Five-point Fundamental Matrix Estimation for Uncalibrated Cameras

Outlines

- Abstract
- Theoretical Background
- Five-point algorithm
- Application – Get more accurate matching points
- Results and Comparison

Abstract

- The proposed minimal solver first estimates a homography from three correspondences assuming that they are co-planar and exploiting their rotational components;
- Then the fundamental matrix is obtained from the homography and two additional point pairs in general position.

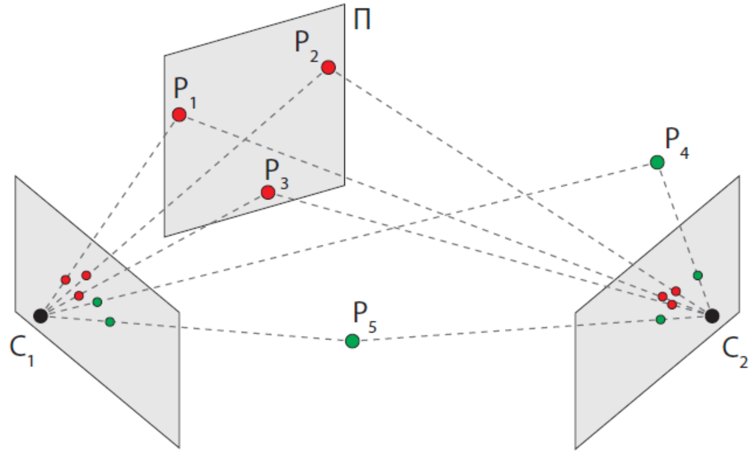


Figure 1: The proposed minimal solver estimates a fundamental matrix between views C_1 and C_2. It first estimates a homography from three correspondences of co-planar points ($\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$) lying on plane π. The fundamental matrix is then obtained from the homography and two additional points ($\mathbf{P}_4$ and $\mathbf{P}_5$) in general position.

Paper: Barath, D., Five-point fundamental matrix estimation for uncalibrated cameras, Conference on Computer Vision and Pattern Recognition, 2018

Theoretical Background – (1) Homography and Affine Correspondences

Affine Correspondences. In this paper, we consider an affine correspondence (AC) as a triplet: $(\mathbf{p}_1, \mathbf{p}_2, \mathbf{A})$, where $\mathbf{p}_1 = [u_1 \quad v_1 \quad 1]^\mathrm{T}$ and $\mathbf{p}_2 = [u_2 \quad v_2 \quad 1]^\mathrm{T}$ are a corresponding homogeneous point pair in the two images (the projections of the 3D points in Fig. 1), and

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

Note that, for perspective cameras, A is the first-order approximation of the related homography matrix:

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

as follows:

$$\begin{aligned} a_1 &= \frac{\partial u_2}{\partial u_1} = \frac{h_1 - h_7 u_2}{s}, & a_2 &= \frac{\partial u_2}{\partial v_1} = \frac{h_2 - h_8 u_2}{s}, \\ a_3 &= \frac{\partial v_2}{\partial u_1} = \frac{h_4 - h_7 v_2}{s}, & a_4 &= \frac{\partial v_2}{\partial v_1} = \frac{h_5 - h_8 v_2}{s}, \end{aligned} \quad (1)$$

where u_i and v_i are the directions in the ith image ($i \in \{1, 2\}$) and $s = u_1 h_7 + v_1 h_8 + h_9$ is the projective depth.

(1) Homography and Affine Correspondences

To form a linear equation system using A, Eqs (1) are multiplied by the common denominator (s), then rearranged as follows:

$$
\begin{aligned}
h_1 - (u_2 + a_1 u_1) h_7 - a_1 v_1 h_8 - a_1 &= 0 \\
h_2 - (u_2 + a_2 v_1) h_8 - a_2 u_1 h_8 - a_2 &= 0 \\
h_4 - (v_2 + a_3 u_1) h_7 - a_3 v_1 h_8 - a_3 &= 0 \\
h_5 - (v_2 + a_4 v_1) h_8 - a_4 u_1 h_8 - a_4 &= 0
\end{aligned} \qquad (2)
$$

The relationship of a homography and point correspondences is $Hp_1 \sim p_2$, so we can get:

$$
\begin{aligned}
u_1 h_1 + v_1 h_2 + h_3 - u_1 u_2 h_7 - v_1 u_2 h_8 &= u_2 \\
u_1 h_4 + v_1 h_5 + h_6 - u_1 v_2 h_7 - v_1 v_2 h_8 &= v_2
\end{aligned} \qquad (3)
$$

(2) Affine Transformation Model

Define an affine transformation model as a combination of linear transformations as follows:

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} s_u & w \\ 0 & s_v \end{bmatrix} = \\ & \begin{bmatrix} s_u \cos(\alpha) & w\cos(\alpha) - s_v \sin(\alpha) \\ s_u \sin(\alpha) & w \sin(\alpha) + s_v \cos(\alpha) \end{bmatrix},\end{aligned} \tag{4}$$

Then, using Eqs (4) substituting the components in Eqs (2), the following equations we can get:

$$\begin{aligned} h_1 - u_2 h_7 - u_1 c_\alpha s_u h_7 - v_1 c_\alpha s_u h_8 - c_\alpha s_u &= 0, \\ h_2 - u_2 h_8 + v_1 c_\alpha w h_8 - v_1 s_\alpha s_v h_8 - & \\ u_1 c_\alpha w h_8 + u_1 s_\alpha s_v h_8 - c_\alpha w + s_\alpha s_v &= 0, \\ h_4 - v_2 h_7 - u_1 s_\alpha s_u h_7 - v_1 s_\alpha s_u h_8 - s_\alpha s_u &= 0, \\ h_5 - v_2 h_8 - v_1 s_\alpha w h_8 - v_1 c_\alpha s_v h_8 - & \\ u_1 s_\alpha w h_8 - u_1 c_\alpha s_v h_8 - s_\alpha w - c_\alpha s_v &= 0, \end{aligned} \tag{5}$$

Step 1 - Homography Estimation

Assume three co-planar point correspondences $\mathbf{p}_{1,i} = [u_{1,i} \quad v_{1,i} \quad 1]^{\mathrm{T}}$, $\mathbf{p}_{2,i} = [u_{2,i} \quad v_{2,i} \quad 1]^{\mathrm{T}}$ $(i \in [1,3])$ and the related rotation components α_i, obtained by e.g. SIFT, to be known. The objective is to find homography $\mathbf{H}$ for which $\mathbf{H}\mathbf{p}_{1,i} \sim \mathbf{p}_{2,i}$ and also satisfies Eqs. 5.

Forming $Hp_{1,i} \sim p_{2,i}$ (Eq. (3)) for all correspondences as a homogeneous linear system $Bh = 0$, coefficient matrix B is of size 6 x 9 and $h = [h1\ h2\ h3\ h4\ h5\ h6\ h7\ h8\ h9]^T$ is the vector of unknown parameters. The null-space of B is three-dimensional, therefore the final solution is calculated as a linear combination of the three null-vectors as follows:

$$\mathbf{h} = \beta\mathbf{b} + \gamma\mathbf{c} + \delta\mathbf{d}, \qquad (6)$$

Replacing each h_j by $\beta b_j+\gamma c_j+\delta d_j$, we can get:

$$
\begin{aligned}
(b_1 - u_2 b_7)\beta + (c_1 - u_2 c_7)\gamma - (u_1 c_\alpha b_7 + v_1 c_\alpha b_8) s_u \beta - \\
(u_1 c_\alpha d_7 + v_1 c_\alpha d_8 + c_\alpha) s_u + (u_1 c_\alpha c_7 + v_1 c_\alpha c_8) s_u \gamma - \\
d_1 - u_2 d_7 = 0,
\end{aligned}
$$

$$
\begin{aligned}
(b_4 - v_2 b_7)\beta + (c_4 - v_2 c_7)\gamma - (u_1 s_\alpha b_7 + v_1 s_\alpha b_8) s_u \beta - \\
(u_1 s_\alpha d_7 + v_1 s_\alpha d_8 + s_\alpha) s_u - (u_1 s_\alpha c_7 + v_1 s_\alpha c_8) s_u \gamma - \\
d_4 - v_2 d_7 = 0.
\end{aligned}
$$

$$
[\beta \quad \gamma \quad s_{u,1} \quad s_{u,1}\beta \quad s_{u,1}\gamma \quad s_{u,2} \quad s_{u,2}\beta \quad s_{u,2}\gamma]^{\mathrm{T}}.
$$

β and γ can be solved as follows：

$$
\begin{aligned}
\beta = \quad & (-c_{\alpha_2}c_1d_7v_{2,2}s_{\alpha_1} + c_{\alpha_2}c_4d_7u_{2,1}s_{\alpha_1} + c_{\alpha_2}c_7d_1v_{2,2}s_{\alpha_1} \\
& -c_{\alpha_2}c_7d_4u_{2,1}s_{\alpha_1} - c_{\alpha_2}c_{\alpha_1}c_4d_7v_{2,1} + c_{\alpha_2}c_{\alpha_1}c_4d_7v_{2,2} \\
& +c_{\alpha_2}c_{\alpha_1}c_7d_4v_{2,1} - c_{\alpha_2}c_{\alpha_1}c_7d_4v_{2,2} - c_1d_7u_{2,1}s_{\alpha_2}s_{\alpha_1} \\
& +c_1d_7u_{2,2}s_{\alpha_2}s_{\alpha_1} + c_7d_1u_{2,1}s_{\alpha_2}s_{\alpha_1} - c_7d_1u_{2,2}s_{\alpha_2}s_{\alpha_1} \\
& +c_{\alpha_1}c_1d_7v_{2,1}s_{\alpha_2} - c_{\alpha_1}c_4d_7u_{2,2}s_{\alpha_2} - c_{\alpha_1}c_7d_1v_{2,1}s_{\alpha_2} \\
& +c_{\alpha_1}c_7d_4u_{2,2}s_{\alpha_2} + c_{\alpha_2}c_1d_4s_{\alpha_1} - c_{\alpha_2}c_4d_1s_{\alpha_1} \\
& -c_{\alpha_1}c_1d_4s_{\alpha_2} + c_{\alpha_1}c_4d_1s_{\alpha_2})/ \\
& (c_{\alpha_2}b_1c_7v_{2,2}s_{\alpha_1} + c_{\alpha_2}b_4c_7u_{2,1}s_{\alpha_1} + c_{\alpha_2}b_7c_1v_{2,2}s_{\alpha_1} \\
& -c_{\alpha_2}b_7c_4u_{2,1}s_{\alpha_1} - c_{\alpha_2}c_{\alpha_1}b_4c_7v_{2,1} + c_{\alpha_2}c_{\alpha_1}b_4c_7v_{2,2} \\
& +c_{\alpha_2}c_{\alpha_1}b_7c_4v_{2,1} - c_{\alpha_2}c_{\alpha_1}b_7c_4v_{2,2} - b_1c_7u_{2,1}s_{\alpha_1}s_{\alpha_2} \\
& +b_1c_7u_{2,2}s_{\alpha_1}s_{\alpha_2} + b_7c_1u_{2,1}s_{\alpha_1}s_{\alpha_2} - b_7c_1u_{2,2}s_{\alpha_1}s_{\alpha_2} \\
& +c_{\alpha_1}b_1c_7v_{2,1}s_{\alpha_2} - c_{\alpha_1}b_4c_7u_{2,2}s_{\alpha_2} - c_{\alpha_1}b_7c_1v_{2,1}s_{\alpha_2} \\
& +c_{\alpha_1}b_7c_4u_{2,2}s_{\alpha_2} + c_{\alpha_2}b_1c_4s_{\alpha_1} - c_{\alpha_2}b_4c_1s_{\alpha_1} \\
& -c_{\alpha_1}b_1c_4s_{\alpha_2} + c_{\alpha_1}b_4c_1s_{\alpha_2}),
\end{aligned}
$$

$$
\begin{aligned}
\gamma = \quad & -(-c_{\alpha_2}b_1d_7v_{2,2}s_{\alpha_1} + c_{\alpha_2}b_4d_7u_{2,1}s_{\alpha_1} + c_{\alpha_2}b_7d_1v_{2,2}s_{\alpha_1} \\
& -c_{\alpha_2}b_7d_4u_{2,1}s_{\alpha_1} - c_{\alpha_2}c_{\alpha_1}b_4d_7v_{2,1} + c_{\alpha_2}c_{\alpha_1}b_4d_7v_{2,2} \\
& +c_{\alpha_2}c_{\alpha_1}b_7d_4v_{2,1} - c_{\alpha_2}c_{\alpha_1}b_7d_4v_{2,2} - b_1d_7u_{2,1}s_{\alpha_1}s_{\alpha_2} \\
& +b_1d_7u_{2,2}s_{\alpha_1}s_{\alpha_2} + b_7d_1u_{2,1}s_{\alpha_1}s_{\alpha_2} - b_7d_1u_{2,2}s_{\alpha_1}s_{\alpha_2} \\
& +c_{\alpha_1}b_1d_7v_{2,1}s_{\alpha_2} - c_{\alpha_1}b_4d_7u_{2,2}s_{\alpha_2} - c_{\alpha_1}b_7d_1v_{2,1}s_{\alpha_2} \\
& +c_{\alpha_1}b_7d_4u_{2,2}s_{\alpha_2} + c_{\alpha_2}b_1d_4s_{\alpha_1} - c_{\alpha_2}b_4d_1s_{\alpha_1} \\
& -c_{\alpha_1}b_1d_4s_{\alpha_2} + c_{\alpha_1}b_4d_1s_{\alpha_2})/ \\
& (-c_{\alpha_2}b_1c_7v_{2,2}s_{\alpha_1} + c_{\alpha_2}b_4c_7u_{2,1}s_{\alpha_1} + c_{\alpha_2}b_7c_1v_{2,2}s_{\alpha_1} \\
& -c_{\alpha_2}b_7c_4u_{2,1}s_{\alpha_1} - c_{\alpha_2}c_{\alpha_1}b_4c_7v_{2,1} + c_{\alpha_2}c_{\alpha_1}b_4c_7v_{2,2} \\
& +c_{\alpha_2}c_{\alpha_1}b_7c_4v_{2,1} - c_{\alpha_2}c_{\alpha_1}b_7c_4v_{2,2} - b_1c_7u_{2,1}s_{\alpha_1}s_{\alpha_2} \\
& +b_1c_7u_{2,2}s_{\alpha_1}s_{\alpha_2} + b_7c_1u_{2,1}s_{\alpha_1}s_{\alpha_2} - b_7c_1u_{2,2}s_{\alpha_1}s_{\alpha_2} \\
& +c_{\alpha_1}b_1c_7v_{2,1}s_{\alpha_2} - c_{\alpha_1}b_4c_7u_{2,2}s_{\alpha_2} - c_{\alpha_1}b_7c_1v_{2,1}s_{\alpha_2} \\
& +c_{\alpha_1}b_7c_4u_{2,2}s_{\alpha_2} + c_{\alpha_2}b_1c_4s_{\alpha_1} - c_{\alpha_2}b_4c_1s_{\alpha_1} \\
& -c_{\alpha_1}b_1c_4s_{\alpha_2} + c_{\alpha_1}b_4c_1s_{\alpha_2}).
\end{aligned}
$$

Step 2 - Fundamental Matrix Estimation from Five Points

Here, suppose that homography H estimated in the previously described way, and two additional point correspondences are given. The objective is to estimate fundamental matrix F both with H and the two correspondences and det(F) = 0 holds.

First, using *hallucinated point* technique generating five point correspondences using H. The five generated and two given point pairs yield seven linear equations through $p_{2,i}^T F p_{1,i} = 0 (i \in [1, 7])$

Combining them, the following homogeneous linear system is given: Df = 0.

$$\mathbf{D} = \begin{bmatrix} u_{1,1}u_{2,1} & v_{1,1}u_{2,1} & u_{2,1} & u_{1,1}v_{2,1} & v_{1,1}v_{2,1} & v_{2,1} & u_{1,1} & v_{1,1} & 1 \\ & & & & \dots & & & & \\ u_{1,7}u_{2,7} & v_{1,7}u_{2,7} & u_{2,7} & u_{1,7}v_{2,7} & v_{1,7}v_{2,7} & v_{2,7} & u_{1,7} & v_{1,7} & 1 \end{bmatrix}.$$

$$f = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9]^T$$

Fundamental Matrix Estimation from Five Points

The null-space of matrix D is two-dimensional and the solution is calculated as the linear combination of the two null-vectors:

$$F = \epsilon e + \eta g \quad (7) \qquad\qquad \eta = 1 - \epsilon,$$

Substituting Eq (7) into def(F) = 0 leads to a cubic polynomial equation. And the possible solutions for ε are obtained as the real roots of the polynomial. The resulting fundamental matrices are finally calculated by substituting each ε to Eq (7).

Application – Get more accurate matching points

- First, using *SIFT* algorithm get some point correspondences and rotation features (Already done and save to files);

- Then using the *Five-point* method to estimate the fundamental matrix *F*;

- Using the estimated *F* matrix can calculate the *Mean Symmetric Epipolar Distance* (MSED);

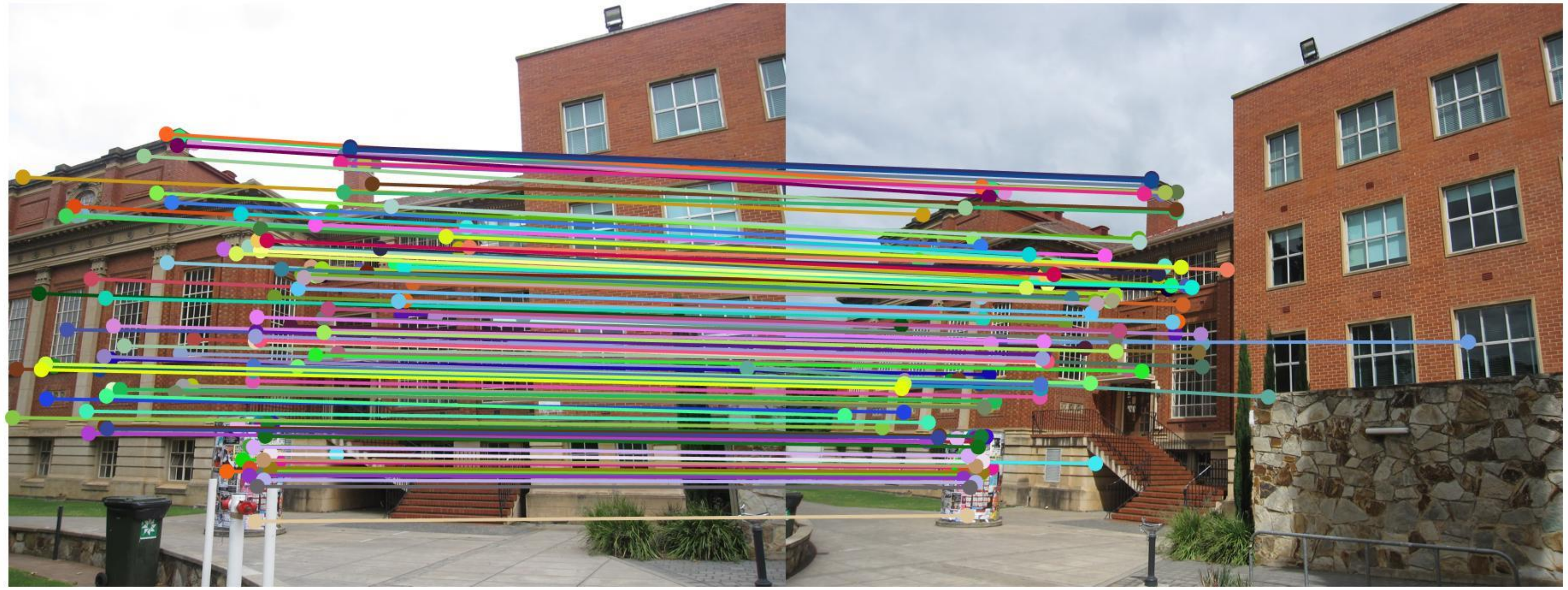
$$\frac{1}{2} \sum_{(\mathbf{p}_1, \mathbf{p}_2) \in \mathcal{P}_\mathrm{R}} \frac{\mathbf{F}\mathbf{p}_1}{\sqrt{(\mathbf{F}\mathbf{p}_1)_1^2 + (\mathbf{F}\mathbf{p}_1)_2^2}} + \frac{\mathbf{p}_2^\mathrm{T}\mathbf{F}}{\sqrt{(\mathbf{p}_2^\mathrm{T}\mathbf{F})_1^2 + (\mathbf{p}_2^\mathrm{T}\mathbf{F})_2^2}}, \tag{8}$$

where $\mathcal{P}_\mathrm{R}$ is the set of reference correspondences.

- The MSED is the error criterion, that can be combined with *RANSAC* to get more accurate matching results.

5-point

- Number of points: 331
- Number of found inliers: 143 (Average)
- Number of iterations = 457 (Average)



7-point

- Number of points: 331
- Number of found inliers: 153 (Average)
- Number of iterations = 363 (Average)

8-point

- Number of points: 331
- Number of found inliers: 177 (Average)
- Number of iterations = 115 (Average)

Results and Comparison

- From the results, we can see, the five-point method gets fewest feature points, and it needs more time to iteration than other methods.

- The seven-point method gets more features than the five-point and less than eight-point method. And it needs less time than five-point.

- The eight-point method gets the most feature points than the other two methods, and it costs the least amount of time.

Thank you !