JOINT INSTITUTE
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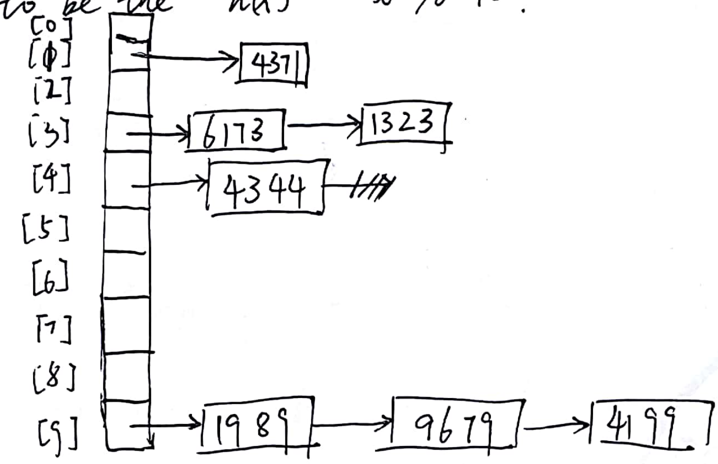
Assignment 3

1. (a) ~~4371~~ every number will turn to the least significant number of it to be the $h(x) = x \% 10$.



(b) For linear probing, $h_i(key) = (h(key) + i) \% n$.

9679	4371	1989	1323	6173	4344				4199
0	1	2	3	4	5	6	7	8	9

(c) For quadratic probing, $h_i(key) = (h(key) + i^2) \% n$.

9679	4371		1323	6173	4344	~~1989~~		1989 ~~9679~~	4199
0	1	2	3	4	5	6	7	8	9

(d) For double hashing, $h_i(key) = (h(key) + i * (g(key))) \% n$

$h(key) = x \% 10$, $g(key) = (7-x) \% 7$

$h_i(key) = (x \% 10 + i * (7-x) \% 7) \% 10$ (I emailed Dr. Qian, h_2 is 9)

$h_1(6173) = (3 + (-6173) \% 7) \% 10 = 4$

$h_1(4344) = (4 + (-4344) \% 7) \% 10 = 7$

$h_1(9679) = (9 + (-9679) \% 7) \% 10 = 1$, $h_2(9679) = 3$, $h_3(9679) = 5$

$h_1(1989) = (9 + (-1989) \% 7) \% 10 = 5$, $h_2(1989) = 1$, $h_3(1989) = 7$, $h_4(1989) = 3$,

	4371		1323	6173	9679		4344		4199
0	1	2	3	4	5	6	7	8	9

$h_5(1989) = 9$, a circle! so $\boxed{\text{1989 can not be incerted to it.}}$

2. ~~(a)~~

org :	4371	1323	6173	4199	4344	9679	1989
%19 :	1	12	17	0	12	8	13

(a) ∵ 4344 % 19 = 1323 % 19, & 1323 is the first,

0	1	...	8	...	12	13	...	17	...
4199	4371		9679		4344 → 1323	1989		6173	

(b) For linear probing, $h_i(key) = (h(key) + i) \% 19$

∵ $h_0(4344) = h_0(1323)$, so $h_1(4344) = 13 = h_0(1989)$

$h_2(4344) = 14$ is ok.

0	1	...	8	...	12	13	14	...	17	...
4199	4371		9679		1323	1989	4344		6173	

(c) For quadratic probing, $h_i(key) = (h(key) + i^2) \% 19$

∵ 1323 is still before 4344,

$h_1(4344) = (12 + 1) \% 19 = 13 = h_0(1989)$,

while 1989 is after 4344,

$h_1(1989) = 14$

0	1	...	8	...	12	13	14	...	17
4199	4371		9679		1323	4344	1989		6173

(d) For double hashing,

$h_i(key) = (x \% 19 + i * (-x) \% 7) \% 19$,

∵ 1323 is still before 4344,

$h_1(4344) = (12 + (-4344) \% 7) \% 19 = 15$, No contradiction.

0	1	...	8	...	12	13	...	15	...	17	...
4199	4371		9679		1323	1989		4344		6173	

3.

$$u(L) = \frac{1}{2}\left[1 + \left(\frac{1}{1-L}\right)^2\right] \leq 8.5 \Rightarrow L \leq \frac{3}{4}$$
$$s(L) = \frac{1}{2}\left[1 + \frac{1}{1-L}\right] \leq 3 \Rightarrow L \leq \frac{4}{5}$$
$$\Rightarrow L \leq \frac{3}{4}$$

$L = \frac{|S|}{n} \leq \frac{3}{4}$, $n \geq |S| \cdot \frac{4}{3} \geq 600 \cdot \frac{4}{3} = 800$

So a proper hash table size is larger than 800.

4. ① every Full node has a ~~father~~ parent, except the root node.

② every Full node ~~has a~~ has two children.

③ every leave has one and only one ~~father~~ parent.

④ every Un-Full and Un-leave node has one parent and one Child

Assume # leave = A,

Full node = B,

Un-Full, Un-leave node = C,

one Un-Un means one unfilled arm, one parent

one leave means ~~one~~ two unfilled arm. one parent.

So #un-filled arm = ~~2AB~~ 2A + C

parent = A + B + C $\boxed{-1}$ = # filled arm.

except the root node

while # un filled arm + # filled arm = # nodes × 2

nodes = A + B + C

$\Rightarrow (2A + C) + (A + B + C - 1) = (A + B + C) \times 2$

$\Rightarrow A - 1 = B$

So # full nodes + 1 = # leaves Proved!