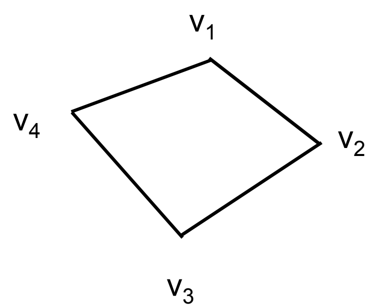
Polygon

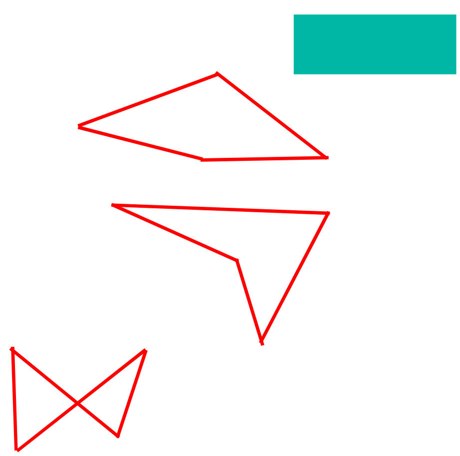
Collection of points connected with lines

- Vertices: v_1, v_2, v_3, v_4
- Edges:
  $e_1 = v_1v_2$
  $e_2 = v_2v_3$
  $e_3 = v_3v_4$
  $e_4 = v_4v_1$

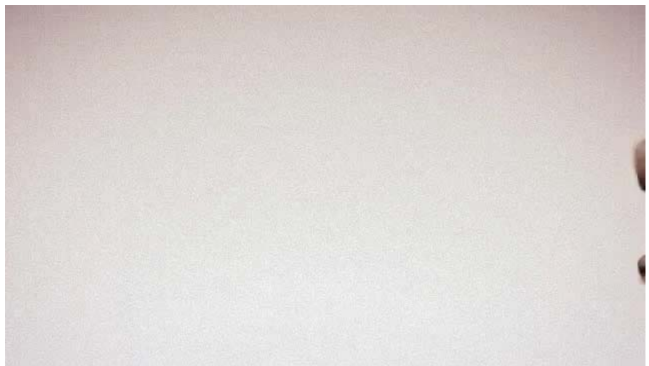


Polygons

- Open / closed
- Planar / non-planar
- Filled / wireframe
- Convex / concave
- Simple / non-simple



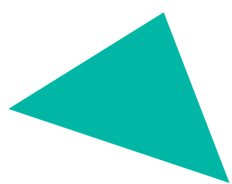
Guerrilla CG Tutorial 01: The Polygon



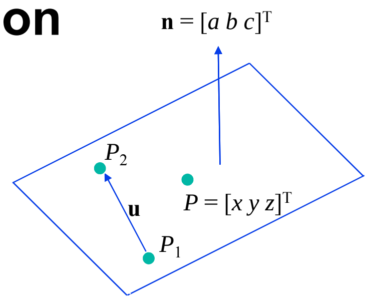
Triangles

The most common primitive

- Simple
- Convex
- Planar



Plane Equation



Normal / point form

$F(x, y, z) = ax + by + cz + d = \mathbf{n} \bullet P + d$

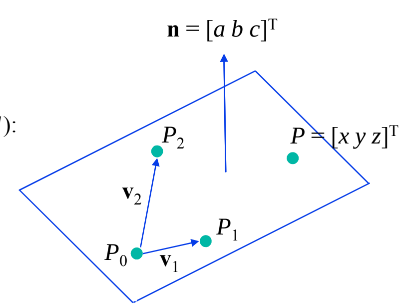
For points on plane, $F(x, y, z) = 0$

Observation: Let's take an arbitrary vector $\mathbf{u}$ that lies on the plane which can be defined by two points; e.g., P_1, P_2 on the plane.

$\mathbf{u} = P_2 - P_1$

$$\left.\begin{array}{l} \mathbf{n} \bullet P_1 + d = 0 \\ \mathbf{n} \bullet P_2 + d = 0 \end{array}\right\} \Rightarrow \mathbf{n} \bullet (P_2 - P_1) = 0 \Rightarrow \mathbf{n} \bullet \mathbf{u} = 0 \Rightarrow \boxed{\mathbf{n} \perp \mathbf{u}}$$

Computing Normal / Point Form From 3 Points



$F(x, y, z) = ax + by + cz + d = \mathbf{n} \bullet P + d$

Points on Plane $F(x, y, z) = 0$

First way (4 equations in unknowns a, b, c, d):

$\mathbf{n} \bullet P_0 + d = 0$

$\mathbf{n} \bullet P_1 + d = 0$

$\mathbf{n} \bullet P_2 + d = 0$

$|\mathbf{n}| = 1$ (arbitrary choice)

Second way:

$\mathbf{n}$ is normal to the plane

Let's find a normal vector:

$\mathbf{n} = (P_1 - P_0) \times (P_2 - P_0) = \mathbf{v}_1 \times \mathbf{v}_2$

Compute d :

$d = -\mathbf{n} \bullet P_0$

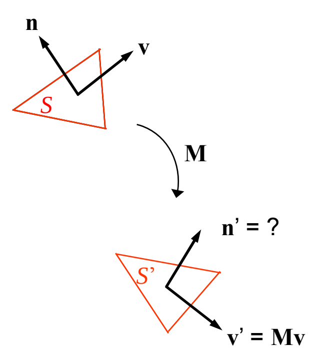
Transforming Normals

Normal vectors are transformed along with vertices and polygons.

- How do you transform a normal ?
- What about unit magnitude ?

Deriving Transformation of Normals

Normal to S: $\mathbf{n} = [n_x, n_y, n_z, 0]^T$
Tangent to S: $\mathbf{v} = [v_x, v_y, v_z, 0]^T$



$S' = \mathbf{M}S \Rightarrow \mathbf{v}' = \mathbf{M}\mathbf{v}$

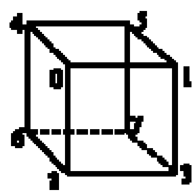
What is $\mathbf{n}'$?

$$\begin{aligned} 0 = \mathbf{n}\cdot\mathbf{v} &= \mathbf{n}^T\mathbf{v} \\ &= \mathbf{n}^T(\mathbf{M}^{-1}\mathbf{M})\mathbf{v} \\ &= (\mathbf{n}^T\mathbf{M}^{-1})(\mathbf{M}\mathbf{v}) \\ &= (\mathbf{M}^{-T}\mathbf{n})^T(\mathbf{M}\mathbf{v}) \\ &= (\mathbf{M}^{-T}\mathbf{n})\cdot(\mathbf{M}\mathbf{v}) = \mathbf{n}'\cdot\mathbf{v}' = 0 \end{aligned}$$

Therefore, $\mathbf{n}' = \mathbf{M}^{-T}\mathbf{n}$

Polygonal Models / Data Structures

Indexed face set



faces

#	vertex list
0	0,2,3,1
1	1,3,7,5
2	5,7,6,4
3	4,6,2,0
4	4,0,1,5
5	2,6,7,3

vertex list

#	x,y,z
0	0,1,1
1	1,1,1
2	0,0,1
3	1,0,1
4	0,1,0
5	1,1,0
6	0,0,0
7	1,0,0

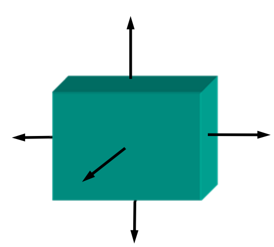
Polygon Attributes

Per vertex

- Color
- Texture coordinates

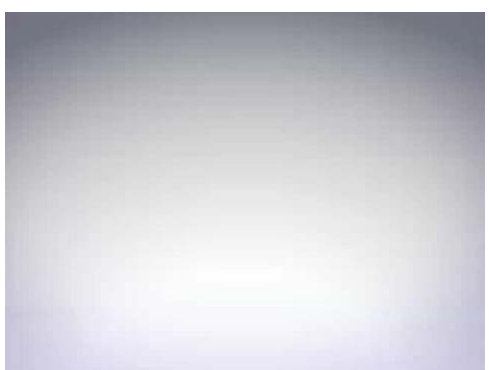
Per vertex or per face

- Color
- Normal



Guerrilla CG Tutorial 02:
Multisided and Intersecting Polygons

Guerrilla CG Tutorial 05:
Objects



Guerrilla CG Tutorial 11: Hierarchies

Guerrilla CG Tutorial 12: Hierarchies – Building a Robot

Guerrilla CG Tutorial 06: Primitives (Blocking Models)