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An extended car-following model with consideration of vehicle to vehicle communication of two conflicting streams

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HIGHLIGHTS

- This paper proposed a car-following model with the consideration of vehicle to vehicle communication of two conflicting streams.
- The effects of guiding space range on driving behaviors are analyzed.
- The effects of guiding density of the traffic flow on driving behaviors are analyzed.
- The effects of guiding speed limitation on driving behaviors are analyzed.
- The effects of length of the safety interval on driving behaviors are analyzed.

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ABSTRACT

In this paper, we propose a car-following model to explore the influences of V2V communication on the driving behavior at un-signalized intersections with two crossing streams and to explore how the speed guidance strategy affects the operation efficiency. The numerical results illustrate that the benefits of the guidance strategy could be enhanced by lengthening the guiding space range and increasing the maximum speed limitation, and that the guidance strategy is more suitable under low to medium traffic density and small safety interval condition.

1. Introduction

Traffic congestion at intersections is a severe issue and even a major cause of urban traffic and environment problems. To date, the development of vehicle to vehicle (V2V) communication system, also known as the car to car (C2C) and inter-vehicle (IC) communication, makes it possible for a driver to know the actual driving conditions of surrounding vehicles. Therefore, under V2V communication condition, drivers could be guided to adjust their driving behavior with the help of speed guidance to improve the operational efficiency. Then, the micro driving behavior will be affected greatly. For example, at the un-signalized intersections, V2V communication can provide drivers information about the traffic conditions on both two crossing streets. Thus, vehicles could search the proper gap in the crossing stream, and go through the intersection more efficiently and smoothly by accelerating or decelerating in advance.

To explore the impacts of V2V communication on traffic flow, researchers developed some models to study the properties of traffic flow in the traffic system with V2V communication. Ngoduy et al. [1] proposed a continuum approach to model

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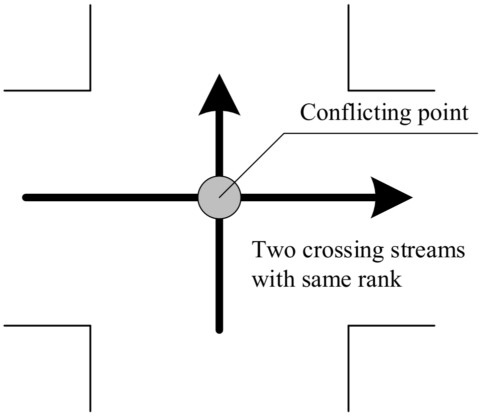


Fig. 1. Un-signalized intersection with two crossing streams.

the dynamics of cooperative traffic flow where the equipped vehicle can issue and receive a warning message when there is downstream congestion. Knorr et al. [2] analyzed mixed traffic flow consisting of vehicles equipped with wireless inter-vehicle communication devices and non-equipped vehicles. Improvement of traffic flow was observed even for a ratio of equipped vehicles as low as five percent by the proposed optimization strategy for equipped vehicles. These models investigated the effect of V2V communication from the macroscopic driving behavior aspect. From the microscopic driving behavior aspect, Tang [3] explored the impacts of V2V communication on the micro driving behavior when large perturbation (e.g., accident) occurs in the traffic system. A new car following model with V2V communication was constructed to study the driving behavior under an accident. However, these studies cannot describe the micro driving behavior with the consideration of V2V communication under daily traffic condition.

Since 1950s, many traffic flow models have been established to analyze various complex traffic phenomena [4–6]. The traffic flow models can be divided into two groups: microscopic models [7–31] and macroscopic models [32–56]. Although the existing micro model can describe the micro properties of the traffic flow, such as lane-changing, overtaking, etc., they do not explicitly reproduce the impact of speed guidance strategy at un-signalized intersections with two crossing streams on the traffic flow and its operation efficiency.

In this paper, we employ a car-following model with the consideration of V2V communication to explore the effects of the speed guidance on the car-following behavior at un-signalized intersections with two conflicting streams.

2. Model

An un-signalized intersection with two crossing streams is considered as the research object, as illustrated in Fig. 1. Before extending the existing car-following model, we give the following assumptions:

(1) The two crossing streams have the same rank within one intersection.
(2) All vehicles should follow the first come first service (FCFS) principle, which the vehicles touch the stop line first could drive through first. (such as All-way STOP sign)
(3) The vehicle should wait at the stop line when the crossing stream exists at the conflicting point.
(4) All the vehicles are equipped with wireless inter-vehicle communication devices and known the actual position and speed of all the vehicles approaching the intersection.

The existing car-following models on a single lane can be written as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = f\left(v_n(t), \Delta x_n(t), \Delta v_n(t), \ldots\right), \tag{1}$$

where $v_n(t)$ is the velocity of the vehicle n at time t, m/s; $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ is the space headway between vehicle n and vehicle $n-1$ at time t, m; $\Delta v_n(t) = v_{n-1}(t) - v_n(t)$ is the velocity difference between vehicle n and vehicle $n-1$ at time t, m/s; $f(\bullet)$ is the nth vehicle's stimulus function determined by the speed, headway, relative speed and other factors.

With the different expression of $f(\bullet)$, Eq. (1) can be divided into the Optimal Velocity (OV) version [21], Full Velocity Difference (FVD) version [57], and Full Velocity and Acceleration Difference (FVAD) version [22]. Among these models, the FVD has widely been extended to investigate the driving behavior. Therefore, the extended car-following model proposed in this paper is based on the FVD model [57], given by:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \lambda \Delta v_n(t), \tag{2}$$

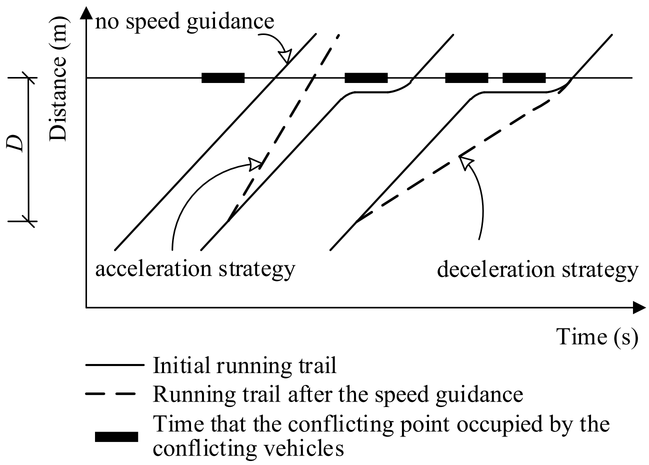


Fig. 2. Schematic diagram of speed guidance strategies.

where $V(.)$ is the optimal velocity function of the vehicle n, m/s, which can be selected as that proposed by Helbing and Tilch [58], as shown in Eq. (3); κ and λ are sensitivity parameters.

$$V\left(\Delta x_n(t)\right) = V_1 + V_2 \tanh\left(C_1\left(\Delta x_n(t) - l_c\right) - C_2\right), \tag{3}$$

where l_c is the length of the vehicle, m; V_1, V_2, C_1, C_2 are parameters.

Under the V2V environment, the full operation information of the two crossing streams could be obtained, including the position, current speed, and acceleration/deceleration of all vehicles within the V2V communication range. Therefore, vehicles could search the proper gap in the crossing stream and change the driving behavior in advance according to the following strategy (see Fig. 2).

Step 1: estimating the time that the vehicle arrives the conflicting point under current running velocity, given by:

$$T_n = t + \frac{d_n(t)}{v_n(t)}, \tag{4}$$

where T_n is the time that the vehicle n arrives the conflicting point, s; $d_n(t)$ is the distance between vehicle n and the stop line at time t, m.

Step 2: judging whether there exist vehicles from crossing street at the conflicting point at the time T_n. If no, go to Step 3; otherwise, go to Step 4.

$$C_n(t) = \sum_{T_n > T_m} \left\lfloor \frac{\delta}{T_n - T_m} \right\rfloor, \tag{5}$$

where $C_n(t)$ is a judgment variable indicating whether there exist conflicting vehicles when the nth vehicle arrives the conflicting point, if $C_n(t) \geq 1$, conflicting vehicle exists, otherwise, no conflicting vehicle; δ is the safety interval, s; T_m is the time that the vehicle m at the crossing street arrives the conflicting point, s;

Step 3: no guidance will be made, and go to Step 6. The car-following model could be given by:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \lambda \Delta v_n(t), x_n(t) > D \quad \text{or} \quad C_n(t) = 0, \tag{6}$$

where D is the guidance spacing range, m.

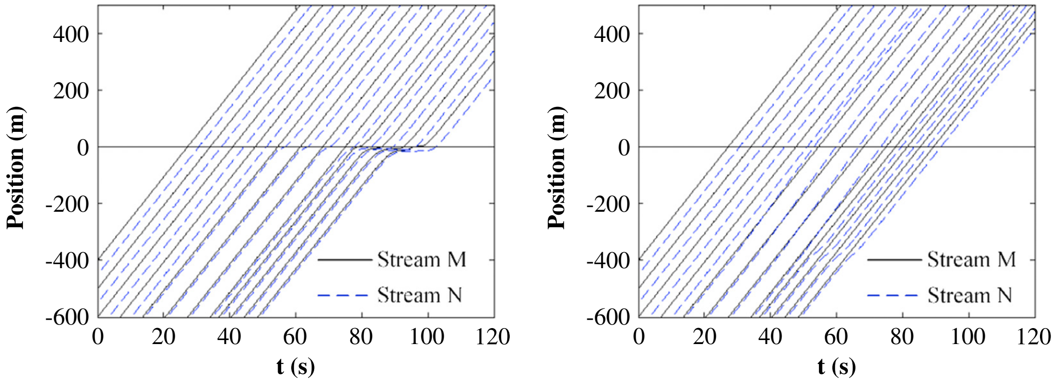
Step 4: considering the possibility of acceleration strategy. If there is an available gap, which is longer than the safety interval, ahead within the limitation of the maximum velocity and the ahead vehicle (see Eq. (7)), the acceleration strategy could be selected, as shown in Eq. (8), and go to Step 6. Otherwise, go to Step 5.

$$P_n(t) = \sum_{T_n > T_m > T_n^{\min} + \delta} \left\lfloor \frac{T_m - T_{m-1}}{2\delta} \right\rfloor, \tag{7}$$

where $P_n(t)$ is possibility of acceleration strategy for the nth vehicle, $P_n(t) \geq 1$-yes, 0-no; $T_n^{\min}$ is the earliest time that the nth vehicle arrives the conflicting point with the limitation of the maximum velocity and the ahead vehicle, s.

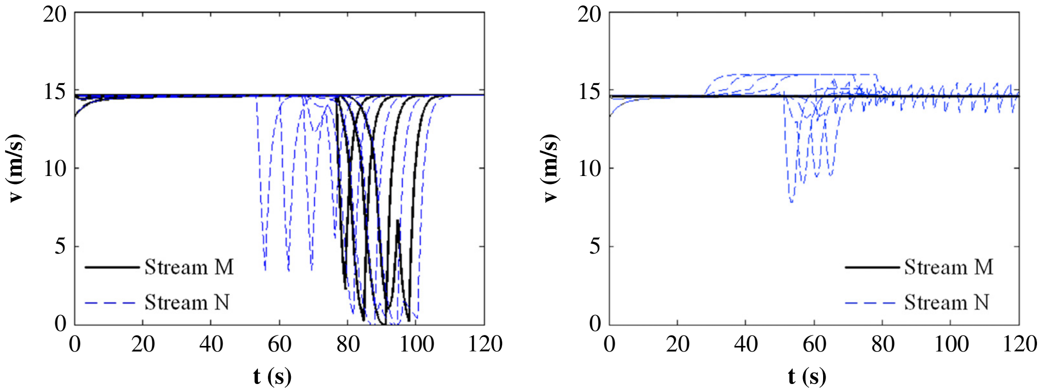
$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(V_{\max} - v_n(t)\right), \qquad C_n(t) \geq 1, \qquad P_n(t) \geq 1, \tag{8}$$

where $V_{\max}$ is the maximum limited velocity, m/s;



(a) FVD model. (b) Proposed model.

Fig. 3. Each vehicle's running trail.



(a) FVD model. (b) Proposed model.

Fig. 4. Each vehicle's velocity.

Step 5: using the deceleration strategy, and following the conflicting vehicle on the crossing street, as shown in Eq. (9).

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa \left(V\left(\Delta x_{n,m}(t)\right) - v_n(t)\right) + \lambda \Delta v_{n,m}(t), \qquad C_n(t) \geq 1, \qquad P_n(t) = 0, \tag{9}$$

where $\Delta x_{n,m}(t) = x_m(t) - x_n(t)$ is the relative space difference between the distance from vehicle m at the crossing street to the conflicting point and the distance from vehicle n to the conflicting point at time t (using the conflicting point as the coordinate origin), m; $\Delta v_{n,m}(t) = v_m(t) - v_n(t)$ is the velocity difference between vehicle m and vehicle n at time t, m/s.

Step 6: end.

Based on the above analysis, the driving behavior of each vehicle with the consideration of V2V communication at unsignalized intersections can be described as follows:

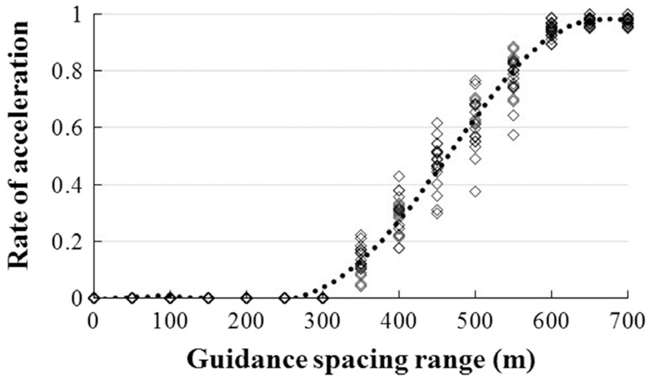
$$\begin{cases} \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa \left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \lambda \Delta v_n(t), & x_n(t) > D \quad \text{or} \quad C_n(t) = 0 \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa \left(V_{\max} - v_n(t)\right), \qquad C_n(t) \geq 1, \qquad P_n(t) \geq 1 & \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa \left(V\left(\Delta x_{n,m}(t)\right) - v_n(t)\right) + \lambda \Delta v_{n,m}(t), & C_n(t) \geq 1, \qquad P_n(t) = 0 \end{cases} \tag{10}$$

3. Numerical tests

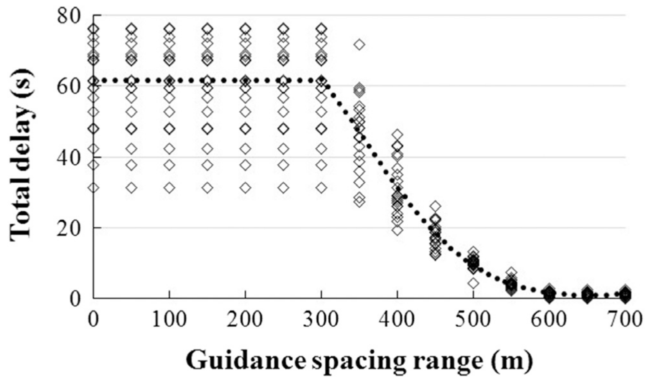
In this section, Eq. (10) is employed to investigate the effects of V2V communication on the car-following behavior for two conflicting streams. It is difficult to obtain the analytical solution of Eq. (10), because it consists of multi ODEs that are non-autonomous systems and many parameters of the proposed are discontinuous. Therefore, extensive numerical schemes

Table 1
Initial position, space headway, and velocity of vehicles.

No.	Stream M			Stream N		
	Position	Headway	Velocity	Position	Headway	Velocity
1	−400		V(100)	−450		V(100)
2	−500	100	V(100)	−550	100	V(100)
3	−600	100	V(100)	−650	100	V(100)
4	−700	100	V(100)	−750	100	V(100)
5	−800	100	V(100)	−810	60	V(60)
6	−900	100	V(100)	−910	100	V(100)
7	−1000	100	V(100)	−1010	100	V(100)
8	−1100	100	V(100)	−1110	100	V(100)
9	−1130	30	V(30)	−1140	30	V(30)
10	−1190	60	V(60)	−1200	60	V(60)
11	−1250	60	V(60)	−1260	60	V(60)
12	−1310	60	V(60)	−1320	60	V(60)



(a) Rate of acceleration.



(b) Total delay.

Fig. 5. Effects of guiding space range.

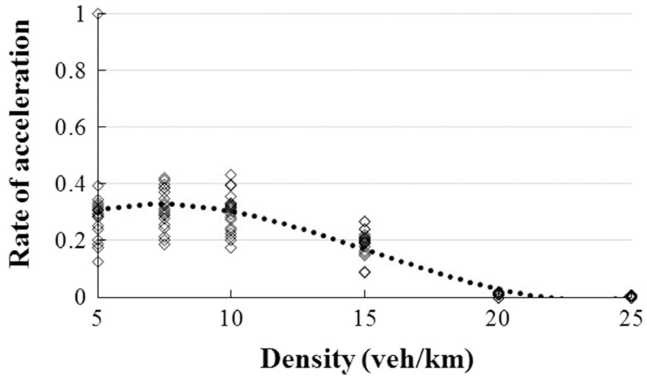
are used to discretize Eq. (10). The Euler forward difference is selected to calculate the speed and position of each vehicle, given by:

$$v_n(t+\Delta t) = v_n(t) + \Delta t \frac{dv_n(t)}{dt} \tag{11}$$

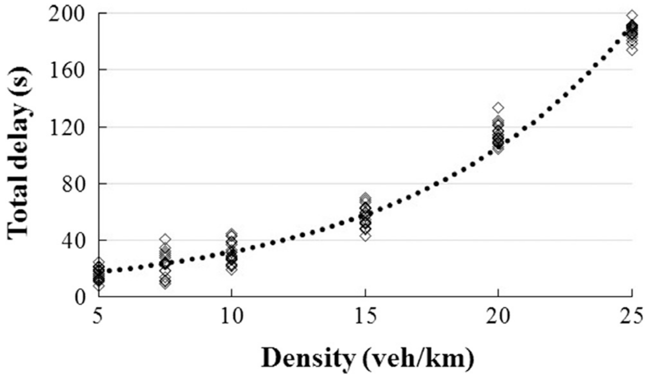
$$x_n(t+\Delta t) = x_n(t) + v_n(t)\Delta t + \frac{1}{2}\frac{dv_n(t)}{dt}(\Delta t)^2 \tag{12}$$

where Δt is the length of the time-step.

The following analysis were divided into two aspects. Firstly, the effect of the guidance strategies on microscopic traffic flow at intersections was explored by comparing the vehicles' running trail between the proposed model and the traditional model, in which the acceleration and deceleration strategies could be visually displayed. Secondly, the sensitivity analysis



(a) Rate of acceleration.



(b) Total delay.

Fig. 6. Effects of traffic flow density.

was produced with the consideration of some key factors of the proposed model, which includes the guidance spacing range, the density of the traffic flow, the speed limitation, and the length of the safety interval.

3.1. Comparison between the proposed model and traditional model

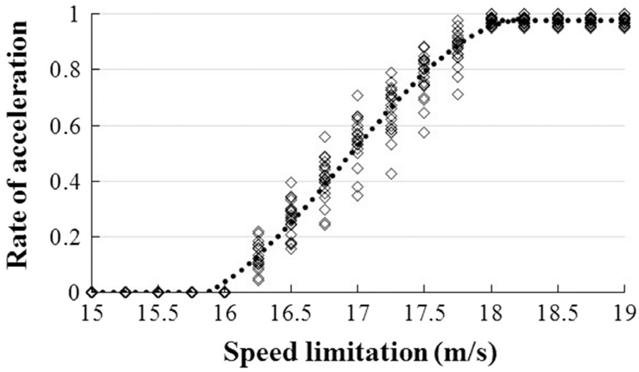
A small numerical test with 24 vehicles on the two crossing streets (12 vehicles on each) was used to explore and display the effect of the proposed car-following model. The initial position, space headway, and velocity of the vehicles on the two crossing street are listed in Table 1. The parameters used in the model are defined as follows: $\Delta t = 0.1$ s, $\kappa = 0.41\ \mathrm{s}^{-1}$, $\lambda = 0.2\ \mathrm{s}^{-1}$, $l_c = 9$ m, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13\ \mathrm{m}^{-1}$, $C_2 = 1.57\ \mathrm{m}^{-1}$, $\delta = 2$ s, $V_{\max} = 16.67$ m/s, $D = 400$ m.

Using the proposed model and the related parameters, each vehicle's position and velocity was obtained, as shown in Figs. 3 and 4, respectively. The positive effects of speeding guidance could be observed.

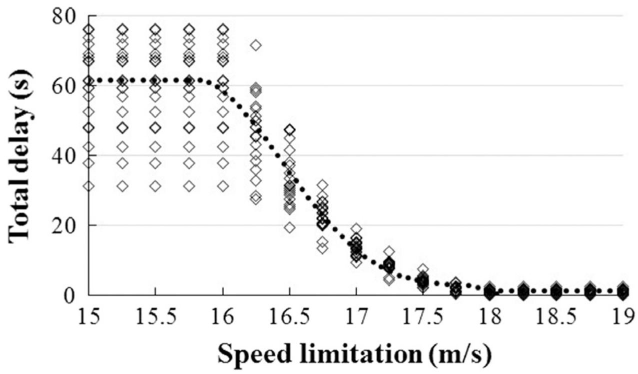
(1) Since the first 4 vehicles on each stream are stagger, they could go through the intersection without stopping under both tradition FVD model and the proposed model.
(2) Since the 5th, 6th, 7th, and 8th vehicle on stream N the was conflicted with the 5th, 6th, 7th, and 8th vehicle on stream M, respectively, they have to decelerate before arriving the intersection to avoid conflicting under the tradition FVD model condition. However, under the proposed model condition, these vehicles successfully utilize the gap ahead by using the acceleration strategy, which can reduce the vehicular delay and improve the traffic efficiency.
(3) The 9th–12th vehicles decelerated to zero and passed the intersection one by one under the tradition FVD model condition due to the high density of the traffic flow. However, under the proposed model condition, these vehicles successfully passed the intersection without stopping by using the deceleration strategy. It reduced the number of stops and improve the traffic efficiency.

3.2. Sensitivity analysis

This section further investigates the impact of different traffic and control condition on effectiveness the guidance strategy. For each test scenario, 200 vehicles on the two crossing streets (100 vehicles on each) was used. The initial headway



(a) Rate of acceleration.



(b) Total delay.

Fig. 7. Effects of speed limitation.

distribution is random and obeyed negative exponential distribution. To overcome the stochastic nature, 20 runs has been used in each testing scenario for fair comparison. Other parameters were kept the same as in Section 3.1.

From the comparison in Section 3.1, one may observe that the deceleration strategy is a last ditch plan. The acceleration strategy is a better choice which could resolve the conflicting without adding vehicular delay. Therefore, two performance indices was used to evaluate the performance, including the rate of the usage of acceleration strategy for conflicting vehicles and the total delay.

(1) Guidance spacing range

The space range is set from 0 to 700 m. The performance evaluation results are shown in Fig. 5. Overall, with the increasing of the guidance spacing range, more vehicles can using the acceleration strategy when the conflict between the two crossing streams occurs. It is because vehicles could make the acceleration more early. Under low guidance spacing range condition ($D \leq 300$ m), no vehicles using the acceleration strategy. It is due to the fact that accomplishing the acceleration strategy needs an essential minimum distance for vehicles to get the gap ahead. On the contrary, approximately all the vehicles facing the conflict will use the acceleration strategy if the guidance spacing range is enough long. Accordingly, the delay can be reduced to a quite low level.

(2) Density of the traffic flow

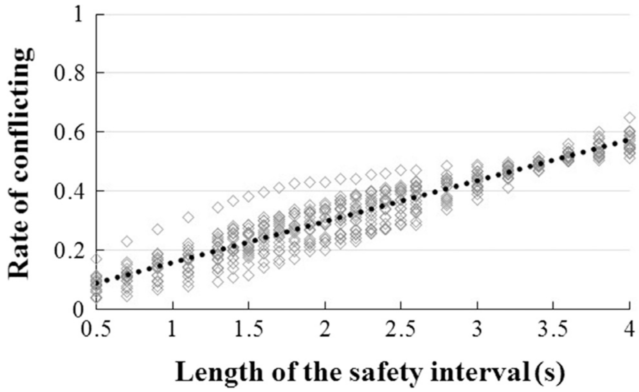
The initial density of the traffic flow is set from 5 to 25 Veh/km. The performance evaluation results are shown in Fig. 6. Obviously, the delay increases rapidly with the destiny. However, one should observe that the highest rate of acceleration does not occur at the lowest traffic density condition. With the increasing of the density, the rate of acceleration will first increase slightly and then decrease.

(3) Speed limitation

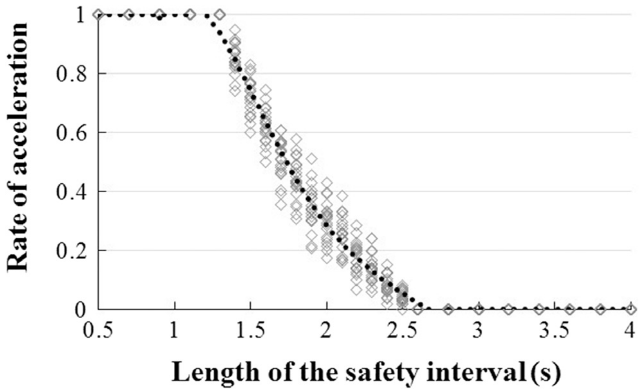
The maximum speed is set from 15 to 19 m/s. The performance evaluation results are shown in Fig. 7. A slight increase in the speed limitation can result in a significant improvement in enhancing operation efficiency and reducing delay. It indicates that the speed guidance strategy is sensitive to the maximum speed limitation. Therefore, a transiently increasing of the speed should be permitted.

(4) Length of the safety interval

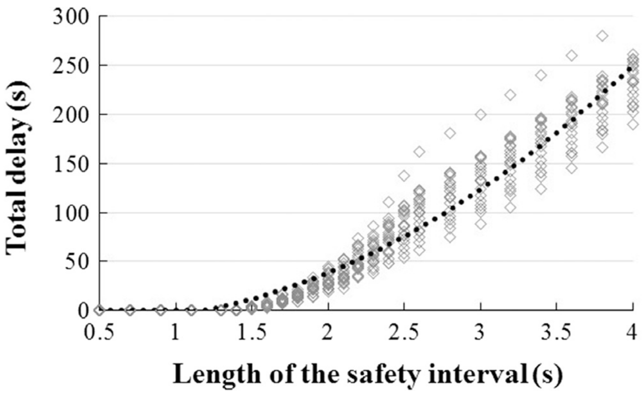
The length of the safety interval is set from 0.5 to 4 s. The performance evaluation results are shown in Fig. 8. The length of the safety interval affect the driving behavior from two aspects. Firstly, the rate of the conflicting between the two traffic flows linearly increases with the increasing of the length of the safety interval, as illustrated in Fig. 8(a). Therefore, more



(a) Rate of conflicting.



(b) Rate of acceleration.



(c) Total delay.

Fig. 8. Effects of length of the safety interval.

vehicles should be guided. Secondly, the rate of the acceleration decreases with the increasing of the length of the safety interval, as illustrated in Fig. 8(b). It is due to the fact that long safety interval makes the available gap hand to get. Therefore, as the above dual effects, the delay increases rapidly with the increase of the length of the safety interval.

4. Conclusions

The main motivation of this paper is to analyze the influences of V2V communication on the driving behavior at un-signalized intersection with two crossing streams and to investigate an extended car-following model to explore the how the speed guidance strategy affects the operation efficiency. The numerical results show that the proposed model can qualitatively describe its effects on microscopic traffic flow at un-signalized intersections.

In the numerical, some vehicles could successfully utilize the gap ahead by using the acceleration strategy, while some other vehicles could passed the intersection without stopping by using the deceleration strategy. Taking the V2V communication in to account can reduce the vehicular delay and the number of stops, and improve the performance operational efficiency at intersections. The benefits of the guidance strategy could be enhanced by lengthening the guiding space range and increasing the maximum speed limitation. Moreover, the guidance strategy is more suitable under low to medium traffic density and small safety interval condition. However, we only analyzed the benefits from the operation efficiency aspect. The safety and emissions problem have not been discussed which could be introduced in future work.

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