Risk Assessment Sensitivities for Very Low Probability Events with Severe Consequences

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Abstract — Modern aerospace systems are typically designed to satisfy numerous very stringent performance requirements. The risks posed thus encompass very low probability events with severe consequences. Long-term operational experience with these systems may produce no occurrences of the events, yet the risk of the severe consequence remains and good decision making requires understanding of the risk. With no observed events, classical methods cannot be used to assess these risks quantitatively. Conditional methods may however be used very effectively to produce assessments of the risk when no actual events have been observed, without the use of questionable assumptions.

This report develops a quantitative risk assessment for astronaut bone fracture during an extended microgravity exposure using these conditional methods. The microgravity environment experienced by astronauts in space is well known to reduce astronaut bone mineral density, approximating osteopenic and osteoporotic conditions that significantly increases risk of fracture for the elderly here on earth. The risk of bone fracture for astronauts working in a microgravity environment is believed to similarly increase. At this point, it is entirely unknown what the effects of a broken bone would be in a space environment, whether healing could occur or not, and how life threatening a fracture would be in the space environment. To date, no astronaut has ever broken a bone during microgravity space operations. Conditional methods are used to develop a quantitative risk assessment for consideration of extending mission durations for the International Space Station, and for risk assessments for long duration missions to Mars. The effect of a hypothetical bone fracture is further investigated as to how a single event can affect the risk assessment. [1] [2] [3]

[1] 978-1-4244-3888-4/10/$25.00 ©2010 IEEE.
[2] IEEEAC paper #1239, Version 3, Updated January 5, 2010
[3] This report is a significant update of a previous work [1]. It provides a more complete analytical development, expands the discussion of application of the required numerical methods, and expands it to an exploration of the sensitivity of the results should event data become available.

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1. INTRODUCTION

Risk assessment essentially answers one simple question:

Based on the available data and information, how sure can we be that the risk of realizing an unacceptable consequence is higher than some acceptable level?

If the risk assessment produces a sufficiently high assurance that the risk of realizing an unacceptable consequence is too high, then the decision is made to expend resources to mitigate the risk. The object of mitigation is to reduce the assurance that the risk is unacceptably high. This reduction of risk assurance level is equivalent to increasing the assurance that the risk of realizing an unacceptable consequence is below an acceptable level. The assurance level is measured using probability, and is produced by inference from processing the available data using a statistical procedure. Mitigation of risks with severe consequences can be very expensive for relatively minor reductions in the consequences. Decisions for mitigation then become very important, and assessments of the risk to be used in the decision should use every piece of data or information possible.

Performing a risk assessment to determine the assurance level for risks for very low probability events with severe consequences can be challenging. For such rare risks, very few events should be observed, if any, during testing and operation of the system. Aerospace engineers face this dilemma often when dealing with very stringent probabilistic requirements, such as those encountered in safety, reliability, and other of the engineering specialties.

Systems for space travel, nuclear power plants, air transportation, and information security among others are subject to risks of events that may have very severe consequences. These systems are designed by engineers intentionally to keep the probability of realizing these events very low.

An example of such a low probability severe consequence risk is the risk of astronaut bone fracture during microgravity environment space missions. Bone fractures on earth can be life threatening; in the microgravity environment of a space mission, there is no reason to suggest that they could be any less life threatening. The consequence of an astronaut bone fracture in a microgravity environment could be very severe [2] even beyond the severity of a bone fracture on earth. Clearly, an astronaut with a broken bone will most likely be less effective in performance of tasks for the mission. The healing process of a broken bone in a space environment is not well understood [3], and it is not known if or how a broken bone will heal in a microgravity environment. Returning an astronaut with a bone fracture to earth, exposing that astronaut to the strong g-forces and vibrations of reentry, could be very perilous to the astronaut's survival not to mention very painful.

To make matters worse, astronauts have been observed to experience loss of bone mineral density as a function of exposure to the microgravity environment during space missions [4]. The bone mineral density loss characterized by the diseases of osteoporosis and osteopenia due to aging on earth is well known to increase risk of fracture. Of course, the microgravity environment encountered by astronauts in orbiting spacecraft may be more benign towards bone fractures than the one gravity of the earth. Many fractures for the osteoporotic elderly on earth occur as a result of falls. Astronauts do not fall in microgravity, yet very small forces can propel an astronaut into a bulkhead with appreciable speed. Astronauts also move around large masses in a microgravity environment, which still retain substantial inertia that could fracture a weakened bone. Compressive fractures also trouble the earthbound elderly, due simply to the force of gravity on weakened bones. Orbiting astronauts should not experience such compressive fractures. Regardless of whether astronauts in microgravity environments are subject to more or less risk of experiencing a bone fracture than when on earth, medical care for life threatening fractures is much more quickly and readily available here on earth.

To date, no astronaut has ever broken a bone during microgravity exposure. As of May 2005, there were 977 astronaut missions with microgravity exposures of up to 434 days. Intuitively, this suggests that the risk of fracture during microgravity exposure may be low, but how low? Classical statistical recipes to infer point estimates of this risk require actual observed bone fracture events. Classical statistical methods thus cannot be used to assess risk for the very low probability of the severe consequence of an astronaut bone fracture during a microgravity exposure. Other questions are also of interest when considering moon and Mars missions: how much will risk of bone fracture increase for astronauts on these longer missions? Suppose that International Space Station mission durations were to be extended from 180 days to a full year? How much would the risk of astronaut bone fracture increase?

This report focuses on the aspect of this problem related to the decision on risk mitigation. Statistical decision theory approaches the assessment of risk using Bayesian methods [5], not classical statistical recipes that produce point estimates. Bayesian methods naturally process all observed event data, all censored data (such as non-observance of an event over some period), and any information concerning the risk to produce a full model for the uncertainty about the risk. From this uncertainty model, the assurance level needed for any risk mitigation decision may be computed by integration. Where classical statistical recipes cannot process the observations that there have been 977 astronaut microgravity environment missions without a bone fracture, Bayesian methods use this information naturally to produce a quantitative risk assessment.

With a quantitative risk assessment based on a preponderance of censored data, the question arises as to what effects a single bone fracture event might have on risk. This report continues with a sensitivity analysis considering hypothetical single bone fracture events added to the data set, evaluating the impacts of the single event on the risk assessment.

2. METHODS

The fundamental purpose of any risk assessment is to enable a decision whether to mitigate the risk or not. Statistical decision theory uses Bayesian methods to infer from the available data an uncertainty model for the risk of interest. The assurance that the risk is above or below some specified level is computed as inferred from the available data by integration of this uncertainty model.

To obtain the uncertainty model for the risk of interest, it is necessary to first choose a probability distribution model to represent the uncertainty about the underlying process that would produce observed events. The choice of probability distribution model should be based on the physics of the phenomenon that would produce observed events. The risk of a broken bone can be considered as a failure process, where a broken bone is considered a failure of the human skeletal system. The most general uncertainty model for a failure process is the Weibull model [6] [7]. The density function for the Weibull model is displayed in equation (1).

$$pd(t_f \mid t_1, \eta, \beta) = \left(\frac{\beta}{\eta}\right)\left(\frac{t_f - t_1}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_f - t_1}{\eta}\right)^{\beta}} \quad (1)$$

The Weibull density function, which has a location parameter t_1, a scale parameter η, and a shape parameter β. The Weibull model is a rather nice model for reliability related problems in that the parameters all have physical meanings. This is not the case for many probability distribution models. The location parameter t_1 represents the time before which failures for this report, a bone fracture) cannot occur, and is called the failure-free time. The scale parameter η is the time at which 63.2% of all failures will have occurred, and is called the critical life. The shape parameter β is an indicator of failure mode. Values of $\beta < 1$ indicate an infant mortality failure mode. Values of $\beta = 1$ indicate a useful life failure mode. Values $1 < \beta < 4$ indicate an early wearout failure mode. And, values of $\beta > 4$ indicate an old age failure mode.

An interesting historical note, and an important caveat relative to this density formulation: Weibull's original paper [6] published in September 1951 provided a distribution function that would produce the density function in equation (2).

$$pd(t_f \mid t_1, \lambda, \alpha) = \left(\frac{\alpha}{\lambda}\right)\left(t_f - t_1\right)^{\alpha-1} e^{-\frac{\left(t_f - t_1\right)^{\alpha}}{\lambda}} \quad (2)$$

In discussions of Weibull's paper [7] published in June 1952, Weibull noted that his original distribution function was incorrect by stating that the "…parentheses are an awkward misprint." Correction of this misprint produces the density function in equation (1).

The significance of this typographical error is profound. First, equation (2) cannot be properly reparameterized to produce equation (1); the density function in equation (2) is fundamentally flawed since neither λ nor α can be classed as proper location, scale, or shape parameter. Second, textbooks [8] [9] exist that use the incorrect density function in equation (2) for the Weibull model. And, third, there are statistical software packages and tools [10] [11] that use the incorrect density function in equation (2) for the Weibull model. These textbooks and statistical software packages and tools apparently did not reference the discussion of Weibull's paper in 1952 in the literature where Weibull admitted the typographical error. The caveat for the reader of this report is that whenever encountering any work using the Weibull model, and when considering any software package or tool, it is imperative to verify that the implementation of the Weibull model uses a form expressible as equation (1). The results obtained in any analytical work or through use of a software package that uses a form expressible as equation (2) will be pathological.

For this work presented in this report, the location parameter t_1 in equation (1) is set to zero, eliminating it from all uses of equation (1). There exists no reason to believe that an astronaut could not fracture a bone at the beginning of microgravity operations.

The Bayesian method develops the joint uncertainty model for the two Weibull parameters, η and β, based on the available data, through a simple yet powerful equation based on Bayes' Law [12].

$$pd\left(\eta, \beta \mid data\right) \propto pd\left(data \mid \eta, \beta\right) pd\left(\eta, \beta\right) \quad (3)$$

In equation (3), $pd(data|\eta,\beta)$ is the *likelihood*. When the data is limited to only fracture event times, this is the same likelihood function used in calculating maximum likelihood estimates. $pd(\eta,\beta)$ is the joint *prior* density for η and β. The joint *prior* density is selected to model of knowledge or ignorance of η and β before obtaining the data. The proportionality in the equation is insignificant – the proportionality constant can always be calculated by integrating over all values of η and β. $pd(\eta,\beta|data)$, the joint density of η and β given the data, is called the joint *posterior* density. [4]

Selection of the *prior* model, to model knowledge or ignorance of the joint density of η and β before obtaining the data, can pose some difficulties. Some decision makers feel that using a priori knowledge of the parameters somehow prejudices the results, casting the pall of a rigged decision subject to second-guesses. Beyond that, for many uncertainty models that might be selected for the data for various problems, the parameters have no useful physical meaning, and thus no reason exists to have any a priori knowledge. To address both of these difficulties, it is possible to use a *prior* density model that imparts no a priori knowledge of η and β. These models for the *prior* are called variously non-informative *priors* [12], ignorance *priors* [13], Jeffrey's *priors* [12], maximum entropy *priors* [14], and reference *priors* [15]. As discussed in each reference, these uncertainty models provide the least possible information about the parameters in the inference, and thus provide maximum objectivity to the statistical inference of the *posterior* model. [15] provides a list of these objective models for most of the commonly used phenomenological models.

The joint *prior* density model is structured such that η and β are independent; this merely means that $pd(\eta,\beta) \propto pd(\eta)pd(\beta)$. The uncertainty models that provide the utmost objectivity for the *posterior*, based on using the

[4] The earliest literature on Bayesian methods used the terms "*a posteriori*" and "*a priori*" density models respectively for "*posterior*" and "*prior*" density models. Virtually all of the literature from the 20th and 21st centuries use the more colloquial terms used in this report.

Weibull model for the failure process uncertainty, are presented in equations (4).

$$pd(\eta) \propto \frac{1}{\eta}; \quad pd(\beta) \propto \frac{1}{\beta}$$
$$pd(\eta,\beta) \propto \left(\frac{1}{\eta}\right)\left(\frac{1}{\beta}\right) \tag{4}$$

Now, given as data N_f fracture event times and N_s survivors (mission durations when no fracture occurred), the *posterior* density model is formed using the Weibull distribution with t_{fi} being the i^{th} fracture time, and t_{sj} being the j^{th} survivor time using equation (3).

$$pd(\eta,\beta \mid data) \propto \left(\prod_{i=1}^{N_f}\left(\frac{\beta}{\eta}\right)\left(\frac{t_{fi}}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_{fi}}{\eta}\right)^{\beta}}\right) * \left(\prod_{j=1}^{N_s} e^{-\left(\frac{t_{sj}}{\eta}\right)^{\beta}}\right) * \left(\frac{1}{\eta}\right) * \left(\frac{1}{\beta}\right) \tag{5}$$

In equation (5), the first term to the right of the proportion sign is the likelihood for the fracture event data, the second is the likelihood for the survivor data, and the two remaining terms are the Jeffrey's *priors* for η and β. One very nice feature of conditional inferential methods apparent in equation (5) that is that survivor data can be used directly via the likelihood [16]. Mission durations when no fracture has occurred comprise very important information that should not be neglected in the *posterior*. For the problem central to this report, there are only survivor data; there have been no astronaut fracture events at all. Conditional inferential methods provide solutions for these data sets; such solutions are not possible without questionable assumptions using classical methods.

Using the Weibull model for the uncertainty for when an astronaut might fracture a bone during a microgravity space mission, the risk of an astronaut bone fracture during a microgravity duration T_M is simply the cumulative probability.

$$R_{T_M} = P\left(T_f \le T_M \mid \eta,\beta\right) = 1 - e^{-\left(\frac{T_M}{\eta}\right)^{\beta}} \tag{6}$$

In equation (6) however, η and β are uncertain with the model formulated in equation (5) based on the data. The joint uncertainty model for η and β based on the data in equation (5) transforms equation (6) into an uncertainty model for the risk of bone fracture during a mission of duration T_M based solely on the data. Equation (7) shows how the joint uncertainty for η and β is marginalized out to provide the uncertainty for this risk as inferred by the data.

$$pd\left(R_{T_M} \mid t_f, t_s\right) \propto \int_0^{\infty}\int_0^{\infty}\left[R_{T_M} * pd\left(\eta,\beta \mid t_f, t_s\right)\right] d\eta\, d\beta \tag{7}$$

To complete the risk assessment, it is necessary to calculate the assurance that the risk of a bone fracture during a microgravity duration T_M is below some acceptable level R_{acc} as in equation (8).

$$P\left(R_{T_M} \le R_{acc} \mid t_f, t_s\right) = \int_0^{R_{acc}} pd\left(R_{T_M} \mid t_f, t_s\right) dR_{T_M} \tag{8}$$

Substitutions of equations (5) and (6) into equations (7) and (8) produce rather long and messy equations. As will be demonstrated later in this report, these substitutions are not really necessary. However, it is important to note that these substitutions produce integrands that are not analytically integrable.

The solution to evaluating the integral in equation (8) is to use numerical methods to approximate the integral, specifically Monte Carlo methods. By Monte Carlo sampling the joint uncertainty model for η and β in equation (5) N times, it is possible to approximate the integral in equation (8) in two simple steps. The first step is to evaluate equation (6) at each joint sample of η and β, obtaining N samples of the actual risk of bone fracture during the mission. The second step is to count the number of resulting samples less than R_{acc}, and to divide this count by N. With reasonably large values of N, the approximation for the integral to obtain the assurance level can have several digits of accuracy. Numerical approximation accuracy via Monte Carlo methods of three decimal digits, to an assurance level accuracy of 0.1%, is more than enough for most mitigation decisions.

One difficulty with this process remains: ordinary Monte Carlo methods require sampling of recognizable probability distribution models. The *posterior* model formulated in equation (5) is not a recognizable probability distribution model. This is very typical of conditional formulations for most real world risk assessments, whether for low probability events or not. Between the non-integrability of equations (7) and (8), and the non-recognizability of the uncertainty model in equation (5), most engineers abandon conditional approaches altogether and resort to qualitative risk assessments. The solution to this difficulty is to use Markov Chain Monte Carlo (MCMC) methods to sample the *posterior* model in equation (5) [17]. MCMC methods allow full range sampling of non-integrable, improper, and unrecognizable distributions given a formulation for the

joint density [18]. The formulation of the *posterior* model in equation (5) is all that is required to obtain a full quantitative risk assessment for the very low probability of an astronaut bone fracture during a microgravity mission.

3. RESULTS

Astronaut Microgravity Exposure Data

As mentioned earlier, no astronaut has ever fractured a bone during a microgravity exposure. As of May 2005, when this risk assessment was performed, 977 astronaut microgravity exposures had occurred without any bone fractures. The count today is over 1,000 astronaut microgravity missions, and still no astronaut has fractured a bone during microgravity exposure. Figure 1 shows the durations of these 977 astronaut microgravity exposures that include all NASA, Chinese, and Russian flights.

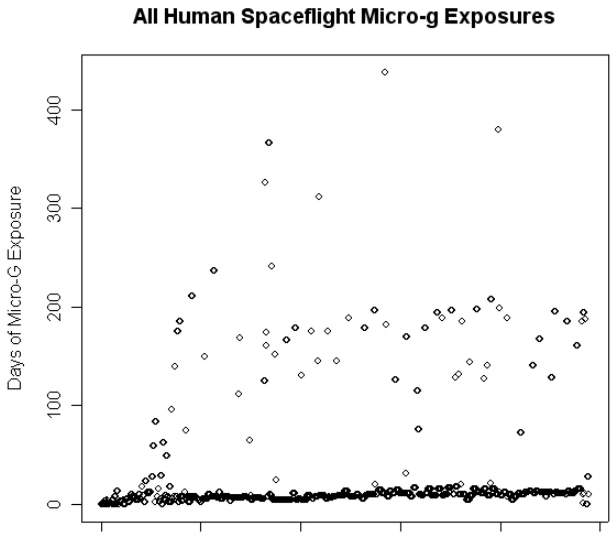

Figure 1 — The scatterplot of historical astronaut exposures to micro-gravity (May 2005) range from hours to over 400 days.

In Figure 1, the longest microgravity exposure was 434 days. Most of the exposures of more than 100 days were International Space Station or Russian space station missions, and the many crowding around 10-14 days are from space shuttle missions.

Formulation of the posterior model

The *posterior* model using these data from Figure 1 is a simplified version of equation (5), there are no bone fracture times to appear in the likelihood.

$$pd\left(\eta,\beta \mid t_s\right) \propto \left(\prod_{j=1}^{977} e^{-\left(\frac{t_{s_j}}{\eta}\right)^{\beta}}\right) * \left(\frac{1}{\eta}\right) * \left(\frac{1}{\beta}\right) \quad (9)$$

Monte Carlo Sampling

This relatively simple formula in equation (9) for the *posterior* model is neither integrable nor proper. When using Markov Chain Monte Carlo sampling, it is common to experience difficulty tuning the Markov chain to obtain good random sampling for non-integrable, improper *posterior* models. Indeed, the Markov chain for the *posterior* model in equation (9) for this risk assessment did not achieve stability for good sampling. The solution to obtain a stable Markov chain is relatively simple, but requires some engineering judgment based on the specific risk assessment at hand. The solution is to use pseudo-ignorance *prior* models [19], essentially truncating the *prior* model at reasonable points.

For this risk assessment, it is expected that as the bone mineral density loss increases due to increases in mission duration, the risk of fracture increases. Values of the Weibull β parameter greater than unity produce an increasing hazard rate with increasing mission duration. The hazard rate is a measure of the instantaneous probability of fracture as a function of mission duration. Values of the Weibull β parameter greater than about 20 reduce the variance to an unreasonably small value. A small variance would lead to most bone fractures occurring all at the same time, and this is not reasonable. The Weibull η parameter is the critical life value, the mission duration at which 63.2% of all fractures will have occurred. Mission durations with microgravity exposures of more than one year are not being considered. If the Weibull η parameter has a value of more than ten years, the risk of fracture for mission durations of one year or less is essentially negligible. Creating and using a pseudo-ignorance *prior* model by limiting Weibull β parameter values to between unity and 20, and limiting Weibull η parameter values to less than 3,650 days (ten years), produces a nice stable Markov Chain allowing good random sampling of η and β based on the 977 missions without an astronaut bone fracture. Figure 2 shows 100,000 joint samples of η and β obtained from the *posterior* model in equation (9), applying the pseudo-ignorance *prior*.

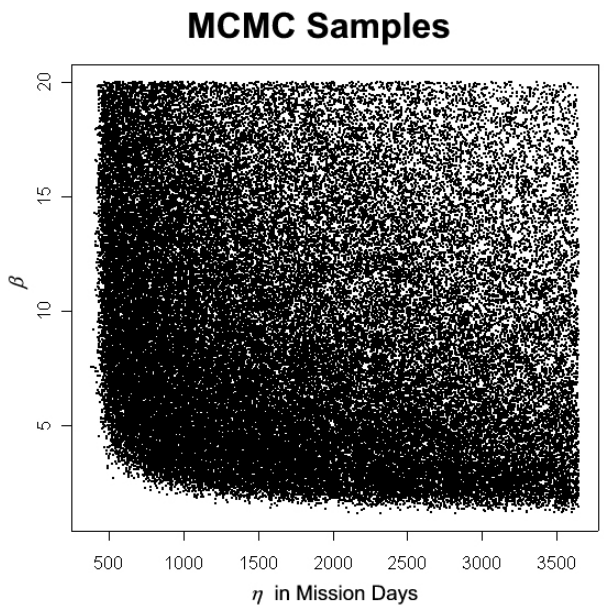

Figure 2 — 100,000 joint samples of η and β obtained from the *posterior* model in equation (9) using MCMC.

Risk Assessment Results

To fully understand how assurance of the risk of bone fracture behaves as a function of mission duration, it is useful to parameterize the risk from equation (8) for different assurance levels R_{acc} as a function of mission duration. Figure 3 demonstrates risk of bone fracture as a function of mission durations out to 1,000 days of microgravity exposure, at assurance levels of 5, 25, 50, 75, and 95%.

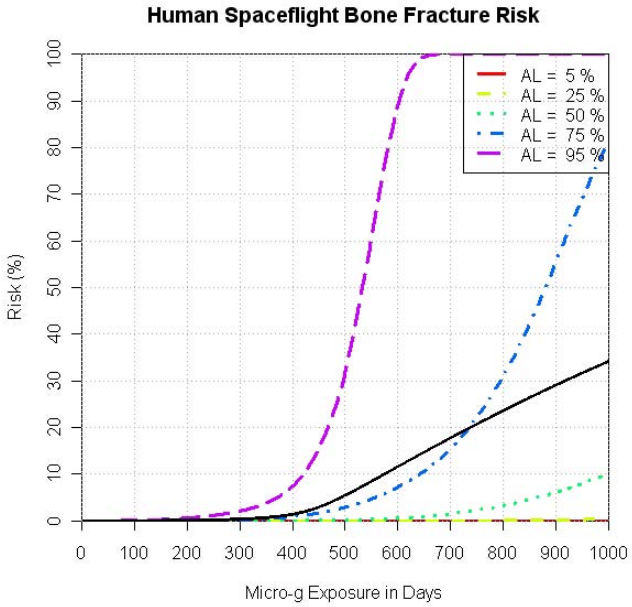

Figure 3 — Risk of bone fracture as a function of microgravity exposure at assurance levels of 5, 25, 50, 75, and 95% based on 977 exposures with no fracture.

The way to interpret Figure 3 is that for the 95% assurance line (long dashed, purple), there is 95% assurance (or probability) based on the data that the bone fracture risk is less than the ordinate value at abscissa value of microgravity exposure; e.g., there is a 95% probability based on the data that the risk of bone fracture for a 500 day microgravity exposure is less than 30%. The solid black line in Figure 3 is the mean risk of bone fracture based on the data as a function of microgravity exposure. Note that the mean risk estimate is misleading. Clearly the assurance level for the mean risk varies as a function of microgravity exposure. Prior to durations of 740 days, the mean risk is at quantiles greater than 75% (above the dash-dot blue line). At durations greater than 740 days, the mean risk is less than 75% (below the dash-dot blue line. This confirms that risk should never be specified using means; there is no way to tell what the assurance level will be to enable good mitigation decisions.

Considering the question of increase of risk associated with an increase of an International Space Station mission from 180 days to 365 days, the comparison is nicely displayed in Figure 4. Figure 4 shows bars that range from the 5th to the 95th quantile, with color saturation proportional to the probability density of the risk.

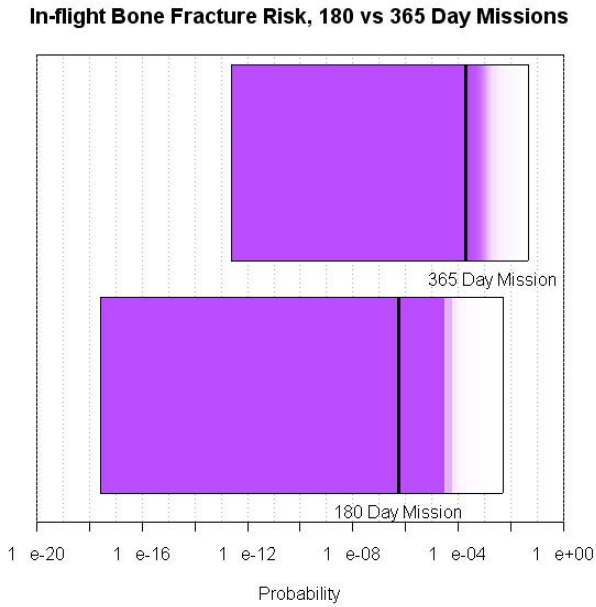

Figure 4 — Conditional inferential methods quantify the increase in bone fracture risk, based on the data, when extending the 180 day space station mission to 365 days.

The heavy black line in Figure 4 is the 50th percentile assurance level. At the 50% assurance level, the risk of bone fracture increases almost three orders of magnitude for a twofold increase of mission duration. This sounds like a dramatic increase in risk, but should be tempered by the fact that for the one year mission, the data provides a 50% assurance that the risk of bone fracture is still less than 3

hundreds of one percent, not much of a risk at all. Note also that the deepest color saturation of the bars is towards smaller risk values. This is very good news, most of the probability density is at very low risk values.

Risk Sensitivity Analysis

The risk results presented in figures 3 and 4 provide bounds on the risk distributions when no actual event data are available. While the numbers of censored data, the survivors, was rather high, it is of interest to study what happens to these risk studies if a bone fracture event were to occur, and see how these bounds on risk will change.

To see these effects, two new sets of data were created by appending to the existing set of survivor data a single fracture event at 50 days and a single fracture event at 300 days. The conditional inferential and MCMC procedure described in this report was executed for each to examine the changes to the risk distributions based on these new data sets.

The first question to consider is what effect if any with the new fracture event have on the MCMC samples. The entire principle behind using conditional inferential methods with objective *priors* is to assure that the results depend on the data. Figures 5 and 6 show respectively the MCMC samples obtained for the data sets including the 50 day fracture event and the 300 day fracture event.

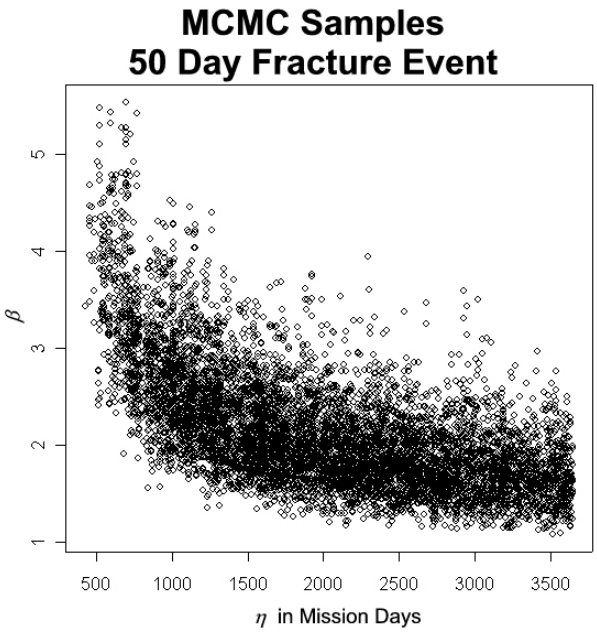

Figure 5 — A single 50 day fracture event added to the 977 survivor data adds significant structure to the joint density of the MCMC samples obtained.

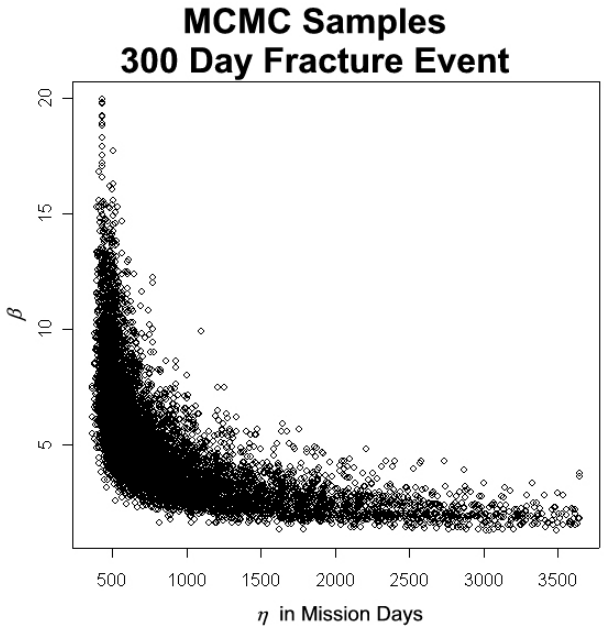

Figure 6 — A single 300 day fracture event added to the 977 survivor data adds even more structure to the joint density of the MCMC samples obtained.

By comparing Figures 5 and 6 with Figure 3, it is clear how dramatically the data drives the MCMC samples obtained. The next question to be considered must be how the risk assurance for fracture risk changes. Figures 7 and 8 show the assurance level risk parameterization resulting from the samples in figures 5 and 6 respectively.

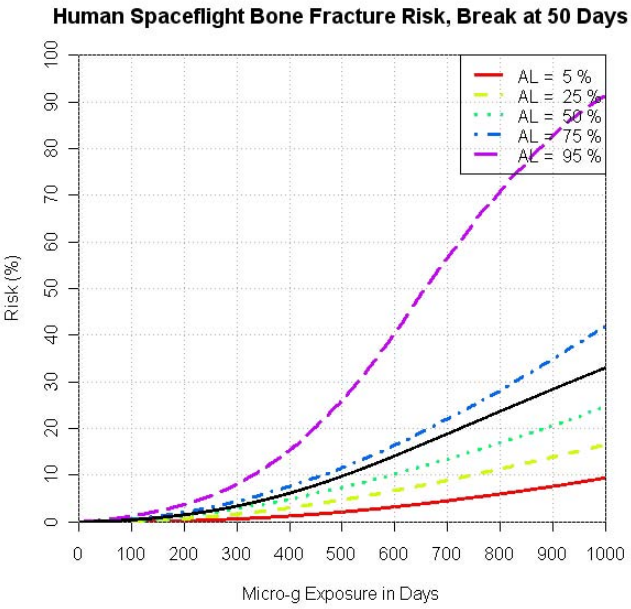

Figure 7 — A single fracture event at 50 days dramatically changes the risk assurance parameterization when added to the 977 survivor data.

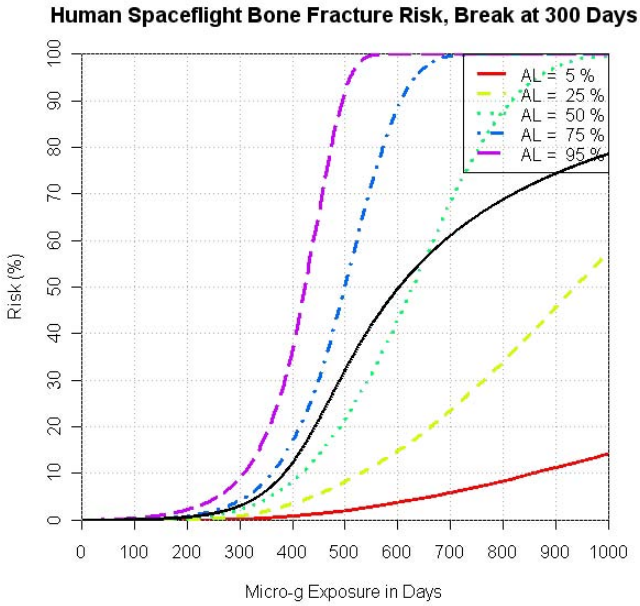

Figure 8 — A single fracture event at 300 days alters the risk assurance parameterization more subtly than an event at 50 days when added to the 977 survivor data.

Intuitively, a fracture event at 50 days would be expected to increase the risks quite a bit at the same assurance levels compared with figure 3. Surprisingly, at 50 days of microgravity exposure, when including the 50 day fracture event, there is a 95% assurance that the risk is less than 1%. This is true even out to 100 days. At 300 days of microgravity exposure, where in figure 4 there was a 95% assurance that the risk was less than 2.5%, with the 50 day fracture event it only increased at the 95% assurance level to 8%. Clearly, while there is some increase, it is not terribly dramatic for a microgravity exposure of one year.

There are two other interesting comparisons between figures 3 and 7. The first is that the mean risk assurance line in figure 7 no longer indicates a radically changing skew as it did in figure 3. This suggests that a single early fracture event may add more risk prediction stability. The second is the risk assurance at 1,000 mission days is at higher risk levels at low assurances, and lower risk levels at high assurance levels. The mean risk at 1,000 mission days in figure 3 is at 35%, where in figure 7 it is lower at 32%. An early event with many longer survivor data actually lowers risk levels at the same assurance levels for very long mission durations.

Considering that the longest mission without a fracture event was 434 days, and that there were only six microgravity exposures over 300 days in the original data set, it was not anticipated that a single event at 300 days would have much effect on the results. However, comparisons of figures 3 and 8 show some dramatic effects. At 300 mission days, the 95% risk assurance is at 9%, even more than it is in figure 7 where the 50 day fracture event was added to the data. All of the risk assurance contours in figure 8 show marked increases beyond 300 days of microgravity exposure. The mean risk in figure 8 increased to 78% at 1,000 mission days where it was only 35% in figure 3. This comparison suggests that amongst all the survivor data, a single 300 day fracture event does not significantly change risk for shorter missions, but assures more severe risks for longer missions.

4. CONCLUSIONS

Four important conclusions result from the work presented in this report.

First, while it is impossible to use Classical statistical methods to perform risk assessments when there are no observed events, it is possible to do so using conditional inferential methods without using any questionable assumptions as presented in this report.

Second, the conditional inferential and numerical methods as presented in this report produce distributions of the assurance that the risk exceeds specified levels. These assurance distributions provide a valid and useful means for comparing risks for a number of relevant questions for aerospace systems, as demonstrated in comparing risks for different mission durations for the International Space Station.

Third, an intuition-based estimate of the risk of an event occurring, when it has not been observed during a large number of missions, naturally would be that the risk is very low. The procedures presented in this report establish quantitative values (vice intuitive and sometimes qualitative guesses) for just how low (or how high) the risk actually is. The quantitative risk actually obtained for this problem using the conditional inferential and numerical methods as presented in this report was much lower than most intuition-based guesses, including this report author's. For risks with severe consequences such as an on-orbit bone fracture, this is very good news. It is no longer necessary to make decisions based on intuitive and qualitative guesses for such problems with aerospace systems with very low probability risks with severe consequences.

Fourth, large numbers of survivor data affect quantitative risk assessments in completely non-intuitive ways. The author of this report has speculated that this could explain why aerospace system decision makers occasionally second guess Classical statistical results that only process small numbers of event data. Use of conditional inferential methods to obtain quantitative risk assessments for low probability events with severe consequences take advantage of the substantial information content in the survivor data, instead of ignoring it as many Classical statistical methods require.

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BIOGRAPHY

Mark Powell has practiced Systems Engineering for over 35 years in a wide range of technical environments including DoD, NASA, DOE, and commercial. More than 25 of those years have been in the aerospace arena. His roles in these environments have included project manager, engineering manager, chief systems engineer, and research scientist. His academic affiliations have included the University of Idaho, Stevens Institute of Technology, and the University of Houston, Clear Lake. Mr. Powell maintains an active engineering and management consulting practice throughout North America, Europe, and Asia. Beyond consulting, he is sought frequently as a symposium and conference speaker and for training, workshops, and tutorials on various topics in Systems Engineering, Project Management, and Risk Management. Mr. Powell is an active member of AIAA, Sigma Xi, the International Society for Bayesian Analysis, and the International Council on Systems Engineering, where he serves as Assistant Director for Systems Engineering Processes.