An Interactive Evolutionary Multiobjective Optimization Method Based on Progressively Approximated Value Functions

Kalyanmoy Deb, Ankur Sinha, Pekka J. Korhonen, and Jyrki Wallenius

***Abstract*—This paper suggests a preference-based methodology, which is embedded in an evolutionary multiobjective optimization algorithm to lead a decision maker (DM) to the most preferred solution of her or his choice. The progress toward the most preferred solution is made by accepting preference based information progressively from the DM after every few generations of an evolutionary multiobjective optimization algorithm. This preference information is used to model a strictly monotone value function, which is used for the subsequent iterations of the evolutionary multiobjective optimization (EMO) algorithm. In addition to the development of the value function which satisfies DM's preference information, the proposed progressively interactive EMO-approach utilizes the constructed value function in directing EMO algorithm's search to more preferred solutions. This is accomplished using a preference-based domination principle and utilizing a preference-based termination criterion. Results on two- to five-objective optimization problems using the progressively interactive NSGA-II approach show the simplicity of the proposed approach and its future promise. A parametric study involving the algorithm's parameters reveals interesting insights of parameter interactions and indicates useful parameter values. A number of extensions to this paper are also suggested.**

***Index Terms*—Evolutionary multiobjective optimization (EMO) algorithms, interactive multiobjective optimization algorithm, multiple criteria decision-making, preference-based multiobjective optimization, sequential quadratic programming (SQP).**

I. Introduction

IN EVOLUTIONARY multiobjective optimization (EMO), the target has usually been to find a set of well-converged and well-diversified Pareto-optimal solutions [1], [2]. Once an optimization run is started, usually no further information is elicited from the decision maker (DM). In an *a posteriori EMO approach*, after a set of approximate Pareto-optimal solutions has been found, preference information is elicited from a DM to choose the most preferred solution. As discussed elsewhere [3], [4], EMO procedures are not particularly suitable for handling a large number of objectives (practically, more than three). First, the usual domination principle allows a majority of the population members to become nondominated to each other, thereby not allowing much room for accommodating new solutions in a finite population. This slows down the progress of an EMO algorithm. Second, the representation of a high-dimensional Pareto-optimal front requires an exponentially large number of points, thereby requiring a large population size in running an EMO procedure. Third, the visualization of a high-dimensional front becomes a nontrivial task for decision-making purposes.

To alleviate the above problems associated with the *a posteriori* EMO approach, some EMO researchers have adopted a particular multiple criteria decision-making (MCDM) approach (*a priori approach*) and attempted to find a crowded set of Pareto-optimal points near the most preferred solution. Since the focus is now on finding a small region on the Pareto-optimal front, despite the high dimensionality of the problem, some of the difficulties mentioned above get alleviated and an EMO algorithm becomes suitable again. The cone-domination based EMO [5], biased niching based EMO [6], reference point based EMO approaches [7], [8], the reference direction based EMO [9], the light beam approach based EMO [10] are a few attempts in this direction. Also, Greenwood *et al.* [11] derived a linear value function from a given ranking of a few alternatives and then employed an EMO algorithm to find points which are preferred with respect to the constructed linear value function. In this method, the preference information is used prior to employing the EMO algorithm, thus this qualifies as another *a priori* method. For a recent survey, see [12]. These studies have clearly shown that it is difficult for an EMO algorithm alone to find a good spread of solutions in 5 or 10-objective problems. When solutions around a specific Pareto-optimal point (or around a region) are the target, MCDM-based EMO approaches suggested in these studies can find satisfactory solutions. However, in these approaches, the DM interacts only at the beginning of an EMO run. The DM provides preference information such as one or more reference point(s), one or more reference directions, one or more light beam specifics, etc. An EMO algorithm then

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targets its population to converge near the specific solutions on the Pareto-optimal front.

The above MCDM-based EMO approaches can also be used in an iterative manner with a DM, similar to the way suggested elsewhere [13], [14]. In a *semi-interactive EMO approach*, some preference information (in terms of reference points, reference directions or other means) can be obtained from the DM and an MCDM-based EMO algorithm can be employed to find a set of preferred Pareto-optimal solutions. Thereafter, a few representative preferred solutions can be shown to the DM and a second set of preference information in terms of new reference points or new reference directions can be obtained and a second MCDM-based EMO run can be made. This procedure can be continued till a satisfactory solution is found. This principle has been utilized with the reference direction [9] and light beam approaches [10] to solve some engineering design problems.

However, the integration of preference information within an EMO algorithm can be made in a more effective manner, as shown in a recent study [15]. Instead of keeping the DM waiting, to complete an EMO run (either to find a complete Pareto-optimal front in the *a posteriori* approach or to find a preferred set of Pareto-optimal solutions based on an MCDM principle in an *a priori* approach), the DM can be involved to periodically provide preference information as the EMO iterations are underway. This will be a less time-consuming and simultaneously more flexible approach than the previously suggested ones. In such a *progressively interactive EMO approach* using value functions (PI-EMO-VF), the DM is allowed to modify her/his preference structure as new solutions evolve. Since the DM gets more frequent chances to provide new information, the overall process is more DM-oriented. Moreover, the DM may feel more in-charge and more involved in the overall optimization-cum-decision-making process.

In this paper, we suggest a simplistic framework of a PI-EMO-VF approach based on a couple of earlier progressive multicriterion decision-making approaches [16], [17]. Periodically, the DM is supplied with a handful of currently nondominated points and is asked to rank the points from best to worst. From here on we refer to this instance as a "DM call." Based on this preference information, an optimization problem is formulated and solved to find a suitable value function, which optimally captures DM's preference information. From this iteration till the next DM call, the derived value function is utilized to drive the EMO algorithm in major ways: 1) in determining termination of the overall procedure, and 2) in modifying the domination principle, which directly affects EMO algorithm's convergence and diversity-preserving operators. The PI-EMO-VF concept is integrated with the well-known NSGA-II algorithm [18]. The working of the algorithm is demonstrated on four problems involving two to five objectives. A parameter sensitivity study is also performed to analyze the influence on working of the overall algorithm. Thereafter, the sensitivity of the proposed PI-NSGA-II-VF procedure on the inconsistencies in decision-maker responses is studied. Finally, a number of important and immediate future studies are listed and conclusions are drawn.

II. Past Studies on Progressively Interactive Methods

There exist a plethora of studies involving *a posteriori* and *a priori* EMO approaches. Most methodologies borrow the core decision-making idea from the MCDM literature and integrate it with an EMO algorithm. Since the focus of this paper is not to discuss *a posteriori* or the *a priori* EMO approaches, but to concentrate on procedures requiring more frequent involvements of a DM with an EMO algorithm, we do not provide a review of *a posteriori* and *a priori* approaches, except to encourage the readers to look at a recent survey [12].

Toward the methodologies involving a progressive use of preference information by involving a DM in an evolutionary multiobjective optimization framework, there are not many studies yet. Some recent studies periodically presented to the DM one or more pairs of alternative points found by an EMO algorithm and expected the DM to provide some preference information about the points. The information is then used to derive a weighted value function, which is linear. Phelps and Köksalan [19] optimized the constructed linearly weighted sum of objectives in subsequent iterations using an evolutionary algorithm. In their technique, if the actual value function is nonlinear, the method may not be able to find a linear approximation and may generate an infeasible solution. This creates a need to reformulate the optimization problem by deleting constraints one at a time. Fowler *et al.* [20] have developed an interactive EMO approach based on the idea of using convex preference cones. They use such cones to partially order the population members and further use the order as the fitness function. They have tested their algorithm on multidimensional (upto four dimensions) knapsack problems. Jaszkiewicz [21] selected a set of linear value functions (based on weighted sum of objectives) from a set of randomly created linear value functions, conforming to the preference information supplied by the DM by pairwise comparisons. EMO algorithm's search is then continued with these selective weight vectors. Although the assumption of linear value functions facilitates a quick and easy determination of the value function representing DM's preference information, linear value functions have limitations in handling nonlinear problems, particularly where the most preferred point lies on a nonconvex part of the Pareto-optimal front. Nevertheless, each interactive EMO idea suggested in the above-mentioned studies remains as the main hallmark of these studies.

Branke *et al.* [15] implemented the GRIP [22] methodology in which the DM compares pairs of alternatives and the preference information thus obtained is used to find all possible compatible additive value functions (not necessarily linear). An EMO algorithm (NSGA-II) then used a preference-based dominance relationship and a preference-based diversity preserving operator to find new solutions for the next few generations. Their procedure recommended to make a single pair of comparison after every few generations in order to develop the preference structure. Since this procedure does not generate enough preference information after every call of the DM, the EMO algorithm is likely to keep a wide variety of points from across the Pareto-optimal front in the population. The authors have demonstrated their procedure on a two-objective test problem. To obtain a narrow range of points close to the true preferred Pareto-optimal point, they had to call the DM at every generation of the EMO algorithm. It is not clear how the procedure will perform in higher objective problems, where dominance-based approaches are too slow and a reasonably high level of preference information would be needed to make a fast and focused search using an EMO algorithm. However, the use of preference information in EMO algorithm's operations remains a significant contribution of this paper.

Korhonen *et al.* [16] suggested a progressive, interactive multiobjective optimization algorithm in which the DM is presented with a set of alternatives and is asked to make a set of binary comparisons of the alternatives. From this information, a linear programming problem is solved to identify a class of value functions in which the DM's preference information falls. They considered three classes of value functions for further processing: 1) linear, 2) quasiconcave, and 3) no pre-assumed form. Based on this classification, a dominance structure is defined and either by search or from an existing sample of alternatives, the expected probabilities of finding new and better alternatives are determined. If there is a reasonable probability of finding better points, the algorithm is continued, otherwise the currently judged most preferred point is reported. An extension of this paper [17] used a sampling-based statistical procedure to compute expected probabilities of finding better solutions. It would be an interesting task to replace the sampling procedure by an EMO algorithm for creating new points. After every decision-making event, an EMO algorithm can be employed for a few generations to find a better population of points, if available. Motivated by this paper and recognizing the need for a simplistic yet efficient interactive preference-based approach involving a DM in an EMO framework, we launch this particular study.

III. Proposed Progressively Interactive EMO Using Value Functions (PI-EMO-VF)

In this section, we propose an interactive EMO algorithm, where an approximate value function is generated progressively after every few generations. Here, we study optimization problems of the following form:

$$\begin{array}{ll} \text{Maximize} & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})\}, \\ \text{subject to} & \mathbf{x} \in \mathcal{S}, \end{array} \tag{1}$$

where $\mathbf{x}$ is a solution vector, $\mathcal{S}$ denotes the feasible search space, and $f_i(\mathbf{x})$ is the ith objective function. Usually, the objective functions are in conflict with each other.

A standard EMO algorithm (such as NSGA-II [18], SPEA2 [23], and others) works with a population of points in each iteration. A sparsed set of nondominated points is preferred in a population so that the algorithm progresses toward the Pareto-optimal front and aims at finding a representative set over the entire front. However, in our proposed approach, we are interested in utilizing DM's preference information repeatedly as the algorithm progresses and in directing the search on the corresponding preferred region of the Pareto-optimal front iteratively.

For this purpose, after every τ generations of an EMO algorithm, we provide the DM with η ($\geq$2) well-sparsed nondominated solutions from the current set of nondominated points and expect the DM to provide a complete or partial preference information about superiority or indifference of one solution over the other. In an ideal situation, the DM can provide a complete ranking (from best to worst) of these solutions, but partial preference information is also allowed. In the event that the DM is not able to declare any preferences, the algorithm has the back-tracking ability in search of new and preferred solutions. With the given preference information, we then construct a strictly increasing polynomial value function. The construction procedure involves solution of a single-objective optimization problem. Until the next τ generations, we use the constructed value function to direct the search for additional such preferred solutions. A termination condition is also set up based on the expected progress, which can be made with respect to the constructed value function. In the following, we provide a step-by-step procedure of the proposed progressively interactive EMO using value function (PI-EMO-VF) methodology.

Step 1: Initialize a population Par_0 and set iteration counter $t = 0$. Set $Par_{\text{old}} = Par_0$. Domination of one solution over another is defined based on the usual definition of dominance [24] and an EMO algorithm is executed for τ iterations. The value of t is incremented by one after each iteration.

Step 2: If ($t \mod \tau = 0$), cluster the current nondominated front to choose η widely distributed points; otherwise, proceed to Step 5.

Step 3: Obtain DM's preference information on η points. If the DM is unable to declare a single preferred point in all pairwise comparisons, set $Par_t = Par_{\text{old}}$ and proceed to Step 5 with usual domination principle in EMO operators; otherwise set $Par_{\text{old}} = Par_t$ and proceed with the following operations. Construct a value function $V(\mathbf{f})$ from this information by solving an optimization problem (VFOP), described in Section III-A. If no feasible value function is found satisfying all DM's preference-information, proceed to Step 5 and use the usual domination principle in EMO operators.

Step 4: A termination check (described in Section III-B) is performed based on the expected improvement in solutions from the currently judged best solution based on the value function $V(\mathbf{f})$. If the expected improvement is not significant (with respect to a parameter d_s, defined later), the algorithm is terminated and the current best solution is chosen as the final outcome.

Step 5: The parent population Par_t is used to create a new offspring population Off_t by using a modified domination principle (discussed in Section III-C) based on the current value function $V(\mathbf{f})$ and EMO algorithm's search operators.

Step 6: Populations Par_t and Off_t are used to determine a new population Par_{t+1} using the current value function and EMO algorithm's diversity preserving operator. The iteration counter is incremented as $t \leftarrow t + 1$ and the algorithm proceeds to Step 2.

The above is a generic progressively interactive PI-EMO-VF procedure, which can be combined with any existing EMO algorithm in Step 1 and subsequently in Steps 5 and 6. The PI-EMO-VF algorithm expects the user to set a value of τ, η, and d_s.

In Step 2, points in the best nondominated front are considered and the k-mean clustering algorithm [1], [23] can be used to identify η well-diversified points in the objective space. Other multicriteria decision-making methodologies [25] of selecting points from a set of nondominated points may also be used.

We now provide the details for the specific procedures used in this paper for Steps 3–6.

A. Step 3: Decision Maker's Preference Information and Construction of a Polynomial Value Function

At an instance of a DM call, η points are presented to the DM. The DM is expected to provide some preference information. One of the usual ways of providing such information is to make pairwise comparisons of given points and suggest one of the following two scenarios: 1) a solution is more preferred over the other, or 2) both solutions are incomparable. Based on such preference statements, it is expected that for some pairs (i, j) of points, the ith point is found to be preferred over the jth point, thereby establishing $P_i \succ P_j$ and for some pairs (i, j), they are incomparable, establishing $(P_i \equiv P_j)$. If the DM is unable to provide a clear preference information $(P_i \succ P_j)$ for all pairs, it means that the current set of trade-off solutions are too diverse or the DM is not satisfied with any of these solutions. The algorithm then backtracks to the previous population for which the DM could make a decisive action and instead of using the modified domination principle, the usual domination principle is used to advance the search process. However, in general, it is expected that the DM is able to establish at least one pair satisfying $P_i \succ P_j$. Thus, at the end of DM's preference elicitation task, usually we are likely to have at least one point which lies in the best category and at least one point which lies in the second-best category. In the "complete ranking" situation, the DM may provide a complete ranking of η solutions (say, P_1 being the best, P_2 being the second-best and so on till P_η being the least preferred point).

Given such preference information, the next task is to construct a polynomial value function satisfying the given preference structure. A similar task has been performed for linear utility functions elsewhere [16], [26]. Here, we construct a simple mathematical value function to capture the given preference information of η points.

1) *Polynomial Value Function for Two Objectives:* A value function is formed based on preference information provided by the DM. We first describe the procedure for two objectives and then present the procedure for the generic case. The structure of the value function is fixed as follows:

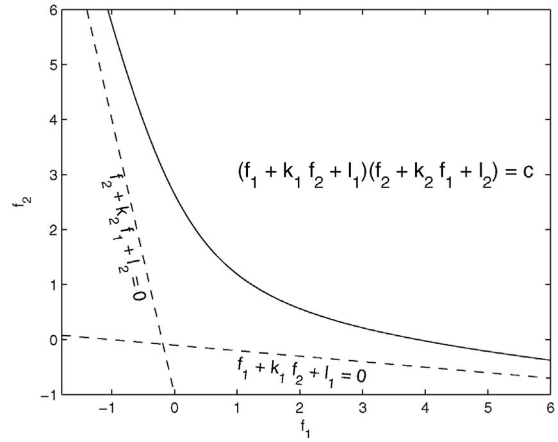

Fig. 1. Proposed value function.

$$\begin{array}{lll} & V(f_1, f_2) = (f_1 + k_1 f_2 + l_1)(f_2 + k_2 f_1 + l_2), & \\ \text{where} & f_1, f_2 & \text{are the objective functions,} \\ \text{and} & k_1, k_2, l_1, l_2 & \text{are the value function parameters.} \end{array} \tag{2}$$

The value function V, for two objectives shown above, is considered to be the product of two linear functions $S_1 : \mathbf{R}^2 \to \mathbf{R}$ and $S_2 : \mathbf{R}^2 \to \mathbf{R}$. The parameters k_1, k_2, l_1, and l_2 are unknown and must be determined from the preference information concerning η points supplied by the DM. For this purpose, we solve the following optimization problem (VFOP):

$$\begin{array}{ll} \text{Maximize} & \epsilon, \\ \text{subject to} & V \text{ is nonnegative at every point } P_i, \\ & V \text{ is strictly increasing at every point } P_i, \\ & V(P_i) - V(P_j) \geq \epsilon \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad \text{satisfying } P_i \succ P_j, \\ & \left|V(P_i) - V(P_j)\right| \leq \delta_V \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad \text{satisfying } P_i \equiv P_j. \end{array} \tag{3}$$

The first two sets of constraints ensure that the derived value function is nonnegative and strictly increasing at all η points. The value function can always be shifted by adding a constant term to the function. Without loss of generality, we construct a value function which assigns a positive value to all the data points. These conditions satisfy the quasiconcavity of the value function—a desired property suggested in the economics literature [27]. The third and fourth sets of constraints ensure that the preference order supplied by the DM is maintained for respective pairs. In order to implement the first two constraint sets, we first sketch the value function $V(f_1, f_2)$ with the desired properties of being nonnegative and strictly increasing. Fig. 1 shows a pair of straight lines represented by $V(f_1, f_2) = 0$ at which either (or both) of the two terms S_1 or S_2 is zero. However, if the chosen points P_i $(i = 1, \ldots, \eta)$ are such that both S_1 and S_2 are nonnegative at these points, the first set of constraints will be satisfied. A generic iso-value curve for which $S_m > 0$ (for $m = 1, 2$) is also depicted in the figure. Thus, the first set of constraints can be satisfied by simply considering $S_m \geq 0$ for $m = 1, 2$. To impose strictly increasing nature of the value function at the chosen points, we can use $\partial V/\partial f_i \geq 0$ for both objectives. For the two-objective case, these two conditions yield $S_2 + k_2 S_1 \geq 0$ and $k_1 S_2 + S_1 \geq 0$.

The fourth constraint set takes into account all pairs of incomparable points. For such pairs of points, we would like to restrict the absolute difference between their value function values to be within a small range (δ_V). To eliminate having another parameter, we suggest $\delta_V = 0.1\epsilon$, such that it is at most 10% of the maximum difference in value functions between $\succ$-class of points.

A little thought will reveal that the above optimization problem attempts to find a value function for which the minimum difference in the value function values between the ordered pairs of points is maximum. Considering all the expressions, we have the following optimization problem:

$$\begin{array}{ll}\text{Maximize} & \epsilon, \\ \text{subject to} & S_m(P_i) \geq 0 \quad i = 1, 2, \ldots, \eta \text{ and } m = 1, 2, \\ & S_2(P_i) + k_2 S_1(P_i) \geq 0 \quad i = 1, 2, \ldots, \eta, \\ & k_1 S_2(P_i) + S_1(P_i) \geq 0 \quad i = 1, 2, \ldots, \eta, \\ & V(P_i) - V(P_j) \geq \epsilon \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad\qquad \text{satisfying } P_i \succ P_j, \\ & \left|V(P_i) - V(P_j)\right| \leq \delta_V \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad\qquad \text{satisfying } P_i \equiv P_j. \end{array} \tag{4}$$

Fig. 2 considers five ($\eta = 5$) hypothetical points for a maximization problem ($P_1 = (3.5, 3.7)$, $P_2 = (2.6, 4.0)$, $P_3 = (5.9, 2.2)$, $P_4 = (0.0, 6.0)$, and $P_5 = (15.0, 0.5)$) and a complete ranking of the points (P_1 being best and P_5 being worst). Due to a complete ranking, we do not have the fourth constraint set. The solution to the above optimization problem results in a value function, the contours (iso-utility curves) of which are drawn in the figure. The value function obtained after the optimization is as follows:

$$V(f_1, f_2) = (f_1 + 4.3229)(f_2 + 0.9401).$$

The asymptotes of this value function are parallel to f_1 and f_2 axes. The optimized value of ϵ is 2.0991. It is interesting to note the preference order and other restrictions are maintained by the obtained value function.

Interestingly, if the DM provides a different preference information: P_1 is preferred over P_2, P_2 is preferred over P_3 and no preference exists among P_3, P_4, and P_5, a different value function will be obtained. We re-optimize the resulting problem with the above preference information on the same set of five points used in Fig. 2 and obtain the following value function:

$$V(f_1, f_2) = (f_1 + 5.9355)(f_2 + 1.6613).$$

Fig. 3 shows the corresponding value function contours. The contour makes a clear distinction between solutions in pairs P_1–P_2 and P_2–P_3 (within an optimized value ϵ), however, there is no distinction among P_3, P_4, and P_5 (with 0.1ϵ), to establish the given preference structure. Since a value function

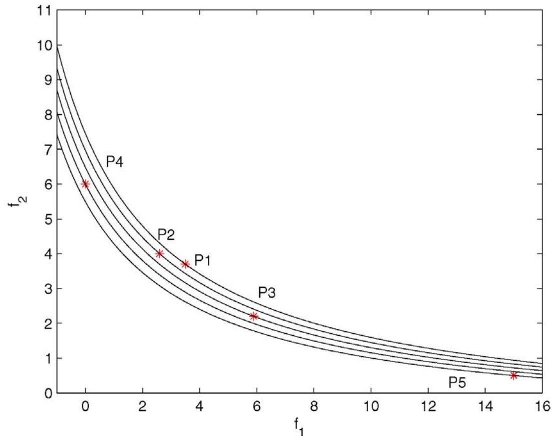

Fig. 2. Value function found by optimization.

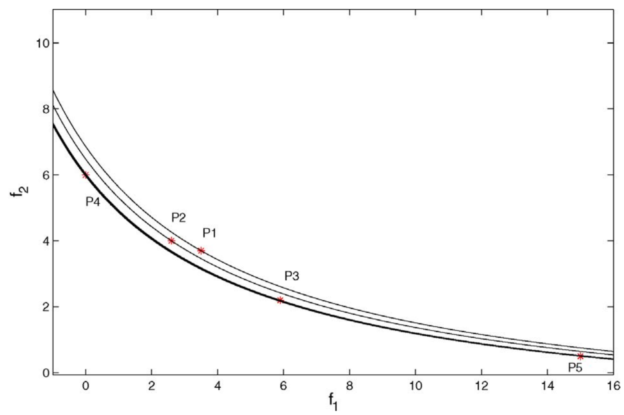

Fig. 3. Revised value function with a different preference information.

maintaining a difference (ϵ) between points in pairs P_1–P_2 and P_2–P_3 is needed and a maximum gap of 10% of ϵ is needed, a somewhat greater ϵ value to that found in the previous case is obtained here. The optimized ϵ value is found to be 2.2645 in this case.

2) *Polynomial Value Function for M Objectives:* The above suggested methodology can be applied to any number of objectives. For a general M objective problem the value function can be written as follows:

$$\begin{array}{rl} V(\mathbf{f}) = & (f_1 + k_{11} f_2 + k_{12} f_3 + \cdots + k_{1(M-1)} f_M + l_1) \times \\ & (f_2 + k_{21} f_3 + k_{22} f_4 + \cdots + k_{2(M-1)} f_1 + l_2) \times \\ & \cdots \\ & (f_M + k_{M1} f_1 + k_{M2} f_4 + \cdots + k_{M(M-1)} f_{M-1} + l_M). \end{array} \tag{5}$$

The above value function can be expressed more elegantly as follows:

$$V(\mathbf{f}) = \prod_{i=1}^{M} \left(\sum_{j=1}^{M} \left[K_{ij} f_j + K_{i(M+1)} \right] \right). \tag{6}$$

Since each term in the value function can be normalized, we can introduce an additional constraint $\sum_{j=1}^{M} K_{ij} = 1$ for each term denoted by i. As discussed below, $K_{ij} \geq 0$ for $j \leq M$ and for each i, however $K_{i(M+1)}$ can take any sign. In the remainder of the paper, we follow the value function definition given in (5).

In the formulation it should be noted that the subscripts of the objective functions change in a cyclic manner as we move from one product term to the next. The number of parameters in the value function is M^2. The optimization problem formulation for the value function suggested above contains $M^2 + 1$ variables (k_{ij} and l_i). The variable ϵ is to be maximized. The second set of constraints (strictly increasing property of V) will introduce nonlinearity. To avoid this, we simplify the above constraints by restricting the strictly increasing property of each term S_k, instead of V itself. The resulting constraints then become $k_{ij} \geq 0$ for all i and j combinations. The optimization problem (VFOP) to determine the parameters of the value function can thus be generalized as follows:

$$\begin{array}{ll}\text{Maximize} & \epsilon, \\ \text{subject to} & S_m(P_i) \geq 0 \quad i = 1, \ldots, \eta \text{ and } m = 1, \ldots, M, \\ & k_{ij} \geq 0 \quad i = 1, \ldots, M \text{ and } j = 1, \ldots, (M-1), \\ & V(P_i) - V(P_j) \geq \epsilon \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad \text{satisfying } P_i \succ P_j \\ & \qquad \text{combinations satisfying } i < j, \\ & \left|V(P_i) - V(P_j)\right| \leq \delta_V \quad \text{for all } (i, j) \text{ pairs} \\ & \qquad \text{satisfying } P_i \equiv P_j. \end{array} \tag{7}$$

In the above problem, the objective function and the first two constraint sets are linear, however the third and fourth constraint sets are polynomial in terms of the problem variables. There are a total of $M\eta + M(M-1)$ linear constraints. However, the number of polynomial constraints depends on the number of pairs for which the preference information is provided by the DM. For a 10-objective ($M = 10$) problem having $\eta = 5$ chosen points, the above problem has 101 variables, 140 linear constraints, and at most 10 ($\binom{5}{2}$) polynomial constraints. Since majority of the constraints are linear, we suggest using a sequential quadratic programming (SQP) algorithm to solve the above problem. The nondifferentiability of the fourth constraint set can be handled by converting each constraint ($|g(\mathbf{x})| \leq \delta_V$) into two constraints [$g(\mathbf{x}) \geq -\delta_V$ and $g(\mathbf{x}) \leq \delta_V$)]. In all our problems, we did not consider the cases involving the fourth constraint and leave such considerations for a later study.

B. Step 4: Termination Criterion

Once the optimal value function V is determined, the EMO algorithm is driven by it in the next τ generations. The value function V can also be used for determining whether the overall optimization procedure should be terminated or not. To implement the idea we identify the best and second-best points P_1 and P_2 from the given set of η points based on the preference information. In the event of more than one point in each of the top two categories (best and second-best classes) which can happen when the "$\equiv$"-class exists, we choose P_1 and P_2 as the points having the highest value function value in each category, respectively.

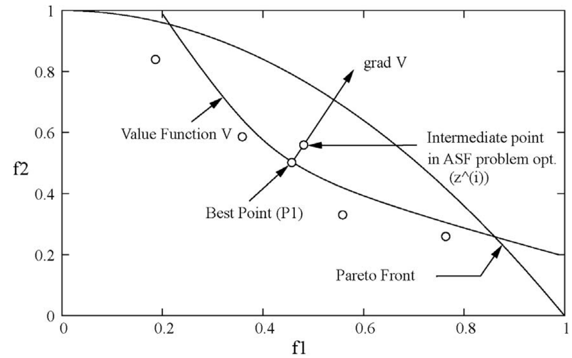

Fig. 4. Local search, when far away from the front, finds a better point more than distance d_s away from the best point. Hence, no termination of the P-EMO.

The constructed value function can provide information about whether any new point P is better than the current best solution (P_1) with respect to the value function. Thus, if we perform a single-objective search along the gradient of the value function (or ∇V) from P_1, we expect to create solutions more preferred than P_1. We can use this principle to develop a termination criterion.

We solve the following achievement scalarizing function (ASF) problem [28] for $P_1 = \mathbf{z}^b$:

$$\begin{array}{ll}\text{Maximize} & \left(\min_{i=1}^{M} \frac{f_i(\mathbf{X}) - z_i^b}{\frac{\partial V}{\partial f_i}}\right) + \rho \sum_{j=1}^{M} \frac{f_j(\mathbf{X}) - z_j^b}{\frac{\partial V}{\partial f_j}}, \\ \text{subject to} & \mathbf{x} \in \mathcal{S}. \end{array} \tag{8}$$

Here, $\mathcal{S}$ denotes the feasible decision variable space of the original problem. The second term with a small ρ (= 10^{-10} is used here) prevents the solution from converging to a weak Pareto-optimal point. Any single-objective optimization method can be used for solving the above problem and the intermediate solutions ($\mathbf{z}^{(i)}$, $i = 1, 2, \ldots$) can be recorded. If at any intermediate point, the Euclidean distance between $\mathbf{z}^{(i)}$ from P_1 is larger than a termination parameter d_s, we stop the ASF optimization task and continue with the EMO algorithm. In this case, we replace P_1 with $\mathbf{z}^{(i)}$. Fig. 4 depicts this scenario. On the other hand, if at the end of the SQP run, the final SQP solution (say, $\mathbf{z}^T$) is not greater than d_s distance away from P_1, we terminate the EMO algorithm and declare $\mathbf{z}^T$ as the final preferred solution. This situation indicates that based on the current value function, there does not exist any solution in the search space which will provide a significantly better solution than P_1. Hence, we can terminate the optimization run. Fig. 5 shows such a situation, warranting a termination of the PI-EMO-VF procedure.

C. Steps 5 and 6: Modified Domination Principle

The utility function V can also be used to modify the domination principle in order to emphasize creation of preferred solutions.

Let us assume that the value function from the most recent decision-making interaction is V. The value function value for the second-best member (P_2 defined in the previous subsection) from the set of η points given to the DM is V_2.

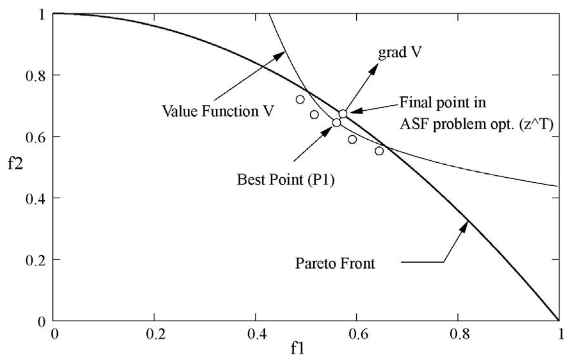

Fig. 5. Local search terminates within distance d_s from the best point. Hence, the P-EMO is terminated.

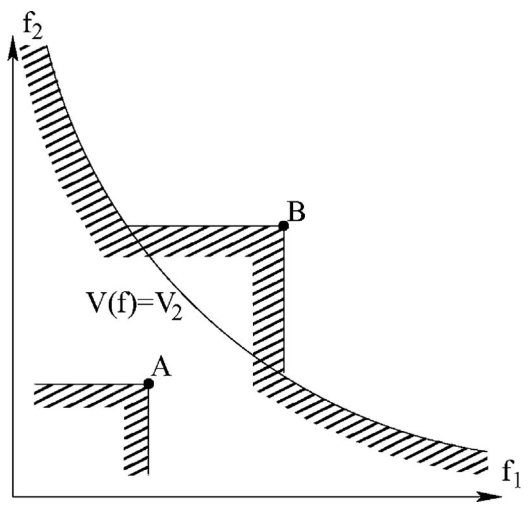

Fig. 6. Dominated regions of two points A and B using the modified definition.

Then, any two feasible solutions $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ can be compared with their objective function values by using the following modified domination criteria.

1) If both solutions have a value function value *less* than V_2, then the two points are compared based on the usual dominance principle.
2) If both solutions have a value function value *more* than V_2, then the two poinmts are compared based on the usual dominance principle.
3) If one has value function value more than V_2 and the other has value function value less than V_2, then the former dominates the latter.

Fig. 6 illustrates regions dominated by points A and B for a hypothetical maximization problem. The value function contour having a value V_2 is shown by the curved line. Point A lies in the region in which the value function is smaller than V_2. The region dominated by point A is shaded. This dominated area is identical to that obtained by the usual domination principle. However, point B lies in the region in which the value function is larger than V_2. For this point, the dominated region is different from that which would be obtained using the usual domination principle. In addition to the usual region of dominance, the dominated region includes all points which have a smaller value function value than V_2.

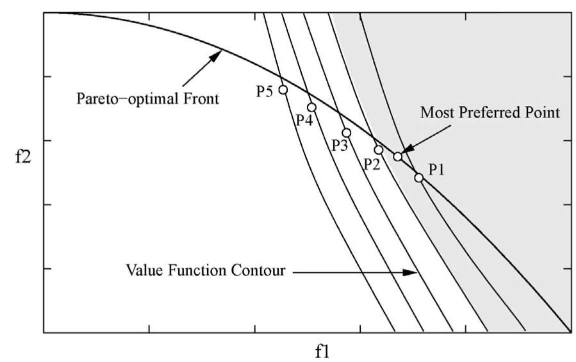

Fig. 7. Scenario in which final preferred point may lie between P_1 and P_2 for a two-objective problem.

We now discuss the reason for choosing the baseline value function value at P_2 (as opposed to at P_1) for defining the modified dominance criterion above. While providing preference information on η points given to the DM, the DM has the knowledge of η points. Consider the scenario in Fig. 7, in which point $\mathbf{z}^*$ may lie between P_1 and P_2. If the value function at P_1 is considered as the baseline value for domination, the most preferred point $\mathbf{z}^*$ will get dominated by points like P_1. In higher objective problems, the most preferred point may lie elsewhere and considering V_2 may also be too stringent. To be more conservative, $V(P_\eta)$ can be considered as the baseline value in the modified domination criterion.

The above modified domination principle can be used in both Steps 5 and 6 for creating the new population Off_t and for selecting the new population Par_{t+1}.

Although we do not handle constrained problems in this paper, the above modified domination principle can be extended for handling constraints. As defined in [18], when both solutions under consideration for a domination check are *feasible*, the above domination principle can simply be used to establish dominance of one over the other. However, if one point is feasible and the other is not, the feasible solution can be declared as dominating the other. Finally, if both points are infeasible, the one having smaller overall constraint violation may be declared as dominating the other. We defer consideration of a constrained PI-EMO-VF to a later study.

IV. PI-NSGA-II-VF Procedure

In the PI-NSGA-II-VF procedure, the first τ generations are performed according to the usual NSGA-II algorithm [18]. Thereafter, we modify the NSGA-II algorithm by using the modified domination principle (discussed in Section III-C) in the elite-preserving operator and also in the tournament selection for creating the offspring population. We also use a different recombination operator in this paper. After a child solution $\mathbf{x}^C$ is created by the SBX (recombination) operator [29], two randomly selected population members $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are chosen and a small fraction of the difference vector is added to the child solution (similar in principle to a differential evolution operator [30]), as follows:

$$\mathbf{x}^C = \mathbf{x}^C + 0.1\left(\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\right). \tag{9}$$

The crowding distance operator of NSGA-II has been replaced with k-mean clustering for maintaining diversity among solutions of the same nondominated front to make the diversity preservation more meaningful for problems having more than two objectives.

The success of EMO algorithms to find a set of diverse trade-off solutions for two and three-objective problems is due to an appropriate balance between their diversity maintaining operators (for example, crowding distance, clustering, or other mechanisms) and their emphasis of nondominated solutions [1]. When preference information is to be implemented in an EMO, the search focus has to shift more toward emphasizing currently preferred solutions, as the target becomes finding a single preferred solution at the end. If a proper balance between these *exploring* and *exploiting* mechanisms are not maintained, the resulting preference-based EMO procedure may not work well and may end up either in a premature convergence to a suboptimal solution or in a random-like search behavior. By modifying the domination principle with preference information, we have emphasized preferred solutions. By using a modified recombination operator for child creation and a clustering operator, instead of crowding distance operator, for a better diversity preservation, we have attempted to make a balance with the enhanced selection pressure toward the preferred solutions. Simulation results of the next section demonstrates this aspect on a number of problems.

The value function optimization problem is solved using the SQP code of KNITRO software [31]. The termination is set if the Karush-Kuhn-Tucker (KKT) error measure computed within KNITRO is less than or equal to 10^{-6}.

For termination check (discussed in Section III-B), we also use the SQP code of KNITRO software and the SQP algorithm is terminated (if not terminated due to d_s distance check from P_1 discussed earlier) when the KKT error measure is less than or equal to 10^{-6}.

V. Results

In this section, we present the results of the PI-NSGA-II-VF procedure on two, three, and five objective test problems. ZDT1 and DTLZ2 test problems are adapted to create maximization problems. In all simulations, we have used the following parameter values:

1) number of points given to the DM for preference information: $\eta = 5$;
2) number of generations between two consecutive DM calls: $\tau = 5$;
3) termination parameter: $d_s = 0.01$;
4) crossover probability and the distribution index for the SBX operator: $p_c = 0.9$ and $\eta_c = 15$;
5) mutation probability: $p_m = 0$;
6) population size: $N = 10M$, where M is the number of objectives.

In the optimization of the VFOP problem [given in (5)], we restrict the bounds of parameters as follows: $0 \leq (k_1, k_2) \leq 1000$ and $-1000 \leq (l_1, l_2) \leq 1000$. In the next section, we perform a parametric study with some of the above parameters. Here, we present the test problems and results obtained with the above setting.

A. Two-Objective Test Problem

Problem 1 is adapted from ZDT1 and has 30 variables [32]

$$\begin{aligned} \text{Maximize} \quad & \mathbf{f}(\mathbf{x}) = \left\{ \begin{array}{c} x_1, \\ \dfrac{10 - \sqrt{x_1 g(\mathbf{x})}}{g(\mathbf{x})} \end{array} \right\}, \\ \text{where} \quad & g(\mathbf{x}) = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i, \\ & 0 \leq x_i \leq 1 \quad \text{for } i = 1, 2, \ldots, 30. \end{aligned} \tag{10}$$

The Pareto-optimal front is given by $f_2 = 10 - \sqrt{f_1}$ and is shown in Fig. 8. The solutions are $\mathbf{x}_i = 0$ for $i = 2, 3, \ldots, 30$ and $x_1 \in [0, 1]$.

This maximization problem has a nonconvex front, therefore if the DM is not interested in the end points, the value function has to be nonlinear. A linear value function will always lead to the end points of the front. In our simulations, we assume a particular value function that acts as a representative of the DM, but the information is not explicitly used in creating new solutions by the operators of the PI-NSGA-II-VF procedure. In such cases, the most preferred point $\mathbf{z}^*$ can be determined from the chosen value function beforehand, thereby enabling us to compare our obtained point with $\mathbf{z}^*$.

In this paper, we assume the following nonlinear value function (which acts as a DM in providing a complete ranking of η solutions at every τ generations):

$$V(f_1, f_2) = \frac{1}{(f_1 - 0.35)^2 + (f_2 - 9.6)^2}. \tag{11}$$

This value function gives the most preferred solution as $\mathbf{z}^* = (0.25, 9.50)$. The contours of this value function are shown in Fig. 8. Since a DM-emulated value function is used to decide on preference of one point to the other in pairwise comparisons, we shall have complete ranking information of all η points in our paper. Thus, we shall not have the fourth set of constraints in determining the value function, as given in (5). In a future study, we shall consider partial preference information and its effect on the constructed value function.

Table I presents the best, median, and worst of 21 different PI-NSGA-II-VF simulations (each starting with a different initial population). The performance (accuracy measure) is computed based on the Euclidean distance of each optimized point with $\mathbf{z}^*$. Note that this accuracy measure is different from the termination criterion used in the PI-NSGA-II-VF procedure. Table II shows minimum, median, and maximum accuracy, the number of overall function evaluations, and the number of DM calls recorded in the 21 runs. The table indicates that the proposed PI-NSGA-II-VF procedure is able

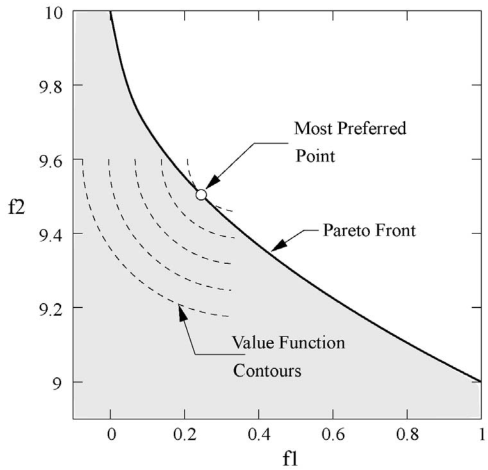

Fig. 8. Contours of the chosen value function (acts as a DM) and the most preferred point corresponding to the value function.

TABLE I
FINAL SOLUTIONS OBTAINED BY PI-NSGA-II-VF FOR THE MODIFIED ZDT1 PROBLEM

	$\mathbf{z}^*$	Best	Median	Worst
f_1	0.2500	0.2498	0.2461	0.2713
f_2	9.5000	9.5002	9.5038	9.4791

TABLE II
DISTANCE OF OBTAINED SOLUTION FROM THE MOST PREFERRED SOLUTION, FUNCTION EVALUATIONS, AND THE NUMBER OF DM CALLS REQUIRED BY THE PI-NSGA-II-VF FOR THE MODIFIED ZDT1 PROBLEM

	Minimum	Median	Maximum
Accuracy	0.0001	0.0062	0.0197
Func. evals.	5408	7372	11 809
# of DM calls	14	19	30

to find a solution close to the final preferred solution. Although the overall number of function evaluations depend on the initial population, for a 30-variable problem these numbers are reasonable.

We now show the working of the PI-NSGA-II-VF approach for a particular run, which required 14 DM calls before termination. Fig. 9 shows the value functions optimized after various DM calls. The first DM call was made after generation 5. Five chosen points (P1 to P5 shown in shaded circles) from the nondominated solutions at generation 5 are shown in the figure. The best and second-best points are close to each other. The strictly increasing requirement of the value function imposed in the optimization process creates an almost linear value function as an optimum choice in this case. The corresponding parameter values of the value function are $k_1 = 998.189$, $k_2 = 0.049$, $l_1 = 369.532$, and $l_2 = 137.170$. The value functions are drawn at the second-best point. After five more generations, the DM is called to provide preference information the second time. The corresponding value function drawn at the second-best point is shown in the figure. Five points used for preference ranking are shown as diamonds.

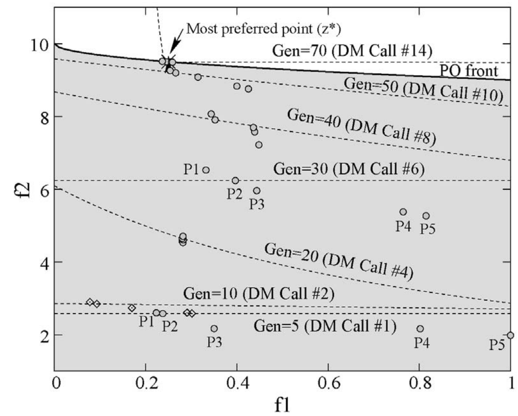

Fig. 9. Evolution of value functions after successive DM calls.

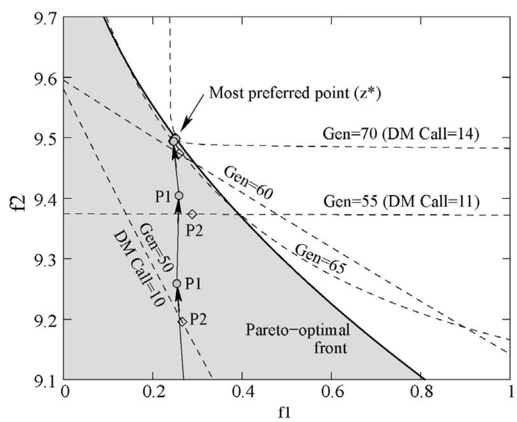

Fig. 10. Value functions near the most preferred point.

The figure shows how the PI-NSGA-II-VF procedure finds better and better points and how progressively the DM calls enable the overall procedure to find refined value functions. Eventually, at the 14th DM call, all five solutions come very close to $\mathbf{z}^*$ and the algorithm terminates with the imposed $d_s = 0.01$ condition.

The optimal parameter values fixing the value functions at various DM calls are shown in Table III. Although no pattern in these parameter values is observed from one DM call to another, every value function thus obtained is strictly increasing and maximizes the maximum difference in value function values between any two chosen points. However, the NSGA-II algorithm with these value functions in five subsequent generations seems to guide the best point toward the most preferred point ($\mathbf{z}^*$) progressively.

Fig. 10 shows the value functions from the 10th DM call onward for clarity. In this figure, the value functions are drawn at the second-best point (shown with a diamond) and the corresponding best point is also shown by a circle. It is interesting to observe how the value functions get modified with generations and how the modified value functions help find better nondominated solutions progressively with the help of modified domination and NSGA-II operators. The final point obtained by the PI-NSGA-II-VF is $(f_1, f_2) = (0.251, 9.499)$,

TABLE III
OPTIMAL PARAMETER VALUES DETERMINING THE VALUE FUNCTION AND CORRESPONDING BEST POINT AT VARIOUS DM CALLS

DM Call	k_1	k_2	l_1	l_2	$P_1 = (f_1, f_2)$
#1	998.189	0.049	369.532	137.170	(0.223, 2.600)
#2	999.998	19.699	114.161	359.199	(0.078, 2.898)
#3	821.797	0.003	−15.116	770.050	(0.260, 4.321)
#4	1000.000	440.133	87.366	393.896	(0.282, 4.706)
#6	804.650	0.033	−99.871	567.481	(0.332, 6.525)
#8	807.395	105.691	−30.880	365.454	(0.344, 8.066)
#10	403.750	49.007	−30.667	290.960	(0.254, 9.259)
#14	0.007	0.006	−0.308	−9.488	(0.251, 9.499)

which is very close to the most preferred point $\mathbf{z}^* = (0.25, 9.5)$ corresponding to the optimum of the DM-emulated value function given in (11).

B. Three-Objective Test Problem

The DTLZ2 test problem [33] is scalable to number of objectives. In the three-objective case, all points (objective vectors) are bounded by two spherical surfaces in the first octant. In the case of minimizing all objectives, the inner surface (close to the origin) becomes the Pareto-optimal front. But here, we maximize each objective of the DTLZ2 problem. Thus, the outer spherical surface becomes the corresponding Pareto-optimal front. An M-objective DTLZ2 problem for maximization is given as follows:

$$\begin{aligned} &\text{Maximize} \quad \mathbf{f}(\mathbf{x}) = \\ &\left\{ \begin{array}{l} (1.0 + g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\cdots\cos(\frac{\pi}{2}x_{M-1}) \\ (1.0 + g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\cdots\sin(\frac{\pi}{2}x_{M-1}) \\ \vdots \\ (1.0 + g(\mathbf{x}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) \\ (1.0 + g(\mathbf{x}))\sin(\frac{\pi}{2}x_1) \end{array} \right\}, \\ &\text{subject to} \quad 0 \le x_i \le 1 \quad \text{for } i = 1, \ldots, 12, \\ &\quad \text{where} \quad g(\mathbf{x}) = \textstyle\sum_{i=3}^{12}(x_i - 0.5)^2. \end{aligned} \tag{12}$$

The Pareto-optimal front for a three-objective DTLZ2 problem is shown in Fig. 11. The points (objective vectors) on the Pareto-optimal front follow the relation: $f_1^2 + f_2^2 + f_3^2 = 3.5^2$. The decision variable values correspond to $x_1 \in [0, 1]$, $x_2 \in [0, 1]$, and $x_i = 0$ or 1 for $i = 3, 4, \ldots, 12$.

To test the working of PI-NSGA-II-VF on this problem, we have replaced the DM by using a linear value function (emulating the DM), as follows:

$$V(f_1, f_2, f_3) = 1.25f_1 + 1.50f_2 + 2.9047f_3. \tag{13}$$

This value function produces the most preferred solution on the Pareto-optimal front as $\mathbf{z}^* = (1.25, 1.50, 2.9047)$.

The PI-NSGA-II-VF is run with $N = 10\times 3$ or 30 population members 21 times, each time with a different random initial population. In terms of the accuracy measure from $\mathbf{z}^*$, Table IV presents the minimum, median and worst performing runs. Table V shows the accuracy, number of overall function evaluations and number of DM calls needed by the procedure.

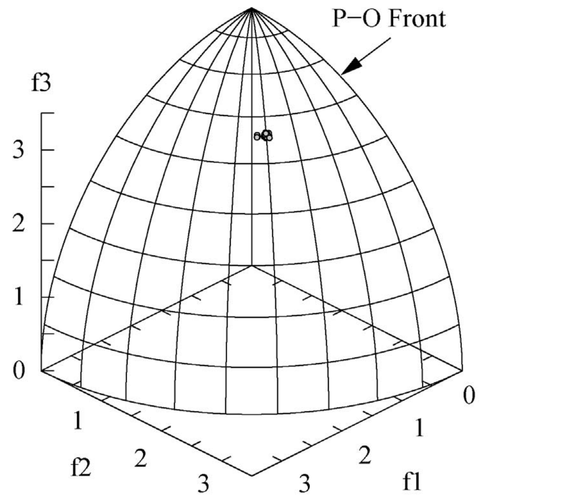

Fig. 11. Final population members after termination of the algorithm for three-objective modified DTLZ2 problem. The complete Pareto-optimal surface is marked as "P-O front."

TABLE IV
FINAL SOLUTIONS OBTAINED BY PI-NSGA-II-VF FOR THE THREE-OBJECTIVE MODIFIED DTLZ2 PROBLEM

	$\mathbf{z}^*$	Best	Median	Worst
f_1	1.2500	1.2459	1.2197	1.3178
f_2	1.5000	1.5050	1.4888	1.4755
f_3	2.9047	2.9039	2.9233	2.8873

TABLE V
DISTANCE OF OBTAINED SOLUTION FROM THE MOST PREFERRED SOLUTION, NUMBER OF FUNCTION EVALUATIONS, AND NUMBER OF DM CALLS REQUIRED BY PI-NSGA-II-VF ON THE THREE-OBJECTIVE MODIFIED DTLZ2 PROBLEM

	Minimum	Median	Maximum
Accuracy	0.0008	0.0115	0.0434
Func. evals.	4200	6222	8982
# of DM calls	17	25	36

It is clear that the obtained points are close to the most preferred point $\mathbf{z}^*$. Fig. 11 shows the population at the final generation of a typical PI-NSGA-II-VF run.

C. Five-Objective Test Problem

We now consider the five-objective ($M = 5$) version of the DTLZ2 problem described in the previous subsection. The Pareto-optimal front is described as $f_1^2+f_2^2+f_3^2+f_4^2+f_5^2 = 3.5^2$.

For this problem, we choose a nonlinear DM-emulated value function, as follows:

$$V(\mathbf{f}) = 1/\sum_{i=1}^{5}(f_i - a_i)^2, \tag{14}$$

where $\mathbf{a} = (1.1, 1.21, 1.43, 1.76, 2.6468)^T$. This value function produces the most preferred point as $\mathbf{z}^* = (1.0, 1.1, 1.3, 1.6, 2.4062)$.

TABLE VI
FINAL OBJECTIVE VALUES OBTAINED FROM PI-NSGA-II-VF FOR THE FIVE-OBJECTIVE MODIFIED DTLZ2 PROBLEM

	$\mathbf{z}^*$	Best	Median	Worst
f_1	1.0000	0.9931	0.9785	0.9455
f_2	1.1000	1.1382	1.0502	1.1467
f_3	1.3000	1.3005	1.3382	1.3208
f_4	1.6000	1.5855	1.5947	1.6349
f_5	2.4062	2.4007	2.4199	2.3714

TABLE VII
DISTANCE OF OBTAINED SOLUTION FROM THE MOST PREFERRED SOLUTION, FUNCTION EVALUATIONS, AND THE NUMBER OF DM CALLS REQUIRED BY PI-NSGA-II-VF FOR THE FIVE-OBJECTIVE MODIFIED DTLZ2 PROBLEM

	Minimum	Median	Maximum
Accuracy	0.0084	0.0240	0.0902
No. of function evaluation	23 126	27 202	41 871
No. of DM calls	57	67	102

Table VI presents the obtained solutions by PI-NSGA-II-VF with 50 population members. Table VII shows the accuracy measure, the number of overall function evaluations, and the number of DM calls. Although the points close to the most preferred point are obtained in each run, the higher dimensionality of the problem requires more function evaluations and DM calls compared to two and three-objective test problems. However, the above results are obtained for a strict termination criterion with $d_s = 0.01$. Smaller number of DM calls and evaluations are expected if this termination criterion is relaxed. We discuss these matters in the next section. It is worth mentioning that the application of a usual EMO (including the original NSGA-II) is reported to face difficulties in converging to the *entire* five-dimensional (5-D) Pareto-optimal front with an identical number of function evaluations in an earlier study [34]. However, since our target is one particular preferred point on the Pareto-optimal front (dictated by a sequence of preference information provided by the DM), our proposed PI-EMO-VF is able to find a near Pareto-optimal solution for the same five-objective optimization problem. This ability of an EMO to handle problems with a large number of objectives demonstrated in this paper remains as an evidence of an advantage of using preference information within an EMO.

D. Five-Objective Test Problem with an Intermediate Change in Decision

Next, we use the same five-objective DTLZ2 problem, but simulate a rather realistic scenario in which the DM changes his/her preference information while the optimization process is on. We use three DM-emulated functions of type given in (14) with the parameters shown in Table VIII. The first DM-emulated function is used to provide preference information for the first 10 DM calls, thereafter for some reason the DM changes his/her mind and the second function is used to provide preference information. Again, after 10 more DM calls, a third DM-emulated function is used. The parameters are quite different from each other, meaning that the corresponding Pareto-optimal solutions lie on different parts of the Pareto-optimal front (shown in the table). Such a simulation will demonstrate the flexibility of the proposed procedure in its ability to find the Pareto-optimal solutions corresponding to the third DM-emulated function, although being distracted by the initial influence of different functions.

TABLE VIII
PARAMETERS OF VALUE FUNCTIONS AND CORRESPONDING DM CALLS IN WHICH THEY ARE USED (CORRESPONDING PARETO-OPTIMAL SOLUTIONS ARE SHOWN)

DM calls	**a**	Corresponding P-O solution
1–10	(2.75, 1.65, 1.10, 1.32, 1.26)	(2.50, 1.50, 1.00, 1.20, 1.14)
11–20	(1.87, 1.54, 1.82, 1.16, 3.18)	(1.70, 1.40, 1.65, 1.05, 2.89)
21–terminate	(1.10, 1.21, 1.43, 1.76, 2.65)	(1.00, 1.10, 1.30, 1.60, 2.40)

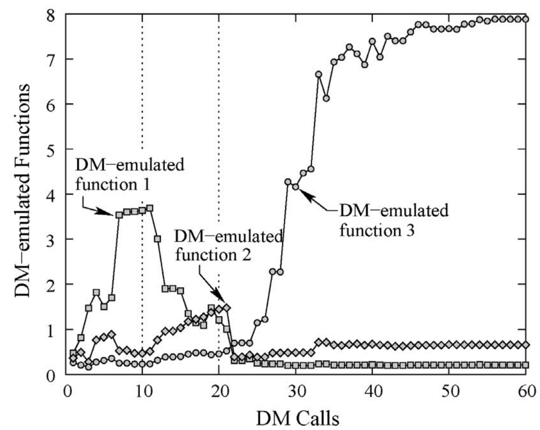

Fig. 12. Values of three DM-emulated functions corresponding to the most preferred solution in each DM call.

We use an identical parameter setting as used in the previous problem. Fig. 12 shows the value of the three DM-emulated functions for the most preferred solution after every DM call. It is clear from the figure that during the first 10 DM calls, the corresponding DM-emulated function value for the most preferred solution steadily increases, as if the DM were interested in maximizing this value function. The fact that the first DM-emulated function value after the 11th call reduces, indicates that the algorithm is able to respond to the new function. Thereafter, again after 21st DM call, the third DM-emulated function is active and the preferred solutions get tuned to this new value function. Eventually, the proposed method is able to converge to the Pareto-optimal solution corresponding to the third DM-emulated function.

VI. PARAMETRIC STUDY

Besides the usual parameters associated with an evolutionary algorithm, such as population size, crossover and mutation probabilities and indices, tournament size, etc., in the proposed PI-NSGA-II-VF we have introduced a few additional parameters that may affect the accuracy and number of DM calls. They are the number of points used in obtaining DM's

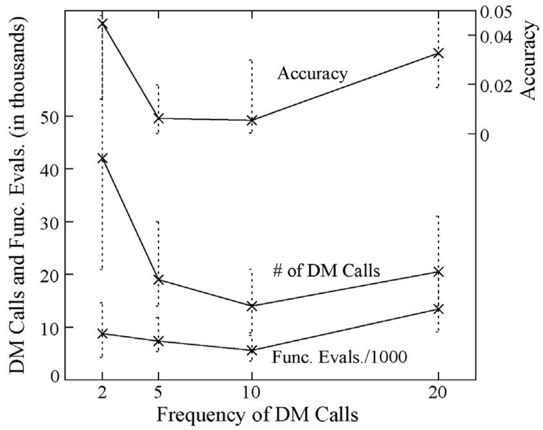

Fig. 13. Three performance measures on modified ZDT1 problem for different τ values.

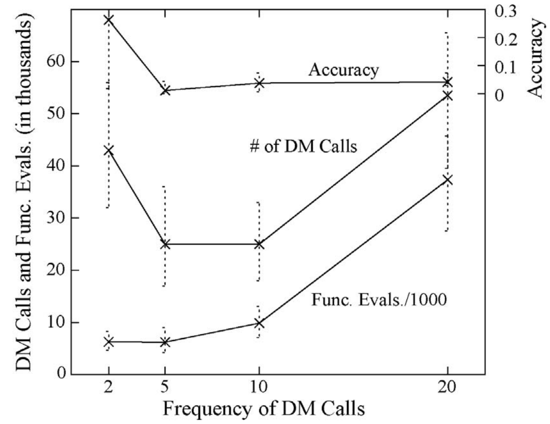

Fig. 14. Three performance measures on three-objective modified DTLZ2 problem for different τ values.

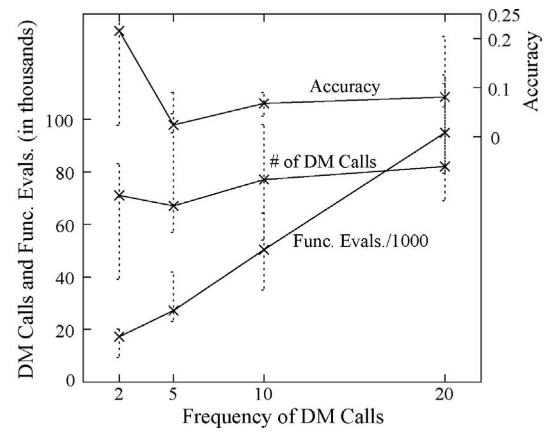

Fig. 15. Three performance measures on five-objective modified DTLZ2 problem for different τ values.

preference information (η), the number of generations between DM calls (τ), termination parameter (d_s), KKT error limit for terminating SQP algorithm in value function optimization and in single-objective optimization used for the termination check, and the parameter ρ used in the ASF function optimization. Of these parameters, the first three have shown to have an effect on the chosen performance measures—accuracy, the number of overall function evaluations, and the number of DM calls. As mentioned earlier, the parameter η is directly related to the maximum number of pairwise comparisons a DM would like to do in a single DM call. Of course, if more points can be compared, a more appropriate value function can be obtained. However, based on a maximum of 10 pairwise comparisons per DM call, we restrict $\eta = 5$ in this paper and do not do a parametric study with this parameter. Thus, in this section, we study the effect of two parameters (τ and d_s), while keeping the other PI-NSGA-II-VF parameters identical to that mentioned in the previous section. In each case, we use the same three test problems.

A. *Effect of Frequency of DM Calls* (τ)

First, we study the effect of τ by considering four different values: 2, 5, 10, and 20 generations. The parameter d_s is kept fixed to 0.01. To investigate the dependence of the performance of the procedure on the initial population, in each case, we run PI-NSGA-II-VF from 21 different initial random populations and plot the best, median, and worst performance measures.

We plot three different performance measures—accuracy, number of DM calls and number of function evaluations obtained for the modified ZDT1 problem in Fig. 13. It is interesting to note that all three median performance measures are best for $\tau = 10$, although $\tau = 5$ also results in a similar accuracy and the number of DM calls. A small value of τ means that DM calls are to be made more frequently. Clearly, this results in higher number of DM calls, as evident from the figure. Frequent DM calls result in more single-objective optimization runs for termination check, thereby increasing the number of overall function evaluations. On the other hand, a large value of τ captures too little preference information to focus the search near the most preferred point, thereby causing a large number of generations to satisfy termination conditions and a large number of DM calls.

Fig. 14 shows the same three performance measures on the three-objective modified DTLZ2 problem. For this problem, the number of DM calls is similar for $\tau = 5$ and 10 generations, whereas accuracy and the number of function evaluations are better for $\tau = 5$ generations. Once again, too small or too large τ is found to be detrimental.

For the five-objective modified DTLZ2 problem, $\tau = 5$ produces optimal median performance on the number of DM calls and accuracy (Fig. 15). However, the overall function evaluations is smaller with smaller τ.

Based on these simulation studies on two, three, and five-objective optimization problems, one can conclude that a value of τ within 5 to 10 generations is better in terms of an overall performance of the PI-NSGA-II-VF procedure. This range of τ provides a good convergence accuracy, requires less function evaluations, and less DM calls to converge near the most preferred point.

B. *Effect of Termination Parameter* d_s

Next, we investigate the effect of the termination parameter d_s on the three performance measures on all three problems. In this paper, we fix $\eta = 5$ and $\tau = 5$. Fig. 16 shows the positive correlation between accuracy and d_s. As d_s is in-

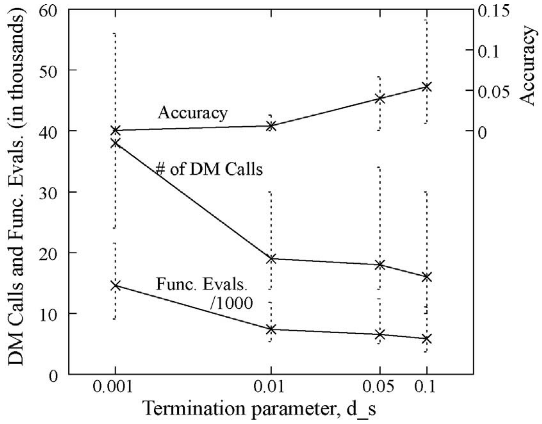

Fig. 16. Three performance measures on modified ZDT1 problem for different d_s values.

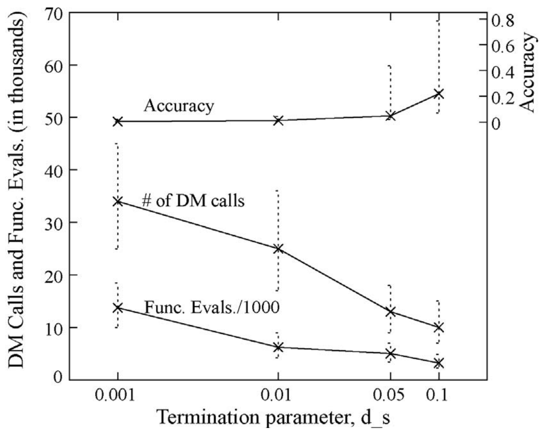

Fig. 17. Three performance measures on three-objective modified DTLZ2 problem for different d_s values.

creased (meaning a relaxed termination), the obtained accuracy (distance from $\mathbf{z}^*$) gets worse. Interestingly, the associated variation in obtained accuracy over number of runs also gets worse. The flip side of increasing d_s is that the number of function evaluations reduces, as a comparatively lesser number of generations are now needed to satisfy the termination condition. Similarly, the number of DM calls also reduces with an increase in d_s.

Similar observations are made for three-objective and five-objective modified DTLZ2 problem, as evident from Figs. 17 and 18, respectively.

These results clearly reveal the behavior of our proposed algorithm with regard to the choice of d_s. Unlike in the parametric study of τ, we find a monotonic variation in performance measures with d_s, however with a tradeoff between accuracy and the number of DM calls (or, the number of function evaluations). This indicates that d_s need not be chosen as an arbitrarily small value. If approximate solutions are acceptable, they can be achieved with a smaller number of function evaluations and DM calls. Fig. 19 shows the tradeoff of these quantities for the modified three and five-objective DTLZ2 problems. The nature of the tradeoff between accuracy and the number of DM calls indicates that $d_s = 0.05$ makes a good compromise between these performance indicators for these problems. A smaller d_s setting calls for substantially

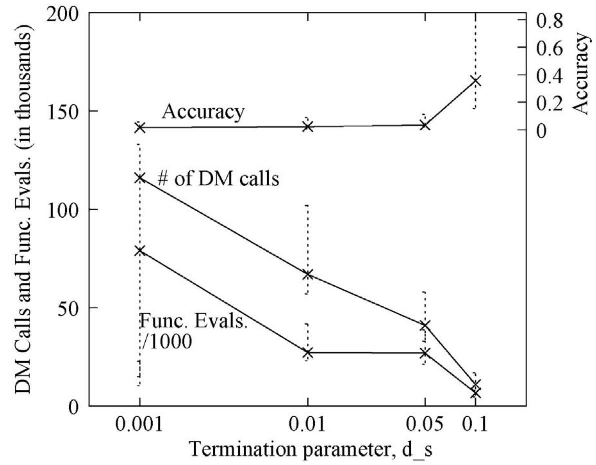

Fig. 18. Three performance measures on modified five-objective modified DTLZ2 problem for different d_s values.

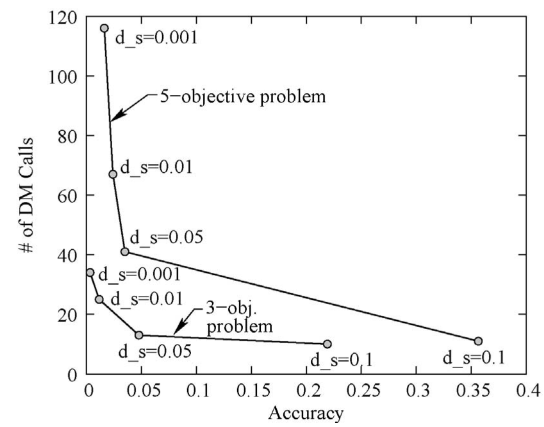

Fig. 19. Tradeoff between accuracy and the number of DM calls for the modified three and five-objective DTLZ problems.

more DM calls, and a larger d_s setting, although reduces the number of DM calls, makes a substantially large deviation from the most preferred solution.

VII. Random Error in Preference Information

In the above simulations, a mathematical value function is used to emulate the preference information to be given by a DM. However, in practice, the DM is a human being. There is bound to be some level of inconsistencies in providing preference information. To simulate the effect of this factor, we consider a DM-emulating value function which is stochastic in nature.

A linear value function similar to the one used before is chosen, but the coefficients of the value function are made stochastic. The stochasticity is reduced with the increase in the number of generations. This has been done to simulate a realistic DM who is likely to make errors during the start of the algorithm when he/she is in the process of learning his/her preferences. Later, the DM is likely to make more consistent decisions. A successful convergence of an algorithm in this case verifies that the algorithm does not get misdirected by inconsistent preference based information during the beginning of the run.

TABLE IX
FINAL SOLUTIONS OBTAINED BY PI-NSGA-II-VF FOR THE THREE-OBJECTIVE MODIFIED DTLZ2 PROBLEM WITH A STOCHASTIC DM-EMULATED VALUE FUNCTION

	$\mathbf{z}^*$	Best	Median	Worst
f_1	1.2500	1.2555	1.2695	1.2902
f_2	1.5000	1.5105	1.5205	1.6437
f_3	2.9047	2.8969	2.8856	2.8078

TABLE X
DISTANCE OF OBTAINED SOLUTION FROM THE MOST PREFERRED SOLUTION, FUNCTION EVALUATIONS, AND THE NUMBER OF DM CALLS BY PI-NSGA-II-VF FOR THE THREE-OBJECTIVE MODIFIED DTLZ2 PROBLEM WITH A STOCHASTIC DM-EMULATED VALUE FUNCTION

	Minimum	Median	Maximum
Accuracy	0.0142	0.0342	0.1779
Functional evaluations	5841	7608	9663
No. of DM calls	24	31	39

The DM-emulated value function used for the three-objective modified DTLZ2 problem is as follows:

$$V(f_1, f_2, f_3) = \text{noise}(1.25, \sigma) f_1 + \text{noise}(1.50, \sigma) f_2 + \text{noise}(2.9047, \sigma) f_3, \quad (15)$$

where σ is set as $\exp(-t/10)$ (t is the generation counter) and noise(μ, σ) refers to a random normal distribution with a mean μ and standard deviation σ. This setting ensures that the standard deviation of the noise around the mean reduces as the number of generations of the algorithm increases. With $\sigma = 0$, this value function gives the most preferred point as $\mathbf{z}^* = (1.25, 1.50, 2.9047)$. At the first instance of DM calls (that is, at $t = \tau = 5$ generations), $\sigma = \exp(-0.5) = 0.606$, meaning a significantly different value function than what is required for the algorithm to converge to the most preferred point.

Table IX shows the best, median, and worst points obtained by the PI-NSGA-II-VF procedure with $\eta = 5$, $\tau = 5$, $d_s = 0.01$, and other parameter values used in Section V-B. Again, 21 different runs were performed from different initial random populations. As clearly shown in Table X, the accuracy for the best and median runs is good, despite the large stochasticities involved in the early stages of the optimization process. Although the number of function evaluations and the number of DM calls are 20 to 40% more compared to that in the deterministic DM-emulated value function case (Table IV), the accuracy of the final point is good. This indicates that the final point is close to the most preferred solution for the deterministic case.

Next, we apply the PI-NSGA-II-VF procedure to the five-objective modified DTLZ2 problem with the following stochastic value function to emulate the DM:

$$V(\mathbf{f}) = \text{noise}(1.0, \sigma) f_1 + \text{noise}(1.1, \sigma) f_2 + \text{noise}(1.3, \sigma) f_3 + \text{noise}(1.6, \sigma) f_4 + \text{noise}(2.4062.\sigma) f_5. \quad (16)$$

The best, median, and worst points obtained by PI-NSGA-II-VF are shown in Table XI. As shown by the performance measures in Table XII, despite somewhat larger function evaluations and number of DM calls, final points obtained by PI-NSGA-II-VF are reasonably close to the most preferred point obtained for the deterministic version of the DM-emulated value function.

TABLE XI
FINAL SOLUTIONS OBTAINED BY PI-NSGA-II-VF FOR THE FIVE-OBJECTIVE MODIFIED DTLZ2 PROBLEM WITH A STOCHASTIC DM-EMULATED VALUE FUNCTION

	$\mathbf{z}^*$	Best	Median	Worst
f_1	1.0000	1.0103	1.0875	1.0557
f_2	1.1000	1.1171	1.1495	1.2394
f_3	1.3000	1.3037	1.3525	1.4915
f_4	1.6000	1.6025	1.5942	1.4977
f_5	2.4062	2.4140	2.4125	2.4889

TABLE XII
DISTANCE OF OBTAINED SOLUTION FROM THE MOST PREFERRED SOLUTION, FUNCTION EVALUATIONS, AND THE NUMBER OF DM CALLS BY PI-NSGA-II-VF FOR THE FIVE-OBJECTIVE MODIFIED DTLZ2 PROBLEM WITH A STOCHASTIC DM-EMULATED VALUE FUNCTION

	Minimum	Median	Maximum
Accuracy	0.0219	0.1137	0.2766
Functional evaluations	33 653	39 264	52 564
No. of DM calls	72	87	136

1) *Effect of Extent of Stochasticity:* In the above study, we used a noise factor on the coefficients of the DM-emulated value function, given as a function of generation counter t as follows: $\sigma = \exp(-t/10)$. As discussed above, at the first DM call with $\tau = 5$, this has an effect of having a standard deviation of 0.606 on each objective. We now investigate the effect of increasing the standard deviation by modifying the σ term as follows:

$$\sigma = s \exp(-t/10), \quad (17)$$

where s is a stochasticity factor. For $s = 1$, we have an identical stochastic effect as in the previous subsection. By using a larger value of s, we can simulate a situation with more inconsistencies in the decision-making process. We use four different values of s: 1, 5, 10, and 100.

With a large value of s, it is expected that the DM-emulated value function provides a different ranking of η points than an ideal ranking (which would have been obtained without the stochasticity effect). We count the number of times the ranking of top three points is different from the ideal ranking of the same three points and tabulate it in Table XIII for a typical run. The corresponding function evaluations and accuracy in the final optimized point from $\mathbf{z}^*$ are also shown in the table. An increase in stochasticity in the decision-making process requires more DM calls and more function evaluations to achieve an acceptable accuracy and termination. Importantly, since all runs are terminated with $d_s = 0.01$ condition, despite large stochasticities involved in the beginning of PI-NSGA-II-VF runs, the algorithm is able to find a point close to the most preferred Pareto-optimal point corresponding to the deterministic version of the DM-emulated value function.

TABLE XIII
EFFECT OF STOCHASTIC FACTOR ON PERFORMANCE MEASURES FOR THREE-OBJECTIVE MODIFIED DTLZ2 PROBLEM

s	Incorrect Ranking	Total DM Calls	Func. Evals.	Accuracy
1	12	20	6786	0.0399
5	17	23	7528	0.0437
10	19	28	8572	0.0498
100	23	31	9176	0.0512

VIII. EXTENSIONS OF CURRENT STUDY

This paper has suggested a simple yet elegant methodology by which the DM's preferences can be incorporated with an EMO algorithm so that the final target is not a complete Pareto-optimal set (as is usual in an EMO application), but a single preferred solution on the Pareto-optimal set. The ideas suggested can be extended in a number of different ways, which we discuss in the following paragraphs.

1) *Incomparable class and constrained problems:* In this paper, we have not simulated the case in which the DM judges some of the η points to be incomparable. Although our optimization problem formulation (5) considers this situation and we have demonstrated its use in constructing the respective value function in Fig. 3, a complete study is needed to implement the idea with the proposed PI-EMO-VF procedure and particularly in analyzing the effect of δ_V in the development of the value function. Since some cases may occur, in which a value function satisfying all DM's preferences is not possible, this paper will also test the specific part (Step 3) of the proposed PI-EMO-VF algorithm. Moreover, we have not tested constrained optimization problems in this paper. The modified constrained domination principle should be used and tested on some challenging problems.
2) *Other value functions:* In this paper, we have restricted the value function to be of certain form (5). Other more generic value function structures can also be considered. Our suggested value function construction procedure results in strictly increasing functions. However, a more generic nonconcave value function may be obtained by using different conditions in the optimization problem formulation.
3) *Robust value functions:* The optimization problem for deriving the value function can include a robustness consideration, in which the insensitivity of the value function coefficients in producing an identical ranking of η points can be ensured. This would be a different way of handling inconsistencies in decision-making.
4) *Other termination conditions:* Our proposed PI-EMO-VF algorithm terminates when there does not exist a far away point with a better value function value than the currently judged preferred point. Although this indicates somewhat the probability of creating better preferred points than the currently judged preferred point, other termination indicators are certainly possible. In this direction, instead of terminating based on Euclidean distance between the two points, the difference in value function values can be checked.
5) *Reduction in DM calls:* One outcome of the parametric study is that by fixing a relaxed termination criterion (relatively larger value of d_s), the number of DM calls can be reduced. However, there are other extensions to this paper which may also reduce the number of DM calls. The basic operators in the suggested algorithm can be extended so that the modified procedure requires a reduced number of overall DM calls. The issue of having more points in each DM call, thereby reducing the overall number of DM calls to achieve a comparable accuracy will constitute an important study. Instead of keeping a fixed interval of τ generations for each DM call, DM call interval can be varied (or self-adapted) based on the extent of improvement achieved from the previous value function. Varying the number of points (η) in each DM call in a self-adaptive manner would be another important task. Since the points early on in the PI-EMO-VF procedure are not expected to be close to the Pareto-optimal front, the number of DM calls and points per call can be made small. Thereafter, when the procedure approaches the Pareto-optimal front, more points can be included per DM call and the frequency of DM calls can be controlled by the observed rate of improvement of the performance of the procedure. Also, it would be an interesting study to ascertain the effect of cumulating the preference information from one decision call to the next and use it in approximating the value function.
6) *Fixed budget of DM calls:* In this paper, we have kept a termination criterion which is related to the extent of improvements in currently judged preferred solution. We then recorded the number of DM calls which were needed until the termination criteria was met. However, a comparative study, in which different algorithms are compared for a fixed number of DM calls may be performed.
7) *Value function based recombination and mutation operators:* In this paper, we have modified the domination principles to emphasize points which have better value function value. However, EMO algorithm's recombination and mutation operators can also be modified based on developed value function. For example, restricting one of the top two currently judged preferred solutions as one parent in the SBX operator may help generate better preferred solutions.
8) *PI-EMO-VF with other EMO algorithms:* In this paper, we have integrated the preference information in NSGA-II algorithm. A natural extension of this paper would be to incorporate the preference handling approach with other popular EMO methodologies, such as SPEA2 [23], PESA [35], and others.

IX. CONCLUSION

In this paper, we have suggested a simple preference-based evolutionary multiobjective optimization (PI-EMO-VF) procedure, which iteratively finds new solutions by using an

EMO algorithm that progressively sends a representative set of trade-off solutions to a DM for obtaining a complete or partial preference ranking. The DM's preference information has been used in the following three ways in developing the new algorithm.

1) First, a strictly increasing value function was derived by solving an optimization problem, which maximizes the value function value between ranked points.
2) Second, the resulting value function was then utilized to redefine the domination principle between the points. The modified domination principle was used to drive the EMO search.
3) Third, the resulting value function was used to design a termination criterion for the PI-EMO-VF algorithm by executing a single-objective search along the gradient direction of the value function.

The above generic preference-based EMO approach has been implemented with the NSGA-II procedure. The PI-NSGA-II-VF procedure has then been applied to three different test-problems involving two, three, and five objectives. By using a DM-emulated utility function, we have shown that the PI-NSGA-II-VF was capable of finding the most preferred solution corresponding to the emulated utility function. A parametric study on the additional parameters has clearly indicated optimal parameter settings. Finally, to simulate inconsistencies, which may arise in providing preference information was simulated by considering a stochastic value function with a noise effect reducing over time. Even in such cases, the PI-NSGA-II-VF has been able to come closer to the most preferred point corresponding to the deterministic version of the DM-emulated value function.

Combining the ideas from EMO algorithms and MCDM seems an encouraging direction for future research in multiobjective optimization. In this paper, we have suggested one particular integration of DM's direct preference information into an EMO algorithm. The method is generic and the obtained results indicate that it is a promising approach. More emphasis must now be placed for developing pragmatic hybrid algorithms for multiobjective optimization and decision-making.

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