Visibility

Do not draw what is invisible

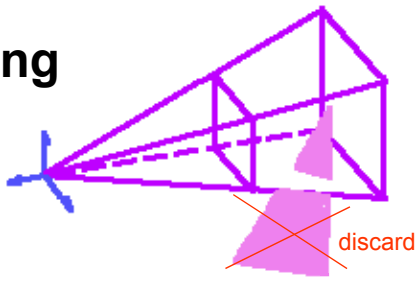
- Due to being outside the view volume
- Due to self-occlusion
- Due to object-to-object occlusion

Visibility

Do not draw what is invisible

- Outside the view volume
  - *Clipping, culling*
- Self-occlusion
- Object-to-object occlusion

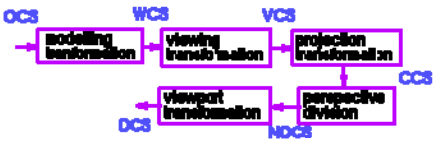
3D Clipping



Keep only what is visible

We can clip in 3 different CSs

1. In the VCS
   What are the six plane equations?
2. In the CCS
   Clipping in homogeneous coordinates
   4D volume bounded by 3D planes
   Still simple and efficient
   Usually done here (!)
   (a.k.a. Clipping Coordinate System)

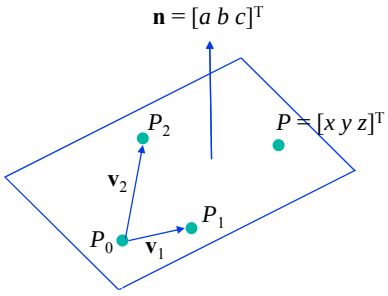


3. In the NDCS
   Singularity at $P_z = 0$

In any case, we must clip lines against planes

Reminder: Planes

Plane equations



- Explicit
  $z = \alpha x + \beta y + \gamma$
- Implicit
  $F(x, y, z) = ax + by + cz + d = \mathbf{n} \bullet P + d$
  Points on Plane: $F(x, y, z) = 0$
- Parametric
  $\text{Plane}(s,t) = P_0 + s(P_1 - P_0) + t(P_2 - P_0)$
  P_0, P_1, P_2 are not collinear
  or
  $\text{Plane}(s,t) = P_0 + s\mathbf{v}_1 + t\mathbf{v}_2$, where $\mathbf{v}_1$, $\mathbf{v}_2$ are basis vectors
  Convex combination defines a triangle:
  $\text{Triangle}(s,t) = (1 - s - t)P_0 + sP_1 + tP_2$, with $s, t \in [0,1]$

Intersection of Line and Plane

Implicit equation for the plane:

$$F(P) = \mathbf{N} \cdot P + D = 0$$

Parametric equation for the line from P_a to P_b:

$$L(t) = P_a + t(P_b - P_a)$$

Plug $L(t)$ into $F(P)$ and solve for $t = t_i$:

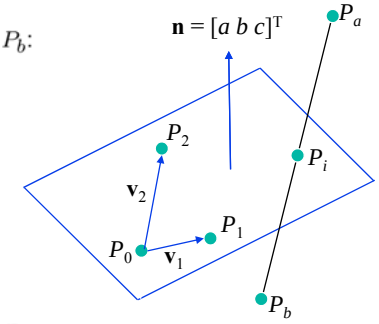
$$\mathbf{N} \cdot [P_a + t_i(P_b - P_a)] = -D$$

Therefore,

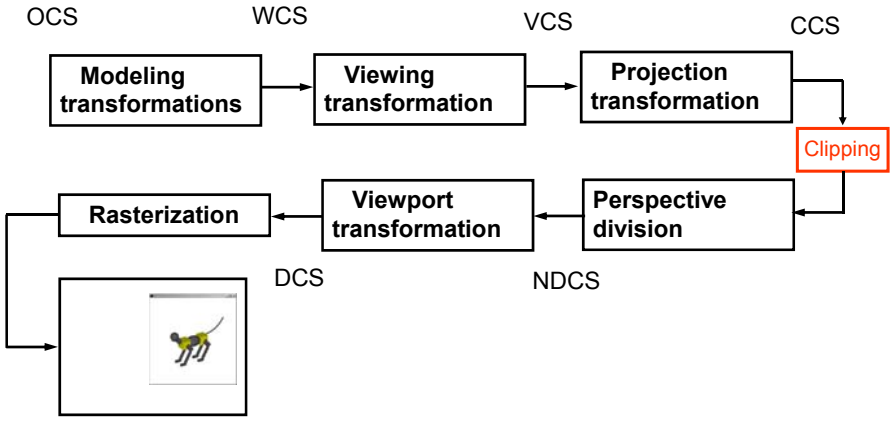
$$t_i = \frac{-D - \mathbf{N} \cdot P_a}{\mathbf{N} \cdot P_b - \mathbf{N} \cdot P_a} = \frac{-F(P_a)}{F(P_b) - F(P_a)}$$

Finally, evaluate $L(t_i)$ for intersection point P_i:

$$P_i = P_a - \frac{F(P_a)(P_b - P_a)}{F(P_b) - F(P_a)} = \frac{P_a F(P_b) - P_b F(P_a)}{F(P_b) - F(P_a)}$$



Z-Buffer Graphics Pipeline



Polygon Culling

When an entire polygon lies outside the view volume, it can be "culled"

- I.e., eliminated from the pipeline
- Especially helpful when many triangles are grouped into an object with an associated bounding volume (e.g., a bounding sphere) which lies outside the view volume (signed distance to plane > sphere radius)

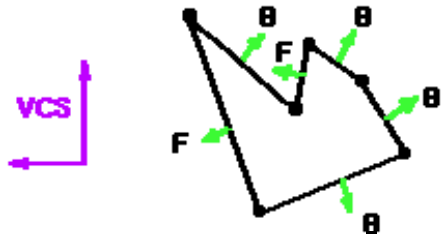
Visibility

Do not draw what is invisible

- Outside the view volume
  - *Clipping, culling*
- Self-occlusion
  - *Backface culling*
- Object-to-object occlusion

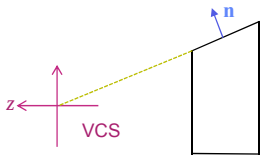
Backface Culling

Remove backfacing polygons



Backface Culling in the VCS

Can we use the z-component of the normal?



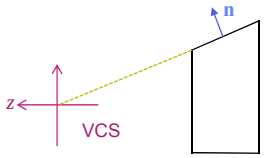
Backface Culling in the VCS

Can we use the z-component of the normal?

Yes, if the projection is orthographic

What about perspective?

No!

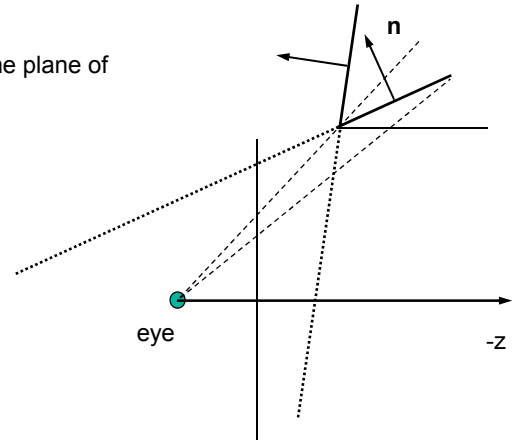


Backface Culling in the VCS

What do we really need to look at?

Answer:

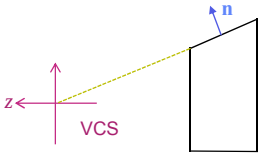
Is the eye above or below the plane of the polygon?



Backface Culling in the VCS

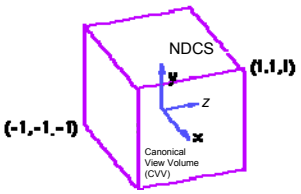
How do we do that?

- Calculate the normal **n** = $[a,b,c]^T$
- Compute *d* in plane equation using any of the vertices of the polygon
  Plane(x,y,z) = ax+by+cz+d = 0
- Compute *Plane(eye-position)* and check the sign
  > 0 above → front facing
  < 0 below (behind) → back facing



Backface Culling in the NDCS

- In NDCS, the z-component of the surface normal does reflect the true visibility, as desired
  - *If the z-component is positive, the normal points away from the eye and the polygon should thus be culled*



Reminder: In the NDCS, the camera is pointing towards the positive *z*-axis.

Visibility

Do not draw what is invisible

- Outside the view volume
  - *Clipping, culling*
- Self-occlusion
  - *Backface culling*
- Object-to-object occlusion
  - *Visibility algorithms*

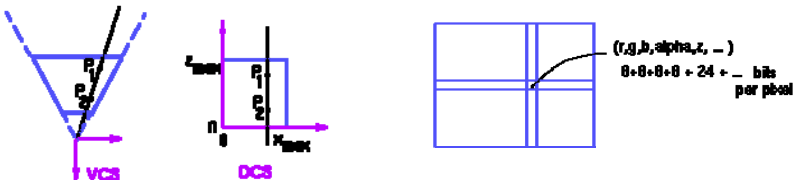
Visibility Algorithms

Visibility algorithms are needed to determine which objects in the scene are obscured by other objects. They are typically classified into two groups:

- Image-space algorithms
  - *operate on display primitives; e.g., pixels, scan-lines*
  - *visibility resolved to the precision of the display*
  - *e.g: Z-buffer, Watkin's algorithm, ray-tracing*
- Object-space algorithms
  - *BSP: binary-space partitions*
  - *variations on the Painter's Algorithm*
  - *worst case: creation of $O(N^2)$ primitives from N original primitives*

Z-Buffer

The Z-buffer keeps depth information about each pixel



Z-Buffer Algorithm

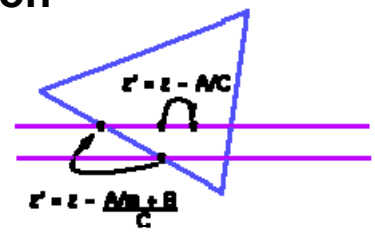
```
for all i,j {
  Depth[i,j] = MAX_DEPTH
  Image[i,j] = BACKGROUND_COLOR
}

for all polygons P {
        for all pixels in P {
                if (Z_pixel < Depth[i,j]) {
                        Image[i,j] = Color_pixel
                        Depth[i,j] = Z_pixel
                }
        }
}
```

Characteristics of the Z-Buffer Algorithm

- *Commonly used*
- *Memory intensive*
- *Hardware implementation common*
- *Handles polygon interpenetration*
- *Jaggies!*

Generating Z Values During Scan Conversion

Method A: From the plane equation



We want $z = f(x,y)$.

Plane equation: $ax + by + c\,z + d = 0$

Solving for z: $z(x,y) = (- ax - by - d) / c$

So $z(x+1,y) = (- a(x+1) - by - d) / c$

$= z(x,y) - a/c$

So along a scanline $z(x+1,y) = z(x,y) - a/c$

Similarly from scanline to scanline $(x,y) \rightarrow (x+1/m, y+1)$ and

$z(x+1/m, y+1) = z(x,y) - (a/m + b)/c$.

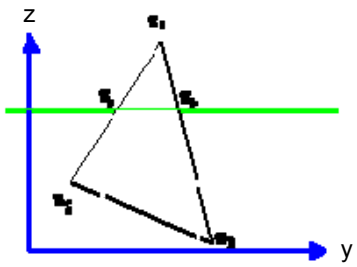
Method B: Bilinear Interpolation

Equivalent to method A, without having to solve for plane equation

- Incrementally interpolates any type of quantity between known values at the vertices
  - *colors (Gouraud shading)*
  - *texture coordinates*
  - *surface normals*

Bilinear Interpolation of Z-Coordinates

It can also be done incrementally



Observe that

$$\frac{z_a - z_2}{z_1 - z_2} = \frac{y_a - y_2}{y_1 - y_2}$$

Therefore

$$z_a = z_2 + (y_a - y_2)\frac{(z_1 - z_2)}{(y_1 - y_2)}$$

A-Buffer

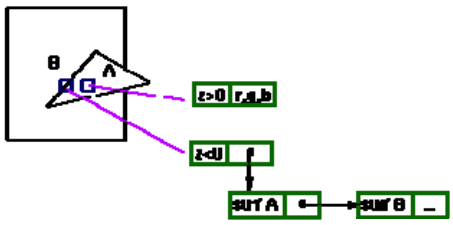
Z-buffer: only one visible surface per pixel

A-buffer: linked list of surfaces

- Antialiased, area-averaged, accumulation buffer

The data for each surface includes:

RGB
z
alpha
area coverage percentage
other surface parameters

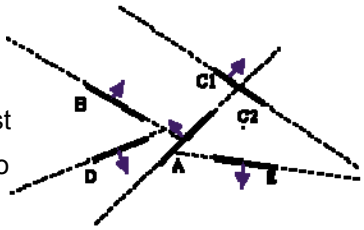


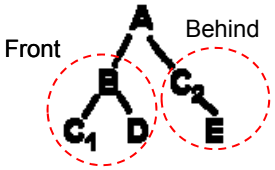
Binary Space Partition (BSP) Trees

- Object space algorithm
- Produces back-to-front ordering
- Preprocess the scene once to build BSP tree
- Traversal of BSP tree is view dependent

Building a BSP Tree

```
BSPtree *BSPmaketree(polygon_list) {
  choose a polygon as the tree root
  for all other polygons
      if polygon is in front, add to front list
      if polygon is behind, add to behind list
      else split polygon and add one part to
           each list
  BSPtree = BSPcombinetree(
      BSPmaketree(front_list),
      root,
      BSPmaketree(behind_list) )
  }
```



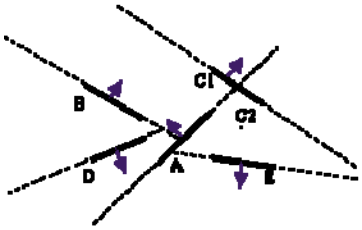


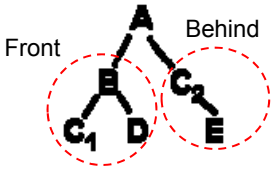
Example tree with A chosen as the root:

Drawing Using the BSP Tree

View dependent

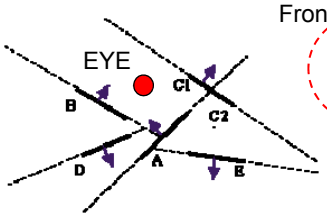
```
DrawTree(BSPtree) {
   if (eye is in front of root) {
        DrawTree(BSPtree->behind)
        DrawPoly(BSPtree->root)
        DrawTree(BSPtree->front)
     } else {
        DrawTree(BSPtree->front)
        DrawPoly(BSPtree->root)
        DrawTree(BSPtree->behind)
     }
   }
}
```

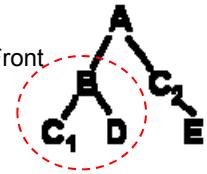




Example

```
DrawTree(BSPtree) {
      if (eye is in front of root) {
          DrawTree(BSPtree->behind)
          DrawPoly(BSPtree->root)
          DrawTree(BSPtree->front)
       } else {
          DrawTree(BSPtree->front)
          DrawPoly(BSPtree->root)
          DrawTree(BSPtree->behind)
      }
    }
}
```





Execution:
Eye in front of A,
draw A->behind,
eye behind C_2,
draw C_2->front,
draw C_2,
draw C_2→behind,
draw E,
draw A,
draw A→front,
Eye in front of B,
draw B→behind,
draw D,
draw B,
draw B→front,
draw C_1

Drawing order: C_2, E, A, D, B, C_1

Depth Sorting Algorithms

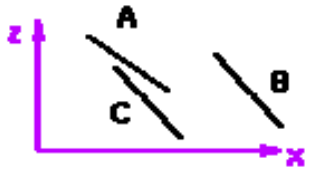
Several object-space algorithms achieve a front-to-back ordering in other ways

These depth-sort algorithms have the following basic steps:

- Sort polygons by *z*
- Resolve ambiguities where *z*-extents overlap
- Scan-convert polygons in back-to-front order

Resolving Ambiguities

- Bounding rectangles do not overlap in *xy*-plane
- A is completely behind C
- C is completely in front of A
- Projections on *xy*-plane do not overlap



If these fail, exchange the order of the surfaces and repeat

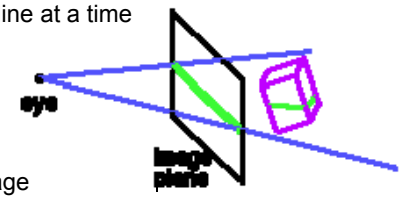
If this still fails, the polygons can be intersected to split one of the polygons if necessary

In the worst case, the algorithm can generate $O(n^2)$ new polygons

Scanline Algorithms

modify scan-conversion to handle multiple polygons

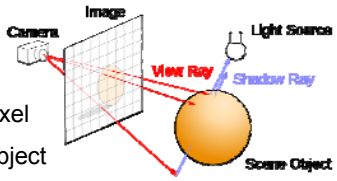
- priority according to Z
- resolve visibility one scanline at a time
- less memory




```
for each scanline (row) in image
  for each pixel in scanline
    determine closest object
    calculate pixel color, draw pixel
  end
end
```

Ray Tracing

We will study this algorithm later



for each pixel on the screen

determine a view ray from the camera through the pixel

find the closest intersection of the view ray with an object

cast off other rays, recursively

calculate pixel color and draw pixel

End

- *View rays cast through image pixels*
- *Solves visibility, some global illumination*
- *Requires efficient intersection tests*
  O(mnN): m x n pixels, N objects