Improving Portfolio Risk Profile with Threshold Accepting

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***Abstract*—The application of the Threshold Accepting (TA) algorithm in portfolio optimisation can reduce portfolio risk compared with a Trust-Region local search algorithm. In a benchmark comparison of several different objective functions combined with different optimisation routines, we show that the TA search algorithm applied to a Conditional Value at Risk (CVaR) objective function yields the lowest Basel III market risk capital requirements. Not only does the TA algorithm outmatch the Trust-Region algorithm in all risk and performance measures, but when combined with a CVaR or 1% VaR objective function, it also achieves the best portfolio risk profile.**

I. Introduction

Recent financial crises have highlighted several weaknesses in the risk management practices of financial institutions. To prevent future negative impacts on the financial market and the economy, financial regulators have enhanced the regulatory framework with major focus on the capital and liquidity standards. In this paper, we apply the heuristic Threshold Accepting (TA) algorithm to a portfolio allocation problem and compare it against the local Trust-Region search algorithm. In contrast to the existing literature, however, we analyse the Basel III market risk capital requirements of the optimised portfolios. We aim to shed more light on the impact of the optimisation approach on the VaR backtesting described by the Basel Comittee. In particular, we examine how a combination of TA optimisation algorithm and VaR and CVaR objective functions can reduce the market risk capital requirements.

In accordance with the market risk framework established in 1995 and revised in 2009, banks can use a standardised or an internal approach to calculate their individual minimum market risk capital requirements [1]. Banks, which want to use their internal Value-at-Risk (VaR) models for calculating their capital requirements, are required to meet a series of quantitative and qualitative standards. Among other criteria, banks are expected to calculate a one-day 99% VaR model based on at least 250 days of historical data [2]. The Basel Committee discusses the possibility of replacing the VaR due to its weaknesses [3] by another risk measure that also captures the tail risk of the distribution, i.e. the Condition-Value-at-Risk (CVaR).

The new regulatory framework increases the banks market risk capital requirements. Hence, it is of increasing importance for financial institutions to find ways to decrease their regulatory capital requirements. One way of doing this is by optimising the market portfolio using downside risk measures, i.e. VaR and CVaR. The use of VaR and CVaR as risk measures in asset allocation often results in more complex, non-linear optimisation problems with multiple local extremes, see i.e. [4].

Heuristic methods can be used to solve such non-linear optimisation problems. [5] with their Threshold Accepting algorithm were among the first to use heuristics in portfolio optimisation. Other heuristic optimisation methods that were also suggested were Particle Swarm Optimisation [6], or Ant Colony Optimisation [7]. A good overview of common heuristic optimisation methods is given by [8] and [9].

When applying an optimisation method to a portfolio allocation problem, the literature usually focuses on the standard portfolio performance measures only, the most prominent one being the Sharpe ratio. However, little is known as to how well different optimised portfolios comply with the Basel III regulations. Given the recent turbulent financial climate, financial institutions are increasingly under pressure to have a sound risk assessment framework.

The remainder of the paper is organised as follows. Section II first introduces the methodology, describing the objective functions and optimisation algorithms used in this paper as well as discussing model evaluation via backtesting. Section III presents the results of the application on empirical data, comparing various optimisation outcomes with respect to their compliance with the Basel III market risk capital requirements. Section IV concludes.

II. Methodology

In the following, we first present the objective functions used for the portfolio optimisation (Section II-A) and then the optimisation algorithms (Section II-B). A description of the backtesting procedure used for model evaluation (Section II-C) concludes this section.

A. Objective Functions

1) Mean-Variance Optimisation: One of the most popular portfolio optimisation methods is the mean-variance (MV) approach by [10]. The portfolio risk of a standard Markowitz

MV portfolio optimisation is determined by minimising the volatility

$$\sigma_p = \sqrt{\mathbf{w}'\mathbf{\Omega}\mathbf{w}} \tag{1}$$

via quadratic programming, where $\mathbf{\Omega}$ is an $M \times M$ covariance matrix of the assets and $\mathbf{w}$ is an $M \times 1$ vector of weights. The objective function is to minimise σ_p with respect to $\mathbf{w}$, subject to $w'\mathbf{1} = 1$ and $0 \leq w_i \leq 1, \forall i$. The portfolio return is given by $\mu_p = \mathbf{w}'\mathbf{r}$ where $\mathbf{r}$ is an $M \times 1$ vector of expected asset returns.

The MV optimisation approach is based on the assumption that investors have a quadratic utility function and that the returns are normally distributed. However, this assumption is questionable (see [11], [12]).

2) Value at Risk and Conditional Value at Risk Optimisation: In our paper, we consider two alternative objective functions, with the primary task to achieve a portfolio performance that is Basel conform with respect to the market risk capital requirements. In the mid-90s, VaR became a very popular risk measure. VaR is defined as the absolute loss in a risk position that is not exceeded with a certain probability in a given time horizon. The VaR objective function with an underlying historical distribution is described by [13] as

$$VaR_{H\alpha} = Q_H(\alpha) \tag{2}$$

where $Q_H(\alpha)$ is the α quantile of the historically distributed returns. We chose the histrorical distribution as empirical asset returns are not well described by a normal distribution [12], e.g. due to their fat tails and an excess peakedness. To account for asymmetric distributions several papers proposed different risk measures, see i.e. [14]–[16]. However, the quantitative risk management literature often suggests the use of empirical distributions [17]–[19]. These have the advantage that they are easy to implement and avoid the estimation of a pre-specified distribution, and leptokurtosis and dependence across assets are already accounted for.

Furthermore, we also consider a CVaR objective function to capture the tail risk of the return distribution. The CVaR objective function with historical distribution is the expected shortfall for the losses exceeding VaR [13]

$$CVaR_{H\alpha} = E[r_P | r_P \leq VaR_{H\alpha}]. \tag{3}$$

where r_P are the portfolio returns. The Conditional-Value-at-Risk (CVaR) is the most common expected tail risk measure, e.g. see overview by [20] or other measures proposed by [14]–[16].

B. Search Algorithm

The portfolio optimisation is performed with the heuristic TA search algorithm and the local Trust-Region search algorithm. The objective function f to be optimised is either the standard MV objective function (Equation (1)), the VaR model (Equation (2)) or the CVaR (Equation (3)) model, with underlying historical distribution at a 1% or 5% significance level.

1) The Local Search Algorithm: The Trust-Region algorithm is a standard local search optimisation process. It is a constraint minimisation method that aims to find the local minimum to a scalar function f subject to constraints. In order to minimise f an approximation function q is used. Function q is a much simpler function of f and is to approximate the behaviour of f in a trusted region or neighbourhood around a current search point x (in our application q is the second order Taylor approximation to f). The neighbourhood is typically a spherical or ellipse area around x [21].

In the search process, trial steps s are drawn from this trust-region. If $f(x+s) < f(x)$ then the current point x is updated to $x + s$; otherwise, x is not updated and the trust-region is shrunk. This trust-region method can fail at saddle points and cause $f(x)$ to stuck at a local optimum [22].

2) The TA Algorithm: [9] generally categorise heuristic methods into three categories: trajectory methods, population-based methods and hybrids. In asset allocation, we face a combinatorial problem. Hence, a logical approach to use for the optimisation process is a trajectory model. Trajectory methods approach new solutions by gradually changing the current solution. The most common trajectory method is TA. It is similar to the local search algorithm which makes the TA a simple, yet powerful optimisation algorithm to use in portfolio optimisation. In contrast to the local search algorithm the TA also accepts solutions that are worse than the current solution. A threshold is determined that defines up to which point a solution is still accepted.

The TA algorithm was applied by [23] to find the global optima of a distribution. In contrast to other stochastic local search algorithms the TA works with different thresholds τ which serve as acceptance criterion for new solutions. The process starts with a random current solution x^c, which is a vector of $M \times 1$ asset weights. Then two elements in x^c are selected and slightly changed to generate a new neighbour solution x^n. One of the two randomly selected elements in x^c is increased by a predefined decimal number while the other element is decreased by the same decimal value. If the generated neighbour solution is above or below a defined upper or lower bound of the asset weights, respectively, the current solution x^c is taken as the new solution x^n. This process is performed in the neighbourhood function N.

The objective function f is calculated for $f(x^c)$ and $f(x^n)$. If the difference between $f(x^c)$ and $f(x^n)$ is less than τ_r, where r is the specified threshold at each round, x^n is the new x^c. Else, if $f(x^n) - f(x^c) > \tau_r$ the new neighbour solution x^n is rejected [5]. Algorithm 1 is based on the TA algorithm presented by [9]. For a detailed analysis of TA, see [24].

Our TA model is calibrated as follows: We first set the parameters for the heuristic optimisation algorithm to nine restarts, ten rounds and 8,000 steps (80,000 iterations in total per round). Then we adjust the portfolio weights by the factor 0.005 [9]. Finally, we use six percentiles that are equally distributed from 0.9 to zero to calculate thresholds as described in [25]. In our implementation, this calibration approach let to fast converging results, as Figure 1 illustrates.

Algorithm 1 Threshold Accepting

set n_{rounds} and n_{steps}
set threshold sequence τ_r
generate initial current solution x^c
for $r = 1 : n_{rounds}$ **do**
  for $r = 1 : n_{steps}$ **do**
    generate $x^n \epsilon N(x^c)$
    compute $\triangle = f(x^n) - f(x^c)$
    if $\triangle < \tau_r$ **then**
      $x^c = x^n$
    end if
  end for
end for

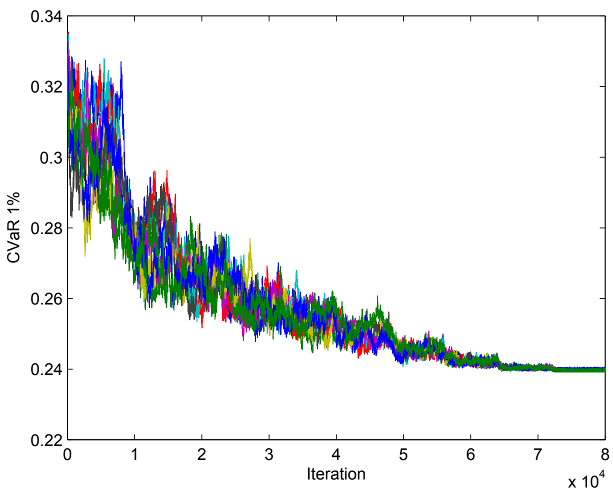


Fig. 1. TA CVaR 1% conversion in the first out-of-sample period.

C. Evaluation - VaR Backtesting

In order to evaluate the banks' market risk models' calculating the capital requirements, the risk models are backtested and accordingly classified into three zones, distinguished by color. Risk models that fall into the green zone have proven themselves to be accurate and qualitative correct. The yellow zone suggests that the model results are questionable in terms of accuracy. The red zone indicates a problem with the backtested risk model [1]. The boundaries for the three zones are deduced by calculating the binomial probabilities for a 99% coverage ratio. For a given sample size, the yellow zone begins at the point where the cumulative probability equals or exceeds 95%. The red zone begins at a cumulative probability of 99.99%. The green zone lies before the yellow zone and hence below a cumulative probability of 95% [1].

From a regulatory perspective [1], the capital requirements are calculated by multiplying a VaR number with a multiplier. The VaR value is the minimum one-day VaR value of either the last day before assessment or the last 60 days one-day VaR average. The multiplier is determined by the zones in which the risk model falls. A multiplier of three stands for the green zone and a multiplier of four for the red zone. For the yellow zone the multiplier lies between three and four depending on the number of violations. The main idea is that the multiplier should be sufficient to transform a model with e.g. 97% coverage to a model with 99% coverage. If the distribution of returns is expected to be normal the fraction of the 99th percentile to the 97th percentile is approximately 1.24, which results in a multiplier of 3.71.

In the quantitative risk management literature, the unconditional coverage test (UC) [26] is commonly applied to test how often the portfolio return exceeded the daily VaR level over time for a given risk model. Hence, the PDF for a random variable X with Bernoulli distribution is [27]

$$f(x, \theta) = \theta^x (1-\theta)^{1-x}$$

where $x \in (0, 1)$ and θ is the fraction of violations. This is, the number of violations T_1 divided by the total number of data points T. T_0 is the number of non-violations. Thus, the likelihood function for a iid Bernoulli sequence is

$$L(\theta) = \left(1 - \frac{T_1}{T}\right)^{T_0} \left(\frac{T_1}{T}\right)^{T_1} \quad .$$

For the unconditional coverage hypothesis that $\theta = \alpha$ the likelihood ratio is derived by:

$$LR_{UC} = -2 \log \left(\frac{\alpha^x (1-\alpha)^{1-x}}{\theta^x (1-\theta)^{1-x}} \right) \quad .$$

The unconditional coverage test uses a χ^2 test with one degree of freedom.

III. Empirical Analysis and Results

The empirical analysis focuses on a portfolio of the constituents of the Dow Jones Industrial Average index. We consider their daily closing prices sampled from 1st January 2003 to 1st January 2013. Log-returns are computed for a total of $T = 2610$ days and $M = 30$ equities. The portfolio optimisation is based on a 1250 days in-sample period. On a rolling window basis with a step size of ten days, the portfolio weights are recalculated. Furthermore, for the assessment of the portfolio outcomes we considered daily 3-month T-Bill rates from 1st January 2003 to 1st January 2013 as the risk-free interest rate. However, we did not consider a target rate in the optimisation process. All financial data are downloaded from DataStream.

We compare all portfolio optimisation results against the actual performance of the Dow Jones index and the "naive" equally weighted (EW) portfolio. The latter two are considered as benchmark models and are included to assess potential outperformance of the optimised portfolios.

In Section III-A, we first compare the local Trust-Region search algorithm with the heuristic TA search algorithm. In Section III-B, we then compare combinations of the optimisation algorithms and objective functions with respect to their impact on the portfolio performance. Finally, in Section III-C, the optimisation models are evaluated based on the Basel III market risk capital requirements.

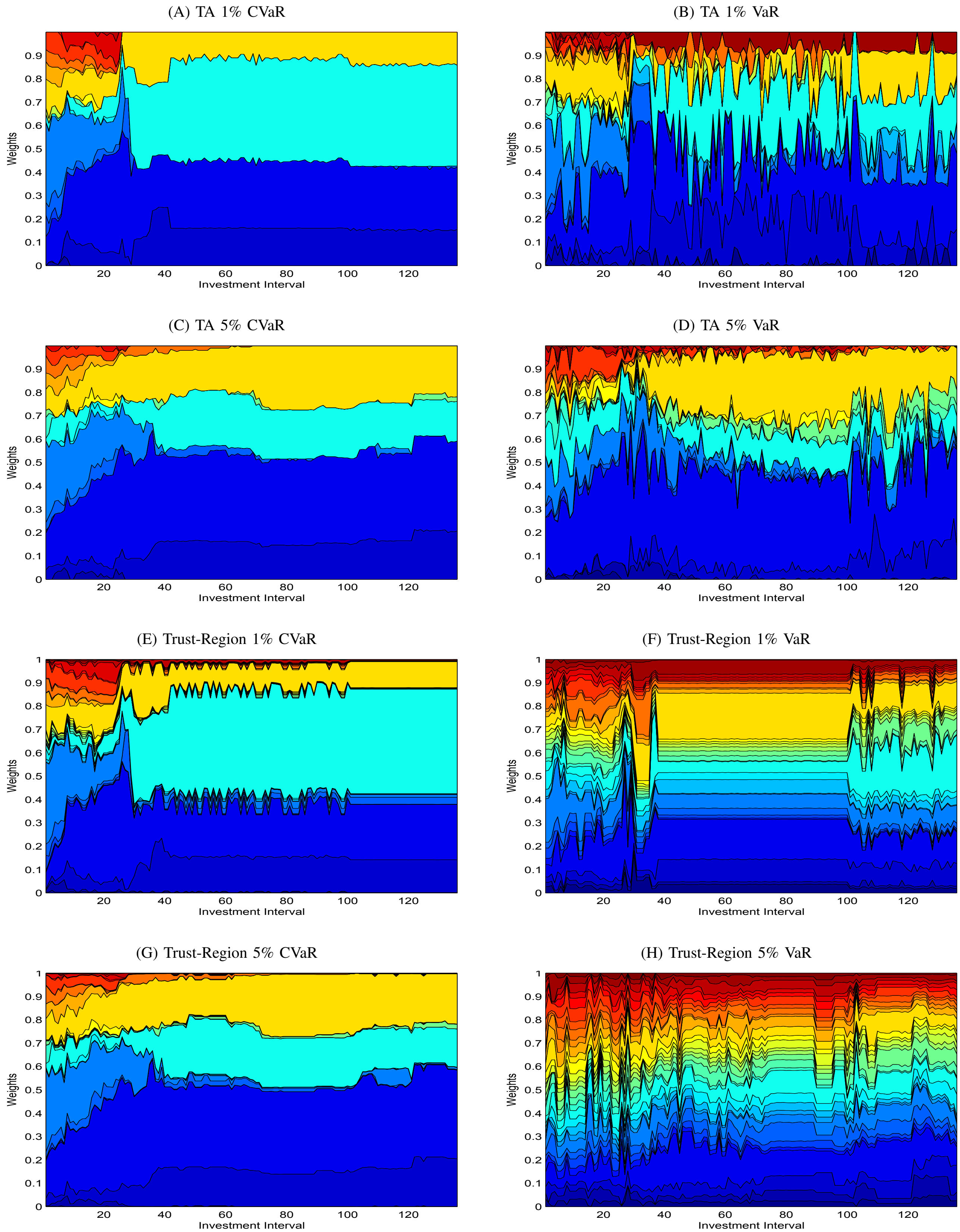


Fig. 2. Comparison of the dynamic portfolio weights for different combinations of the objective function and the optimisation approach.

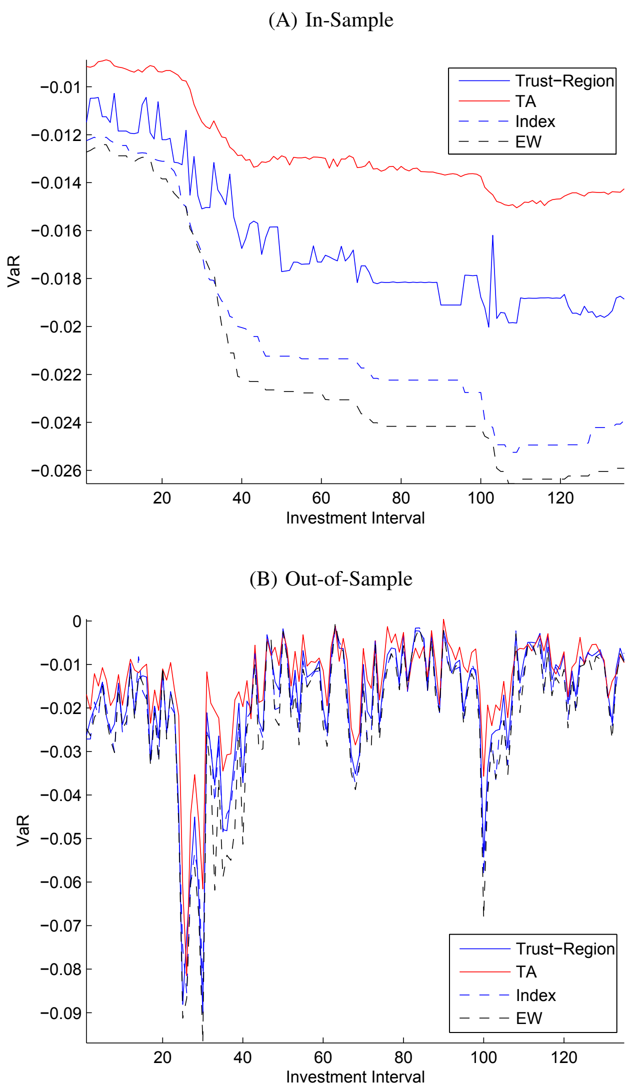


Fig. 3. Comparison between TA and local search with respect to their historical portfolio VaR at 5% significance level in the in-sample (3A) and out-of-sample period (3B).

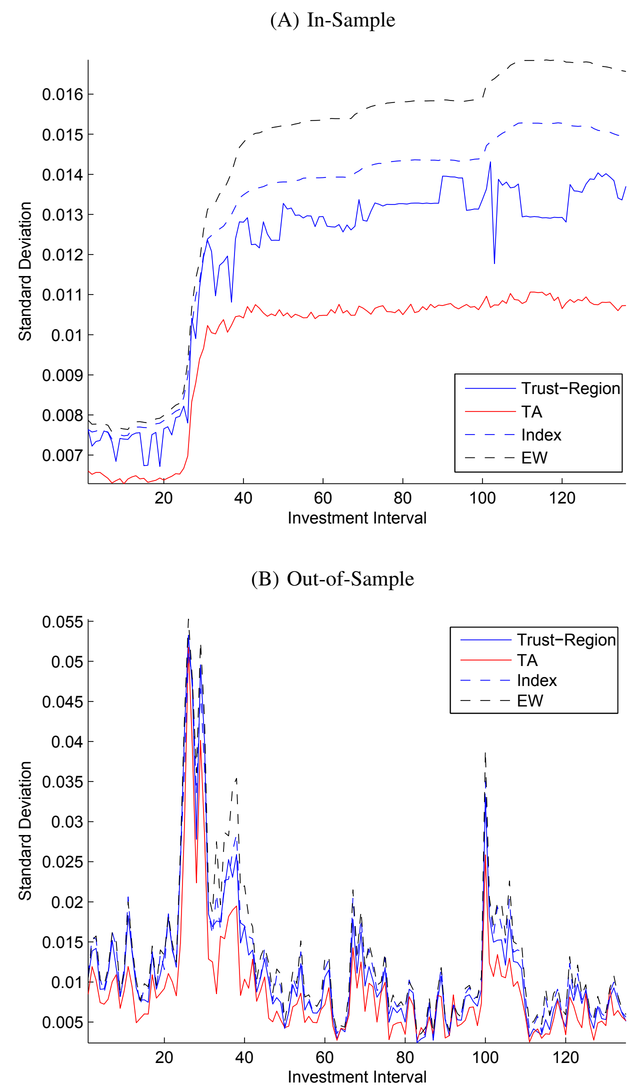


Fig. 4. Comparison between TA and local search with respect to their portfolio volatility in the in-sample (4A) and out-of-sample period (4B).

A. *Comparison of Search Algorithms*

Figure 2 first compares the dynamic portfolio allocation (weight structure) resulting from the $2^3 = 8$ combinations of optimisation algorithms (TA - upper panels, Trust Region - lower panels), objective functions (CVaR - left panels, VaR - right panels) and confidence level (1% or 5%). As discernible, the portfolio composition for CVaR optimised portfolios exhibits far less fluctuation than their VaR analogues, regardless of the confidence level and the optimisation method used. (The standard Markowitz optimisation is included in our analysis but not plotted here as it is widely studied in the literature.)

Figure 3 compares the historical VaR at 5% significance level of the TA algorithm with the Trust-Region search algorithm. Both algorithms use the VaR objective function (Equation (2)) with a 5% significance level and historical return distribution. The optimised portfolios have a lower VaR in the in-sample period than the index and EW portfolio. Moreover, the portfolio optimised with the heuristic algorithm shows to have a lower VaR than the portfolio optimised with the local search algorithm. This finding is observed not only for the in-sample period (Figure 3A) but also for the out-of-sample period (Figure 3B).

Figure 4 compares the standard deviation of the optimisation algorithms when a VaR objective function with underlying historical distribution and 5% significance level is used. The optimised portfolios have a lower standard deviation than the index and the EW portfolio in the in-sample and the out-of-sample period. In both, the in-sample and the out-of-sample period the TA algorithm produces better statistical results than the Trust-Region algorithm (see Figure 4A and Figure 4B). In general, the TA optimised portfolios show better or at least the same results than the portfolios with local search algorithm. This is observed for CVaR and VaR objective functions with

TABLE I
OUT-OF-SAMPLE PORTFOLIO RESULTS

Model		Descriptive Statistics and Performance Measure						
Algorithm	Objective Function	Mean	Standard Deviation	Kurtosis	Skewness	Maximum Draw-Down	1% CVaR Historical	Sharpe Ratio
TA	1% CVaR	3.18E-5	0.0106	14.4803	0.1599	-0.4577	-0.0421	0.0012
	5% CVaR	2.64E-5	0.0104	15.0665	0.2486	-0.4696	-0.0421	0.0007
	MV	-1.19E-5	0.0106	16.5730	0.3189	-0.5100	-0.0431	-0.0030
	1% VaR	8.75E-5	0.0112	14.4697	0.2566	-0.4167	-0.0459	0.0061
	5% VaR	-8.36E-5	0.0111	18.8564	0.4775	-0.6210	-0.0461	-0.0092
TR	1% CVaR	5.05E-6	0.0108	14.5687	0.1968	-0.5066	-0.0437	-0.0013
	5% CVaR	7.17E-6	0.0105	15.1187	0.2367	-0.4945	-0.0426	-0.0012
	MV	-1.09E-5	0.0106	16.6065	0.3200	-0.5082	-0.0431	-0.0029
	1% VaR	-1.48E-4	0.0127	13.5472	-0.0656	-0.7388	-0.0551	-0.0132
	5% VaR	-1.59E-4	0.0141	13.1292	-0.0654	-0.7959	-0.0601	-0.0127
	Index	-1.44E-4	0.0148	10.2530	-0.2179	-0.8407	-0.0614	-0.0204
	EW	-2.09E-4	0.0163	10.3800	-0.2405	-0.9503	-0.0690	-0.0140

historical distribution at 1% and 5% significance level.

B. Comparison of Objective Functions

In this section we focus on the impact of different objective functions on the portfolio measures when a TA or Trust Region search algorithm is used for optimisation. Table I shows for the entire out-of-sample period measures of the portfolio returns and the portfolio performance resulting from the various combinations of the applied algorithm and the objective functions. For completeness, we also provide the results for the index and the EW portfolio.

As expected, the optimised portfolios have a lower standard deviation than the index or the EW portfolio. The lowest standard deviation with 0.0104 can be seen for the TA algorithm with 5% CVaR objective function. The portfolio mean for the TA optimised portfolios are all significantly higher compared with the index and the EW portfolio. The highest performance can be seen for the 1% VaR objective function followed by the 1% CVaR and the 5% CVaR optimised portfolios. Even though, we did not consider a target rate in the optimisation process, the portfolios with TA algorithm and CVaR (1% and 5%) and 1% VaR objective function have a positive

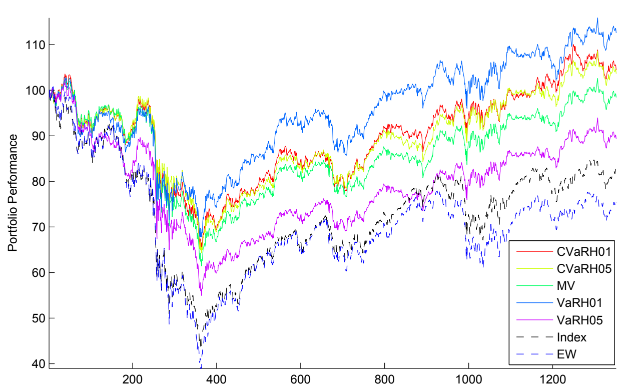


Fig. 5. TA portfolio price development (normalised) in the out-of-sample period.

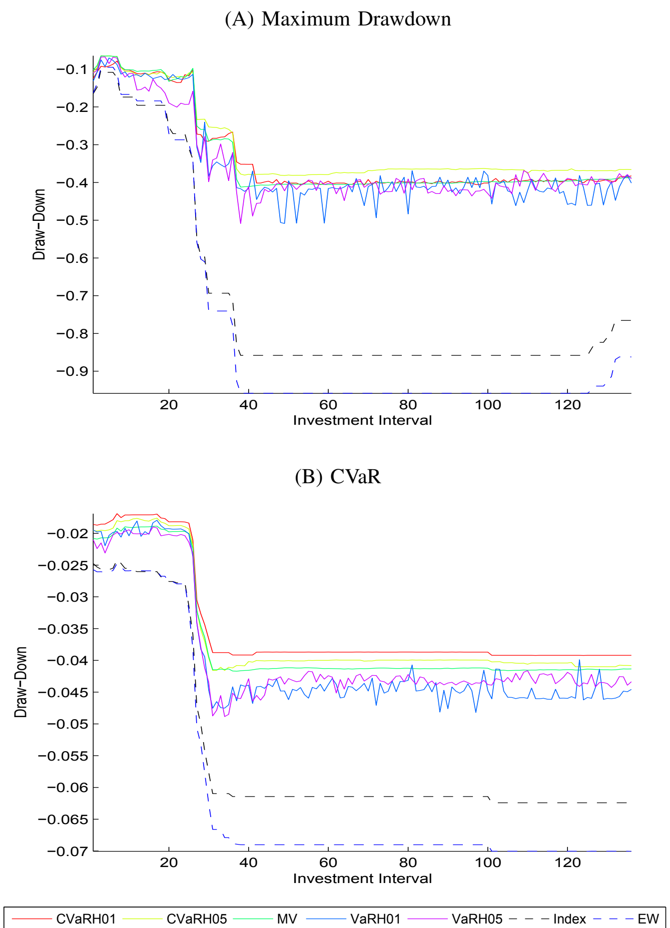


Fig. 6. Maximum draw down (6A) and 1% CVaR (6B) in the in-sample period.

Sharpe ratio, while the other portfolios have a negative ratio. Figure 5 shows the entire out-of-sample portfolio development (normalised) when applying the TA.

Moreover, the TA optimised portfolios all have a positive

TABLE II
VaR Backtesting Results

	Trust-Region		TA	
Objective Function	LR_{UC}	CR	LR_{UC}	CR
MV	0.3471	0.0429	0.3471	0.0429
CVaR 5%	0.7608	0.0416	0.7608	0.0409
CVaR 1%	0.3471	0.0465	1.3194	0.0442
VaR 5%	0.0892	0.0546	0.7608	0.0429
VaR 1%	0.3471	0.0531	1.3194	0.0515
Index	(0.0892)	(0.0630)	(0.0892)	(0.0630)
EW	(0.0892)	(0.0688)	(0.0892)	(0.0688)

skewness. A very high skewness can be seen for the 5% VaR objective function. The 5% VaR objective function is strongly leptokurtical, with a kurtosis of about 18. The 1% VaR objective function has with 14.4697 the lowest kurtosis of all TA optimised portfolios. The lowest overall kurtosis can be seen for the index with 10.2530.

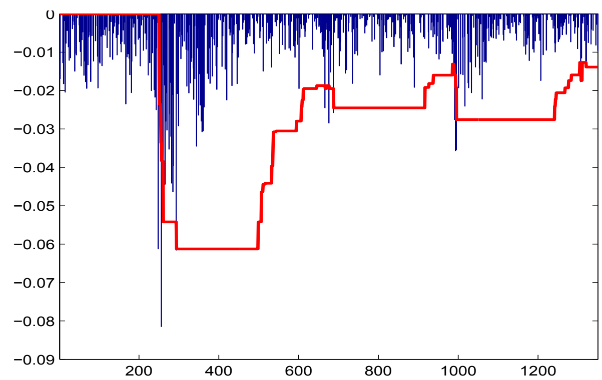
Figure 6A shows the maximum draw-down of the portfolios. All optimised portfolios have a smaller maximum draw-down in the in-sample period compared to the index and the EW portfolio which have substantially high draw-downs. For the out-of-sample period, the optimised portfolios outperform the index and the EW portfolio as Table I shows. The lowest maximum draw-down with -0.4167 is observed for the TA algorithm with a 1% VaR objective function, followed by the 1% CVaR and 5% CVaR optimised portfolio.

Figure 6B displays the 1% historical CVaR of the optimised portfolios. In the in-sample period, the 1% CVaR optimised portfolio clearly outperforms the other portfolios. In the out-of-sample period, Table I generally shows that the portfolios with the TA optimisation algorithm perform better than those with the local search algorithm. The TA also clearly outperforms the benchmark models.

C. VaR Backtesting Results

The risk profile of the portfolios is evaluated by the results computed in the VaR backtesting which calculates the daily VaR values for a 1% significance level based on rolling window with window width of 250 days. Hence, the first daily VaR value is calculated 250 days after the in-sample period. The VaR values are calculated based on the historical distribution of each portfolio. The results are evaluated using LR_{UC} with a χ^2_1 distribution and a 1% significance level. In our study, the Basel III traffic light scheme [1], which is used to calculate the Basel III capital requirements for financial institutions, serves as an assessment criterion.

Table II displays the LR_{UC} ratio and the capital requirement (CR) for a one-day 99% VaR risk model with underlying historical distribution. The CR values are calculated for a portfolio value of one monetary unit, making the results comparable and interpretable for any portfolio market value. The results differ based on the portfolio optimisation method used, i.e. the combination of objective function and algorithm. For a better comparison, we also provide the backtesting results for the benchmark models (in parenthesis, as the results are not obtained by optimisation).



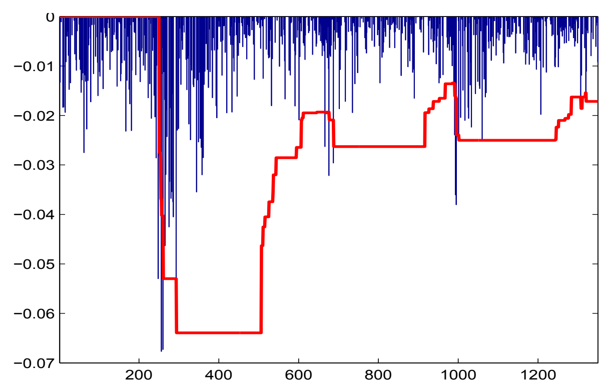


Fig. 7. Daily VaR backtesting in the out-of-sample (Note that the different return series generated by the optimisation approach have a similar btut not identical pattern).

As a χ^2 test with 1% significance level and one degree of freedom is used for the LR_{UC}, the critical value for this χ^2 test is 6.6349. If the values in the table exceed the critical value, the null hypothesis that the fraction of observed violations equals the expected significance level is rejected.

As Table II shows, all models are below the critical χ^2 value. Thus, the backtested VaR model is not rejected for any of the optimised portfolios for the investigated sample period. For all portfolios, the backtested VaR model lies in the green zone. Hence, a multiplier of three is used for the calculations of the capital requirements.

The Trust-Region algorithm yields higher capital requirements than the TA algorithm. The greatest difference with 0.0117 is seen for the historical 5% VaR objective function. For the local search algorithm the capital requirements are 0.0546, while for the TA algorithm the capital requirements are 0.0429.

Also, the 5% VaR objective function for the Trust-Region algorithm is with 0.0546 higher than the 1% VaR objective function with 0.0531 for the Trust-Region algorithm. Both models use the average 60 days VaR for the capital calculation.

However, as Figure 7 shows, the daily negative portfolio returns for the portfolio with 1% VaR objective function are less extreme than for a 5% VaR objective function. Interestingly, the EW optimised portfolio has the highest capital requirements with 0.0688, which is even worse than the index with 0.0630. The backtested VaR risk model has the lowest capital requirements with 0.0409 when a TA search algorithm and a 5% CVaR objective function is used. The second lowest capital requirements with 0.0416 can be observed for the same objective function but for the local search algorithm. With a capital requirement of 0.0429 the 5% VaR and the MV are the third best objective functions to be used for the backtested VaR model. In general, the market risk capital requirements are lowest whenever the TA algorithm is used for the portfolio optimisation process. This is true for all objective functions.

IV. Conclusion

The TA algorithm achieves better risk measures than the local Trust-Region search algorithm whenever a VaR or CVaR objective function with underlying historical distribution is used. This can be observed for 1% and 5% significance levels. Both algorithms produce the same optimisation results when the MV objective function is used. The TA and the Trust-Region algorithm generate better results than the index and the EW portfolio. This is true for the in-sample and out-of-sample period.

The portfolio risk profile can be improved when the TA search algorithm is used with a CVaR (1% or 5%) and 1% VaR objective function. For all VaR and CVaR objective functions, the TA optimised portfolios have substantially better risk measures than the Trust-Region optimised portfolios. The TA portfolios also clearly outperform the benchmarks in any risk measure. The TA portfolio with 5% CVaR objective function exhibits a lower standard deviation than any other portfolio. It also has a better standard deviation than the MV optimised portfolios. In terms of reducing the probability of significant portfolio losses, the TA optimised portfolios also have the lowest maximum draw-down. In both the in-sample and out-of-sample period the TA portfolios with CVaR (1% or 5%) and 1% VaR objective functions clearly surpass the other portfolios. Furthermore, the best portfolio performance in terms of price development in the out-of-sample period is obtained when a 1% VaR objective function or a CVaR objective function is used for the heuristic algorithm. Although no target rate is used in the optimisation process, the heuristic CVaR (1% and 5%) and the 1% VaR optimised portfolios delivered a positive Sharpe ratio in the out-of-sample period.

Regardless of the objective function the local search algorithm has higher capital requirements, while the lowest Basel III capital requirements can be achieved when a 5% CVaR objective function is applied for the portfolio TA optimisation process. In general, the backtested one-day 99% VaR model has the lowest Basel III capital requirements whenever the TA algorithm is used for the portfolio optimisation.

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