

Errata to *Unitarity of The Modular Tensor Categories Associated to Unitary Vertex Operator Algebras, I*

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1. In the published version, the annihilation operator \mathcal{Y}_{ii} is misprinted as $\mathcal{Y}_{\overline{ii}}$. (See for instance Eq. (1.41) (1.42).) The notations in the preprints are correct.
2. The proof of Lem. 3.8-(a) is too frivolous. A more serious argument is as follows. One should first understand in which sense (3.20) holds, i.e.

$$\mathcal{Y}_\alpha(w^{(i)}, f)w^{(j)} = \sum_{s \in \mathbb{Z}_V} \hat{f}(s) \mathcal{Y}_\alpha(w^{(i)}, s)w^{(j)}$$

At the beginning, we only know that it holds when evaluated with any vector of W_k . Therefore, it holds in the algebraic completion $\overline{W_k}$ where the RHS converges in the $\sigma(\overline{W_k}, W_k)$ -topology to the LHS.

Equivalently, if for each $\lambda \geq 0$ we let P_λ be the projection of $\overline{W_k}$ onto its L_0 -weight λ subspace, then

$$P_\lambda \mathcal{Y}_\alpha(w^{(i)}, f)w^{(j)} = \sum_{s \in \mathbb{Z}_V} \hat{f}(s) P_\lambda \mathcal{Y}_\alpha(w^{(i)}, s)w^{(j)}$$

where the RHS is a finite sum. One can also replace P_λ with $P_{\leq \lambda}$ where $P_{\leq \lambda} = \sum_{0 \leq \mu \leq \lambda} P_\mu$, noting that all but finitely many μ in $[0, \lambda]$ satisfy that $P_\mu \neq 0$. Namely

$$P_{\leq \lambda} \mathcal{Y}_\alpha(w^{(i)}, f)w^{(j)} = \sum_{s \in \mathbb{Z}_V} \hat{f}(s) P_{\leq \lambda} \mathcal{Y}_\alpha(w^{(i)}, s)w^{(j)} \quad (1)$$

Now one proves Lem. 3.8-(a) as follows. Similar to (3.21), one shows that

$$\sum_{s \in \mathbb{Z}_V} \|\hat{f}(s) P_{\leq \lambda} \mathcal{Y}_\alpha(w^{(i)}, s)w^{(j)}\|_p \leq M_p \|f\|_{V, |p|+t} \|w^{(j)}\|_{p+r} \quad (2)$$

for all λ , noting that the LHS is a finite sum. This, together with (1), shows that

$$\|P_{\leq \lambda} \mathcal{Y}_\alpha(w^{(i)}, f)w^{(j)}\|_p \leq M_p \|f\|_{V, |p|+t} \|w^{(j)}\|_{p+r}$$

Since λ is arbitrary, we conclude that $\mathcal{Y}_\alpha(w^{(i)}, f)w^{(j)}$, a priori only a vector of $\overline{W_k}$, belongs to \mathcal{H}_k^p (for all p).