Bet Smarter With The Monte Carlo Simulation

By Tzveta Iordanova on October 21, 2010

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In finance, there is a fair amount of uncertainty and risk involved with estimating the future value of figures or amounts due to the wide variety of potential outcomes. Monte Carlo simulation (MCS) is one technique that helps to reduce the uncertainty involved in estimating future outcomes. MCS can be applied to complex, non-linear models or used to evaluate the accuracy and performance of other models. It can also be implemented in risk management, portfolio management, pricing derivatives, strategic planning, project planning, cost modeling and other fields. (To learn more, read *Monte Carlo Simulation With GBM*.)

Definition
MCS is a technique that converts uncertainties in input variables of a model into probability distributions. By combining the distributions and randomly selecting values from them, it recalculates the simulated model many times and brings out the probability of the output.

Basic Characteristics

- MCS allows several inputs to be used at the same time to create the probability distribution of one or more outputs.
- Different types of probability distributions can be assigned to the inputs of the model. When the distribution is unknown, the one that represents the best fit could be chosen.
- The use of random numbers characterizes MCS as a stochastic method. The random numbers have to be independent; no correlation should exist between them.
- MCS generates the output as a range instead of a fixed value and shows how likely the output value is to occur in the range.

Some Frequently Used Probability Distributions in MCS

Normal/Gaussian Distribution - Continuous distribution applied in situations where the mean and the standard deviation are given and the mean represents the most probable value of the variable. It is symmetrical around the mean and is not bounded. (For related reading, see *The Uses And Limits Of Volatility*.)

Lognormal Distribution - Continuous distribution specified by mean and standard deviation. This is appropriate for a variable ranging from zero to infinity, with positive skewness and with normally distributed natural logarithm.

Triangular Distribution - Continuous distribution with fixed minimum and maximum values. It is bounded by the minimum and maximum values and can be either symmetrical (the most probable value = mean = median) or asymmetrical.

Uniform Distribution - Continuous distribution bounded by known minimum and maximum values. In contrast to the triangular distribution, the likelihood of occurrence of the values between the minimum and maximum is the same.

Exponential Distribution - Continuous distribution used to illustrate the time between independent occurrences, provided the rate of occurrences is known. (For more insight, see *Find The Right Fit With Probability Distributions*.)

The Math Behind MCS
Consider that we have a real-valued function g(X) with probability frequency function P(x) (if X is discrete), or probability density function f(x) (if X is continuous). Then we can define the expected value of g(X) in discrete and continuous terms respectively:

$$E(g(X)) = \sum_{-\infty}^{+\infty} g(x)P(x) \text{ , where P(x)>0 and } \sum_{-\infty}^{+\infty} P(x) = 1$$

$$E(g(X)) = \int_{-\infty}^{+\infty} g(x)f(x)dx \text{ , where f(x) >0 and } \int_{-\infty}^{+\infty} f(x)dx = 1$$

Next, make n random drawings of X (x_1, …..,xn) , called trial runs or simulation runs, calculate g(x_1),….g(xn) and find the mean of g(x) of the sample:

$$\overset{\mu}{g}_n(x) = \frac{1}{n}\sum_{i=1}^{n} g(x_i) \text{ , which represents the final simulated value of E(g(X)).}$$

$$\text{Therefore } \overset{\mu}{g}_n(X) = \frac{1}{n}\sum_{i=1}^{n} g(X) \text{ will be the Monte Carlo estimator of E(g(X)).}$$

As $n \to \infty$, $\overset{\mu}{g}_n(X) \to E(g(X))$, thus we are now able to compute the dispersion around the estimated mean with the unbiased variance of $\hat{\mu}_n(X)$:

$$\widehat{Var}(\hat{\mu}_n(X)) = \frac{1}{n}\frac{1}{n-1}\sum_{i=1}^{n}(g(x_i) - \hat{\mu}_n(x))^2 .$$

Simple Example

How will the uncertainty in unit price, unit sales and variable costs affect the EBITD?

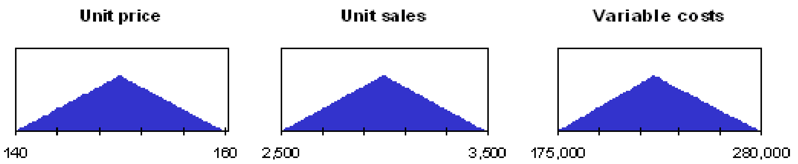
	YEAR					
				4*		
	1	2	3	Min	Most likely	Max
Unit price	160	150	150	140	150	160
Unit Sales	2500	3000	3200	2500	3000	3500
Revenues	**400000**	**450000**	**480000**	**350000**	**450000**	**560000**
Variable costs	200000	225000	240000	175000	225000	280000
Fixed costs	30000	30000	30000	30000	30000	30000
Costs	**230000**	**255000**	**270000**	**205000**	**255000**	**310000**
EBITD	**170000**	**195000**	**210000**	**145000**	**195000**	**250000**

* Forecast

First, we have to create a model:

EBITD = (Unit Price)*(Unit Sales)-(Variable Costs + Fixed Costs)

Let us explain the uncertainty in the inputs - unit price, unit sales and variable costs - using triangular distribution, specified by the respective minimum and maximum values of the inputs from the table.



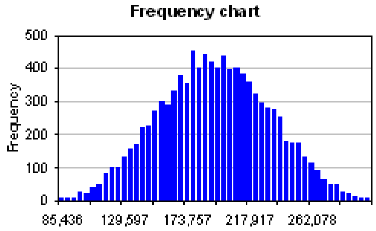
The next step is to start the Monte Carlo simulation - a value from each distribution is randomly picked and EBITD is recalculated many times, each time using a different combination of values for the unit price, unit sales and variable costs.

- 181,005 = (143)*(2 947) - (210,416 + 30,000)
- 203,296 = (150)*(3 034) - (221,804 + 30,000)
- 186,425 = (141)*(3 115) - (222,790 + 30,000)

After 10,000 trial runs, there will be 10,000 estimations of EBITD and summary statistics of the output can be derived:

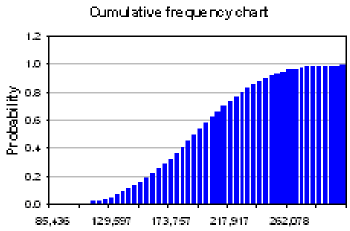
MCS Summary statistics	
Mean	193,629
Median	193,309
Standard Deviation	39,429
Variance	1,554,631,579
Mean Std. Error	394
Skewness	0
Kurtosis	3
Minimum	68,198
Maximum	323,746
Range Width	255,548

We can also add different charts in the analysis of the results for illustration. The frequency chart, for instance, shows the degree of uncertainty in the EBITD, namely the range of the obtained 10,000 values for EBITD and how often they occur.



From both the summary statistics table and the frequency chart, it is apparent that the simulated EBITD is normally distributed. According to the normal distribution, the most probable value is the mean of the distribution, $193,629 in our case, which implies that we can expect EBITD to be slightly below the forecasted most likely value of $195,000.

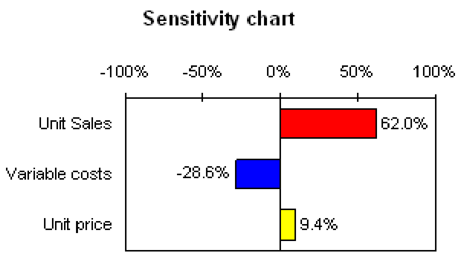
The cumulative frequency chart and the percentile table provide another way to explain the results and are often preferred. They give us the probability that a value will fall within, above or below a given range. The fiftieth percentile, for instance, is $193,309, which means that 50% of the values are $193,309 or less.



MCS Percentiles	
0%	68,198
10%	142,272
30%	172,175
40%	182,976
50%	193,309
60%	203,836
70%	214,730
80%	228,287
90%	245,795
100%	323,746

Sensitivity Chart

A sensitivity chart can be very useful when it comes to analyzing the effect of the inputs on the output. What it says is that unit sales account for 62% of the variance in the simulated EBITD, variable costs for 28.6% and unit price for 9.4%. The correlation between unit sales and EBITD and between unit price and EBITD is positive or an increase in unit sales or unit price will lead to an increase in EBITD. Variable costs and EBITD, on the other hand, are negatively correlated and by decreasing variable costs we will increase EBITD.



A Few Things to Keep in Mind

Beware that defining the uncertainty of an input value by a probability distribution that does not correspond to the real one and sampling from it will give incorrect results.

In addition, the assumption that the input variables are independent might not be valid. Misleading results might come from inputs that are mutually exclusive or if significant correlation is found between two or more input distributions.

Also note that the number of trials should not be too small, as it might not be sufficient to simulate the model, causing clustering of values to occur.

The Bottom Line

The MCS technique is straightforward and flexible. It cannot wipe out uncertainty and risk, but it can make them easier to understand by ascribing probabilistic characteristics to the inputs and outputs of a model. It can be very useful for determining different risks and factors that affect forecasted variables and, therefore, it can lead to more accurate predictions.

by Tzveta Iordanova

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