Supplement Material of the paper "Maximum AoI Minimization for Target Monitoring in Battery-free Wireless Sensor Networks"

Appendix A
Proof of Theorem 2

We only need to prove that Equation (2) is correct by contradiction. Assume there exists another path $P_l'(n, q_1, j, q_2)$ such that its length $Mt'(n, q_1, j, q_2) < Mt(n, q_1, j, q_2)$. Assume that on the path $P_l'(n, q_1, j, q_2)$, the last state of q_2 is q_3. According to the definition of the path on $EN(n)$, there is an edge $(q_3, q_2, action, m) \in \Delta(n)$ and $action$ = 0 or 1. Firstly, we discuss the case when $action$ = 1. Let $Mt'(n, q_1, k, q_3)$ denote the path length on $P_l'(n, q_1, j, q_2)$ from q_1 to q_3, by the definition of $P_l(n, q_1, j, q_2)$, we have $k \geq j\text{-}m$ and $Mt'(n, q_1, k, q_3) \geq Mt'(n, q_1, j\text{-}m, q_3)$. Thus, $Mt'(n, q_1, j, q_2)$ - $m \geq Mt'(n, q_1, j\text{-}m, q_3)$. Also, according to Equation (2), $Mt(n, q_1, j, q_2) \leq Mt(n, q_1, j\text{-}m, q_3) + m$. Therefore, we have $Mt'(n, q_1, j\text{-}m, q_3) \leq Mt'(n, q_1, j, q_2)\text{-}m < Mt(n, q_1, j, q_2)\text{-}m \leq Mt(n, q_1, j\text{-}m, q_3)$, which conflicts with the assumption that $Mt(n, q_1, j\text{-}m, q_3)$ is optimal. The case in which $action$ = 0 can be proved similarly.

Appendix B
Proof of Theorem 5

Firstly, consider an edge $(q_1(n), q_j(n), 1, m) \in \Delta(n)$. As mentioned in Step 1 in Section 3.2.2, $q_1(n)$ represents n's stable voltage $U_s(n)$. When n's voltage reaches $U_s(n)$, after m consecutive active slots, the voltage of n, namely $U_d(n, U_s(n), m) \in [D_p(n, m\text{+}1), D_p(n, m)]$, which is the voltage subinterval represented by state $q_{m+2}(n)$. Thus, we have $j = m\text{+}2$. Also, $|Q(n)| = Df(n, U_s(n)) + 2$. Thus, $1 \leq m \leq Df(n, U_s(n)) = |Q(n)|$ - 2.

Secondly, consider an edge $(q_k(n), q_j(n), 1, m) \in \Delta(n)$, where $k \neq 1$. As mentioned by Step 2 in Section 3.2.2, the energy level of node n at state $q_k(n)$ is regarded as voltage $D_p(n, k\text{-}1)$. According to Property 3, since $U_d(n, U_s(n), k\text{-}1) \leq D_p(n, k\text{-}1) \leq U_d(n, U_s(n), k\text{-}2)$, we have $Df(n, U_d(n, U_s(n), k\text{-}1)) \leq Df(n, D_p(n, k\text{-}1)) \leq Df(n, U_d(n, U_s(n), k\text{-}2))$. Also, $Df(n, U_d(n, U_s(n), k\text{-}1)) = |Q(n)|\text{-}k\text{-}1$ and $Df(n, U_d(n, U_s(n), k\text{-}2)) = |Q(n)|\text{-}k$. Thus, when node n is at state $q_k(n)$, n can be active continuously for at most $|Q(n)|\text{-}k\text{-}1$ or $|Q(n)|\text{-}k$ slots. Thus, the largest value of m is $|Q(n)|\text{-}k\text{-}1$ or $|Q(n)|\text{-}k$. For a certain m, according to Property 3, we have $U_d(n, U_d(n, U_s(n), k\text{-}1), m) \leq U_d(n, D_p(n, k\text{-}1), m) \leq U_d(n, U_d(n, U_s(n), k\text{-}2), m)$. That is, $U_d(n, U_s(n), k\text{+}m\text{-}1) \leq U_d(n, D_p(n, k\text{-}1), m) \leq U_d(n, U_s(n), k\text{+}m\text{-}2)$. Also, we have $U_d(n, U_s(n), k\text{+}m\text{-}1) \in [D_p(n, k\text{+}m), D_p(n, k\text{+}m\text{-}1)]$, which is the voltage interval denoted by $q_{k+m+1}(n)$, $U_d(n, U_s(n), k\text{+}m\text{-}2) \in [D_p(n, k\text{+}m\text{-}1), D_p(n, k\text{+}m\text{-}2)]$, which is the voltage interval denoted by $q_{k+m}(n)$. Therefore, voltage $U_d(n, D_p(n, k\text{-}1), m)$ is within the interval denote by $q_{k+m+1}(n)$ or $q_{k+m}(n)$. As voltage $D_p(n, k\text{-}1)$ is regarded as the energy level of state $q_k(n)$, $j = k\text{+}m$ or $k\text{+}m\text{+}1$. This completes the proof.