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An extended car-following model with consideration of speed guidance at intersections

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HIGHLIGHTS

- This paper proposed a car-following model with the consideration of speed guidance.
- The effects of guiding space range on driving behaviors are analyzed.
- The effects of guiding speed limitation on driving behaviors are analyzed.
- The effects of intelligent vehicles' proportion on driving behaviors are analyzed.

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ABSTRACT

The main motivation of this paper is to analyze the influences of speed guidance strategies on the driving behaviors under four different traffic signalized conditions and to investigate an extended car-following model to explore how the speed guidance affects two different vehicle types that are intelligent vehicles and traditional vehicles during the phase-change periods. The numerical results show that the proposed model can qualitatively describe the impacts of the speed guidance strategies on vehicle's movement trail including the acceleration strategy, smooth braking strategy, and deceleration strategy. Moreover, the benefits of the speed guidance could be enhanced by lengthening the guiding space range, expanding permitted guiding speed range, and increasing the percentage of the intelligent vehicles.

1. Introduction

To date, traffic delay and fuel consumption on roadways have become critical problems in many cities across the world due to increasing traffic volumes. The frequent acceleration, deceleration and stop of vehicles at intersections are critical influence factors to cause a big travel delay and lose of fuel [1–3]. Many traffic signal optimization models, including isolated intersections signal timing methods [4–9] and coordinated signal timing methods [10–14] have been developed to enhance capacity and relieve congestion. However, the travel speed is assumed to be pre-set which may reduce the operation efficiency because real-time travel speed is effected by several factors, including the distance between intersections, driver behavior, vehicle characteristics, traffic volumes, etc.

Speed guidance is an effective approach to overcome this technical difficulty. Based on the recently developed Connected Vehicle (CV) technology, signal controller is allowed to guide vehicles' travel speed dynamically according to the current

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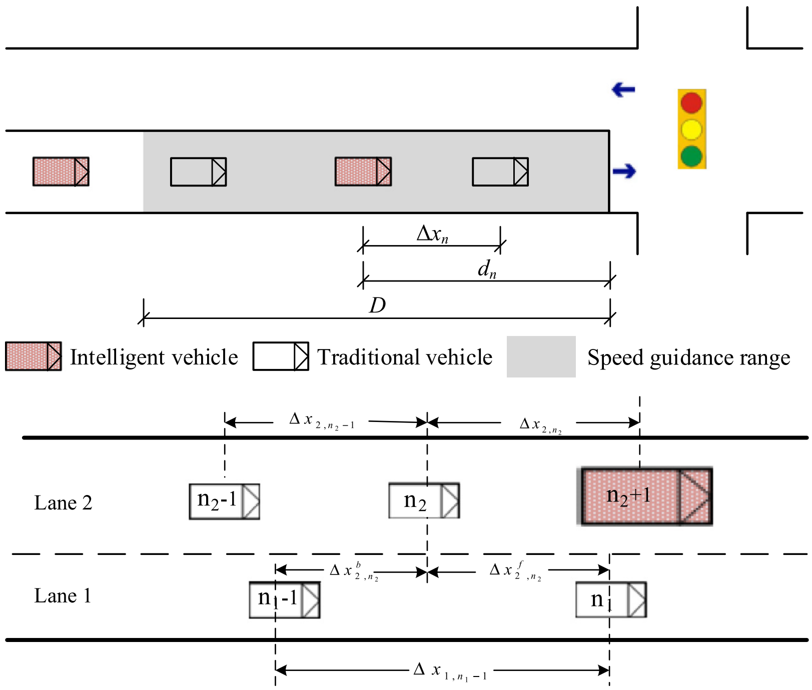


Fig. 1. The scheme of car-following model with speed guidance.

traffic condition and signal timing states [15]. Several studies on speed guidance strategy and its evaluation have been developed to decrease delay and number of stops by guiding vehicles to clear the intersections without stopping at a red light [15–19]. These studies focus on designing the guidance strategy, which cannot describe the impacts of the speed guidance on drivers' car-following behaviors or be directly employed to explore some complex phenomena of the traffic flow.

Since 1950s, many traffic flow models have been established to analyze various complex traffic phenomena, including congestion, phase transition, lane-changing, local clusters, etc. [20–23]. The existing traffic flow models can be divided into two groups: microscopic models [24–44] and macroscopic models [45–59]. Although the existing micro models can describe many micro properties of traffic flow, such as lane-changing, overtaking, etc., they cannot be used to explore the impacts of speed guidance at intersections on traffic flow since this factor is not considered.

In fact, the speed guidance at intersections may have significant influence on the driving behavior. For example, the vehicle may decelerate in advance to avoid the stop at the end of red light, while the vehicle may accelerate to clear the intersection before the end of the green light. In this paper, a car-following model with consideration of the speed guidance at intersections will be developed to study the effects of the speed guidance on the car-following behavior.

2. Model

To investigate the influences of speed guidance strategies on driving behaviors under four different conditions, two categories vehicles are divided as intelligent vehicles and traditional vehicles, as illustrated in Fig. 1.

The intelligent vehicles can communicate with the signal controller and receive speed guidance information, while the traditional vehicles are without aforementioned feature. The drivers' behaviors of intelligent vehicles within the guidance range can be divided into four conditions according to the status of the traffic light, as illustrated in Fig. 2. Condition 1: if vehicles can pass the intersection during the green time at current speed, the speed guidance is not required. Condition 2: if vehicles arrive before the end of green but cannot pass the intersection during the green time at current speed, the acceleration strategy should be used. Condition 3: if vehicles cannot pass the intersection during the green time at the maximum permitted speed and have to stop, the smooth braking strategy should be used to avoid sudden stop or red-light violations due to the driver's misjudgment. Condition 4: if vehicles arrive when the traffic light is red or the traffic light is green but the queue at the approach is not dissipated, the deceleration strategy should be used to avoid stopping and reduce the number of stops.

In order to explore the aforementioned effects of the speed guidance strategies on car-following behaviors of the arrival traffic flow at signalized intersections, the existing car-following models should be extended. Based on the above analysis, the driving behavior of each vehicle can be described as the following five parts:

(1) When the nth vehicle is an intelligent vehicle that runs in the first condition, its motion behavior should be the same as traditional vehicles. Therefore, the full velocity difference model could be used to describe the car-following behaviors,

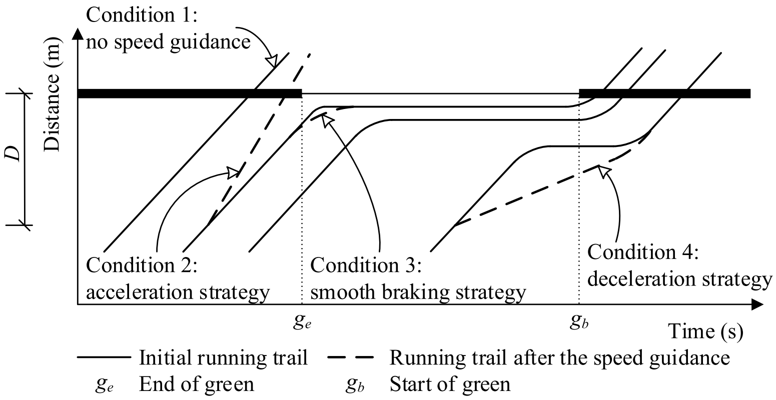


Fig. 2. Four conditions of speed guidance strategies.

given by:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa \left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \lambda \Delta v_n(t), \tag{1}$$

where $v_n(t)$ is the velocity of the vehicle n at time t, m/s; $V(.)$ is the optimal velocity function, m/s; $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ is the space headway between vehicle n and vehicle $n-1$ at time t, m; $\Delta v_n(t) = v_{n-1}(t) - v_n(t)$ is the velocity difference between vehicle n and vehicle $n-1$ at time t, m/s; κ and λ are sensitivity parameters.

The optimal velocity function of the vehicle n can be selected as that proposed by Helbing and Tilch [60], given by:

$$V\left(\Delta x_n(t)\right) = V_1 + V_2 \tanh\left(C_1\left(\Delta x_n(t) - l_c\right) - C_2\right), \tag{2}$$

where l_c is the length of the vehicle, m; V_1, V_2, C_1, C_2 are parameters.

(2) When the nth vehicle is an intelligent vehicle that runs in the second condition, according to the speed guidance strategy, the intelligent vehicle should accelerate to the maximum limited velocity that could ensure the vehicle to clear the intersection before the end of the green light and not to block the following vehicles. However, the intelligent vehicles may be blocked by any other vehicle moving ahead in its course. Therefore, its motion behavior can be formulated as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \begin{cases} \kappa\left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \lambda \Delta v_n(t), & \Delta x_n(t) \le d_n(t) \\ \kappa\left(V_{\max} - v_n(t)\right), & \Delta x_n(t) > d_n(t) \end{cases} \tag{3}$$

where $V_{\max}$ is maximum velocity, m/s; $d_n(t)$ is the distance between vehicle n and the stop line at time t, m.

(3) When the nth vehicle is an intelligent vehicle that runs in the third condition, according to the speed guidance strategy, the driver will be informed in advance that the vehicle cannot go through the intersection in the green time of this signal cycle. Therefore, the sudden stop could be prevented. Its motion behavior can be formulated as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(V\left(d_n(t) - n_n(t)h_d\right) - v_n(t)\right) - \lambda v_n(t) \tag{4}$$

where $n_n(t)$ is the number of stopped vehicles between vehicle n and the stop line at time t; h_d is the space headway for queuing vehicles, m; $(d_n(t) - n_n(t)h_d)$ represents the distance between the intelligent vehicle and the queue tail at time t, m.

(4) When the nth vehicle is an intelligent vehicle that runs in the fourth condition, the vehicle should decelerate to a lower velocity under which the vehicle could exactly reach the queue tail when the last vehicle begins moving. Therefore, the optimal velocity is equal to the distance between the vehicle and the queue tail divided by the length of time that is the interval from the time t to the last vehicle start moving time. Its deceleration could be determined by:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(\frac{d_n(t) - n_n(t)h_d}{g_b + \delta_2 - t} - v_n(t)\right) \tag{5}$$

where g_b is the begin of green, s; δ_2 is the queue clearance time, s.

(5) When the nth vehicle is a traditional vehicle or the nth vehicle locates out of the space range of the speed guidance, it should run as traditional vehicles with the acceleration formulated as Eq. (1).

In summary, we obtain an extended car-following model with the consideration of the speed guidance that can be written as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t}=\begin{cases}\kappa\left(V\left(\Delta x_n(t)\right)-v_n(t)\right)+\lambda\Delta v_n(t), & \left(I_n=0 \text{ or } d_n(t)>D \text{ or } v_n(t)\geq\dfrac{d_n(t)}{g_e-\delta_1-t}\right)\\ \kappa\left(V_{\max}-v_n(t)\right), & \left(I_n=1 \text{ and } d_n(t)\leq D \text{ and } v_n(t)<\dfrac{d_n(t)}{g_e-\delta_1-t}\leq V_{\max} \text{ and } \Delta x_n(t)>d_n(t)\right)\\ \kappa\left(V\left(d_n(t)-n_n(t)h_d\right)-v_n(t)\right)-\lambda v_n(t), & \\ \quad\left(I_n=1 \text{ and } d_n(t)\leq D \text{ and } \dfrac{d_n(t)}{g_e-\delta_1-t}>V_{\max} \text{ and } \dfrac{d_n(t)-n_n(t)h_d}{g_b+\delta_2-t}<V_{\min}\right) & \\ \kappa\left(\dfrac{d_n(t)-n_n(t)h_d}{g_b+\delta_2-t}-v_n(t)\right), & \left(I_n=1 \text{ and } d_n(t)\leq D \text{ and } \dfrac{d_n(t)-n_n(t)h_d}{g_b+\delta_2-t}\geq V_{\min}\right)\end{cases}\tag{6}$$

where I_n is a binary variable indicating whether the nth vehicle is an intelligent vehicle, 1-yes and 0-no; D is the guiding space range, m; g_e is the end of green, s; δ_1 is the safety interval, s; $V_{\max}$ and $V_{\min}$ are maximum and minimum velocities, m/s.

3. Numerical tests

In this section, Eq. (6) is used to study the effects of the speed guidance on the car-following behavior. It is difficult to obtain the analytical solution of Eq. (6), because it consists of multi ODEs that are non-autonomous systems and many parameters of the proposed are discontinuous. Therefore, extensive numerical schemes are used to discretize Eq. (6) and study the impacts of the speed guidance. The Euler forward difference is selected to calculate the speed and position of each vehicle, given by:

$$v_{\mathrm{n}}\left(t+\Delta t\right)=v_{\mathrm{n}}\left(t\right)+\Delta t\frac{\mathrm{d}v_{\mathrm{n}}\left(t\right)}{\mathrm{d}t}\tag{7}$$

$$x_{\mathrm{n}}\left(t+\Delta t\right)=x_{\mathrm{n}}\left(t\right)+v_{\mathrm{n}}\left(t\right)\Delta t+\frac{1}{2}\frac{\mathrm{d}v_{\mathrm{n}}\left(t\right)}{\mathrm{d}t}\left(\Delta t\right)^2\tag{8}$$

where Δt is the length of the time-step.

The parameters are defined as follows: $N=15$, $\Delta t=0.1$ s, $\kappa=0.41$ s^{-1}, $\lambda=0.2$ s^{-1}, $l_c=9$ m, $V_1=6.75$ m/s, $V_2=7.91$ m/s, $C_1=0.13$ m^{-1}, $C_2=1.57$ m^{-1}, $V_{\max}=16.66$ m/s, $V_{\min}=6$ m/s, where N is the number of vehicles.

The following four analysis steps are used to explore the effects of the speed guidance on microscopic traffic flow at intersections.

Step 1: The vehicles' running trail between the proposed car-following model and the traditional model is compared during different periods of the signal cycle. The initial conditions are as follows: $\Delta x_1(0)=\cdots=\Delta x_N(0)=100$ m, $v_1(0)=\cdots=v_N(0)=V(100)$, $x_1(0)=0$ m, $d_1(0)=290$ m, $D=300$ m, $g_b=0$ s, $g_e=40$ s, $C=90$ s, and all the vehicles are intelligent vehicles.

Step 2: The effects of guiding space range (D) on the driving behaviors are analyzed. The guiding space range is changed from 100 m to 420 m. Other conditions are the same as those in Step 1.

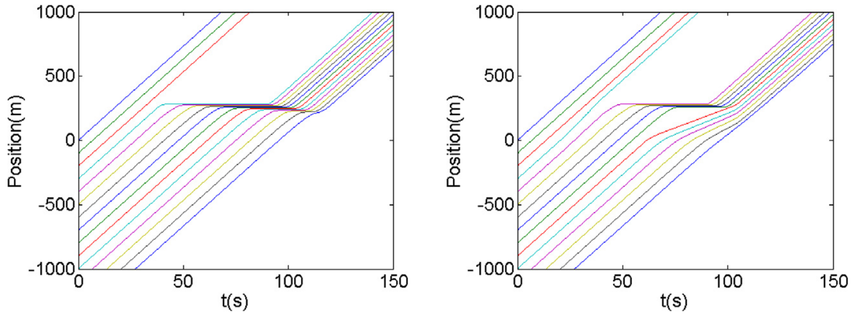
Step 3: The effects of the guiding speed limitation on the driving behaviors are analyzed. Since the maximum speed is usually limited according to the speed limitation of the street, we mainly discuss how the variation of the minimum speed affects the drivers' behavior. The minimum speed ($V_{\min}$) is set from 5 to 10 m/s. Other conditions are the same as those in Step 1.

Step 4: The effects of the percentage of the intelligent vehicles on the driving behaviors are analyzed. Four cases of percentage of the intelligent vehicles are considered: 20%, 46%, 73% and 100%, uniformly distributed in the platoon. Other conditions are the same as those in Step 1.

3.1. Comparison between the proposed model and traditional model

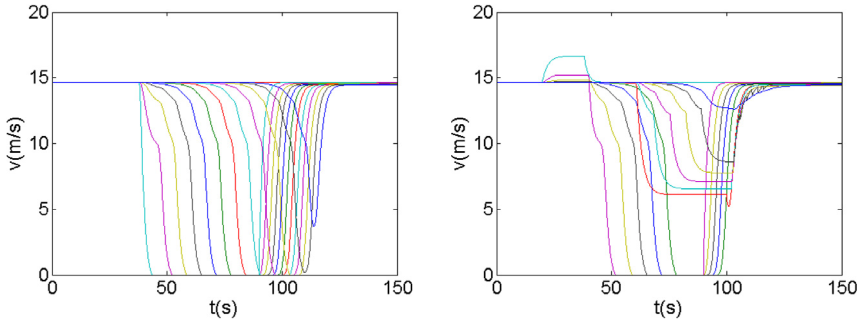
Using the proposed model and the related parameters, we can obtain each vehicle's position and velocity, as illustrated in Figs. 3 and 4, respectively. The positive effects of speeding guidance could be observed.

(1) The 4th vehicle successfully passed the intersection before the end of green by using the acceleration strategy, which can reduce the vehicular delay and improve the traffic efficiency.
(2) The 5th vehicle firstly followed the ahead vehicle (4th vehicle) to accelerate. However, when it enters into the guiding area, the 5th vehicle smoothly braked to stop instead of continuing following the ahead vehicle blindly by using the smooth braking strategy, which can reduce the risk of rear-end crashes and red-light violations.
(3) The 10th–15th vehicles successfully passed the intersection without stopping by using the deceleration strategy. Please note that these vehicles decelerated to a suitable speed instead of decelerated to zero. It can reduce the number of stops, and lower the emissions and fuel consumption of vehicles.



(a) FVD model. (b) Proposed model.

Fig. 3. Each vehicle's running trail.



(a) FVD model. (b) Proposed model.

Fig. 4. Each vehicle's velocity.

3.2. Effects of guiding space range

The benefits of the speed guidance are effected by the guiding space range. Fig. 5(a)–(c) show each vehicle's running trail under the conditions that the guiding space ranges are 150 m, 300 m, and 400 m, respectively. Fig. 5(d) further analyzes the number of vehicles passing the intersection without stopping under different guiding space range.

We can observe that more vehicles can pass the intersection without stopping with the increase of the guiding space range. Moreover, intelligent vehicles have more chance to pass the intersection before the end of green. It is because vehicles could make the acceleration or deceleration strategy more early.

3.3. Effects of the guiding speed limitation

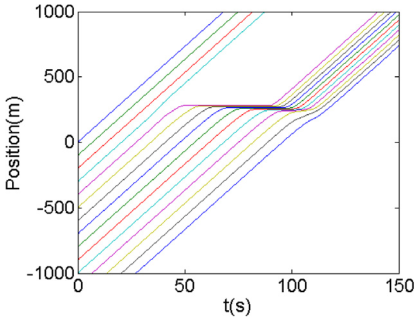
The adjustment range of the travel speed is determined by the guiding speed limitation. Fig. 6(a)–(c) show each vehicle's running trail under the conditions that the minimum permitted speed are 5 m/s, 6 m/s, and 8 m/s, respectively. Fig. 6(d) further analyzes the number of vehicles passing the intersection without stopping under different minimum speed. Higher maximum speed leads to higher possibility of vehicles using the acceleration strategy while lower minimum speed leads to higher percentage of vehicles using the deceleration strategy.

3.4. Effects of the intelligent vehicles' proportion

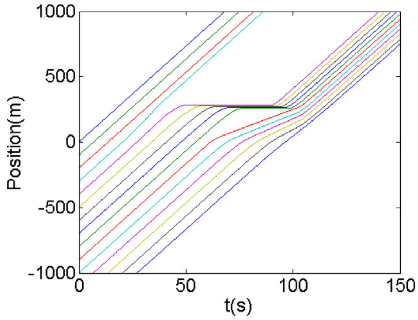
Due to the presence of traditional vehicles, the implement of the speed guidance may be limited. Fig. 7(a)–(d) show each vehicle's running trail under the conditions that the number of the intelligent vehicles within the 15 vehicles are 3 (20%), 7 (46%), 11 (73%) and 15 (100%), respectively.

For acceleration strategy, the usage possibility reduces with the decrease of the percentage of the intelligent vehicles. E.g., if the 4th vehicle is an intelligent vehicle, it can pass the intersection before the end of green. However, in Fig. 7(a), it is a tradition vehicle, so that the acceleration strategy cannot be used. Moreover, the intelligent vehicles may be blocked by the ahead tradition vehicle when aiming to accelerate.

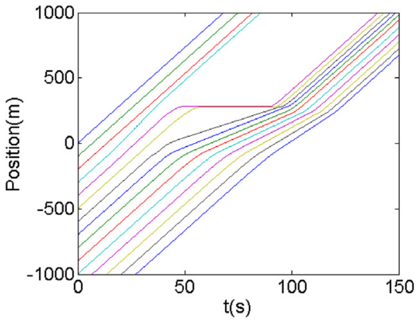
For deceleration strategy, the traditional vehicles will not affect the implement of the speed guidance. Moreover, if only one intelligent vehicle uses the deceleration strategy, the following vehicles have to decelerate, see 10th vehicle (intelligent vehicle) and 11th vehicle (traditional vehicle) in Fig. 7(b) as an example. Though the running trail is not the same as the condition that all the vehicles are intelligent vehicles, the same effects of reducing the number of stops could be achieved.



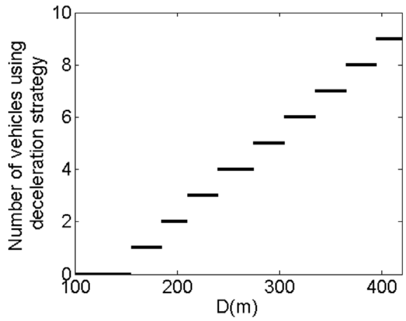
(a) $D = 150$ m.



(b) $D = 300$ m.

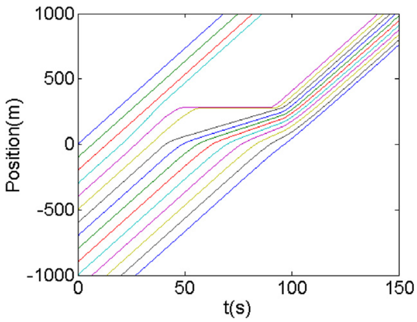


(c) $D = 400$ m.

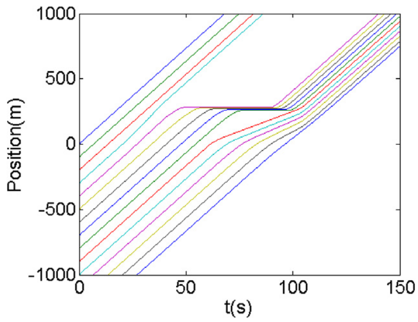


(d) Variation trend.

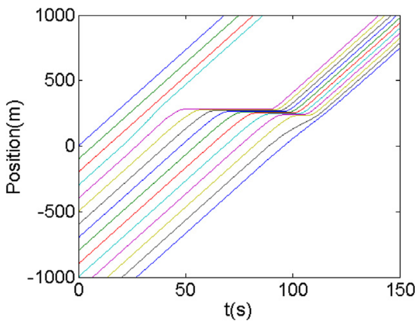
Fig. 5. Effects of guiding space range.



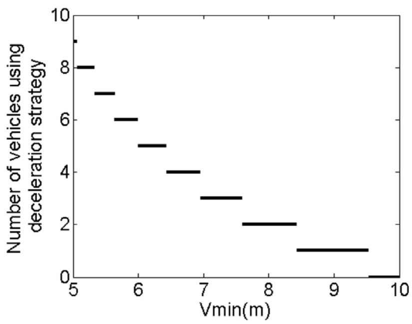
(a) $V_{\min} = 5$ m/s.



(b) $V_{\min} = 6$ m/s.

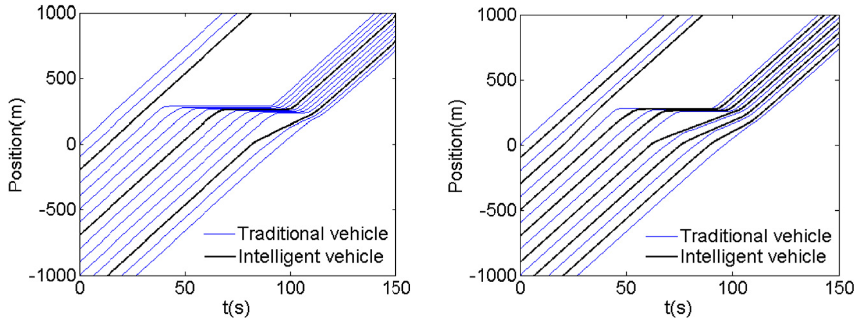


(c) $V_{\min} = 8$ m/s.



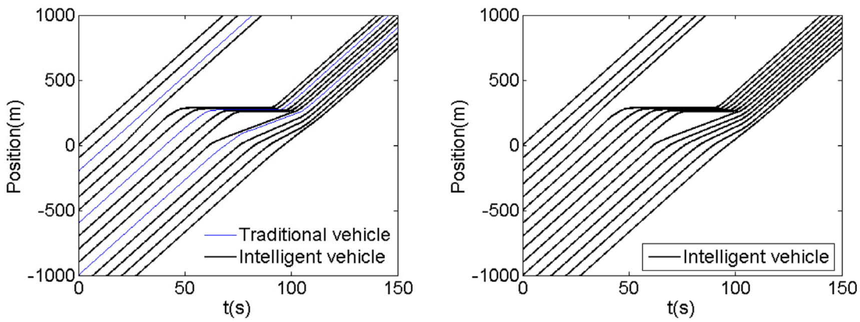
(d) Variation trend.

Fig. 6. Effects of guiding speed limitation.



(a) 3 intelligent vehicles.

(b) 7 intelligent vehicles.



(c) 11 intelligent vehicles.

(d) 15 intelligent vehicles.

Fig. 7. Effects of the intelligent vehicles' proportion.

4. Conclusions

In this paper, we investigate an extended car-following model to study the influences of the speed guidance on the driving behavior at signalized intersections. The numerical results show that the proposed model can qualitatively describe the effects of the speed guidance on microscopic traffic flow at intersections. In the numerical, some vehicles could successfully pass the intersection before the end of green by using the acceleration strategy, while some other vehicles could successfully pass the intersection without sudden stopping by using the deceleration strategy at the end of red time. It is also shown that taking the speed guidance strategies in to account can help drivers to avoid sudden stop or red-light violations due to the driver's misjudgment. Moreover, the benefits of the speed guidance could be enhanced by lengthening the guiding space range, expanding permitted guiding speed range, and increasing the percentage of the intelligent vehicles. However, the signal timing in this model is assumed to be fixed. The emissions and fuel consumption of vehicles and the coordination between the speed guidance and signal control have not been considered which could be introduced in future work.

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