Errata to *Unitarity of The Modular Tensor Categories Associated to Unitary Vertex Operator Algebras, I*

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1. In the published version, the annihilation operator $\mathcal{Y}_{\bar{i}i}$ is misprinted as $\mathcal{Y}_{\bar{i}\bar{i}}$. (See for instance Eq. (1.41) (1.42).) The notations in the preprints are correct.

2. The proof of Lem. 3.8-(a) is too frivolous. A more serious argument is as follows. One should first understand in which sense (3.20) holds, i.e.

$$\mathcal{Y}_\alpha(w^{(i)},f)w^{(j)}=\sum_{s\in\mathbb{Z}_V}\widehat{f}(s)\mathcal{Y}_\alpha(w^{(i)},s)w^{(j)}$$

At the beginning, we only know that it holds when evaluated with any vector of W_k. Therefore, it holds in the algebraic completion $\overline{W_k}$ where the RHS converges in the $\sigma(\overline{W_k},W_k)$-topology to the LHS.

Equivalently, if for each $\lambda\geqslant 0$ we let P_λ be the projection of $\overline{W_k}$ onto its L_0-weight λ subspace, then

$$P_\lambda\mathcal{Y}_\alpha(w^{(i)},f)w^{(j)}=\sum_{s\in\mathbb{Z}_V}\widehat{f}(s)P_\lambda\mathcal{Y}_\alpha(w^{(i)},s)w^{(j)}$$

where the RHS is a finite sum. One can also replace P_λ with $P_{\leqslant\lambda}$ where $P_{\leqslant\lambda}=\sum_{0\leqslant\mu\leqslant\lambda}P_\mu$, noting that all but finitely many μ in $[0,\lambda]$ satisfy that $P_\mu\neq 0$. Namely

$$P_{\leqslant\lambda}\mathcal{Y}_\alpha(w^{(i)},f)w^{(j)}=\sum_{s\in\mathbb{Z}_V}\widehat{f}(s)P_{\leqslant\lambda}\mathcal{Y}_\alpha(w^{(i)},s)w^{(j)}\tag{1}$$

Now one proves Lem. 3.8-(a) as follows. Similar to (3.21), one shows that

$$\sum_{s\in\mathbb{Z}_V}\|\widehat{f}(s)P_{\leqslant\lambda}\mathcal{Y}_\alpha(w^{(i)},s)w^{(j)}\|_p\leqslant M_p|f|_{V,|p|+t}\|w^{(j)}\|_{p+r}\tag{2}$$

for all λ, noting that the LHS is a finite sum. This, together with (1), shows that

$$\|P_{\leqslant\lambda}\mathcal{Y}_\alpha(w^{(i)},f)w^{(j)}\|_p\leqslant M_p|f|_{V,|p|+t}\|w^{(j)}\|_{p+r}$$

Since λ is arbitrary, we conclude that $\mathcal{Y}_\alpha(w^{(i)},f)w^{(j)}$, a priori only a vector of $\overline{W_k}$, belongs to $\mathcal{H}_k^p$ (for all p).