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Interactive fuzzy multiobjective reliability optimization using NSGA-II

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Abstract

In many practical situations where reliability enhancement is involved, the decision making is complicated because of the presence of several mutually conflicting objectives. Presence of multiple objectives in a problem, in principle, gives rise to a set of optimal solutions (largely known as Pareto-optimal solutions), instead of a single optimal solution. This type of problem is known as multiobjective optimization problem (MOOP). In general, a MOOP can be solved using weighted sums or decision-making schemes. An alternative way is to look for the Pareto-optimal front. Many evolutionary algorithms (EAs) like genetic algorithm (GA) have been suggested to solve MOOP, hence termed as multiobjective evolutionary algorithms (MOEAs). Nondominated sorting genetic algorithm (NSGA-II) is one such MOEA which demonstrates the ability to identify a Pareto-optimal front efficiently. Thus, it provides the decision maker (DM) a complete picture of the optimal solution space. This paper presents the reliability optimization of a life-support system in a space capsule where reliability of the system is maximized while minimizing the cost. An interactive fuzzy satisficing method for deriving a Pareto-optimal solution preferred by the DM is presented here. Prior preference of the DM has been taken into account here. Using the concept of fuzzy sets and convex fuzzy decision making a multiobjective fuzzy optimization problem is formulated from the original crisp optimization problem. Different nonlinear membership functions based on the DM's preference have been employed for the fuzzification. Then, NSGA-II is applied to solve the resulting fuzzified MOOP. Resulting Pareto-optimal solution gives the DM variety of alternatives to seek an appropriate solution by modifying parameters interactively according to his/her preference again. Various Pareto-optimal fronts under different preferences of DM have been reported.

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Keywords

Fuzzy optimization, Genetic algorithms, Multiobjective optimization, Pareto-optimal solution, Reliability optimization

1. Introduction

In the broadest sense, *reliability* can be defined as *measure of performance* of systems. As systems have grown more complex, the consequences of their unreliable behavior have become severe in terms of cost, effort, lives etc. and the interest in assessing system reliability along with its improvement has become very important. Unlike conventional optimization methods where it is assumed that all design data are precisely known and objectives are well defined and easy to formulate, many practical optimization problems e.g., reliability optimization, there are incompleteness and unreliability of input information. The reason for unreliability can be many such as; uncertainty in judgments, lack of evidence, etc. Further more, a DM often has vague desires such as, "this objective function should be less than or greater than or equal to this certain value". Fuzzy set theory [1] is effective handling these cases. In reliability optimization problems, it is often required to minimize or maximize several objectives subject to several constraints. Such problem is formulated as multiobjective optimization problem (MOOP). A MOOP can be solved in two ways; first one is to solve it by transforming the MOOP into single objective problem using positive weights (for objectives) and penalties (for constraints), and the other one which is also better is to obtain a Pareto-optimal solution which gives a DM suitable range of choice to adjust trade off between different objectives. Sakawa [2] used the surrogate worth trade off method to a multiobjective formulation of a reliability allocation problem to maximize the system reliability while minimizing the system cost. Huang [3] tackled fuzzy multiobjective optimization decision-making problem on the series reliability system with two objectives. Ravi et al. [4] and [5] implemented simulated annealing (SA) algorithm for several reliability optimization problem. Mahapatra et al. [6] proposed a new fuzzy multiobjective optimization method to solve reliability optimization problem having several conflicting objectives. Genetic algorithms (GAs) are well-known stochastic methods of global optimization based on the evolution theory of Darwin and have successfully been applied in different real-world applications including reliability optimization. Since GAs work with a population of points, a number of Pareto-optimal solutions may be captured using GAs, making it a very powerful tool also for MOOPs. The non-dominated sorting genetic algorithm (NSGA-II) [7] is a well known and extensively used algorithm based on its predecessor NSGA [8]. It is a fast and very efficient Multiobjective evolutionary algorithm (MOEA), which incorporates the features an elitist archive and a rule for adaptation assignment that takes into account both the rank and the distance of each solution regarding others. Daniel et al. [9] has applied and compared the efficiency of NSGA-II with existing methods for different reliability optimization problems without including fuzzy environment. Kishor et al. ([10]- [12]) have applied NSGA-II to solve different reliability optimization problems and compared the results with Huang [3] and Ravi et al. [4, 5].

In this paper an interactive fuzzy decision making approach has been used for solving the multiobjective reliability optimization of a life-support system in a space capsule, where system reliability is maximized while minimizing the cost. In this approach a prior interaction with DM has guided the analyst to obtain the Pareto front of DM's preference. Earlier Ravi et al. [4] and [5] solved the same problem using linear membership functions for both objectives (i.e. system reliability and system cost) This paper incorporates the use of different nonlinear membership functions for the fuzzification of the original crisp optimization problem to obtain the preferred Pareto-optimal front. Resulting fuzzified MOOP has been effectively solved by NSGA-II.

1.1 Mathematical model of problem

Let R_j and C_j be the reliability and cost of j^{th} component of the system and R_s and C_s denote the total reliability and cost of the system. It is often required to consider, in addition to maximization of system reliability, the minimization of the cost. Mathematically, this problem can be expressed as

$$Maximize\;\; R_S(R_1, R_2, R_3, ..., R_n) = \begin{cases} \prod_{j=1}^{n} R_j \; for\ series\ system \\ 1 - \prod_{j=1}^{n} (1 - R_j) \; for\ parallel\ system \\ or \\ combination\ of\ series\ and\ parallel\ system \end{cases}$$

$$Minimize\;\; C_S(R_1, R_2, R_3, ..., R_n) = \sum_{j=1}^{n} C_S(R_j)$$

subject to: $R_{j,min} \le R_j \le 1, \quad \mathrm{R}_{\mathrm{s,\,min}} \le \mathrm{R}_{\mathrm{s}} \le 1$ for j=1,2,…,n

where n represent total number of components in the system while $R_{j,min}$

and $R_{s,min}$ are minimum values for the j^{th} component and system respectively. Here, we consider a complex system which represents a block

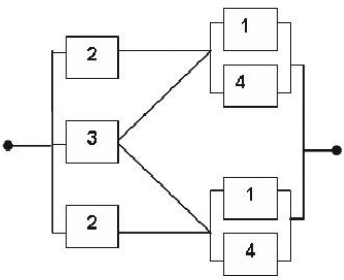


Fig. 1. Block diagram of life-support system in a space capsule

diagram of reliability of a life-support system in a space capsule. Fig. 1 shows the system.

The mathematical model for the life support system in a space capsule can be formulated using the block diagram (see Fig. 1) as follows:

$$\textit{Maximize}\ R_s(R) = 1 - R_3((1-R_1)(1-R_4))^2 - (1-R_3)[1-R_2(1-(1-R_1)(1-R_4))]^2 \qquad (1)$$

$$\textit{Minimize}\ C_s(R) = 2.\sum_{j=1}^{4} K_j R_j^{\acute{a}_j} \qquad (2)$$

$$\textit{subject to}: 0.5 \le R_{j,min} \le R_j \le 1 \quad 0.5 \le R_{s,min} \le R_j \le 1 \quad \textit{for}\ j = 1,2,3,4.$$

where different parameters values are K_j 's as 100, 100, 200, 150 respectively and all $\acute{a}_j$'s equal to 0.6.

1.2 Genetic algorithm

Genetic algorithms have been successfully applied as an optimization technique. GA introduced by Holland [13] and further described by Goldberg [14] and Deb [15], mimics natural selection or *Darwinian Theory* of 'survival of the fittest'. The basic GA *methodology* can be presented in the following form

1. Set population size, tournament size, crossover rate, mutation rate, mutation exponent and elitism size. Set the parameters of the stopping criterion.

2. Initialize the population with random numbers.

3. Compute the fitness function values. Perform selection, crossover, mutation and elitism in order to create a new population.

4. If the stopping criterion is not satisfied, return to Step 3. Otherwise, choose the best individual found as the final solution.

1.3 Nondominated sorting genetic algorithm

There were some major drawbacks in NSGA such as

- High computational complexity of nondominated sorting.
- Lack of elitism.
- Lack of specification of sharing parameter.

Deb et al. [7] proposed an improved version of NSGA [8], called NSGA-II which dealt all the drawbacks of original NSGA. NSGA-II incorporates an archive and a rule for adaptation assignment that takes into account both the rank and the distance of each solution.

Let P^t represents the current population during any generation t, and P_A^t the population which consist of non-dominated solutions archive. The pseudo code for NSGA-II can be stated as follows

Input:

N (Population size)
M (Archive size)
t_{max} (maximum number of generation)

Begin:

- Randomly initialize P_A^t, set $P^0 = \emptyset$, t=0
- while $t < t_{max}$
- $P^t = P^t + P_A^t$
- Assign adaptation to P^t
- P_A^{t+1}={ M best individuals from P^t }
- Mating Pool={ N individuals randomly selected from P_A^{t+1} using a binary tournament}
- P^{t+1}= { N new individuals generated by applying recombination (crossover and mutation) on mating pool
- t= t +1

Output:

Non-dominated solution from P_A^t

2. Methodology

Present methodology to solve a multiobjective reliability optimization problem consists of following steps

Step 1) Interaction:
DM some times wants biasness/preference in the Pareto-optimal front. For example, DM may need a Pareto-optimal front more clustered towards maximum system reliability or minimum cost. Similarly various cases may arise depending upon DM's choice. The analyst observes the choice/choices made by DM and decides accordingly further.

Step 2) Fuzzification:
Let $\tilde{f}_1$ and $\tilde{f}_2$ be the fuzzy region of satisfaction of system reliability (R_s) and system cost (C_s) and $\mu_{\tilde{f}_1}(R_s)$ and $\mu_{\tilde{f}_2}(C_s)$ be their corresponding membership functions respectively, to be decided upon the basis of interaction with DM. R_s and C_s are defined in (1) and (2). The membership functions and can be linear/nonlinear depending upon the preference of DM., this paper discusses cases involving nonlinear membership functions as linear ones has already been taken into account in the literature [4], [5], and [12] for this problem. This paper demonstrates the effectiveness of nonlinear membership functions on obtaining the biased Pareto-optimal front.

Step 3) Problem reformulation:
Conversion of multiobjective crisp reliability optimization problem into fuzzified MOOP of membership functions. Each of the membership function needs to be maximized. The reformulated problem can be mathematically expressed as

$$\text{Maximize } ((\mu_{\tilde{f}_1}(R_s), \mu_{\tilde{f}_2}(C_s)) \tag{3}$$

Subject to: $0.5 \leq Rj \leq$ $j = 1,2,3,4$

Step 4) Solution:
Finding the solution of the resulting MOOP using NSGA-II described in section 1.3. An NSGA-II is then employed to solve the resulting MOOP maximization problem (6) of membership functions. Varying the different parameters (crossover probability (p_c), mutation probability (p_m), population size (N), maximum number of generation (t_{max}), etc.) of NSGA-II different Pareto-optimal fronts can be obtained. Based on rigorous experimentation and tuning of the parameters best optimal front (for each Case) obtained have been reported here.

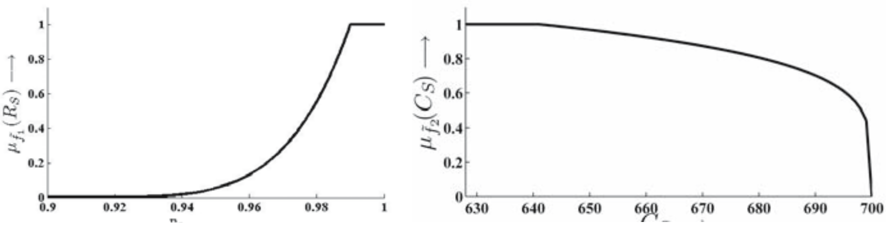
3. Results and discussion

Based on the present methodology (see Section 2) different cases (according to the DM's choice/choices) have been considered here for the reliability optimization of life-support system in a space capsule. For all the cases fuzzy version of the problem formulation is based on the fuzzy goals preferred by the DM.

Case-I: If during interaction phase (i.e. Step 1) DM wants the biasness towards maximizing reliability where reliability can be as close as possible to 1 (unit). Also DM doesn't care about the other objective (i.e. minimizing the cost). Then the membership functions shall be taken as follows:

$$\mu_{\tilde{f}_1}(R_s) = \begin{cases} 0 & if\ R_S \le 0.9 \\ \left((R_S - 0.9)/0.99 - 0.9\right)^5 & if\ 0.9 \le R_S \le 0.99 \\ 1 & if\ R_S \ge 0.99 \end{cases} \tag{4}$$

$$\mu_{\tilde{f}_2}(C_s) = \begin{cases} 1 & if\ C_S \le 641 \\ \left((700 - C_S)/700 - 641\right)^{1/5} & if\ 641 \le C_S \le 700 \\ 0 & if\ C_S \ge 700 \end{cases} \tag{5}$$



Figs. 2(a) and 2(b). Membership functions for R s and C s respectively in Case I

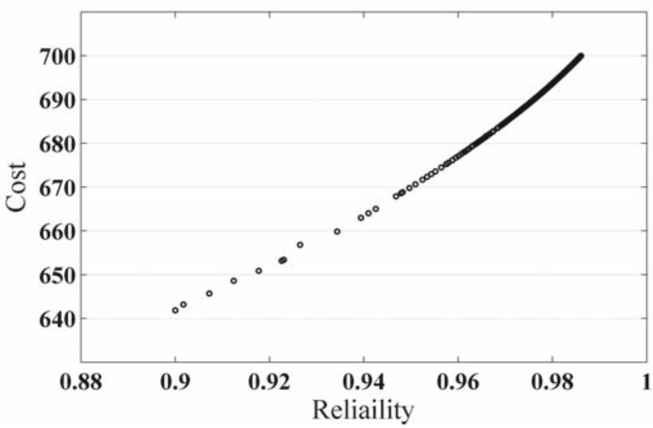


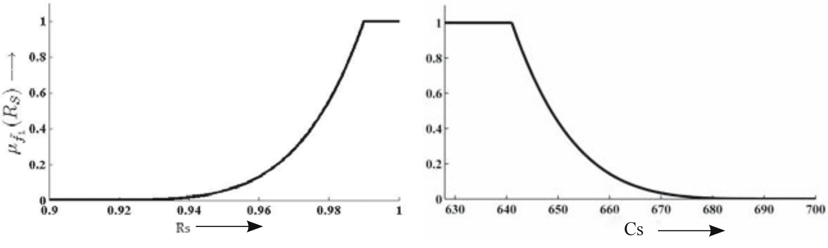
Fig. 3. Pareto-optimal curve obtained for Case I

Case-II: If the DM wishes to be biased towards minimizing the cost where cost can be as close as possible to 641 (unit). DM doesn't care about maximizing the reliability this time. Then the membership functions shall be:

$$\mu_{\tilde{f}_1}(R_s) = \begin{cases} 0 & \text{if } R_S \le 0.9 \\ \left((R_S - 0.9)/0.99 - 0.9\right)^{1/5} & \text{if } 0.9 \le R_S \le 0.99 \\ 1 & \text{if } R_S \ge 0.99 \end{cases} \tag{6}$$

$$\mu_{\tilde{f}_2}(C_s) = \begin{cases} 1 & \text{if } C_S \le 641 \\ \left((700 - C_S)/700 - 641\right)^{5} & \text{if } 641 \le C_S \le 700 \\ 0 & \text{if } C_S \ge 700 \end{cases} \tag{7}$$

i.e. membership function is concave (6) for system reliability and convex (7) for cost as shown in Figs. 4(a) and 4(b) respectively. So on solving the Pareto-optimal front will shift towards the convex part again i.e. towards the minimum cost which was the preference of the DM in this case (see Fig. 5).



Figs. 4(a) and 4(b). Membership functions for Rs and Cs respectively in Case II

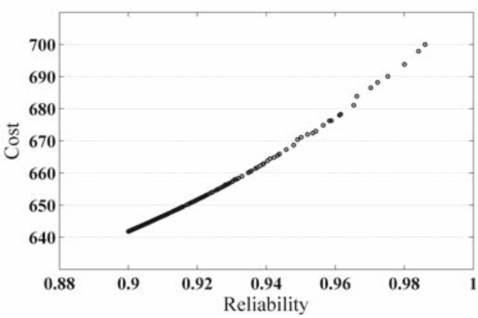
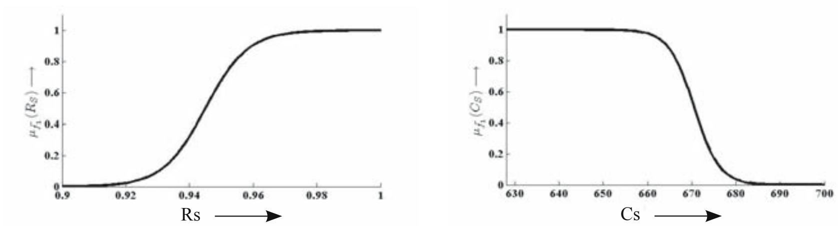


Fig. 5. Pareto-optimal curve obtained for Case II

Case-III: If the DM insists to have no biasness towards any of the objectives (i.e. not to attain maximum reliability or minimum cost but biasness towards the midway between both objectives). Then the both membership functions can be taken as sigmoidal shape (see Figs. 6 (a) and 6(b)) and formulation as follows:

$$\mu_{\tilde{f}_1}(R_s) = \begin{cases} 0 & if \;\; R_S \le 0.9 \\ 1 / \left(1+e^{-150*(R_S-0.945)}\right) & if \;\; 0.9 \le R_S \le 0.99 \\ 1 & if \;\; R_S \ge 0.99 \end{cases} \tag{8}$$

$$\mu_{\tilde{f}_2}(C_s) = \begin{cases} 1 & if \;\; C_S \le 641 \\ 1 / \left(1+e^{0.35*(C_S-670.5)}\right) & if \;\; 641 \le C_S \le 700 \\ 0 & if \;\; C_S \ge 700 \end{cases} \tag{9}$$



Figs. 6(a) and 6(b). Membership functions for Rs and Cs respectively in Case III

Pareto-optimal front in this case is shown in Fig. 7.

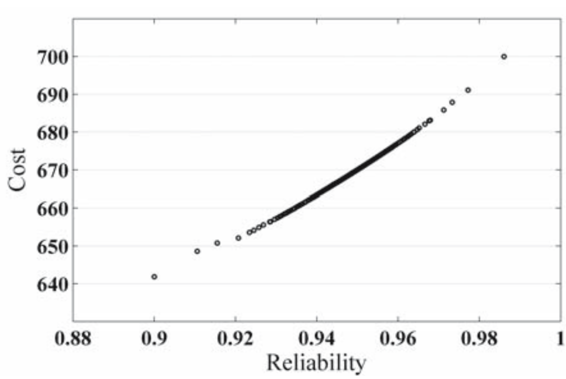


Fig. 7. Pareto-optimal curve obtained for Case III.

Similarly other cases according to the wish of DM can be modeled and preferred Pareto-optimal front can be obtained. Here, to solve all the MOOPs involved the NSGA-II parameters have been taken to be the same to maintain the homogeneity. The population size N=200 and maximum number of generations t_{max} =300 have been taken through various round of experiments. Crossover probability pc is 0.9 and mutation probability pm 0.02 for all the cases discussed in this paper.

4. Conclusion

A real life reliability optimization problem of a life-support system in a space capsule has been discussed here. A very interactive fuzzy satisficing method for deriving a biased Pareto-optimal solution preferred by the DM is presented in this paper. Different nonlinear membership functions have been introduced for fuzzification. Resulting fuzzy MOOP has been solved using NSGA-II.

Earlier approaches such as [4], [5] and [9] have either solved the same optimization problem as crisp formulation [9] or using linear membership functions [4] and [5] for fuzzification not considering the preference of the DM to the extent this paper considers.

Earlier preference/preferences of DM have largely been taken into account using weighted sum method to get the biased Pareto-optimal front. The advantage of the present methodology is that it takes into account different nonlinear membership functions for fuzzification. Thus, it can present some interesting Pareto-optimal front for DM. Further it incorporates the prior preferences of the DM towards the decision making termed as fuzzy goals. In posteriori DM can choose or alter the preference or biasness according to the obtained Pareto-optimal front. This approach is very flexible and interactive as well as very easy to implement. Thus it can be used for any MOOP if the fuzzy goals from the DM are prescribed.

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