F p (equivalent LTL monitoring problem)

PREDICTABILITY IN DISCRETE-EVENT SYSTEMS UNDER PARTIAL OBSERVATION [1]

Sahika Genc, Stéphane Lafortune *

* *Department of Electrical Engineering and Computer Science, University of Michigan, 1301 Beal Avenue, Ann Arbor, MI 48109-2122, USA* sgenc,stephane@eecs.umich.edu

Abstract: This paper studies the problem of predicting occurrences of a significant event in a discrete-event system. The predictability of occurrences of an event in a system is defined in the context of formal languages. The predictability of a language is a stronger condition than the diagnosability of the language. An implementable necessary and sufficient condition for predictability of occurrences of an event in systems modeled by regular languages is presented.

Keywords: Discrete-event systems, prediction, diagnosis.

1. INTRODUCTION

This paper addresses the problem of predicting occurrences of a significant (e.g., fault) event in a discrete-event system (DES). The system under consideration is modeled by a language over an event set. The event set is partitioned into observable events (e.g., sensor readings, changes in sensor readings) and unobservable events, i.e., the events that are not directly recorded by the sensors attached to the system. The objective is to predict occurrences of a possibly unobservable event in the system behavior, based on the strings of observable events. If it is possible to predict occurrences of an event in the system, then depending on the nature of the event the system operator can be warned and the operator may decide to halt the system or otherwise take preventive measures.

To the best of our knowledge, the notion of predictability that is introduced and studied in this paper is different from prior works on other notions of predictability in (Cao, 1989; Buss *et al.*, 1991; Shengbing and Kumar, 2004; Fadel and Holloway, 1999). For instance, the prediction problem considered in (Cao, 1989) is related to the properties of a special type of projection between two languages (sets of trajectories); this is is much more general than our objective, which is to predict occurrences of specific events, but our work is not a special case. The state prediction of coupled automata studied in (Buss *et al.*, 1991) is formulated as computing the state vector of n identical automata after T steps in the operation of the system; the system structure in this work is different from ours. In our case the interest is on a single automaton and event prediction, not state, under partial observation. The notion of prediction considered in (Shengbing and Kumar, 2004) differs from the one in our work in the sense that in (Shengbing and Kumar, 2004) predictability of a system is a necessary condition for diagnosability of the system while in our work diagnosability is a necessary condition for

[1] This research is supported in part by NSF grant CCR-0325571 and by ONR grant N00014-03-1-0232. The first author wishes to acknowledge support from a Barbour Fellowship from the Horace H. Rackham School of Graduate Studies at the University of Michigan.

predictability. The prediction problem studied in (Fadel and Holloway, 1999) considers issuing a warning when it is *likely* for a fault to happen in the future evolution of the system; in our work, if the occurrence of an event is predictable in a language, then it is *certain* that the event will occur. Also, in (Fadel and Holloway, 1999), it is possible that false *fault prediction warnings* are issued; in our work, no false positives are issued.

The problem of prediction studied in this paper is inspired by the problem of fault diagnosis for DES. The problem of fault diagnosis for DES has received considerable attention in the last decade (see the references in (Sampath *et al.*, 1996)) and diagnosis methodologies based on the use of discrete-event models have been successfully used in a variety of technological systems ranging from document processing systems to intelligent transportation systems. A discrete-event process called *diagnoser* introduced in (Sampath *et al.*, 1996) is of particular relevance to the present work. Later in the paper, the diagnoser is used to derive a necessary and sufficient condition for predictability in systems modeled by *regular* languages.

The rest of the paper is organized as follows. In Section 2, the notation and frequently used terms are introduced. In Section 3, the predictability of occurrences of an event in a system is defined in the context of formal languages. The predictability property of a language is a stronger condition than the diagnosability of the language as defined in (Sampath *et al.*, 1996). In Section 4, it is shown that in the case of regular languages, there exists a necessary and sufficient condition for predicting occurrences of an event in the language in the form of a test on diagnosers. In Section 5, a summary of the results in the paper is presented, and concluding remarks are given. Omitted proofs are available in (Genc, 2006).

2. PRELIMINARIES

Let Σ be a finite set of events. A *string* is a finite-length sequence of events in Σ. The set of all strings formed by events in Σ is denoted by Σ^*. The set Σ^* is also called the Kleene-closure of Σ. Any subset of Σ^* is called a *language* over Σ. The *prefix-closure* of language L is denoted by $\overline{L}$ and defined as $\overline{L} = \{s \in \Sigma^* : \exists t \in L \text{ such that } st \in L\}$. Given a string $s \in L$, L/s is called the *post-language* of L after s and defined as $L/s = \{t \in \Sigma^* : \exists st \in L\}$. L is live if every string in L can be extended to another string in L. Let L be a language over $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$, where Σ_o and Σ_{uo} denote the observable and unobservable events, respectively. The projection of strings from L to Σ_o^* is denoted by P. Given a string $s \in L$, $P(s)$ is obtained by removing unobservable events (elements of Σ_{uo}) in s. The inverse projection of a string $s_o \in \Sigma_o^*$ with respect to L is the set of strings in L whose projection is equal to s_o.

Given an event $\sigma \in \Sigma$ and a string $s \in \Sigma^*$, we use the set notation $\sigma \in s$ to say that σ appears at least once in s. Let L be a prefix-closed and live language over Σ. Given an event $\sigma \in \Sigma$ and L, $\mathcal{S}(\sigma, L)$ is the set of strings in L that ends with σ. Formally,

$$\mathcal{S}(\sigma, L) = \{s\sigma \in L : \ s \in \Sigma^*, \ \sigma \in \Sigma\}.$$

3. PROBLEM STATEMENT

In this section, we define the problem of predicting occurrences of an event in a system that is under partial observation. We model the system as a language L over an event set Σ. The event to be predicted may be an unobservable event or an observable one. First, we present an illustrative example to introduce the notion of predictability. Then, we give the formal definition for predictability of the occurrence of an event. We conclude the section by comparing the diagnosability of a language L as defined in (Sampath *et al.*, 1996) to the predictability of L.

Roughly speaking, the occurrence of an event in a language is predictable if it is possible to infer about future occurrences of the event based on the observable record of strings that do not contain the event to be predicted. Consider any string s in $\mathcal{S}(\sigma_p, L)$ where σ_p is the event to be predicted. We wish to find a prefix t of s such that t does not contain σ_p and all the *long-enough* continuations in L of the strings with the same projection as t contain σ_p. If there is at least one such t, then the occurrence of σ_p is predictable in L.

Consider the prefix-closed, live language generated by the automaton shown in Fig. 1. The language generated is

$$L = \overline{aabcpc^* + abpc^* + bpac^* + ac^*}, \qquad (1)$$

where $\Sigma_{uo} = \{a, p\}$ and $\Sigma_o = \{b, c\}$. Let p be the event to be predicted. The set of strings that end with p is

$$\mathcal{S}(p, L) = \{aabcp, abp, bp\}. \qquad (2)$$

In order to show that p is predictable in L, we must find an $n \in \mathbb{N}$ and a $t \in \overline{s}$ for all $s \in \mathcal{S}(p, L)$ such that $p \notin t$ and for all u and its continuations $v \in L/u$ **if**

- u records the same string of observable events as t, i.e., $P(t) = P(u)$, and
- u does not contain p, i.e. $p \notin u$, and
- v is of length greater than $n \in \mathbb{N}$, i.e. $\| v \| \geq n$,

then v contains p.

Let us start with $s = aabcp \in \mathcal{S}(p, L)$. Then $t \in \overline{aabc}$. Suppose that $t = aa$. Then, $P^{-1}(aa) \cap (\Sigma \setminus \{p\})^* \cap L = \{\epsilon, a, aa\}$. If $u = a$, then $L/u = \overline{abcpc^* + bpc^* + c^*}$. Since $p \notin c^*$, there is a continuation of u that does not contain p. Then, there exists a string which records the same string of observable events as t and not all of its continuations contain p. Thus, $t = aa$ is a wrong choice to prove the predictability of p. Suppose that $t = aab$. For all $u \in P^{-1}(aab) \cap (\Sigma \setminus \{p\})^* \cap L = \{aab, ab, b\}$ and for all $v \in L/u$ such that $\| v \| \geq 2$, then v contains p. Thus, $t = aab$ is a right choice for $s \in aabcp \in \mathcal{S}(p, L)$. Similarly, it can be verified that $t = ab$ and $t = b$ work for $s = abp$ and $s = bp$ in $\mathcal{S}(p, L)$, respectively.

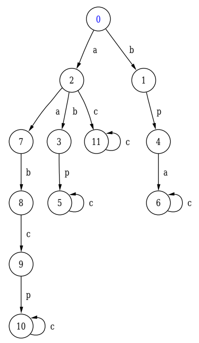

Fig. 1. G.

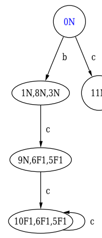

Fig. 2. D_G.

Based on the above discussion, we formally define the notion of predictability.

Definition 1. Given L a prefix-closed, live language over Σ, occurrences of event $\sigma_p \in \Sigma$ are *predictable* in L with respect to P if

$$(\exists n \in \mathbb{N})(\forall s \in \mathcal{S}(\sigma_p, L))(\exists t \in \overline{s})[(\sigma_p \notin t) \wedge \mathbf{P}]$$

where

$$\mathbf{P}: \ (\forall u \in L)(\forall v \in L/u)[(P(u) = P(t)) \wedge (\sigma_p \notin u) \wedge (\| v \| \geq n) \Rightarrow (\sigma_p \in v)].$$

3.1 Diagnosability vs. Predictability

The predictability of occurrences of an event σ_p in a prefix closed and live language L is stronger than the diagnosability of L with respect to σ_p. We consider the diagnosability as defined in (Sampath *et al.*, 1996) in the context of formal languages. Roughly speaking, L is diagnosable with respect to σ_p if it is possible to *detect* occurrences of σ_p with a finite delay. For the sake of completeness, we recall in Definition 2 the formal definition of diagnosability.

Definition 2. A prefix-closed and live language is diagnosable with respect to P and σ_p if

$$(\exists n \in \mathbb{N})(\forall s \in \mathcal{S}(\sigma_p, L))(\forall t \in L/s)[\| t \| \geq n \Rightarrow \mathbf{D}]$$

where

$$\mathbf{D}: \omega \in P^{-1}P(st) \cap L \Rightarrow \sigma_p \in \omega.$$

We now present an illustrative example where a language is diagnosable with respect to an event but the occurrence of the event is not predictable. We consider the language generated by the automaton shown in Fig. 3. The language is

$$L = \overline{eac^* + abepd^* + abcd^* + aebpdd^*} \qquad (3)$$

where $\Sigma_o = \{a, b, c, d\}$ and $\Sigma_{uo} = \{e, p\}$.

In this case, the occurrence of p is not predictable. Let $s = abep \in \mathcal{S}(p, L)$. Then, $t \in \overline{abe}$. For any $t \in \overline{abe}$, we always have have the string $abcd^n$ where $n \geq 0$, which does not contain p. Thus, there does not exist a t so that Definition 1 is satisfied for p. However, the occurrence of p (an unobservable event) can be detected with a finite delay. After the observation of abd, we are certain that p has occurred at least once. Thus, L is diagnosable with respect to σ but the occurrence of σ is not predictable in L.

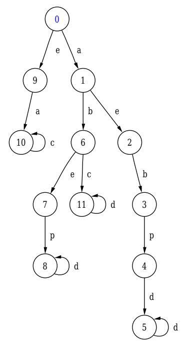

Fig. 3. G.

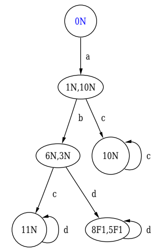

Fig. 4. D_G.

The following proposition follows directly from the above definitions.

Proposition 3. Given a prefix-closed and live language $L \subseteq \Sigma^*$, if occurrences of $\sigma_p \in \Sigma$ are predictable in L with respect to P, then L is diagnosable with respect to P and σ_p.

4. VERIFICATION OF PREDICTABILITY FOR REGULAR LANGUAGES

In this section, we consider systems modeled by regular languages. Regular languages are the languages that are accepted (or generated) by Finite State Automata (FSA). An FSA is a four-tuple

$$G = (Q, \Sigma, \delta, q_0) \qquad (4)$$

where Q is the set of states, Σ is the finite set of events, $\delta : Q \times \Sigma \to Q$ is the state transition function and q_0 is the initial state.

The necessary and sufficient condition (presented later in this section) for predictability is based on a discrete-event process called *diagnoser*. The diagnoser is an FSA built for the system with respect to a projection P onto the set of observable events and to a given event. Let $G = (Q, \Sigma, \delta, q_0)$ be an FSA that generates language L. We denote by D_G the diagnoser built for G and $\sigma_p \in \Sigma$. The diagnoser D_G is of the form

$$D_G = (Q_D, \Sigma_o, \delta_D, q_{D,0}, \sigma_p), \tag{5}$$

where Q_D is the set of diagnoser states, $\delta_D : Q_D \times \Sigma_o \to Q_D$ is the diagnoser state transition function, $q_{D,0} \in Q_D$ is the initial diagnoser state. The diagnoser state space Q_D is a subset of $2^{Q\times\{N,F1\}}$. State $q_D \in Q_D$ is of the form

$$q_D = \{(q_1, l_1), \ldots, (q_n, l_n)\}, \tag{6}$$

where $q_i \in Q$ and $l_i \in \{N, F1\}$ for $i = 1, \ldots, n$.

Let q_D and q'_D be two diagnoser states in Q_D such that q'_D is reached from q_D by $\sigma_o \in \Sigma_o$, i.e., $q'_D = \delta_D(q_D, \sigma_o)$ is defined. Let

$$q_D = \{(q_1, l_1), \ldots, (q_m, l_m)\}$$

and

$$q'_D = \{(q'_1, l'_1), \ldots, (q'_n, l'_n)\}.$$

For all $i \in \{1, \ldots, n\}$, there exists $j \in \{1, 2, \ldots, m\}$ such that

$$q'_i = \delta(q_j, s), \tag{7}$$

where $s = t\sigma_o$ and $t \in \Sigma^*_{uo}$, and

$$l'_i = \begin{cases} F1, & \text{if } l_j = F1 \text{ or } (\sigma_p \in s), \\ N, & \text{if } l_j = N \text{ and } (\sigma_p \notin s). \end{cases} \tag{8}$$

We say that a diagnoser state $q_D = \{(q_1, l_1), \ldots, (q_m, l_m)\} \in Q_D$ for $m \in \mathbb{N}$ is *normal* if $l_j = N$ for all $j = 1, \ldots, m$; *certain* if $l_j = F1$ for all $j = 1, \ldots, m$; and *uncertain* if there exist $l_j = N$ and $l_i = F1$ for some $i, j \in \{1, \ldots, m\}$. We denote by $Q^N_D \subseteq Q_D$ the set of diagnoser states that are normal, by $Q^U_D \subseteq Q_D$ the set of diagnoser states that are uncertain, and by $Q^C_D \subseteq Q_D$ the set of diagnoser states that are certain.

Consider FSA G in Fig. 1. Let $\Sigma_{uo} = \{a, p\}$. The diagnoser [2] for G and p is as shown in Fig. 2. The diagnoser state $\{1N, 8N, 3N\}$ is normal, $\{9N, 6F1, 5F1\}$ is uncertain, and $\{10F1, 6F1, 5F1\}$ is certain.

We define an accessibility operation on an FSA to find the accessible part of an FSA from a state.

Definition 4. Let $G = (Q, \Sigma, \delta, q_0)$ and $q \in Q$. The *accessible* part of G with respect to q is denoted by $Ac(G, q)$ and is

$$Ac(G, q) = (Q_{ac}, \Sigma, \delta_{ac}, q), \tag{9}$$

where $Q_{ac} = \{q' \in Q : (\exists s \in \Sigma^*)(\delta(q, s) = q' \text{ is defined})\}$, and $\delta_{ac} = \delta\,|_{Q_{ac}\times\Sigma\to Q_{ac}}$.

[2] Diagnosers shown in this paper are built using UMDES-LIB software (Lafortune, n.d.).

Let $G = (Q, \Sigma, \delta, q_0)$. We say that a set of states $\{q_1, q_2, \ldots, q_n\} \subseteq Q$ and a string $\sigma_1\sigma_2\ldots\sigma_n \in \Sigma^*$ form a *cycle* if $q_{i+1} = \delta(q_i, \sigma_i),\ i = 1, 2, \ldots, n-1$ and $q_1 = \delta(q_n, \sigma_n)$.

In the rest of this section, we assume the system satisfies the following: *If $\{q_1, q_2, \ldots, q_n\} \subseteq Q$ and $\sigma_1\sigma_2\ldots\sigma_n \in \Sigma^*$ form a cycle, then there exists at least one observable event σ_j in $\{\sigma_1, \ldots, \sigma_n\} \subseteq \Sigma$.* That is, G does not contain a cycle in which states are connected with unobservable events only.

Lemma 5 below states that if there is a cycle in D_G that contains a certain diagnoser state, then all the diagnoser states in the cycle are certain (since the $F1$ label propagates). Lemma 6 states that if there is a cycle in D_G that is formed by uncertain or normal states, then there exists a corresponding cycle in G such that all the states in the cycle have normal labels in the cycle in D_G.

Lemma 5. Let $G = (Q, \Sigma, \delta, q_0)$ be an FSA that generates L such that L is prefix-closed and live and let $D_G = (Q_D, \Sigma_o, \delta_D, q_{D,0}, \sigma_p)$ be the diagnoser for G and σ_p. Suppose $\{q_{D,1}, \ldots, q_{D,n}\} \subseteq Q_D$ and $\sigma_{o,1}\ldots\sigma_{o,n} \in \Sigma^*_o$ form a cycle in D_G where $n \in \mathbb{N}$. If there exists $i \in \{1, 2, \ldots, n\}$ such that $q_{D,i} \in Q^C_D$, then $q_{D,j} \in Q^C_D$ for all $j = 1, 2, \ldots, n$.

Lemma 6. Let $G = (Q, \Sigma, \delta, q_0)$ be an FSA that generates L such that L is prefix-closed and live, and let $D_G = (Q_D, \Sigma_o, \delta_D, q_{D,0}, \sigma_p)$ be the diagnoser for G and σ_p. Suppose $\{q_{D,1}, \ldots, q_{D,n}\} \subseteq Q_D$ and $\sigma_{o,1}\ldots\sigma_{o,n} \in \Sigma^*_o$ form a cycle in D_G where $n \in \mathbb{N}$ and $q_{D,i}$ is in Q^U_D or Q^N_D for all $i = 1, 2, \ldots, n$. Then, there exists $(q_i, l_i) \in q_{D,i}$ for $i = 1, 2, \ldots, n$, such that $q_{i+1} = \delta(q_i, s_i)$ for $i = 1, 2, \ldots, n-1$ and $q_1 = \delta(q_n, s_n)$ where $s_i \in \Sigma^*$, $P(s_i) = \sigma_{o,i}$, and $l_i = N$ for $i = 1, 2, \ldots, n$.

Let F_D be the set of normal diagnoser states that possess an immediate successor that is not normal. Formally, $F_D = \{x_D \in Q^N_D : \exists\ y_D = \delta_D(x_D, \sigma_o) \text{ such that } \sigma_o \in \Sigma_o \text{ and } y_D \notin Q^N_D\}$. Lemma 7 states that any uncertain or certain diagnoser state is reached from a diagnoser state in F_D.

Lemma 7. Let $G = (Q, \Sigma, \delta, q_0)$ be an FSA that generates L such that L is prefix-closed and live, and let $D_G = (Q_D, \Sigma_o, \delta_D, q_{D,0}, \sigma_f)$ be the diagnoser for G and σ_p. Let $x_{D,i} = \delta_D(x_{D,i-1}, \sigma_{o,i})$ for $i = 1, 2, \ldots, m$ where $m \in \mathbb{N}$, $x_{D,i}$ is a diagnoser state, $\sigma_{o,i}$ is an observable event for $i = 1, 2, \ldots, m$, and $x_{D,0}$ is the initial diagnoser state. If $x_{D,m}$ is in Q^U_D or Q^C_D, then there exists $M \leq m$ such that $x_{D,M} \in F_D$.

Proof 8. The proof is by induction on the sequence of observable events.

Base ($m = 1$): In this case, $x_{D,m} = x_{D,1} \notin Q_D^N$ and $x_{D,1} = \delta_D(x_{D,0}, \sigma_{o,1})$. Since $x_{D,0}$ is the initial diagnoser state, by definition it is normal. If the immediate successor $x_{D,1}$ of $x_{D,0}$ is not a normal diagnoser state, then $x_{D,0} \in F_D$. This completes the proof of induction base.

Hypothesis ($m = M'$): If $x_{D,M'} \notin Q_D^N$, then there exists $M \leq M'$ such that $x_{D,M} \in F_D$.

Step (m=M'+1): We need to show that if $x_{D,M'+1} \notin Q_D^N$, then there exists $M \leq M' + 1$ such that $x_{D,M} \in F_D$. We consider two cases: (i) $x_{D,M'} \in Q_D^N$, and (ii) $x_{D,M'} \notin Q_D^N$. In the first case, if $x_{D,M'} \in Q_D^N$, then $x_{D,M'}$ is in F_D by definition. For the other case, if $x_{D,M'} \notin Q_D^N$, then by the induction hypothesis there exists $M \leq M' < M'+1$ such that $x_{D,M}$ is in F_D. This completes the proof of the induction step.□

In the following theorem, we state the necessary and sufficient condition for predictability of occurrences of an event. The condition is based on analyzing the cycles in the diagnoser.

Theorem 9. Let $G = (Q, \Sigma, \delta, q_0)$ be an FSA that generates L where L is prefix-closed and live. Let $D_G = (Q_D, \Sigma_o, \delta_D, q_{D,0}, \sigma_p)$ be the diagnoser for G and σ_p. The occurrences of σ_p are predictable in L with respect to P iff for all $q_D \in F_D$, condition $\mathbf{C}$ holds, where

$\mathbf{C}$: *all cycles in $Ac(D_G, q_D)$ are cycles of certain diagnoser states.*

Proof 10. The proof is in two parts.

($\Rightarrow$): We prove that if σ_p is predictable in L, then for all $q_D \in F_D$ the only cycles in $Ac(D_G, q_D)$ are cycles of certain diagnoser states. The proof is by contradiction.

Suppose that there exists $q_D \in F_D$ such that $Ac(D_G, q_D)$ contains a cycle formed by $\{x_{D,1}, \ldots, x_{D,m}\}$ and $\sigma_{o,1} \ldots \sigma_{o,m} \in \Sigma_o^*$ where $x_{D,i} \notin Q_D^C$ for some $i \in \{1, 2, \ldots, m\}$.

By Lemma 5, if there exists a diagnoser state $x_{D,i}$ in the cycle such that $x_{D,i}$ is not a certain diagnoser state, then none of the other diagnoser states in the cycle are certain. Thus, $x_{D,i} \notin Q_D^C$ for all $i = 1, 2, \ldots, m$.

By Lemma 6, corresponding to the cycle of diagnoser states in the diagnoser, there exists a cycle in G such that each state in that cycle is labeled with N in the cycle in the diagnoser. Suppose that the cycle in G is formed by $\{x_1, \ldots, x_m\}$ and $s_1 \ldots s_m \in \Sigma^*$ where $(x_i, N) \in x_{D,i}$ and $s_i \in \Sigma^*$ such that $P(s_i) = \sigma_{o,i}$ for $i = 1, 2, \ldots, m$.

Let $q_D \in F_D$ be reached from the initial diagnoser state $q_{D,0}$ by $s_o \in \Sigma_o^*$. Since q_D is in F_D, then there exists $s \in \mathcal{S}(\sigma_p, L)$ such that $P(s) = s_o$. Moreover, since σ_p is predictable in L, then by definition of predictability, there exists $t \in \overline{s}$ such that $(\sigma_p \notin t) \wedge \mathbf{P}$. We now prove that there exists a u such that $P(u) = P(t)$ and $\sigma_p \notin u$, and for all continuations v of u if v is of length greater than any $n \in \mathbb{N}$, then v does not contain σ_p.

Pick a diagnoser state in the cycle. Without loss of generality pick $x_{D,1}$. Then, we pick the state in the diagnoser state which has label N and is a part of the corresponding cycle in G. Let (x_1, l_1) be that state in $x_{D,1}$, with $l_1 = N$.

Suppose that $x_{D,1}$ is reached from q_D by executing $s'_o \in \Sigma_o^*$. Then, $x_{D,1} = \delta_D(q_{D,0}, s_o s'_o)$. Let u and $u' \in L/u$ be such that $P(uu') = s_o s'_o$ and $x_1 = \delta(q_0, uu')$. Since $l_1 = N$, then neither u nor u' contain σ_p. Since x_1 is in the corresponding cycle in G, then $x_1 = \delta(q_0, uu'(s_1 s_2 \ldots s_m)^k)$ for $k \in \mathbb{N}$.

Let $v = u'(s_1 s_2 \ldots s_m)^k$ where k is an integer such that $\parallel v \parallel \geq n$. Then, there exists u and $v \in L/u$ such that $P(u) = P(t)$ (since $P(t) = s_o$), $\sigma \notin u$, $\parallel v \parallel \geq n$ and $\sigma_p \notin v$ (since $l_1 = N$). This violates the condition $\mathbf{P}$ in the definition of predictability. Thus, there is a contradiction. This completes one part of the proof.

($\Leftarrow$): We prove that if for all $q_D \in F_D$ the only cycles in $Ac(D_G, q_D)$ are cycles of certain diagnoser states, then σ_p is predictable in L.

Pick any $s \in \mathcal{S}(\sigma_p, L)$. Let $q = \delta(q_0, s) \in Q$. Then, pick any $s_{uo}\sigma_o \in L/s$ such that $s_{uo} \in \Sigma_{uo}^*$ and $\sigma_o \in \Sigma_o$. Let $y = \delta(q, s_{uo}\sigma_o) \in Q$. Suppose that $P(s) = s_o \in \Sigma_o^*$. Then, let $x_D = \delta_D(q_{D,0}, s_o)$ and $y_D = \delta_D(x_D, \sigma_o)$ in Q_D. Then, there exists $(y, l_y) \in y_D$ where $l_y = F1$. Thus, $y_D \in Q_D^U \cup Q_D^C$. We now consider the following two cases: (i) $x_D \in Q_D^N$, thus, $x_D \in F_D$, and (ii) $x_D \in Q_D^U \cup Q_D^C$.

Case (i). Since $x_D \in Q_D^N$ and $y_D \notin Q_D^N$, then $x_D \in F_D$. We choose $t = s$. For all u such that $P(u) = P(t)$, $P(u) = s_o$. Since the only cycles in $Ac(D_G, x_D)$ are cycles of certain states, then for all $v \in L/u$, v contains σ_p.

Case (ii). If $x_D \in Q_D^U \cup Q_D^C$, i.e., x_D is not normal, then we wish to find a normal diagnoser state in F_D from which x_D is reached. By Lemma 7, there exists a diagnoser state w_D reachable from the initial diagnoser state, x_D is accessible from w_D, and w_D is in F_D. Then, since F_D consists of normal diagnoser states, w_D is in Q_D^N. Thus, the proof of Case (ii) reduces to the case of (i) in which we substitute $w_D \in Q_D^N$ for $x_D \in Q_D^N$. This completes the second part of the proof. □

Consider the FSA in Fig. 1 and the corresponding diagnoser in Fig. 2 where $\Sigma_{uo} = \{a, p\}$ and $\Sigma_o = \{b, c\}$, and $F_D = \{\{1N, 8N, 3N\}\}$. The accessible FSA from $\{1N, 8N, 3N\}$ contains only one cycle formed by $\{10F1, 6F1, 5F1\}$ which is a certain diagnoser state. Thus, the occurrence of p is predictable. If we consider the FSA in Fig. 3 and the corresponding diagnoser in Fig. 4 where $\Sigma_o = \{a, b, c, d\}$ and $\Sigma_{uo} = \{e, p\}$, then, $F_D = \{\{6N, 3N\}\}$. The accessible FSA from $\{6N, 3N\}$ contains two cycles one of which contains a normal diagnoser state. Here, the occurrence of p is not predictable.

We now show that it is sufficient to test condition **C** in Theorem 9 on certain subsets of F_D to guarantee that this condition holds for all states in F_D.

Corollary 11. Let $x_D, y_D \in F_D$ such that $y_D = f_D(x_D, s_o)$ is defined for some $s_o \in \Sigma_o^*$. Then, condition **C** holds for all $q_D \in F_D$ iff **C** holds for all $q_D \in F_D \setminus \{y_D\}$.

In view of Corollary 11, let us call a subset of F_D "**C**-*sufficient*" if testing condition **C** in Theorem 9 on this subset is sufficient to guarantee that **C** holds for all $q_D \in F_D$. Denote by S_{F_D} the set of all **C**-*sufficient* subsets of F_D. Let $Min(S_{F_D})$ denote all subsets of F_D in S_{F_D} that have minimum cardinality.

Proposition 12. $Min(S_{F_D})$ is not a singleton in general.

Define a relation between x_D and y_D in F_D as follows: $x_D \sim y_D \Leftrightarrow \exists s_o, t_o \in o^*$ *such that* $y_D = \delta_D(x_D, s_o)$ *and* $x_D = \delta_D(y_D, s_o)$. That is, two states in F_D are related if both of them appear in a cycle in the diagnoser.

Proposition 13. The relation $\sim$ is an equivalence relation.

We now work on the equivalence classes (induced by $\sim$) in F_D instead of the states in F_D. Let E_D be the equivalence classes of F_D for the relation $\sim$. Denote by S_{E_D} the set of all **C**-*sufficient* subsets of E_D. Let $Min(S_{E_D})$ denote all sets in S_{E_D} that have minimum cardinality. Theorem 14 states that there is only one **C**-*sufficient* subset of E_D with the minimum cardinality.

Theorem 14. $Min(S_{E_D})$ is a singleton.

We have developed an algorithm for finding this unique element in $Min(S_{E_D})$. In view of Corollary 11 and Theorem 14, the necessary and sufficient condition for predictability in Theorem 9 becomes: "Condition **C** holds for all $q_D \in Min(S_{E_D})$." In general, $Min(S_{E_D}) \subseteq F_D$, thus resulting in computational savings once $Min(S_{E_D})$ has been computed.

5. CONCLUSION

We have defined the new property of predictability of the occurrence of a significant event (e.g., fault) based on the current record of observable events. We have shown a necessary and sufficient condition for predictability in the case of systems modeled by regular languages. We have presented a test to verify the predictability property based on diagnosers. An alternate test of polynomial-time complexity (in the number of system states) is presented in (Genc, 2006). The study of predictability is inspired and motivated by the study of fault diagnosis. Our long term goal is to form an integrated theory of diagnosis and prediction in the framework of formal languages.

REFERENCES

Buss, Samuel R., Christos Papadimitriou and John Tsitsiklis (1991). On the predictability of coupled automata: an allegory about chaos. *Complex Systems* **5**, 525–539.

Cao, Xi-Ren (1989). The predictability of discrete event systems. *IEEE Trans. Automatic Control* **34**(11), 1168–1171.

Fadel, H.K. and L.E. Holloway (1999). Using SPC and template monitoring method for fault detection and prediction in discrete event manufacturing systems. In: *Proceedings of the 1999 IEEE International Symposium on Intelligent Control/Intelligent Systems and Semiotics*. pp. 150 – 155.

Genc, Sahika (2006). On diagnosis and predictability of partially-observed discrete-event systems. Ph.D. Thesis. University of Michigan, Electrical Engineering: Systems.

Lafortune, S. (n.d.). Umdes-lib software library. http://www.eecs.umich.edu/umdes/toolboxes.html.

Sampath, M., R. Sengupta, S. Lafortune, K. Sinnamohideen and D. Teneketzis (1996). Failure diagnosis using discrete event models. *IEEE Trans. Control Systems Technology* **4**(2), 105–124.

Shengbing, Jiang and Ratnesh Kumar (2004). Failure diagnosis of discrete-event systems with linear-time temporal logic specifications. *IEEE Trans. Automatic Control* **49(6)**, 934 – 945.