Algorithm 0 sample inhomogeneous poisson process with rate $\Omega(t)-\lambda(t)h(t-l(t))$

$G=0, G_1, G_2, \ldots\ldots$

Let $X_{pre}=\tau_{pre}=0$, $X_{next}=G_1$, $i=1$, $\widetilde{G}=\{\}$

while(1){

Propose $\tau_{cur} \sim \Omega(\tau) \mid \tau>\tau_{pre}$

if($X_{pre}+\tau_{cur}<X_{next}$) {

- sample $\lambda(X_{pre}+\tau_{cur})$, the GP evaluated at $X_{pre}+\tau_{cur}$
- with probability $1-\frac{\lambda(Xpre+\ \tau cur)}{\hat{\lambda}}$ accept i.e. $\widetilde{G}=\widetilde{G} \cup \{X_{pre}+\tau_{cur}\}$, else reject i.e. $\widetilde{G}=\widetilde{G}$
- set $\tau_{pre}=\tau_{cur}$

}

else {

- $X_{pre}=X_{next}$
- $\tau_{pre}=0$
- $i=i+1$
- if($i<len(G)$) $X_{next}=G_i$ else break;

}

}

Algorithm 1 Blocked Gibbs sampler for GP-modulated renewal process on the interval [0,T] when hazard is unbounded

Input: Set of event times G, set of thinned Times $\widetilde{G}_{pre}$ and l instantiated at $G \cup \widetilde{G}_{pre}$.

Output: A new set of thinned times $\widetilde{G}_{new}$, and a new instantiation $l_{G \cup \widetilde{G}_{new}}$ of GP on $G \cup \widetilde{G}_{new}$.

1. Let $X_{pre}= \tau_{pre}=0$, $X_{next}=G_1$, $i=1$, $\widetilde{G}_{new}=\{\}$, $l_{\widetilde{G}_{new}}=\{\}$, $A=\{\}$, $l_A=\{\}$

while(1){

Propose $\tau_{cur} \sim \Omega(\tau) \mid \tau > \tau_{pre}$

if($X_{pre}+ \tau_{cur}<X_{next}$) {

- sample $\lambda(X_{pre}+ \tau_{cur})$, the GP evaluated at $X_{pre}+\tau_{cur}$,
  i.e. $l_{X_{pre}+\tau_{cur}} \mid l_{G \cup \widetilde{G}_{pre} \cup A}$
- Set $A=A \cup \{ X_{pre}+ \tau_{cur} \}$, $l_A= l_A \cup \{ l_{X_{pre}+\tau_{cur}}\}$
- with probability $1-\frac{\lambda(X_{pre}+\tau_{cur})}{\hat{\lambda}}$
  accept Set $\widetilde{G}_{new}=\widetilde{G}_{new} \cup \{ X_{pre}+ \tau_{cur} \}$, $l_{\widetilde{G}_{new}}= l_{\widetilde{G}_{new}} \cup \{ l_{X_{pre}+\tau_{cur}} \}$
  else
  reject do nothing
- set $\tau_{pre}=\tau_{cur}$

}

else {

- $X_{pre}=X_{next}$
- $\tau_{pre}=0$
- $i=i+1$

- if(i<len(G)) $X_{next}=G_i$ else break;

}

}

2. Let $\widetilde{G}_{new}$, $\iota_{\widetilde{G}_{new}}$ be the resulting new set. Discard $\widetilde{G}_{pre}$, $\iota_{\widetilde{G}_{pre}}$.

3. Resample $\iota_{G \cup \widetilde{G}_{new}}$ using elliptical sampling.