Homework 3: Recursion, Tree Recursion

hw03.zip (hw03.zip)

Due by 11:59pm on Thursday, September 23

Instructions

Download hw03.zip (hw03.zip). Inside the archive, you will find a file called hw03.py (hw03.py), along with a copy of the `ok` autograder.

Submission: When you are done, submit with `python3 ok --submit`. You may submit more than once before the deadline; only the final submission will be scored. Check that you have successfully submitted your code on okpy.org (https://okpy.org/). See Lab 0 (/lab/lab00#submitting-the-assignment) for more instructions on submitting assignments.

Using Ok: If you have any questions about using Ok, please refer to this guide. (/articles/using-ok)

Readings: You might find the following references useful:

- Section 1.7 (http://composingprograms.com/pages/17-recursive-functions.html)

Grading: Homework is graded based on correctness. Each incorrect problem will decrease the total score by one point. There is a homework recovery policy as stated in the syllabus. **This homework is out of 2 points.**

Required Questions

Getting Started Videos

Q1: Num eights

Write a recursive function `num_eights` that takes a positive integer `pos` and returns the number of times the digit 8 appears in `pos`.

Important: Use recursion; the tests will fail if you use any assignment statements. (You can however use function definitions if you so wish.)

```
def num_eights(pos):
    """Returns the number of times 8 appears as a digit of pos.

    >>> num_eights(3)
    0
    >>> num_eights(8)
    1
    >>> num_eights(88888888)
    8
    >>> num_eights(2638)
    1
    >>> num_eights(86380)
    2
    >>> num_eights(12345)
    0
    >>> from construct_check import check
    >>> # ban all assignment statements
    >>> check(HW_SOURCE_FILE, 'num_eights',
    ...       ['Assign', 'AnnAssign', 'AugAssign', 'NamedExpr'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q num_eights
```

Q2: Ping-pong

The ping-pong sequence counts up starting from 1 and is always either counting up or counting down. At element `k`, the direction switches if `k` is a multiple of 8 or contains the digit 8. The first 30 elements of the ping-pong sequence are listed below, with direction swaps marked using brackets at the 8th, 16th, 18th, 24th, and 28th elements:

Index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**[8]**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**[16]**	**17**	**[18]**	**19**	**20**	**21**	**22**	**23**
PingPong Value	1	2	3	4	5	6	7	[8]	7	6	5	4	3	2	1	[0]	1	[2]	1	0	-1	-2	-3

Index (cont.)	**[24]**	**25**	**26**	**27**	**[28]**	**29**	**30**
PingPong Value	[-4]	-3	-2	-1	[0]	-1	-2

Implement a function `pingpong` that returns the nth element of the ping-pong sequence *without using any assignment statements*. (You are allowed to use function definitions.)

You may use the function `num_eights`, which you defined in the previous question.

Important: Use recursion; the tests will fail if you use any assignment statements. (You can however use function definitions if you so wish.)

> **Hint:** If you're stuck, first try implementing `pingpong` using assignment statements and a `while` statement. Then, to convert this into a recursive solution, write a helper function that has a parameter for each variable that changes values in the body of the while loop.

```
def pingpong(n):
    """Return the nth element of the ping-pong sequence.

    >>> pingpong(8)
    8
    >>> pingpong(10)
    6
    >>> pingpong(15)
    1
    >>> pingpong(21)
    -1
    >>> pingpong(22)
    -2
    >>> pingpong(30)
    -2
    >>> pingpong(68)
    0
    >>> pingpong(69)
    -1
    >>> pingpong(80)
    0
    >>> pingpong(81)
    1
    >>> pingpong(82)
    0
    >>> pingpong(100)
    -6
    >>> from construct_check import check
    >>> # ban assignment statements
    >>> check(HW_SOURCE_FILE, 'pingpong',
    ...       ['Assign', 'AnnAssign', 'AugAssign', 'NamedExpr'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q pingpong
```

Q3: Missing Digits

Write the recursive function `missing_digits` that takes a number `n` that is sorted in non-decreasing order (for example, `12289` is valid but `15362` and `98764` are not). It returns the number of missing digits in `n`. A missing digit is a number between the first and last digit of `n` of a that is not in `n`.

Important: Use recursion; the tests will fail if you use any loops.

```
def missing_digits(n):
    """Given a number a that is in sorted, non-decreasing order,
    return the number of missing digits in n. A missing digit is
    a number between the first and last digit of a that is not in n.
    >>> missing_digits(1248) # 3, 5, 6, 7
    4
    >>> missing_digits(19) # 2, 3, 4, 5, 6, 7, 8
    7
    >>> missing_digits(1122) # No missing numbers
    0
    >>> missing_digits(123456) # No missing numbers
    0
    >>> missing_digits(3558) # 4, 6, 7
    3
    >>> missing_digits(35578) # 4, 6
    2
    >>> missing_digits(12456) # 3
    1
    >>> missing_digits(16789) # 2, 3, 4, 5
    4
    >>> missing_digits(4) # No missing numbers between 4 and 4
    0
    >>> from construct_check import check
    >>> # ban while or for loops
    >>> check(HW_SOURCE_FILE, 'missing_digits', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q missing_digits
```

Q4: Count coins

Given a positive integer `change`, a set of coins makes change for `change` if the sum of the values of the coins is `change`. Here we will use standard US Coin values: 1, 5, 10, 25. For example, the following sets make change for `15`:

- 15 1-cent coins
- 10 1-cent, 1 5-cent coins
- 5 1-cent, 2 5-cent coins
- 5 1-cent, 1 10-cent coins
- 3 5-cent coins
- 1 5-cent, 1 10-cent coin

Thus, there are 6 ways to make change for `15`. Write a **recursive** function `count_coins` that takes a positive integer `change` and returns the number of ways to make change for `change` using coins.

You can use either of the functions given to you:

- `ascending_coin` will return the next larger coin denomination from the input, i.e. `ascending_coin(5)` is `10`.
- `descending_coin` will return the next smaller coin denomination from the input, i.e. `descending_coin(5)` is `1`.

There are two main ways in which you can approach this problem. One way uses `ascending_coin`, and another uses `descending_coin`.

Important: Use recursion; the tests will fail if you use loops.

> **Hint:** Refer the implementation (http://composingprograms.com/pages/17-recursive-functions.html#example-partitions) of `count_partitions` for an example of how to count the ways to sum up to a final value with smaller parts. If you need to keep track of more than one value across recursive calls, consider writing a helper function.

```
def ascending_coin(coin):
    """Returns the next ascending coin in order.
    >>> ascending_coin(1)
    5
    >>> ascending_coin(5)
    10
    >>> ascending_coin(10)
    25
    >>> ascending_coin(2) # Other values return None
    """
    if coin == 1:
        return 5
    elif coin == 5:
        return 10
    elif coin == 10:
        return 25

def descending_coin(coin):
    """Returns the next descending coin in order.
    >>> descending_coin(25)
    10
    >>> descending_coin(10)
    5
    >>> descending_coin(5)
    1
    >>> descending_coin(2) # Other values return None
    """
    if coin == 25:
        return 10
    elif coin == 10:
        return 5
    elif coin == 5:
        return 1

def count_coins(change):
    """Return the number of ways to make change using coins of value of 1, 5, 10, 25.
    >>> count_coins(15)
    6
    >>> count_coins(10)
    4
    >>> count_coins(20)
    9
    >>> count_coins(100) # How many ways to make change for a dollar?
    242
    >>> count_coins(200)
    1463
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(HW_SOURCE_FILE, 'count_coins', ['While', 'For'])
    True
    """
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

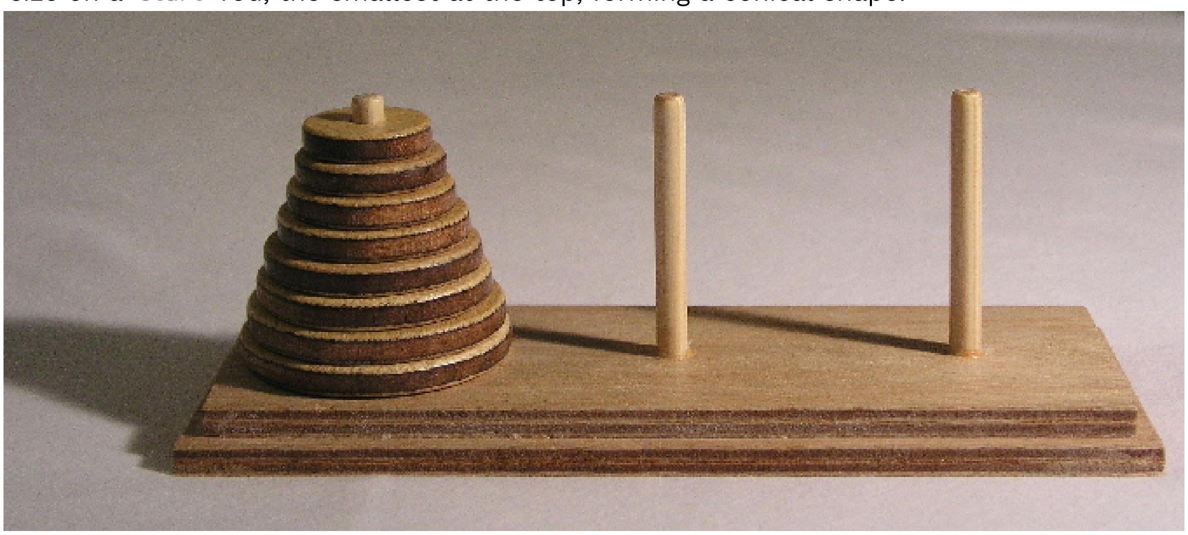
```
python3 ok -q count_coins
```

Just for fun Questions

These questions are out of scope for 61a. You can try them if you want an extra challenge, but they're just puzzles that are not required or recommended at all. Almost all students will skip them, and that's fine.

Q5: Towers of Hanoi

A classic puzzle called the Towers of Hanoi is a game that consists of three rods, and a number of disks of different sizes which can slide onto any rod. The puzzle starts with `n` disks in a neat stack in ascending order of size on a `start` rod, the smallest at the top, forming a conical shape.



The objective of the puzzle is to move the entire stack to an `end` rod, obeying the following rules:

- Only one disk may be moved at a time.
- Each move consists of taking the top (smallest) disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
- No disk may be placed on top of a smaller disk.

Complete the definition of `move_stack`, which prints out the steps required to move `n` disks from the `start` rod to the `end` rod without violating the rules. The provided `print_move` function will print out the step to move a single disk from the given `origin` to the given `destination`.

Hint: Draw out a few games with various `n` on a piece of paper and try to find a pattern of disk movements that applies to any `n`. In your solution, take the recursive leap of faith whenever you need to move any amount of disks less than `n` from one rod to another. If you need more help, see the following hints.

Hint 1

Hint 2

Hint 3

```
def print_move(origin, destination):
    """Print instructions to move a disk."""
    print("Move the top disk from rod", origin, "to rod", destination)

def move_stack(n, start, end):
    """Print the moves required to move n disks on the start pole to the end
    pole without violating the rules of Towers of Hanoi.

    n -- number of disks
    start -- a pole position, either 1, 2, or 3
    end -- a pole position, either 1, 2, or 3

    There are exactly three poles, and start and end must be different. Assume
    that the start pole has at least n disks of increasing size, and the end
    pole is either empty or has a top disk larger than the top n start disks.

    >>> move_stack(1, 1, 3)
    Move the top disk from rod 1 to rod 3
    >>> move_stack(2, 1, 3)
    Move the top disk from rod 1 to rod 2
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 2 to rod 3
    >>> move_stack(3, 1, 3)
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 1 to rod 2
    Move the top disk from rod 3 to rod 2
    Move the top disk from rod 1 to rod 3
    Move the top disk from rod 2 to rod 1
    Move the top disk from rod 2 to rod 3
    Move the top disk from rod 1 to rod 3
    """
    assert 1 <= start <= 3 and 1 <= end <= 3 and start != end, "Bad start/end"
    "*** YOUR CODE HERE ***"
```

Use Ok to test your code:

```
python3 ok -q move_stack
```

Q6: Anonymous factorial

> This question demonstrates that it's possible to write recursive functions without assigning them a name in the global frame.

The recursive factorial function can be written as a single expression by using a conditional expression (http://docs.python.org/py3k/reference/expressions.html#conditional-expressions).

```
>>> fact = lambda n: 1 if n == 1 else mul(n, fact(sub(n, 1)))
>>> fact(5)
120
```

However, this implementation relies on the fact (no pun intended) that `fact` has a name, to which we refer in the body of `fact`. To write a recursive function, we have always given it a name using a `def` or assignment statement so that we can refer to the function within its own body. In this question, your job is to define `fact` recursively without giving it a name!

Write an expression that computes `n` factorial using only call expressions, conditional expressions, and `lambda` expressions (no assignment or `def` statements).

> **Note:** You are not allowed to use `make_anonymous_factorial` in your return expression.

The `sub` and `mul` functions from the `operator` module are the only built-in functions required to solve this problem.

```
from operator import sub, mul

def make_anonymous_factorial():
    """Return the value of an expression that computes factorial.

    >>> make_anonymous_factorial()(5)
    120
    >>> from construct_check import check
    >>> # ban any assignments or recursion
    >>> check(HW_SOURCE_FILE, 'make_anonymous_factorial',
    ...     ['Assign', 'AnnAssign', 'AugAssign', 'NamedExpr', 'FunctionDef', 'Recursion'])
    True
    """
    return 'YOUR_EXPRESSION_HERE'
```

Use Ok to test your code:

```
python3 ok -q make_anonymous_factorial
```