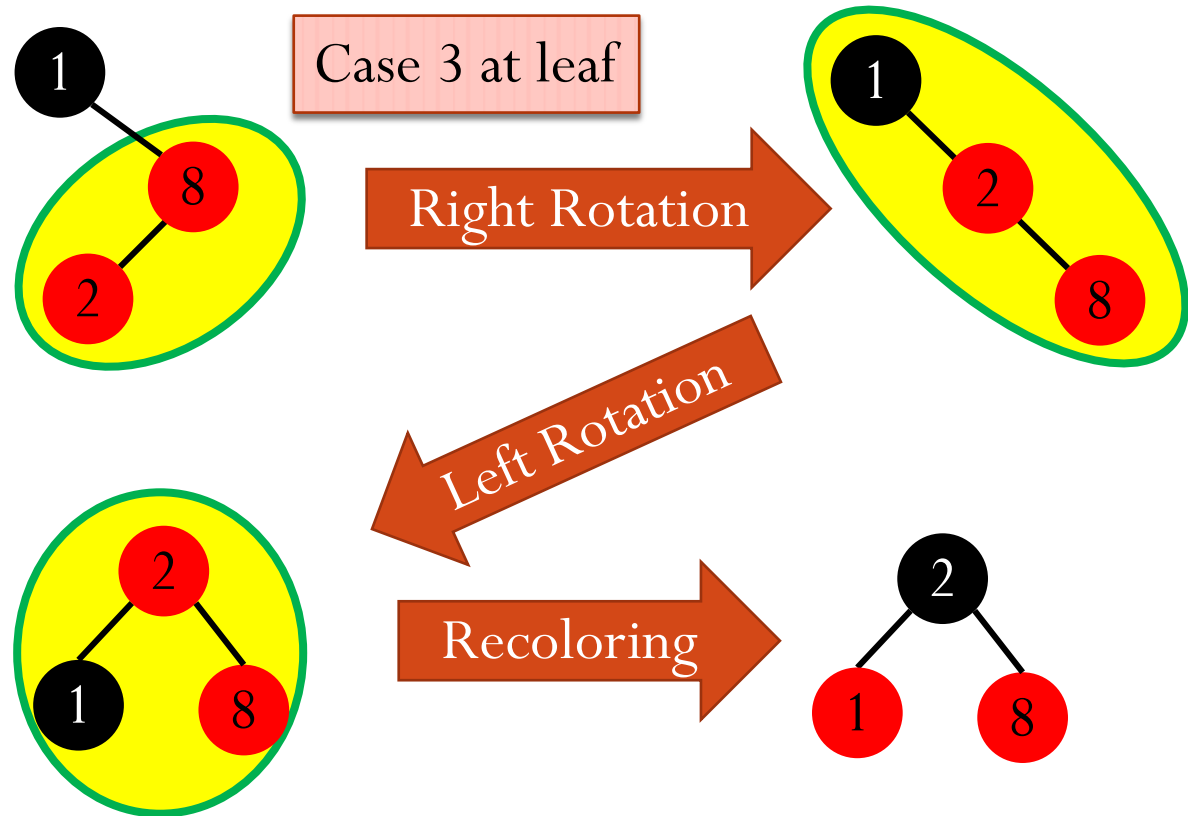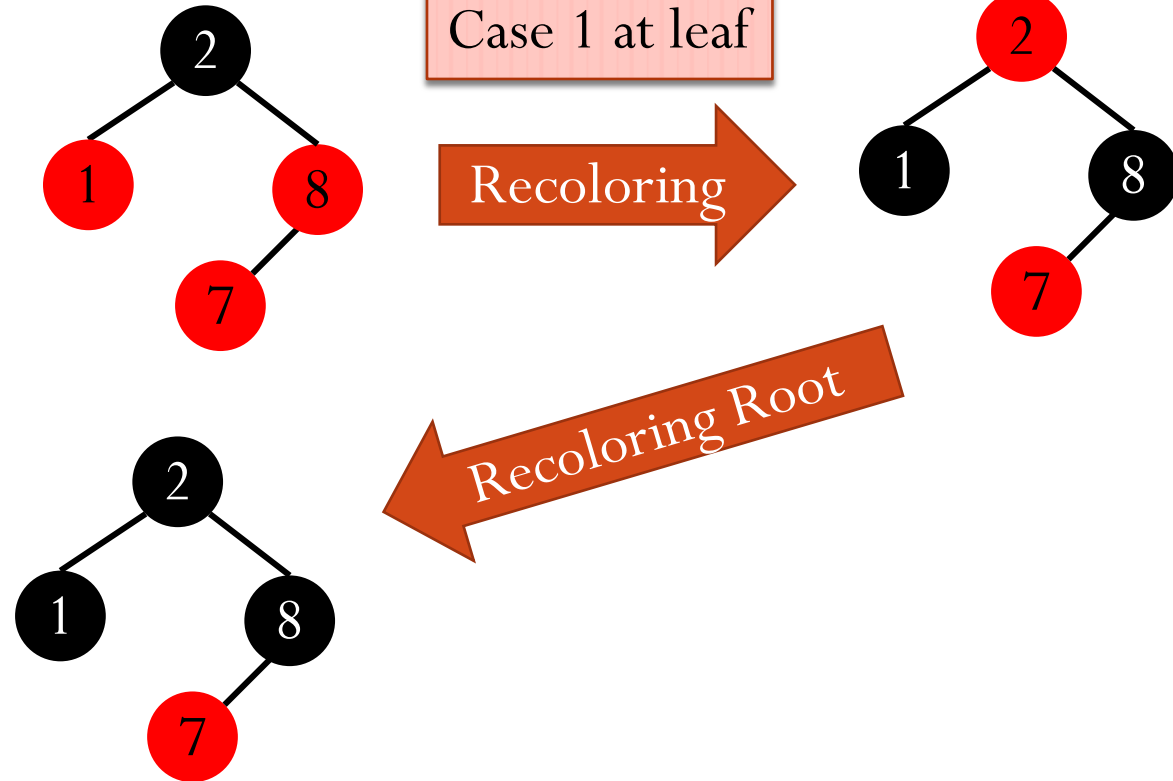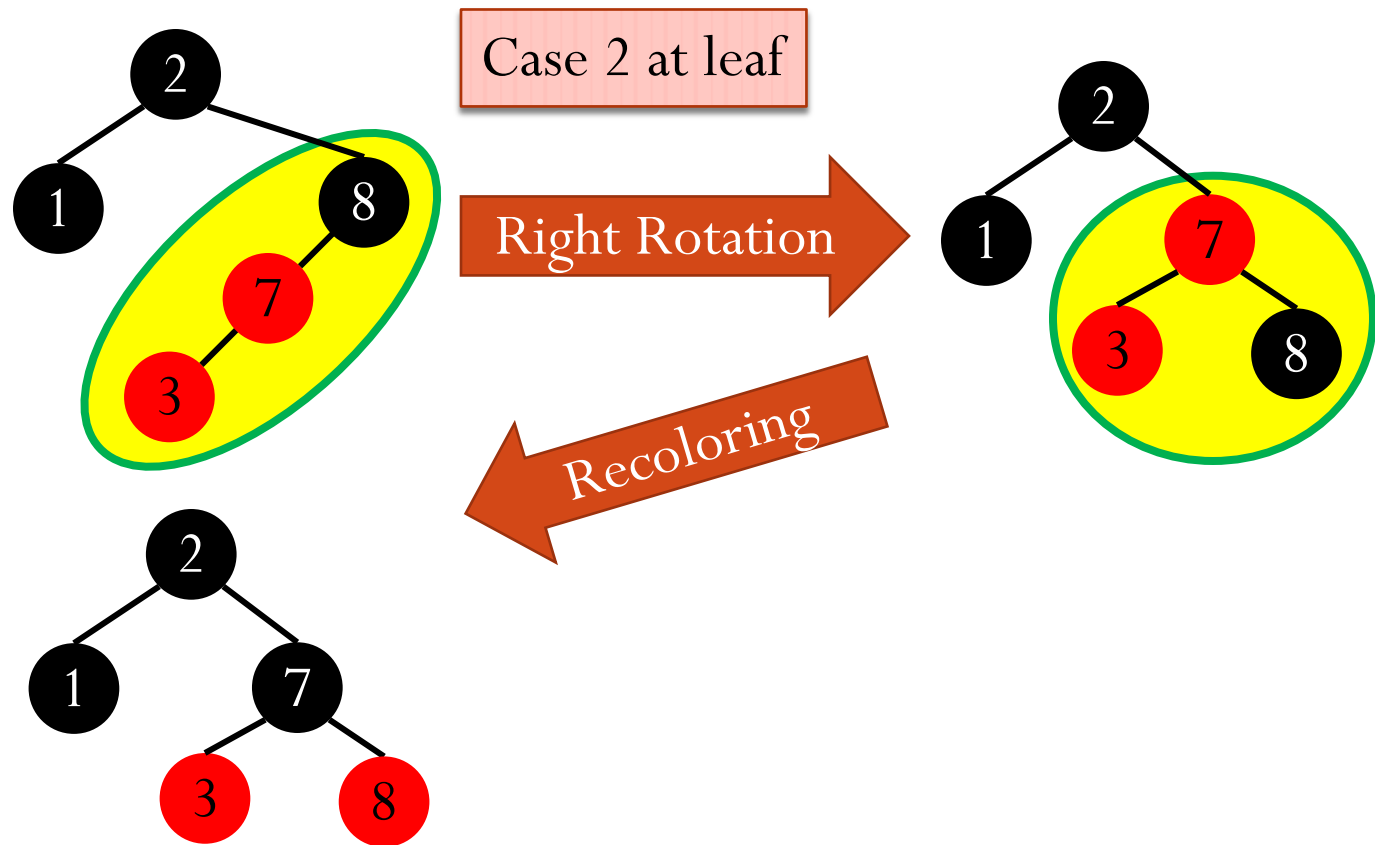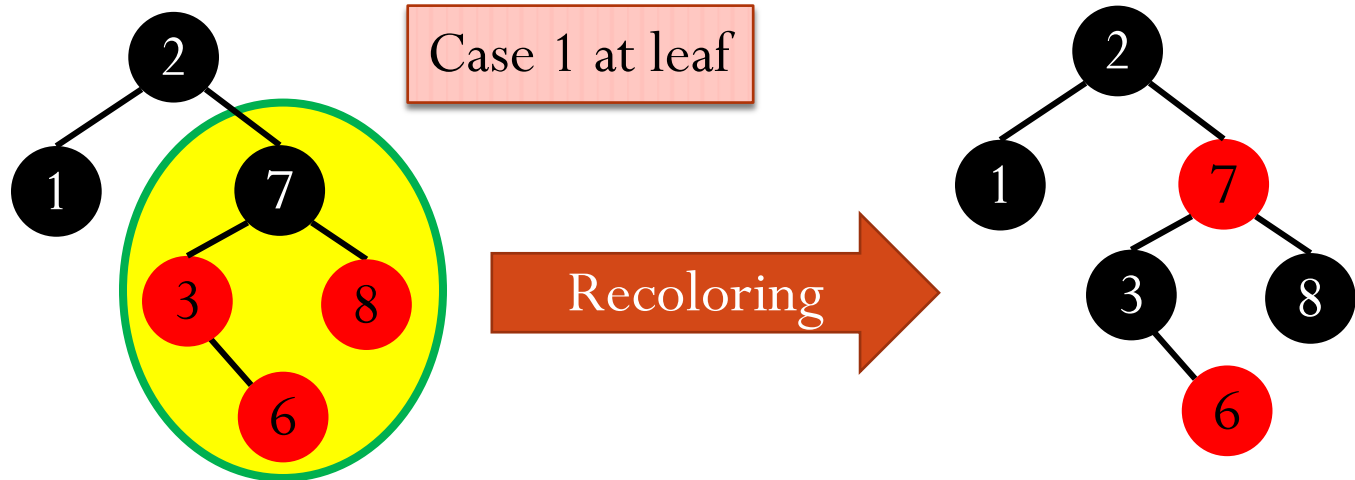
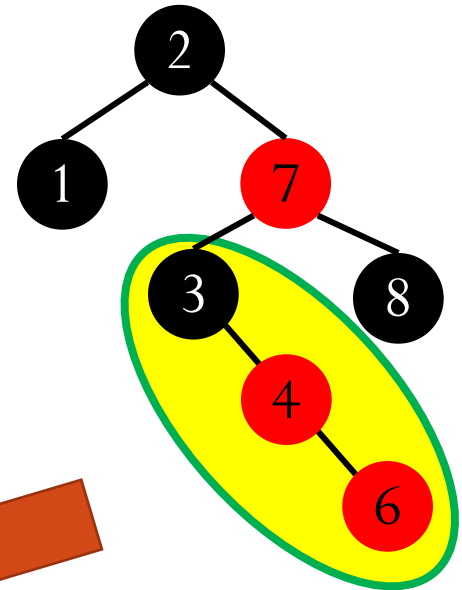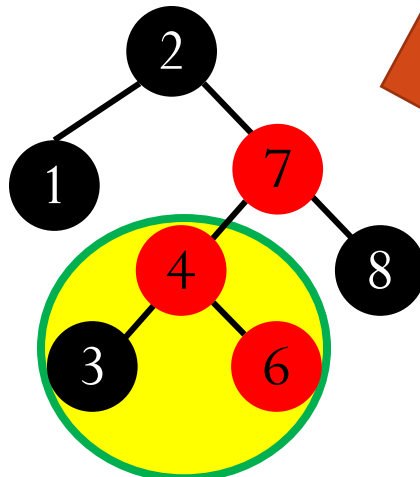Recoloring

Example (cont.)
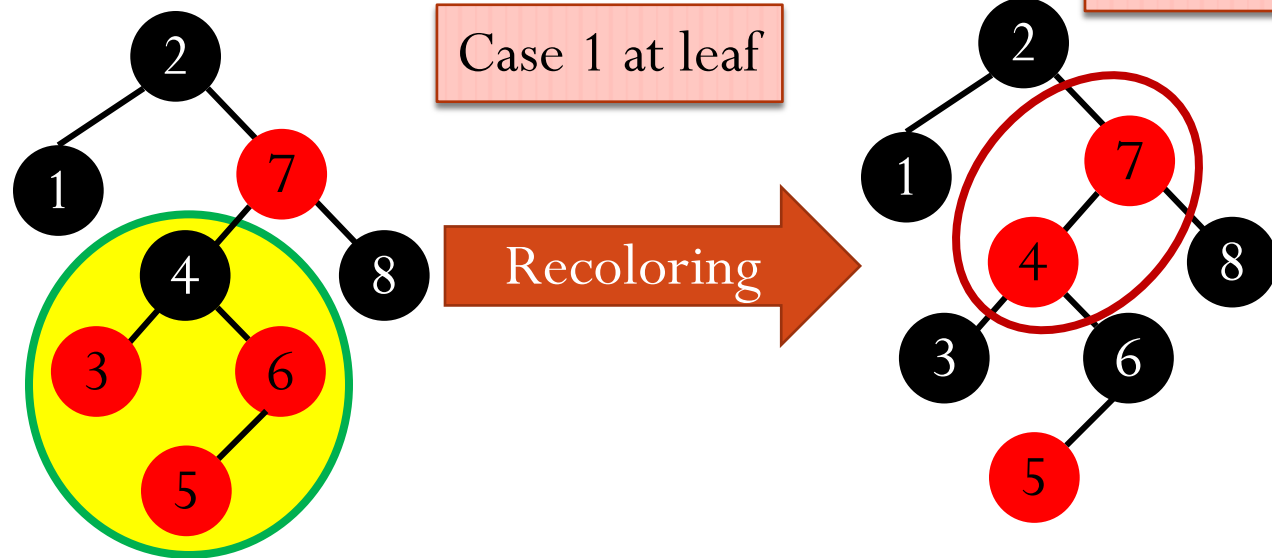
- Insert 4

Case 3 at leaf

Right Rotation

Left Rotation

Recoloring

Example (cont.)

- Insert 5

Case 1 at leaf
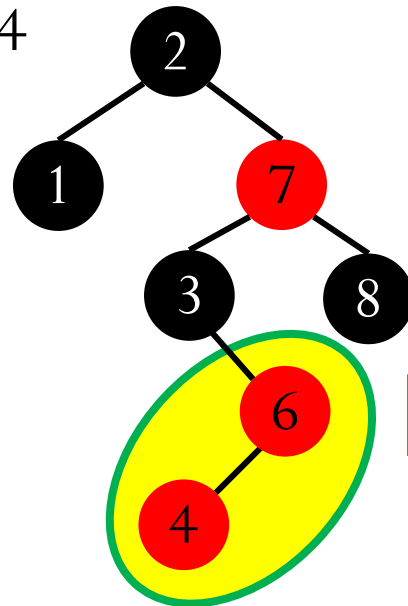
Recoloring
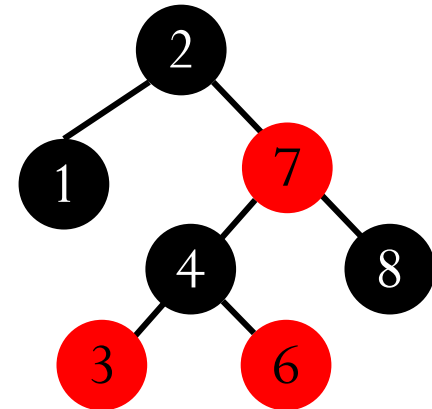
Case 3 at internal node

Example (cont.)

Case 3 at internal node

Right Rotation

Left Rotation

Recoloring

17