

VE281 Writing Assignment One

Ex. 1

The strategy is:

Input: maximum integer N

Output: correct integer x

$begin \leftarrow 1, end \leftarrow N$

while True **do**

$x \leftarrow \lfloor (begin + end)/2 \rfloor$

$result \leftarrow$ make a guess with x

if $result =$ “equal to” **then break**

else if $result =$ “less than” **then** $end \leftarrow x - 1$

else $begin \leftarrow x + 1$

end if

end while

In the worst case, I never directly guess the correct number in the process of the strategy. After a guess of N , the next range of numbers become $(N - 1)/2$. When $N = 2^m - 1$, the last guess will contain $2^2 - 1 = 3$ numbers, and I'll get the correct one after the guesses. So the number of guess in the worst case is $m - 1$.

In the average case, if there are M numbers left, I've got the probability of $1/M$ to guess the number directly. Let the probability of guessing n times be P_n ,

$$P_1 = \frac{1}{2^m - 1}$$

$$P_n = \frac{1 - P_{n-1}}{2^{m-n+1} - 1} = \frac{2^{n-1}}{2^m - 1}$$

Then we can get the equation

$$T_m = \sum_{n=1}^{m-1} nP_n = \frac{1 \cdot 2^0 + 2 \cdot 2^1 + \cdots + n \cdot 2^{m-2}}{2^m - 1} = \frac{(m-2)2^{m-1} + 1}{2^m - 1}$$

Ex. 2

In the best situation, if $n = 2^m$, only cm steps (c is a constant) is necessary. So $f(n) = \log n$.

Ex. 3

$$\lim_{n \rightarrow \infty} \frac{n^{100}}{1.001^n} = \lim_{n \rightarrow \infty} \frac{100n^{99}}{1.001^n \ln 1.001} = \dots = \lim_{n \rightarrow \infty} \frac{100!}{1.001^n \ln^{100} 1.001} = 0$$

So $n^{100} = O(1.001^n)$ is false.

Ex. 4

Since $f_1(n) = \Theta(g_1(n))$, $f_2(n) = \Theta(g_2(n))$, we know

$$c_1 g_1(n) \leq f_1(n) \leq c_2 g_1(n), n \geq n_1$$

$$c_3 g_2(n) \leq f_2(n) \leq c_4 g_2(n), n \geq n_2$$

Then

$$c_1 g_1(n) + c_3 g_2(n) \leq f_1(n) + f_2(n) \leq c_2 g_1(n) + c_4 g_2(n), n \geq \max\{n_1, n_2\}$$

Now we should find $h(n)$ so that

$$\lim_{n \rightarrow \infty} \frac{c_1 g_1(n) + c_3 g_2(n)}{h(n)} = C_1$$

$$\lim_{n \rightarrow \infty} \frac{c_2 g_2(n) + c_4 g_2(n)}{h(n)} = C_2$$

So

$$h(n) = \max\{g_1(n), g_2(n)\}$$

Ex. 5

The outer loop will be called $\lfloor 1 + \log_a n \rfloor$ times, in k^{th} loop, $i = a^k$, where $k \in [0, \lfloor \log_a n \rfloor] \cap \mathbb{Z}$.

The inner loop will be called ib times in each loop.

$$T = b \sum_{k=0}^{\lfloor \log_a n \rfloor} a^k = \frac{b(1 - a^{\lfloor \log_a n \rfloor + 1})}{1 - a}$$

Ex. 6

(a)

$$T_1(n) = \lceil n/2 \rceil$$

$$T_1(n) = \lceil n/20 \rceil$$

$$T_3(n) = \lceil 1 + \log_2 n \rceil$$

(b)

$$C_1(n) = 4n + 2\lceil n/2 \rceil$$

$$C_2(n) = 10 + 4n + \lceil n/20 \rceil$$

$$C_3(n) = 40 + 4n + 2\lceil 1 + \log_2 n \rceil$$

- (c) Method 1 is the cheapest, it costs \$40.
- (d) Method 2 is the cheapest, it costs \$529.
- (e) Method 3 is the cheapest, it costs \$8256.