Global Sensitivity Analysis of Predictor Models in Software Engineering

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May 20, 2007

Introduction

Predictor Models in Software Engineering

- Predictor models describe situations in software engineering
- Various types, aims, ...
- Examples
  - COCOMO (costs)
  - Musa-Okumoto (reliability)
  - Fault-proneness

Problem

- The models themselves can be complex
- Their use needs a lot of effort
- How can I analyse those models themeselves?
- How can I simplify them?
- How do I improve their predictive power?

Example: Fischer-Wagner Model

- Reliability model used at Siemens COM
- Two parameters estimated by failure data
- Failure probability of a fault: $p_a = p_1 \cdot d^{(a-1)}$
- Geometrical distribution : $F_a(t) = 1 - (1 - p_a)^t$
- Cummulated failures up to t:
  $\mu(t) = \sum_{a=1}^{\infty} 1 - (1 - p_1 \cdot d^{(a-1)})$
- Input parameters
  - p_1: Highest failure probability of a fault
  - d: Complexity of the system $]0; 1[$
  - t: Execution time (incidents)
  - *inf*: approximation of ∞
- Output: $\mu(t)$

More details: S. Wagner, H. Fischer. Ada-Europe, 2006

Questions About the Model

- How do the input parameters influence the output?
- Which parameter(s) do I have to estimate best to get a good prediction?
- Are there insignificant parameters that I can remove?

Introduction

Global Sensitivity Analysis

Approach for SE

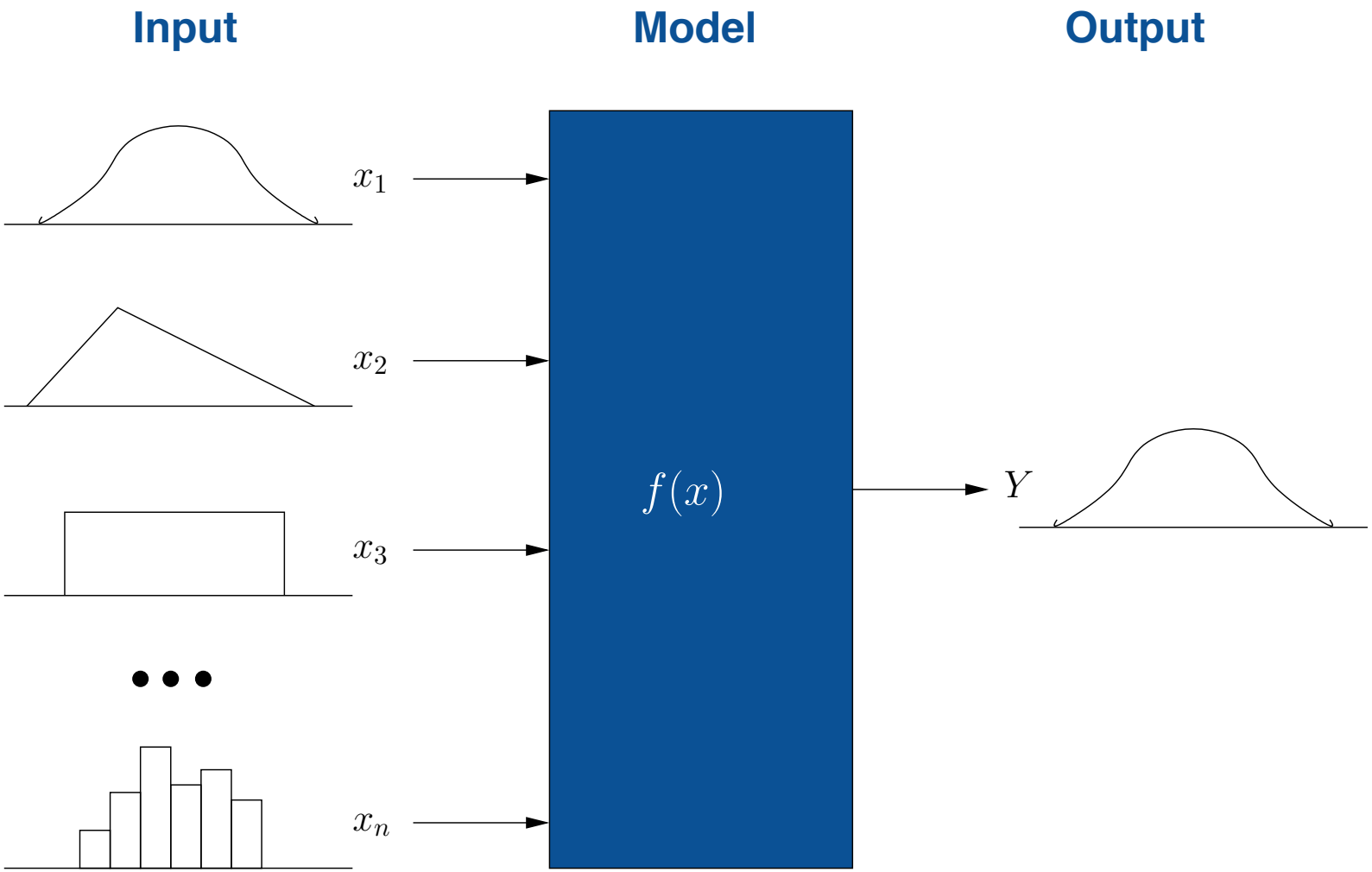
Summary

Global Sensitivity Analysis

Definition

- Saltelli (2000): "Sensitivity analysis studies the relationships between information flowing in and out of the model."
- How do the input parameters influence the output?
- How can this influence by quantified?
- Global properties
  - Inclusion of influence of shape and scale
  - Multidimensional averaging

Method



(Based on slide of S. Tarantola)

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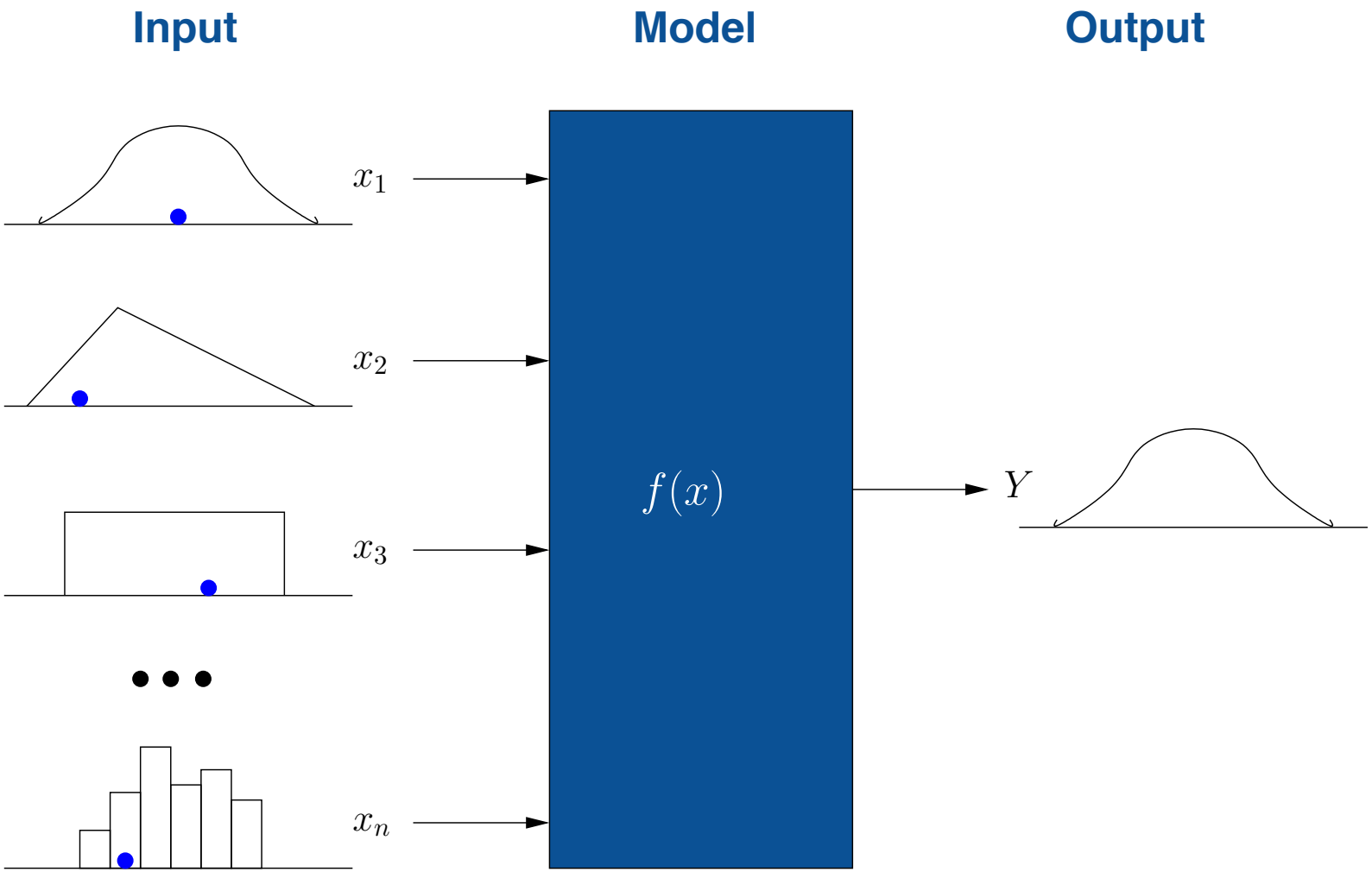

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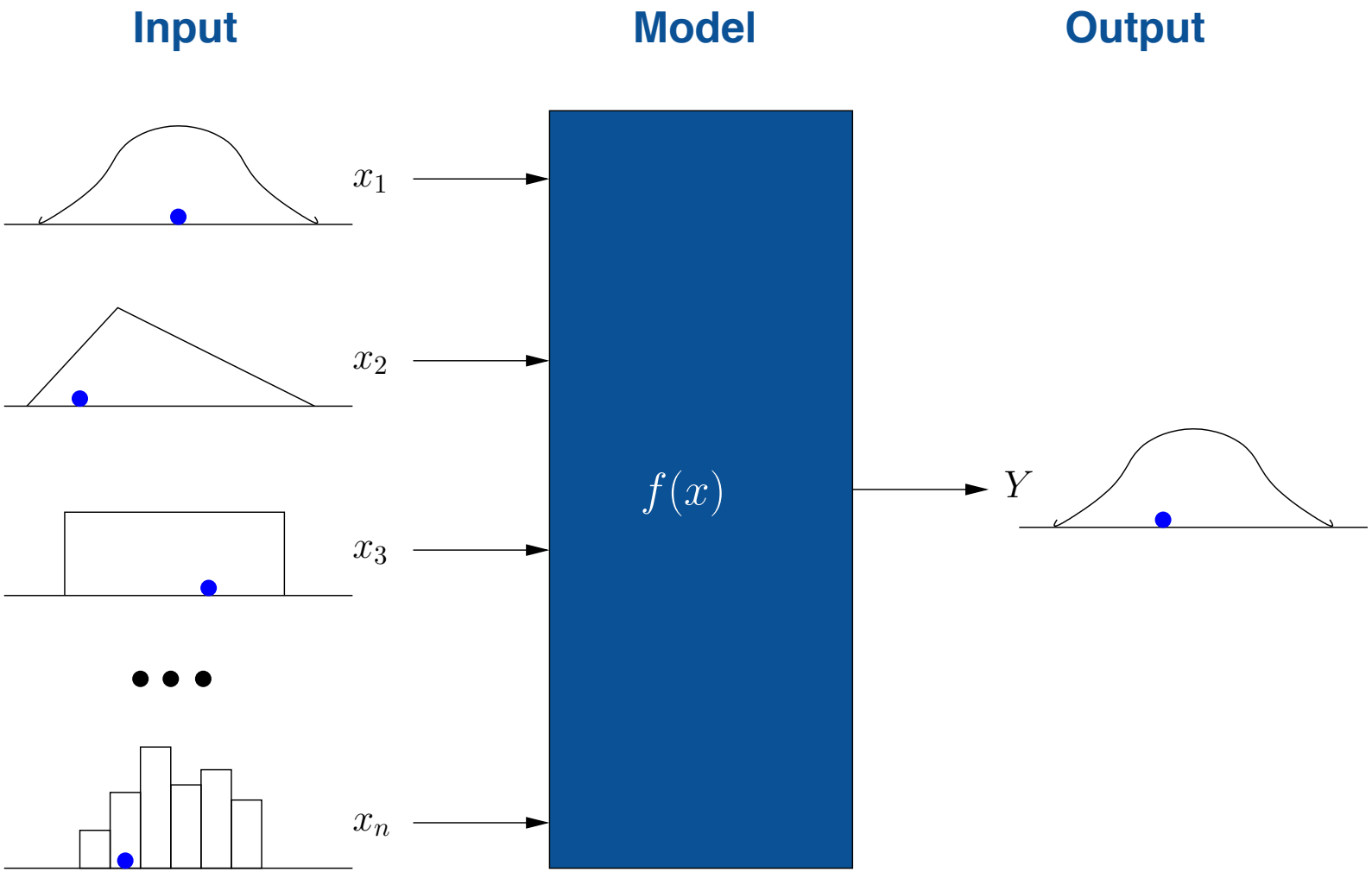

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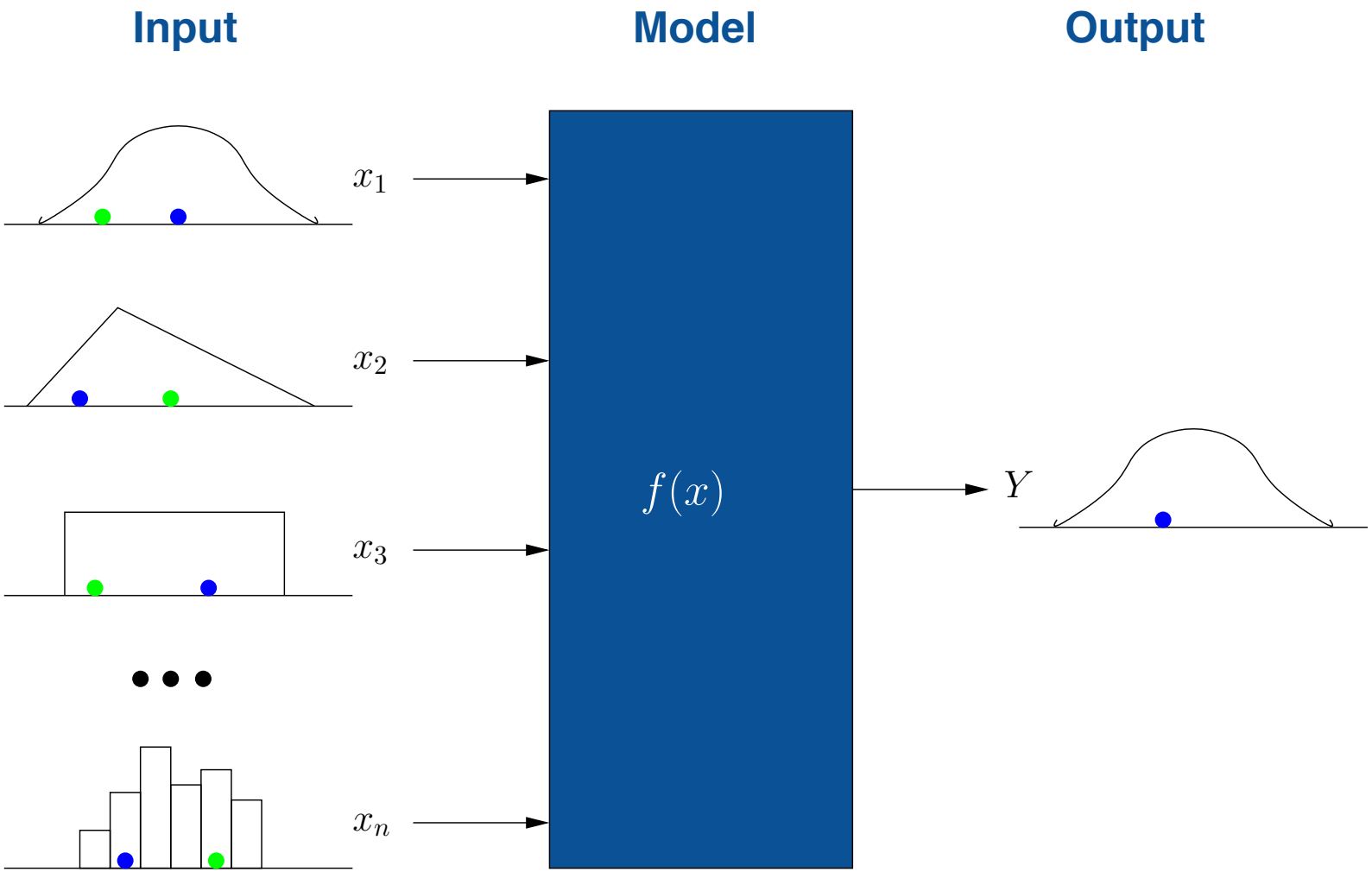

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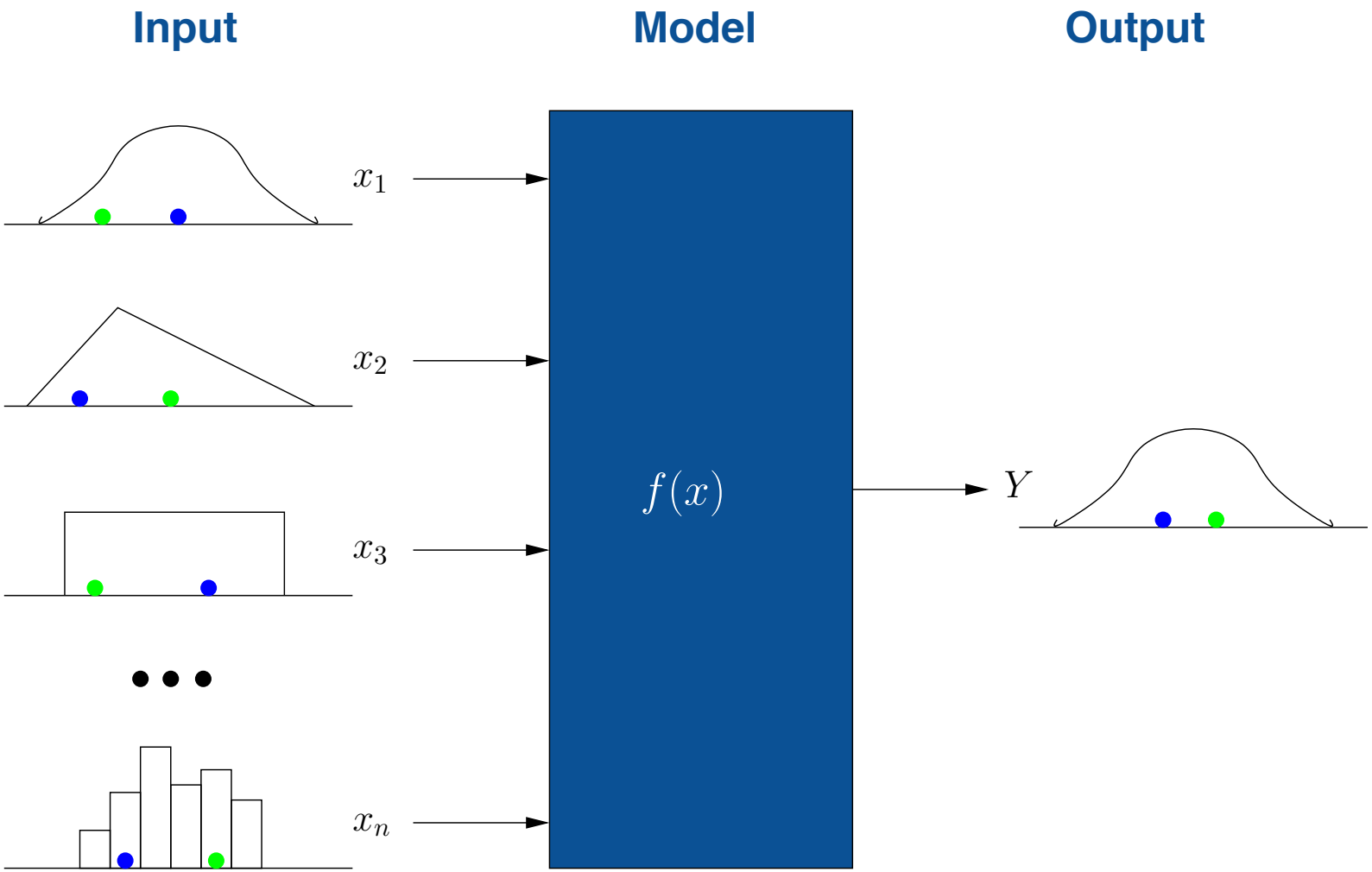

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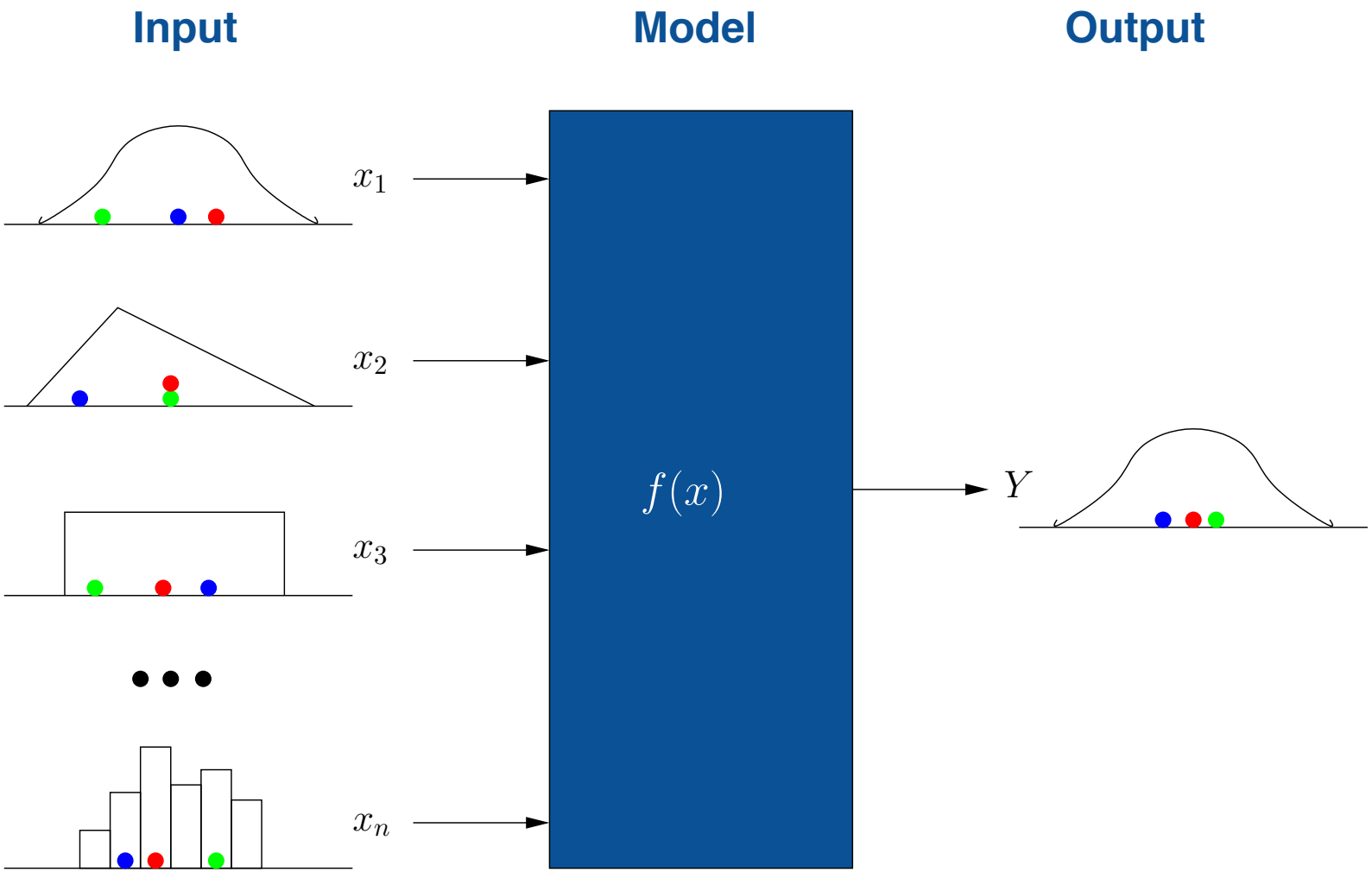

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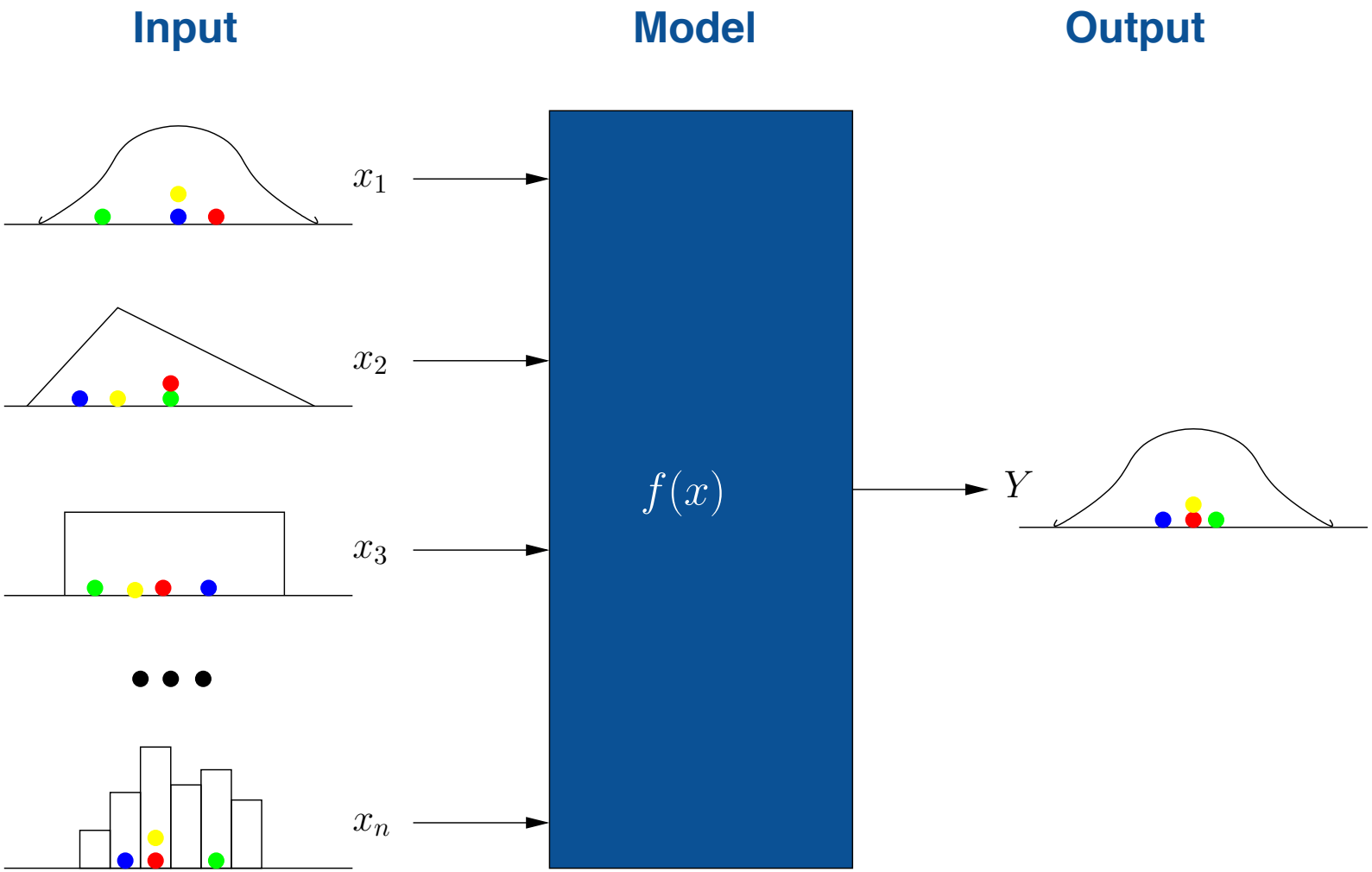

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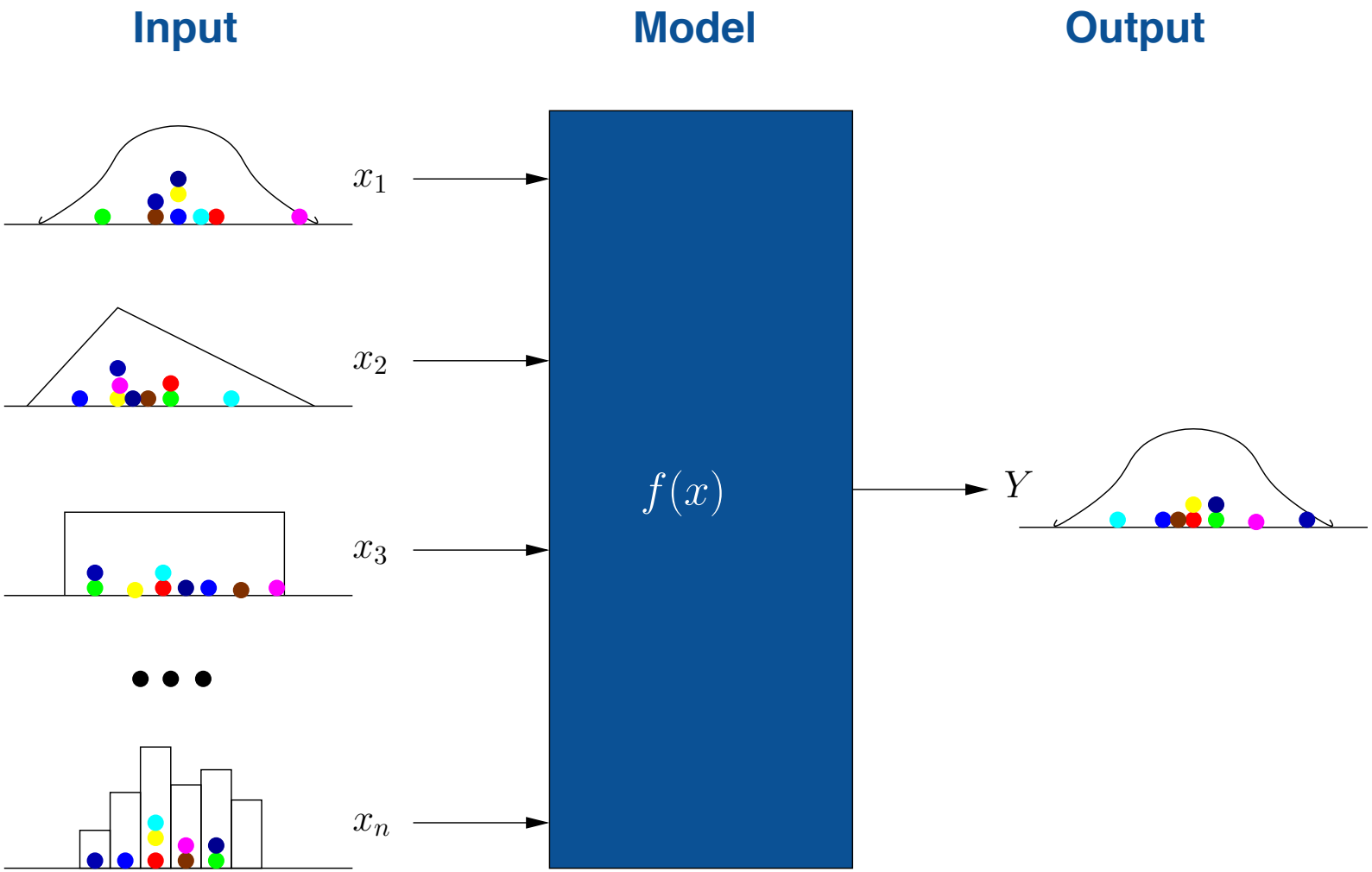

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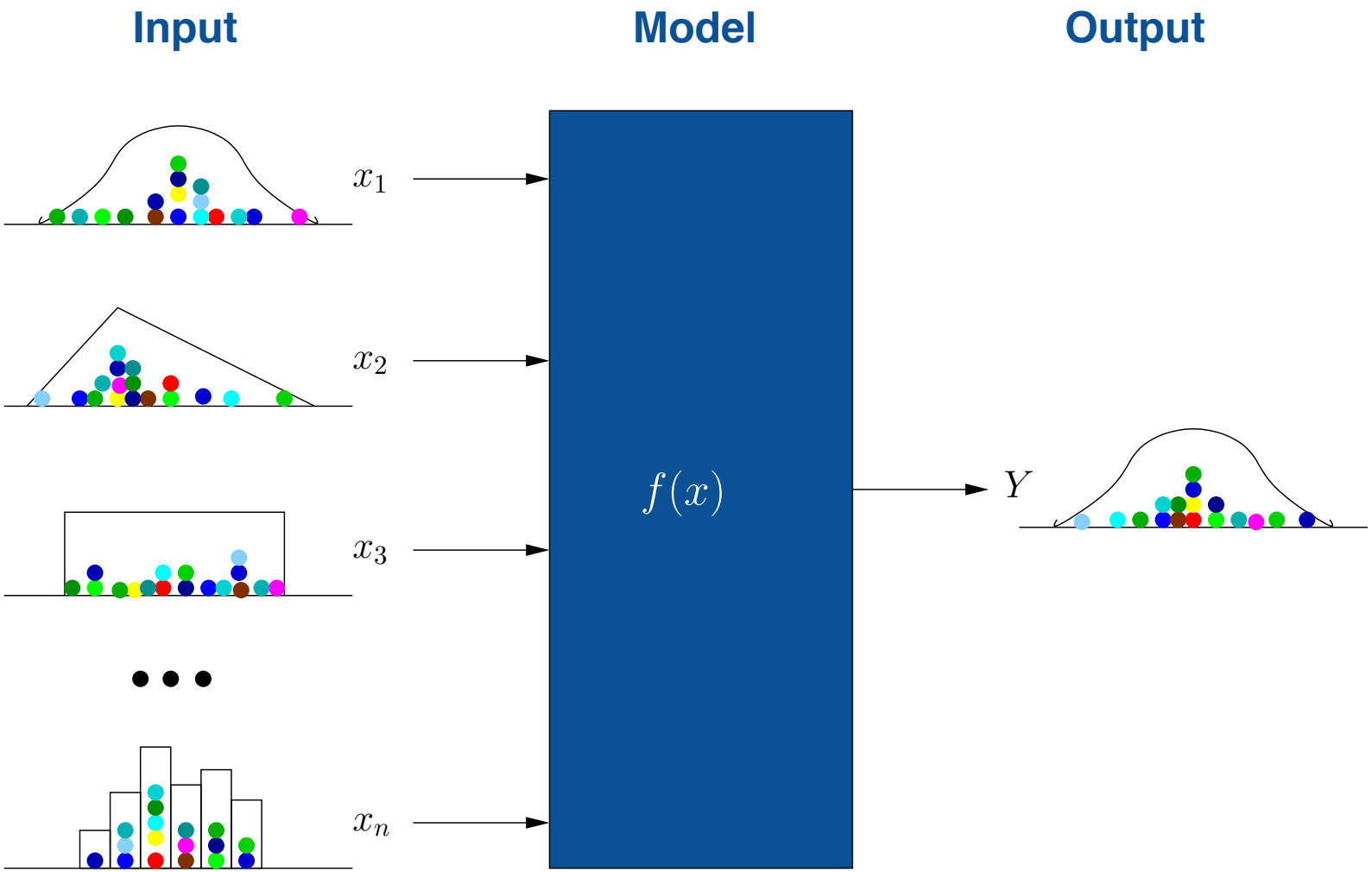

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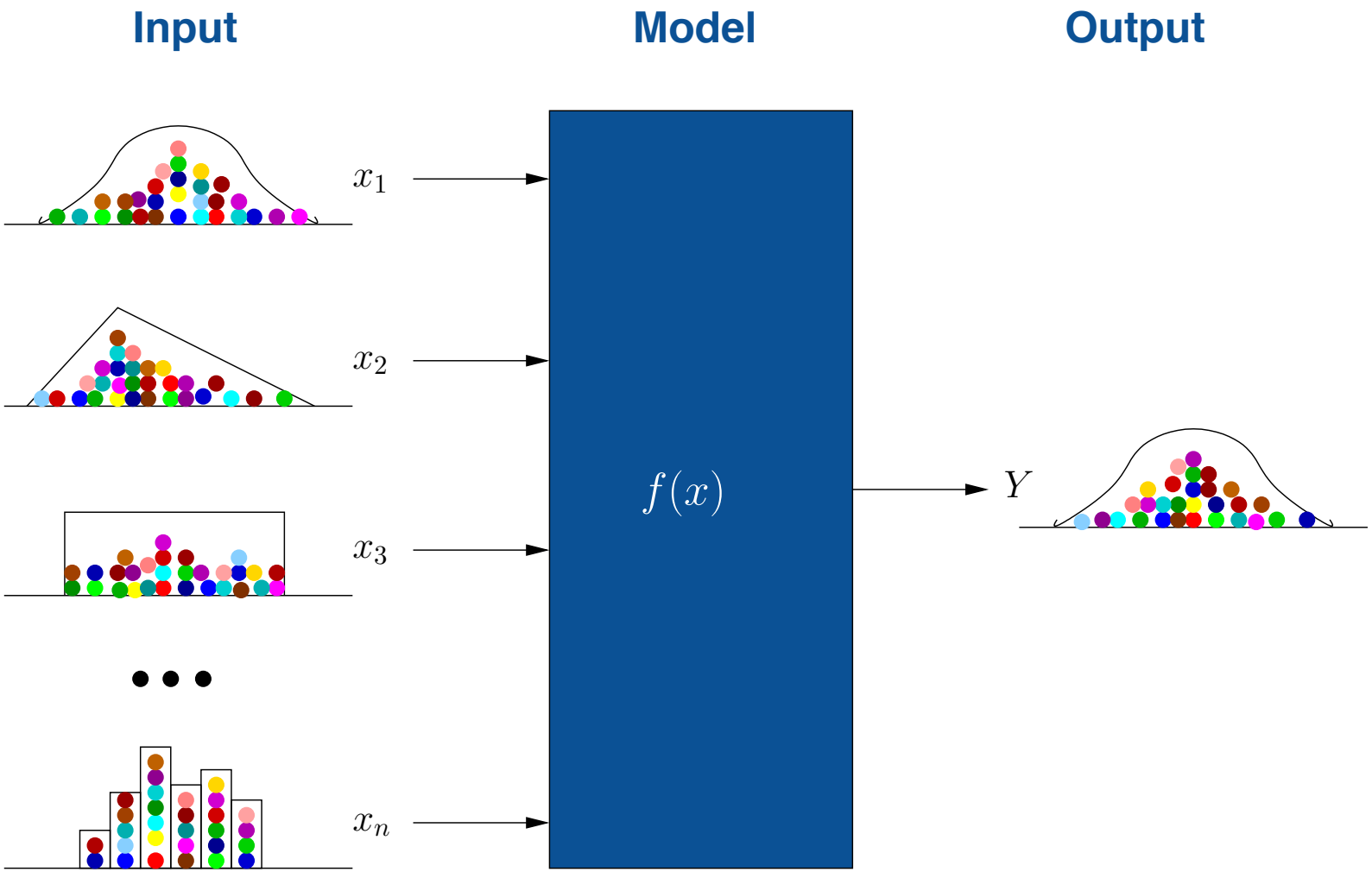

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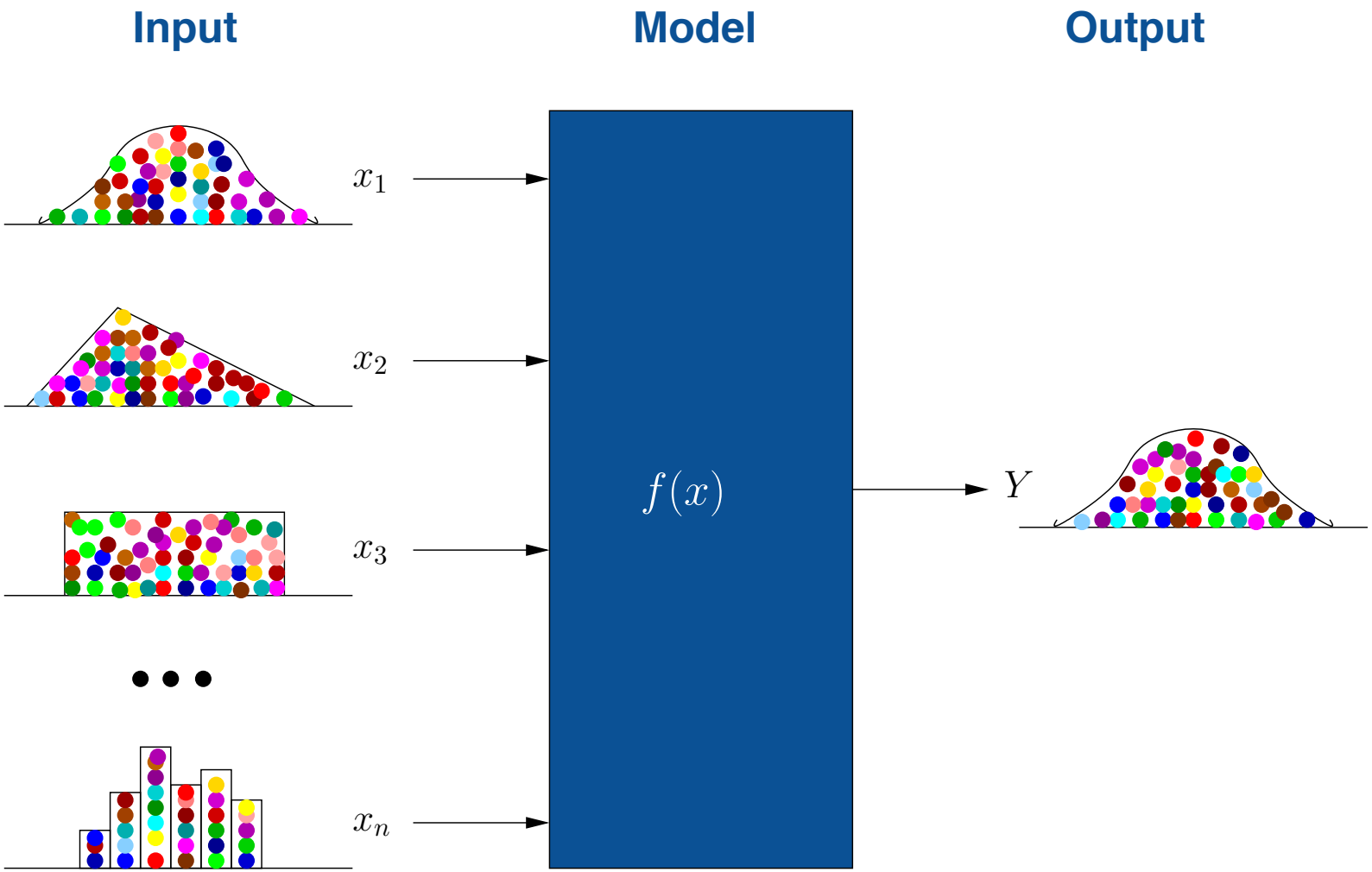

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Variance-based Methods

- Global analysis usually variance-based
- "model-free"
- Analysis on the basis of **sensitivity indices**
  - *Main* or *first-order effect*
  - *Higher-order effects*
  - *Total effect*
- Decomposition in *main effects* and *interactions*

$$y = f(x) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{i} \sum_{j>i} f_{ij}(x_i, x_j) + \ldots + f_{1,2,\ldots,k}(x_1, x_2, \ldots, x_k)$$

Variance-based Methods

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For example $k = 3$:

$$f(x) = f_0 + f_1(x_1) + f_2(x_2) + f_3(x_3) + f_{12}(x_1, x_2)$$

$$+ f_{13}(x_1, x_3) + f_{23}(x_2, x_3) + f_{123}(x_1, x_2, x_3)$$

Main Effect

Decomposition of the variance V of $f(x)$

$$\mathrm{Var}(y) = V = \sum_{i=1}^{k} V_i + \sum_i \sum_j V_{ij} \sum_i \sum_j \sum_k V_{ijk} + \ldots + V_{1,2,\ldots k}$$

First-order sensitivity coefficient

$$S_i = \frac{V_i}{V}$$

(Factors Priorisation)

Higher-Order Effects

Higher-order indices analog

$$S_{i_1 \ldots i_o} = V_{i_1 \ldots i_o} / V$$

With $k = 4$:

2. order: $S_{12}, S_{13}, S_{14}, S_{23}, S_{24}, S_{34}$
3. order: $S_{123}, S_{124}, S_{134}, S_{234}$
4. order: S_{1234}

Total Effect

- Sum of the first- and higher-order effects of x_i
- Removal of the non-relevant parts

$$S_1 + S_2 + S_3 + S_4 + S_{12} + S_{13} + S_{14} + S_{23} + S_{24} + S_{34} + S_{123} + S_{124} + S_{134} + S_{234} + S_{1234} = 1$$

(Factors Fixing)

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$$S_{T1} = S_1 + S_{12} + S_{13} + S_{14} + S_{123} + S_{124} + S_{134} + S_{1234}$$

(Factors Fixing)

Total Effect

- Sum of the first- and higher-order effects of x_i
- Removal of the non-relevant parts

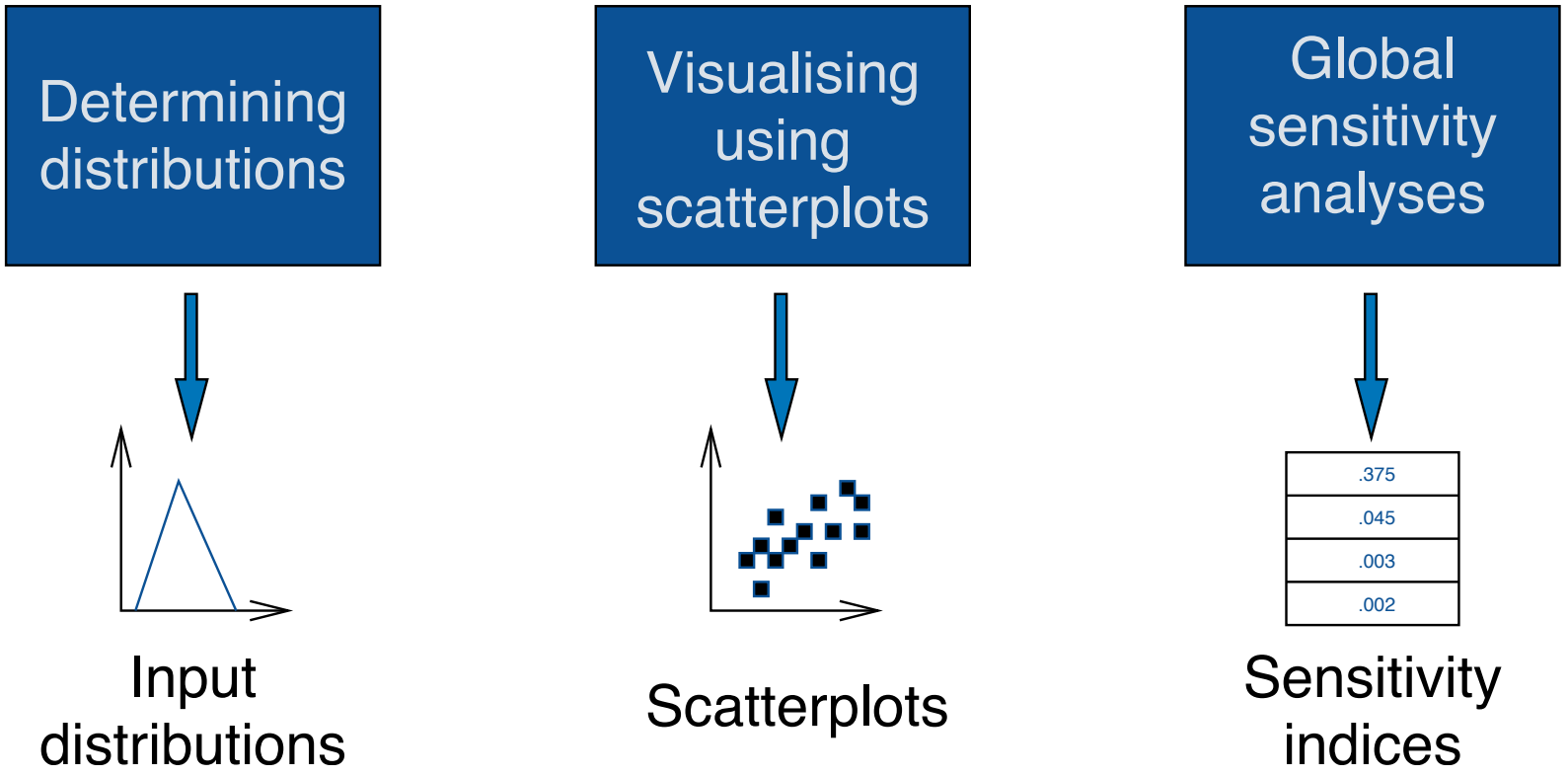
$$S_1 + \cancel{S_2} + \cancel{S_3} + \cancel{S_4} + S_{12} + S_{13} + S_{14} + \cancel{S_{23}} + \cancel{S_{24}} + \cancel{S_{34}} + S_{123} + S_{124} + S_{134} + \cancel{S_{234}} + S_{1234} = 1$$

$$S_{T1} = S_1 + S_{12} + S_{13} + S_{14} + S_{123} + S_{124} + S_{134} + S_{1234}$$
$$S_{T2} = S_2 + S_{12} + S_{23} + S_{24} + S_{123} + S_{124} + S_{234} + S_{1234}$$

(Factors Fixing)

Approach for SE

Overview of the Approach



Determining Distributions

- Distribution of the values of the input parameteters
- Derived from
  - scientific literature
  - physical boundaries
  - expert opinion
  - surveys
  - experiments

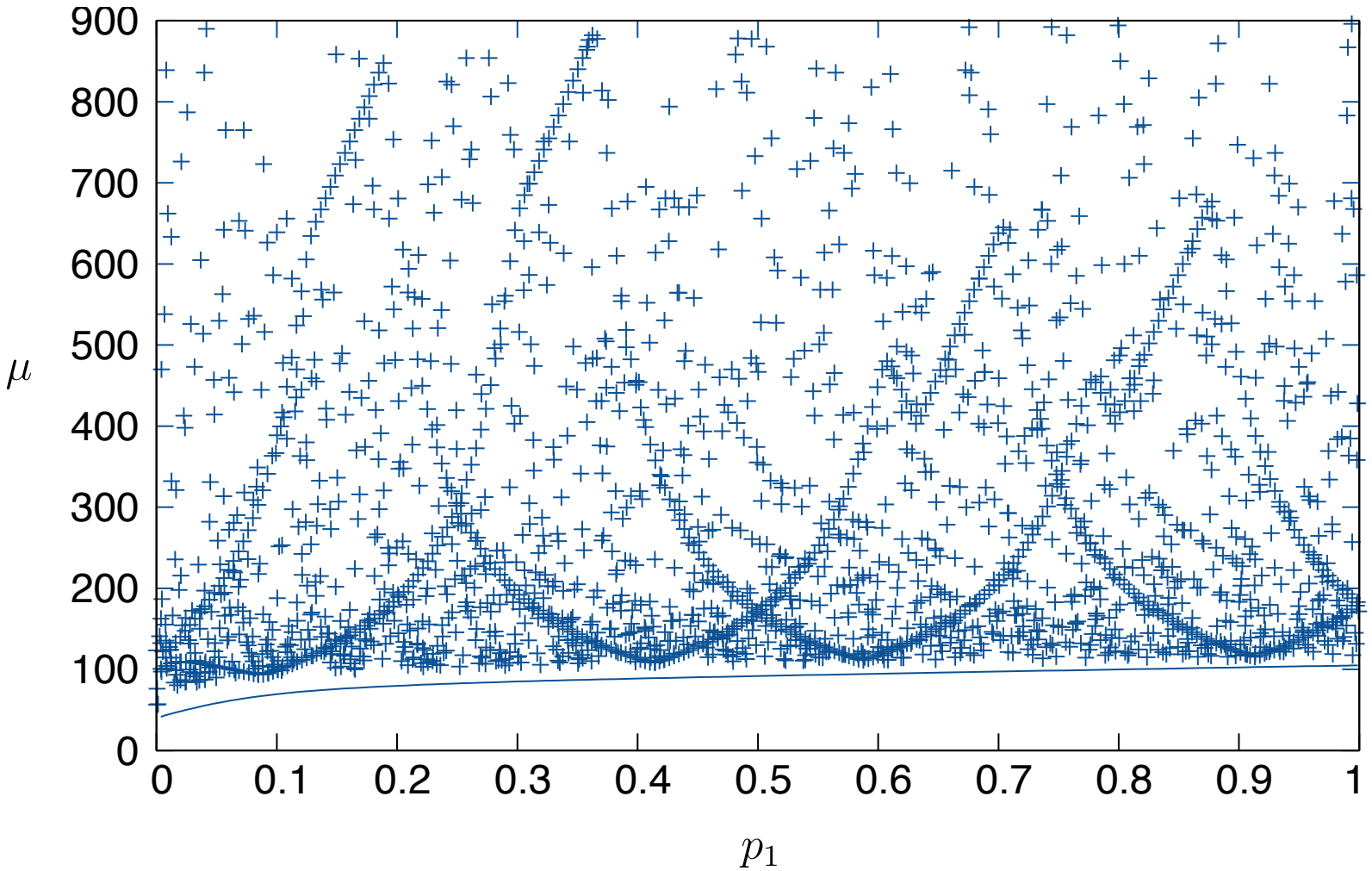
Example: Distributions

- Based on expert opinion
- General parameter
  - p_1 uniformly distributed between 0 and 1
  - d uniformly distributed between 0.9 and 1
- Dependent on application
  - *inf* e.g.. uniformly distributed between 50 and 500
  - For t e.g. $t \sim \mathcal{N}(1000, 200)$

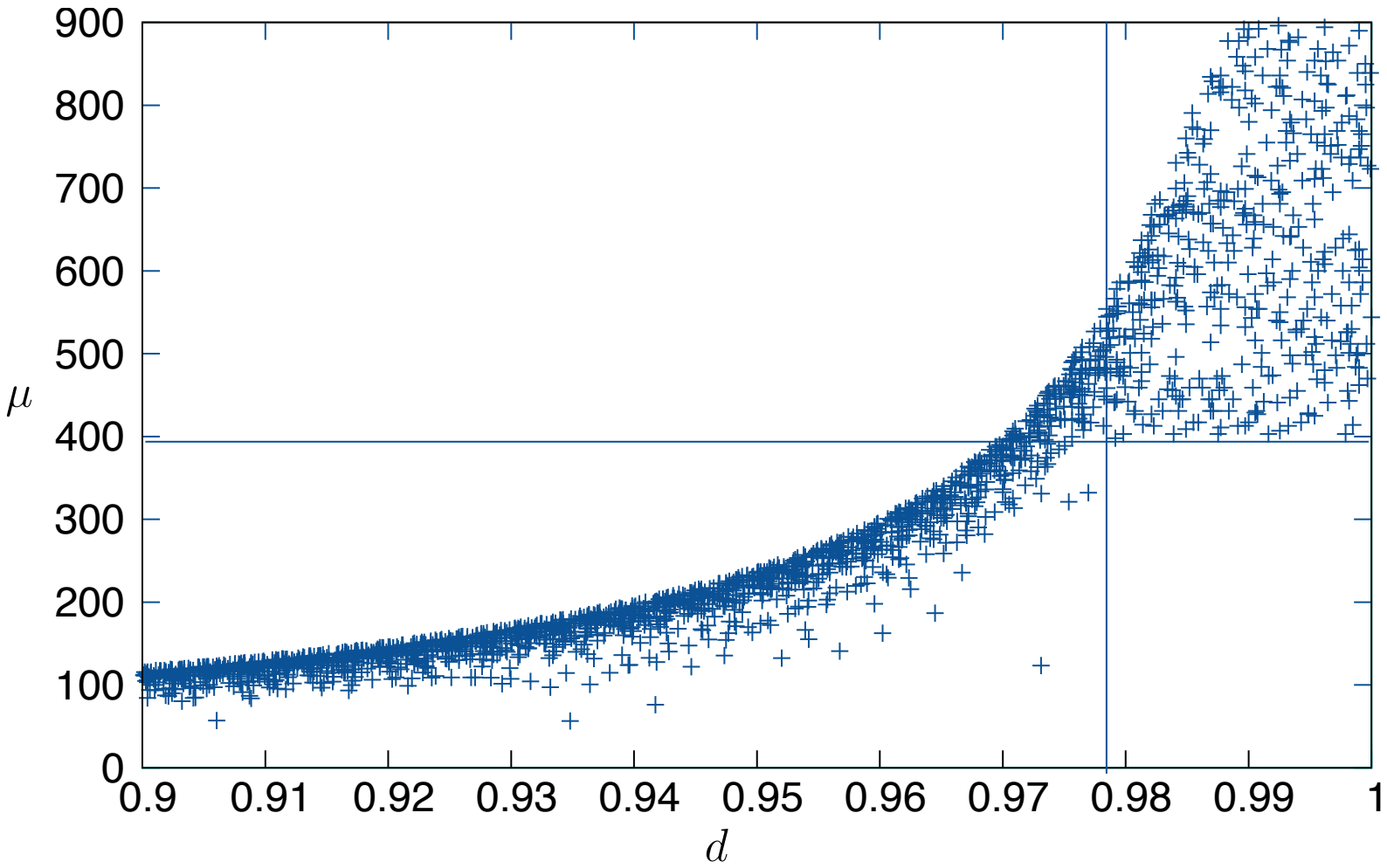
Visualising Using Scatterplots

- Sampling using the distributions
- Pairwise relationship between factors
- Detection of errors in the model
- Indications of influences

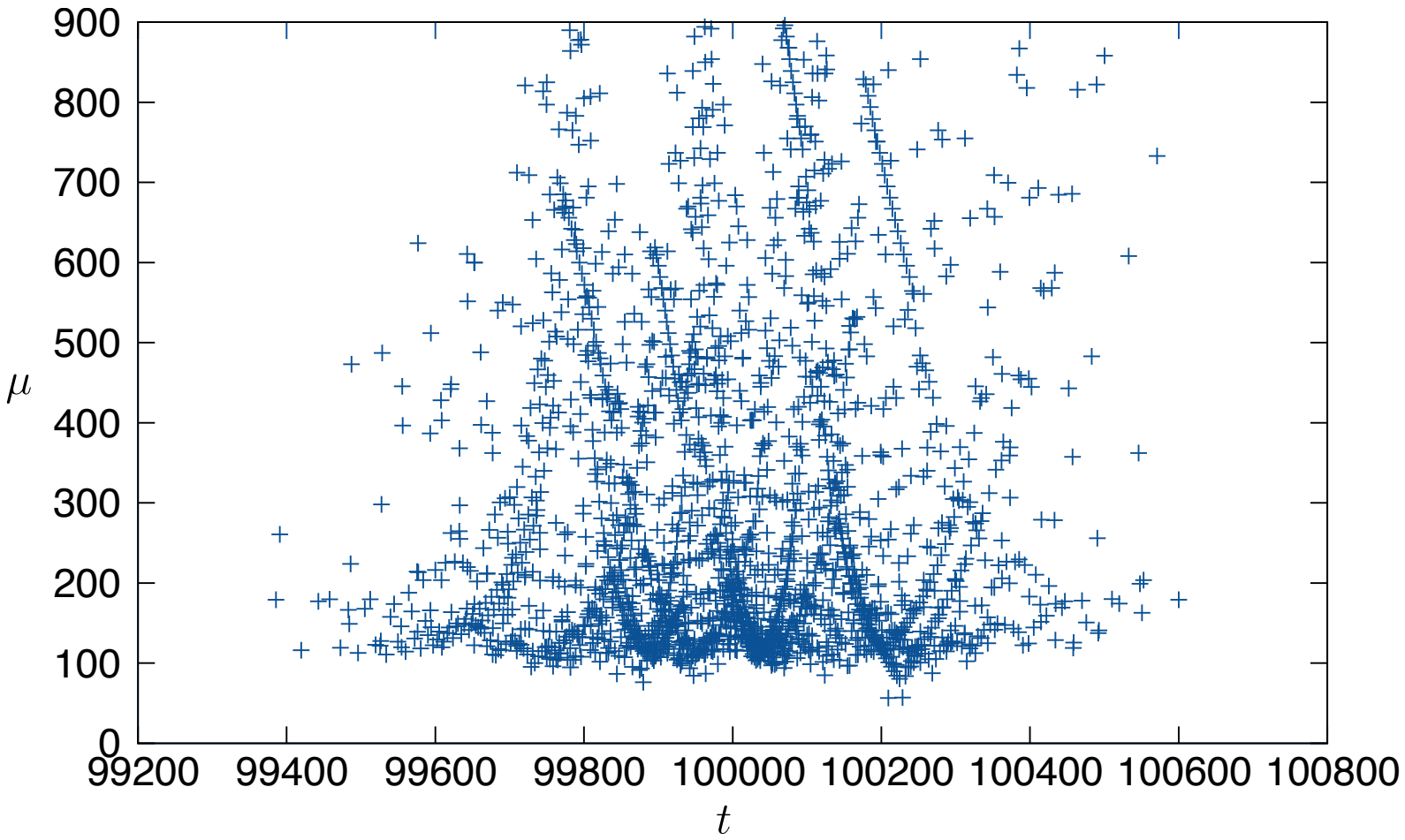
p_1 and μ



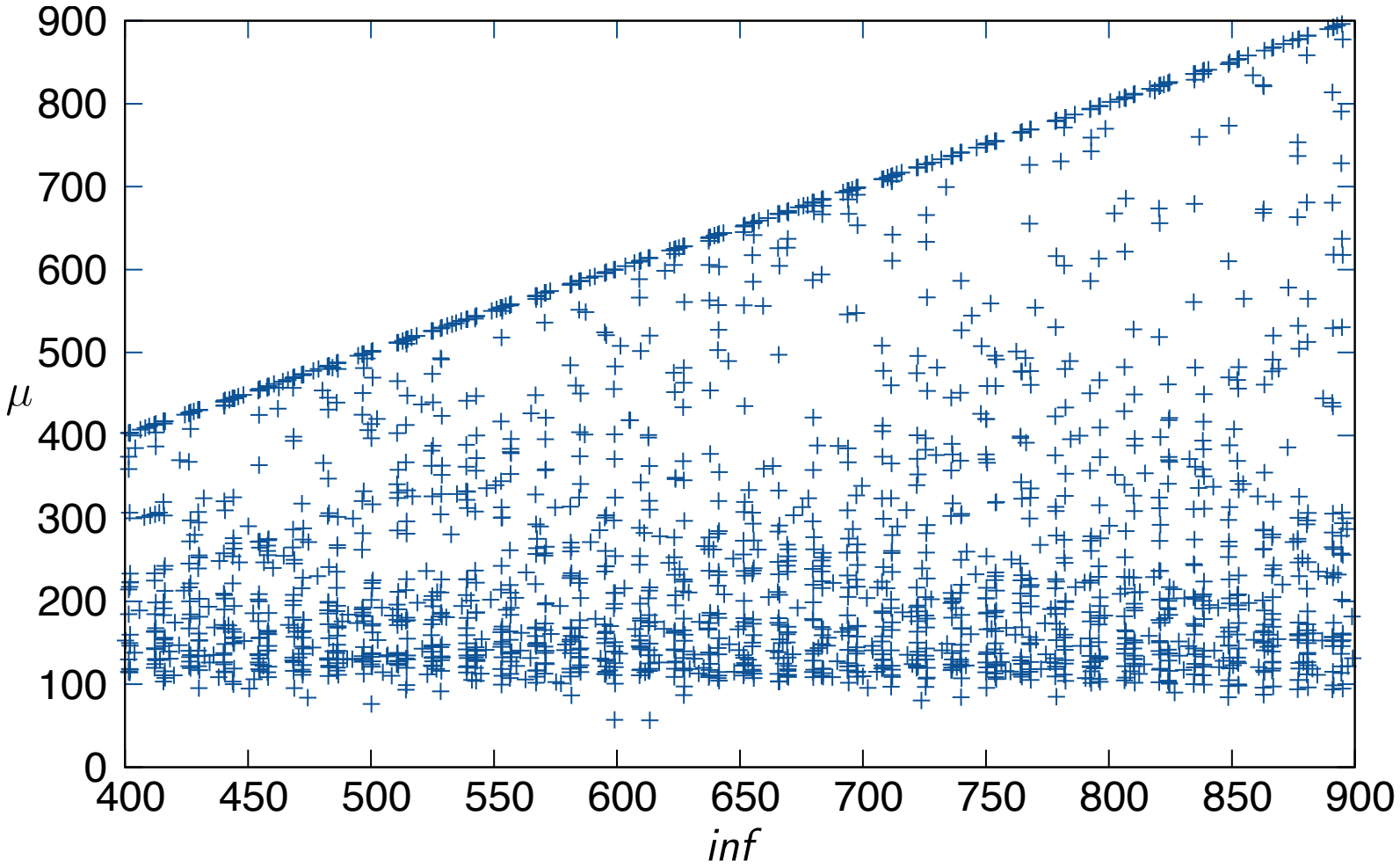
d and μ



t and μ



inf and μ



Applying Global SA

- Various methods for calculating indices
- FAST (Fourier Amplitude Sensitivity Test) gives quantitative results
- Uses sampled data
- Tool-support: Simlab

Main Effect

t	6.88e-5
p_1	0.0090
d	0.9026
inf	0.0158

Indizes

Main Effect

t	6.88e-5
p_1	0.0090
d	0.9026
inf	0.0158

Total Effect

t	0.005788
p_1	0.022365
d	0.975155
inf	0.086735

Summary

Conclusions

- Sensitivity analysis useful tool for predictor models
- SA can help to
  - find errors in the models
  - simplify the models
  - identify interactions between input parameters
  - identify parameters that should be investigated more
  - get more robust predictions
- Experience with
  - reliability model
  - QA economics model
  - process model
  - expert system for IT tools
- Good tool-support available (Simlab: `http://simlab.jrc.cec.eu.int/`)