

### The Closure Property of Data Types

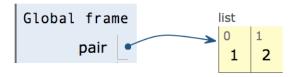
- A method for combining data values satisfies the *closure property* if:

  The result of combination can itself be combined using the same method
- Closure is powerful because it permits us to create hierarchical structures
- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

## Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element Each box either contains a primitive value or points to a compound value

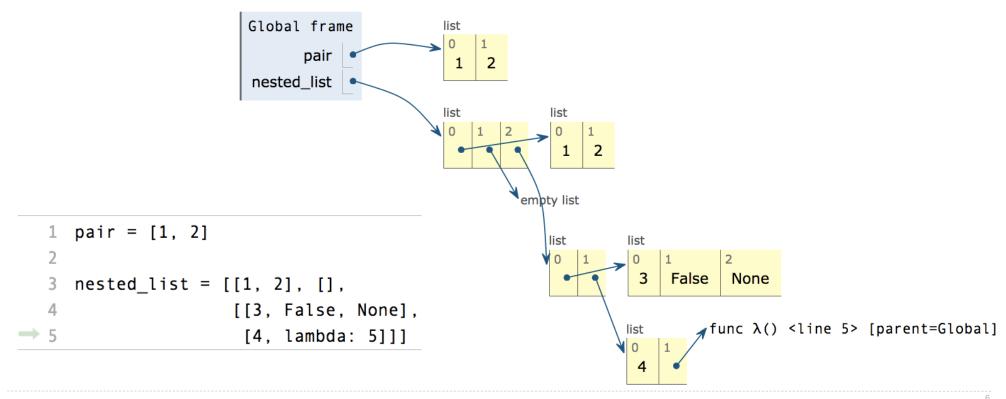


$$pair = [1, 2]$$

5

## Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element Each box either contains a primitive value or points to a compound value



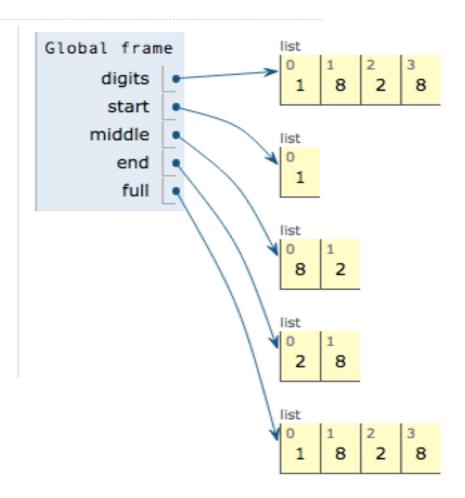
# Slicing

(Demo)

## **Slicing Creates New Values**

```
1 digits = [1, 8, 2, 8]
2 start = digits[:1]
3 middle = digits[1:3]
4 end = digits[2:]

> 5 full = digits[:]
```



pythontutor.com/composingprograms.html#code=digits%20%3D%20figits[%3A]]%0Astart%20%3D%20digits[%3A]]%0A

**Processing Container Values** 

#### Sequence Aggregation

Several built-in functions take iterable arguments and aggregate them into a value

• sum(iterable[, start]) -> value

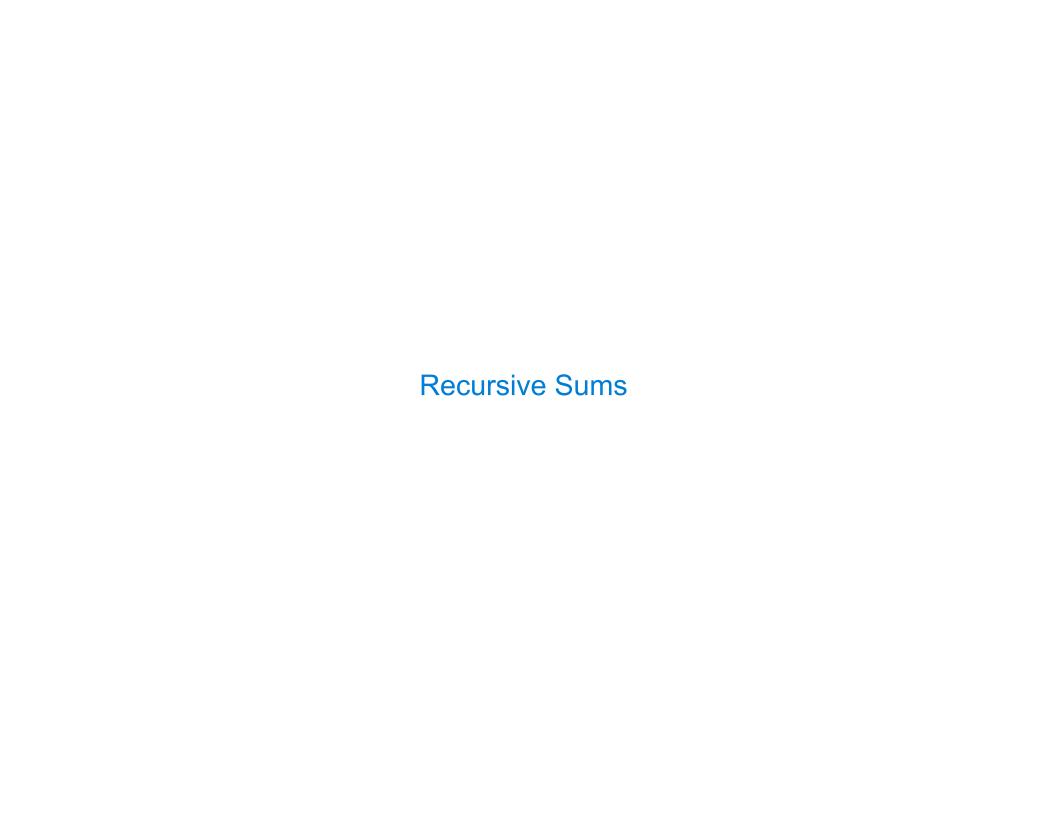
Return the sum of an iterable (not of strings) plus the value of parameter 'start' (which defaults to 0). When the iterable is empty, return start.

• max(iterable[, key=func]) -> value
 max(a, b, c, ...[, key=func]) -> value

With a single iterable argument, return its largest item. With two or more arguments, return the largest argument.

all(iterable) -> bool

Return True if bool(x) is True for all values x in the iterable. If the iterable is empty, return True.



```
Sum (recursively)

def mysum(L):
    if (L == []):
        return 0
    else:
        return L[0] + mysum( L[1:] )

mysum( [2, 4, 1, 5] )

2 + mysum( [4, 1, 5] )

4 + mysum( [1, 5] )

1 + mysum( [5] )

5 + mysum( [] )
```

```
# --- DRILL ---
# Write an iterative function that takes as input
# integer "n" and returns the sum of the first "n"
# integers: sum(5) returns 1+2+3+4+5
```

```
# --- DRILL ---
# Write an iterative function that takes as input
# integer "n" and returns the sum of the first "n"
# integers: sum(5) returns 1+2+3+4+5

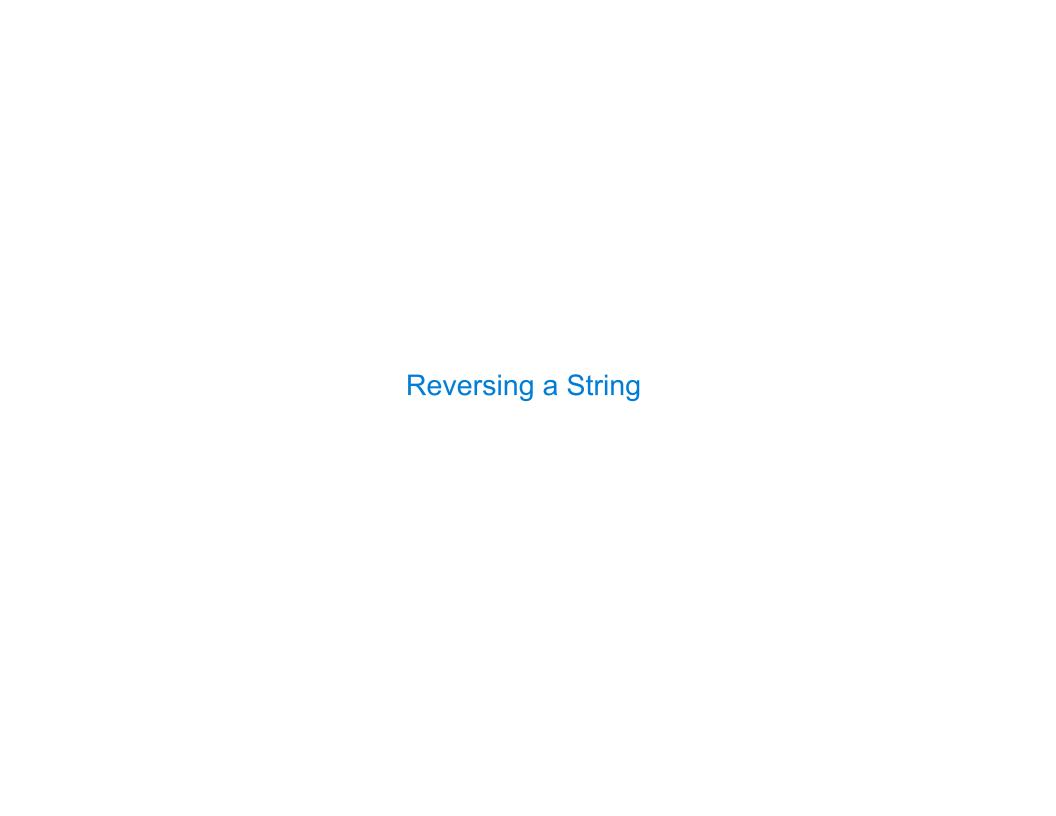
def sum_iter(n):
    sum = 0
    for i in range(0,n+1):
        sum = sum + i

    return( sum )
```

```
# --- DRILL ---
# Write a recursive function that takes as input
# integer "n" and returns the sum of the first "n"
# integers: sum(5) returns 1+2+3+4+5
```

```
# --- DRILL ---
# Write a recursive function that takes as input
# integer "n" and returns the sum of the first "n"
# integers: sum(5) returns 1+2+3+4+5

def sum_rec(n):
    if( n == 0 ):
        return(0)
    else:
        return n + sum_rec(n-1)
```



```
Reversing a List (recursively)

reverse("ward") = "draw"

reverse("ward") = reverse("ard") + "w"

reverse("ard") = reverse("rd") + "a"

reverse("rd") = reverse("d") + "r"

reverse("d") = "d"
```

```
Reversing a List (recursively)

reverse("ward") = "draw"

reverse("ward") = reverse("ard") + "w"

reverse("ard") = "d" + "r" + "a"
```

```
Reversing a List (recursively)

def reverse(s):
    if len(s) == 1:
        return s
    else:
        return reverse(s[1:]) + s[0]
```