Analysis of Traffic Properties of Commuters in a Speed-Limit Corridor with Toll Station under Microscopic Method

Tao Wang[1]; Tie-Qiao Tang[2]; Jian Zhang[3]; and Peng Li[4]

Abstract: This study applies the full velocity difference (FVD) model to investigate commuters' driving process in a speed limit corridor with a toll station and explains how some factors affect the outflow of the traffic corridor system from the microscopic perspective. First, a micromodel system is constructed to depict the commute process in a traffic corridor with a toll station considering the speed limit. Second, a metastable state is defined in the paper for deriving the properties of commute problem conveniently. Finally, numerical tests are conducted to investigate some factors' effects on traffic properties. This study concludes that many traffic properties will be convergent when the commuter's number increases, and that the length of the road, the speed limit, and the service time of the toll station have effects on the time headway of the origin at the equilibrium state. The results show how the outflow forms in a speed-limit corridor with a toll station from the microscopic perspective and provide insights for policymaking. **DOI: 10.1061/JTEPBS.0000319.** *© 2020 American Society of Civil Engineers.*

Author keywords: Traffic corridor; Full velocity difference (FVD) model; Metastable state; Time headway.

Introduction

Considerable models about morning commute have been developed with the aim of relieving traffic congestion, which dates back to Vickrey (1969). In this seminal study, commuters travel on a single origin–destination pair road with a bottleneck, and they attempt to minimize their travel costs, which include travel time cost and an early/late penalty. After the classical bottleneck model, many extended bottleneck models have sprung up in the transportation research field (e.g., Hendrickson and Kocur 1981; Arnott et al. 1990; Xiao et al. 2013; Li et al. 2014; Xiao et al. 2014, 2015a, b; Wu and Huang 2015; Li et al. 2017), where the models focused on explaining the significance of schedule delay for departure time choice or analyzing the bottleneck from the view of economics. In addition, Yang and Huang (1997) have proposed a time-varying pricing model based on optimal control theory to alleviate congestion. Huang and Wu (2014) have analyzed some strategies on the road considering highway and transit. These bottleneck models are meaningful in providing policies to ease traffic congestion and make decisions on departure time. However, the bottleneck models cannot describe the dynamic properties of traffic flow behind bottlenecks because of their simplicity. Newell (1988) has developed a model for the morning commute in which a fixed number of identical commuters travel on a road with fixed length and demand subject to Lighthill-Whitham-Richards (LWR) model (Lighthill and Whitham 1955; Richards 1956), and derived the analytical solutions of social optimum (SO) and user optimum (UO) situations. Numerous efforts have been devoted to extending Newell's work (e.g., Kuwahara 1990; Arnott et al. 1993; Lago and Daganzo 2007; Akamatsu et al. 2015; Vincent et al. 2016). De Palma and Arnott (2012) have analyzed a model of early morning traffic congestion, which is a special case of Newell's model, deduced a closed-form analytical solution for the SO problem and a quasi-analytical solution for the user equilibrium (UE) problem, and provided some economic explanations, but they did not consider the part of late arrival. Li and Huang (2017) have extended this work (De Palma and Arnott 2012) to investigate the SO solution and UE solution in a single-entry corridor with late arrival and deduced some important analytical results with a novel approach. Moreover, some studies (Li et al. 2017; Li and Huang 2018; Li et al. 2018) have extensively considered the influences of some significant factors including staggered shifts, continuous time value, and the evening commute.

Prior studies, either the basic bottleneck models or Newell's work (and its extended works) are milestones for investigating morning commute problems. The basic bottleneck model has been extended for studying commuters' heterogeneity, road's heterogeneity, and different travel patterns (Kuwahara 1990; Arnott et al. 1993; Lago and Daganzo 2007; Akamatsu et al. 2015; Du and Wang 2014; Wang and Du 2016; Vincent et al. 2016), which thoroughly reveals the traffic properties in a complicated traffic system. Newell's work and its extensions further explained how the traffic wave and traffic density affect the outflow of the traffic system, and deduced numerous meaningful results under UE and SO patterns. Most of those efforts pay close attention to investigating traffic parameters (e.g., inflow rate, outflow rate, density, and speed) from the macroscopic point of view. However, it is important to

[1]Ph.D. Candidate, School of Transportation Science and Engineering, Beihang Univ., Beijing 100191, China. Email: taowang@buaa.edu.cn

[2]Professor, School of Transportation Science and Engineering, Beihang Univ., Beijing 100191, China (corresponding author). Email: tieqiaotang@buaa.edu.cn

[3]Ph.D. Candidate, School of Transportation Science and Engineering, Beihang Univ., Beijing 100191, China. ORCID: https://orcid.org/0000-0003-1344-8085. Email: jian_zhang@buaa.edu.cn

[4]Ph.D. Candidate, Dept. of Supply Chain Management, Rutgers Business School, Rutgers Univ., Newark, NJ 07102. Email: peng.li.scm@rutgers.edu

Note. This manuscript was submitted on September 26, 2018; approved on August 12, 2019; published online on January 13, 2020. Discussion period open until June 13, 2020; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Transportation Engineering, Part A: Systems***, © ASCE, ISSN 2473-2907.

investigate how the inflow rate forms from the perspective of drivers' behavior.

As a factor of importance in the traffic system, driving behavior has attracted many scholars' attention. Researchers have studied driving behavior in many ways. One of the most popular methods is the car-following model. This model, which dates back to 1953 (Pipes 1953), describes the processes by which drivers follow each other, and each vehicle's motion complies with the fundamental law of kinematics. Chandler et al. (1958) have proposed the well-known car-following model that is called the Gazis-Herman-Rothery (GHR) model (Gazis et al. 1961). The GHR model's core idea is that the vehicle's acceleration is proportional to the relative velocity between the vehicle and the one ahead of it, and a number of attempts (Herman and Potts 1959; Edie 1961; May and Keller 1967; Heyes and Ashworth 1972; Ceder and May 1976) were made to calibrate the parameters of the GHR model. Based on the GHR model, Herman and Potts (1959) proposed a model considering the impact of the front vehicle braking on the rear vehicle. Another important model is the collision avoidance (CA) model. The original CA model was proposed by Kometani and Sasaki (1959). Later, Gipps (1981) and other researchers calibrated and refined the previously mentioned models and proposed some new insightful studies. Bando et al. (1995) have proposed the optimal velocity model (OVM) and explored the model's stability. However, Helbing and Tilch (1998) have found that some vehicles will run with unrealistic accelerations and deceleration under the OVM, and then proposed the generalized force model (GFM). Jiang et al. (2001) have developed the full velocity difference (FVD) model, which also avoids the irrational acceleration and deceleration in simulations. In recent years many car-following models (Zhao et al. 2016; Li et al. 2016, 2017) have been proposed to describe different traffic scenes or from different perspectives.

Most researchers have explored the morning commute problem by the macroscopic method (i.e., bottleneck model, LWR model, etc.). However, these findings can neither demonstrate how commuters' driving behaviors influence the traffic features in the morning commute nor clarify the inflow pattern under UE state from the perspective of individual commuter. Scholars (Correia and Silva 2011; Silvano and Bang 2016; Tan et al. 2017) have found that the speed limit has significant impacts on the road features in an urban area. In this paper, we first introduce the speed limit into the morning commute problem from the microscopic perspective. Second, the metastable state is developed to study the basic properties of the traffic corridor. Some numerical tests are studied to verify the analytical results and explore how many factors (e.g., the service time of toll station, the limit speed, the road length, etc.) affect the time headway at the origin under the UE state. Finally, we summarize a brief conclusion and future research.

Microscopic Method for Analyzing the Simple Traffic Corridor

Commute Process

Consider a traffic station depicted in Fig. 1 with a toll station. The origin represents a toll station with constant service rate and the destination a city business center. All commuters have same travel path in the one-to-one corridor.

The commute process on the simple traffic corridor can be divided into two parts:

1. Commuters depart from the same origin and pass through a toll station with a fixed time interval (i.e., the service rate of the toll station).

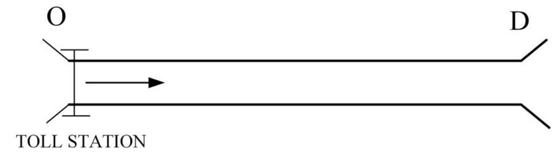

Fig. 1. Sketch of one-to-one traffic corridor.

2. Commuters depart with a fixed rate (unless their time headways are longer than the service time of the toll station) and drive their cars to the destination under the car-following model.

In Part 1, if the departure rate in origin exceeds the service rate, vehicles will queue to pass through the toll station. Otherwise, the opposite is true. In Part 2, commuters tend to start up with a fixed departure rate (unless the inflow rate is less than the service rate) and drive their cars to the destination. The service rate of the toll station is always lower than the capacity of the road. Let t^T denote the duration of passing through the toll station, $3,600/t^T$ as the service rate of toll station. Some definitions of commute process are listed in the "Notation" section.

When the nth commuter reaches the toll station, they must wait to pass through the toll station if the front cars have not finished the charging process. Otherwise, the nth commuter only spends t^T to pass through the toll station. The formulation of the commute process is given by

$$t_n^T = \max\{t_n^O, t_{n-1}^T\} + t^T \tag{1}$$

$$t_n^w = \max\{0, t_{n-1}^T - t_n^O\} \tag{2}$$

When the nth commuter has finished the payment at toll station, they drive to the destination under the ruler of car-following behavior. Based on the drive environment, such as velocity, relative velocity, and relative spacing with front vehicle, the commuters might change their drive behaviors. Vehicles pull out the toll station with a relatively large time interval, $\Delta t_n^D (\geq t^T)$, which can ensure that each commuter can speed up quickly at the origin. Commuters will finish the acceleration, cruise, and park process with the car-following model and the speed limit.

Traffic Properties of the Traffic Corridor

When the commuter is driving on the road, they will change the vehicle's velocity based on the complicated information collected from the drive environment, including relative spacing, relative speed, and other factors. After changing the velocity, the commuter will keep the state for a while because no one has the energy to push or release the gas pedal all the time and it is difficult for drivers to react to nuance in traffic flow. Some studies (Tang et al. 2015, 2016) about bounded rationality in driving behaviors have explained that changing acceleration frequently has an adverse impact on traffic flow and that drivers are not supposed to change their acceleration if the absolute value of acceleration calculated by the car-following model is less than a threshold. Based on the literature (Tang et al. 2015), this study defines a metastable state in this paper. Commuters' motion can be divided into two steps.

1. One commuter decides how fast their car should be according to the information that he/she collects and accelerates the car to the corresponding velocity.
2. The commuter does not change the vehicle's speed until the acceleration calculated by the car-following model is bigger than a threshold.

The state in which commuters do not change their velocity in Step 2 is called the metastable state in this paper. Based on previously mentioned contents, commuters' acceleration is formulated in Eq. (3) (Tang et al. 2015). In this equation, ε represents a threshold parameter; $a_n(t)$ denotes the nth commuter's actual acceleration at time t; and $A_n(t)$ is the nth commuter's acceleration calculated by the car-following model. Thus, $a_n(t)$ can be formulated as follows:

$$a_n(t) = \begin{cases} 0, & \text{if} |a_n(t)| \leq \varepsilon \\ A_n(t), & \text{if} |a_n(t)| > \varepsilon \end{cases} \tag{3}$$

Some notations for analyzing commuters' motions are listed in the "Notation" section, and some assumptions must be proposed in this paper as follows:

1. No overtaking in the traffic corridor system;
2. The original state of the traffic flow has a fine continuity, i.e., the time headway at origin cannot be quite long, and this paper does not consider the parking process because of its uncertainty and complexity;
3. Vehicles' speed cannot be greater than the speed limit of the road; and
4. Vehicles will cruise if they have run for a long time, their velocity change slightly, i.e., the influence of perturbation is not considered in this paper.

Without loss of generality, this paper analyzes commuters' properties on the road based on the FVD model (Jiang et al. 2001). In the FVD model, each vehicle's acceleration is determined by its velocity, relative velocity, and relative spacing

$$\frac{dv_n(t)}{dt} = \kappa[V(\Delta x_n(t)) - v_n(t)] + \lambda \Delta v_n(t) \tag{4}$$

where $V(\cdot)$ is an increasing function, but the second derivative of the function is less than zero, which depends on the relative spacing. It is similar to the k-v relation in the fundamental diagram. The λ is the reaction parameter, which is equal to zero when commuter's relative spacing is larger than l_R. This means that commuter cannot get the value of relative velocity by physiological perception if the relative spacing is long enough.

Imagine that when the nth commuter just passes x they can make their decision considering much information, and the traffic flow under the coordinate (n, x) can reach the metastable state. Let $v_n^s(x)$ denote the nth commuter's speed when the traffic flow reaches the metastable state around x. It is the first metastable state for the nth commuter after they pass x.

With respect to the first commuter, their driving behaviors have nothing to do with the relative velocity or relative spacing (the reference is the destination) when they cannot see the destination. When the first vehicle speeds up to a metastable value, the commuter need not consider the term composed by relative velocity, and the speed at metastable state when x is relatively large can be formulated as follows. Thus, the model of the first commuter's speed in a metastable at Position x can be built as follows:

$$v_1^s(x) = V(\eta L) = v^R \tag{5}$$

where $0 < \eta < 1$ is a parameter that depends on L and the road's speed limit. For the second commuter, their driving behaviors are related to the relative spacing and relative speed. After responding to the relative spacing and relative velocity, the second commuter will speed up or slow down to the metastable state. When the second commuter just reaches the metastable state, their acceleration is equal to zero. Combined with Eq. (4), it can be obtained that the second commuter's velocity at the metastable state is related to their optimal velocity and the first commuter's velocity at the starting point of Step 2. We consider the case in which the location x is relatively large, at which the first commuter runs with the speed $v_1^s(x)$. At this time, $v_1^s(x)$ can be formulated as follows:

$$v_2^s(x) = \frac{\kappa V(\Delta x_2) + \lambda v_1^s(x)}{\kappa + \lambda} \tag{6}$$

When $x \geq \max\{x_n^c\}$, vehicles cruise on the road. The duration of the metastable state of the (n–1)th commuter is longer than the time headway of the nth commuter. Similarly, the $v_{n-1}^s(x)$ is given by

$$v_n^s(x) = \frac{\kappa V(\Delta x_n) + \lambda v_{n-1}^s(x)}{\kappa + \lambda} (n \geq 2) \tag{7}$$

From Eqs. (5)–(7), it can be concluded that commuter's speed at a metastable state is related to the relative spacing and the leader's speed. However, a different original state (inflow rate) and a different location have effects on the speed at the metastable state. This paper will analyze the properties of commuter's speed at a metastable state when the location x is relatively large under an original state with fine continuity.

Lemma 1. When $x \geq \max\{x_n^c\}$, each commuter' velocity at metastable state is greater than a positive constant.

Proof. When $x \geq \max\{x_n^c\}$, all commuters have finished their acceleration process. Eqs. (4)–(7) indicate that the commuter's velocity at the metastable state depends on their leader's velocity and the relative spacing. No one can stop as long as the first commuter runs at a large speed, because all commuters must follow their leader vehicle. If the nth commuter runs at a low speed, Δx_n, $V(\Delta x_n)$, and $v_n^s(x)$ increase, i.e., the vehicle must accelerate. Therefore, when $x \geq \max\{x_n^c\}$

$$v_n^s(x) \geq v^c \tag{8}$$

where v^c is a positive constant.

Lemma 2. When $x \geq \max\{x_n^c\}$, each commuter's velocity at the metastable state is less than or equal to their leader's.

Proof. When $x \geq \max\{x_n^c\}$, vehicles cruise on the road. When the nth commuter passes the location x, the (n–1)th commuter still runs with the speed $v_n^s(x)$. According to Assumption 1

$$v_1^s(x) \geq v_2^s(x) \geq \cdots \geq v_n^s(x) \tag{9}$$

Proposition 1. When $x \geq \max\{x_n^c\}$, $\{v_n^s(x)\}$, $V(\Delta x_n)$ and $\{h_n^t\}$ at the starting point of Step 2 are convergent.

Proof. Using Lemma 1 and 2, it is easy to prove that $\{v_n^s(x)\}$ is convergent when $x \geq \max\{x_n^c\}$. If there are enough commuters, the (n–1)th commuter's velocity at the metastable state is approximately equal to the nth commuter's, that is

$$v_{n-1}^s(x) \approx v_n^s(x) \tag{10}$$

Substituting Eq. (10) into Eq. (7)

$$v_n^s(x) = \frac{\kappa V(\Delta x_n) + \lambda v_{n-1}^s(x)}{\kappa + \lambda} \approx \frac{\kappa V(\Delta x_n) + \lambda v_n^s(x)}{\kappa + \lambda} \tag{11}$$

Simplifying Eq. (11)

$$V(\Delta x_n) \approx v_n^s(x) \tag{12}$$

Recall that $V(\cdot)$ is an increasing function that depends on Δx_n. If there are infinite commuters, $V(\Delta x_n)$ is convergent to a constant. Consequently, the nth commuter's time headway and spacing headway are convergent to a constant. Based on the previous analysis, it is obvious that each commuter's relative spacing drops with their

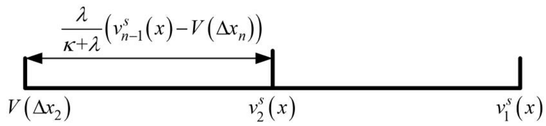

Fig. 2. Relation of $v_n^s(x)$ and $V(\cdot)$.

number and is convergent to a constant when the commuters are at the metastable state.

Proposition 1 indicates that at the destination commuters' speed, time headway, and spacing headway are convergent. It is a significant conclusion that can reveal the basic properties of the traffic corridor. In the numerical tests, the factors that influence the convergence speed and convergence value will be investigated.

Lemma 3. When $x \geq \max\{x_n^c\}$, $\Delta v_n^s(x)$ is proportion to $(v_{n-1}^s(x) - V(\Delta x_n))$ at the starting point of Step 2. The proportionality coefficient is equal to $\frac{\kappa}{\kappa+\lambda}$, where $V(\Delta x_n) \leq v_n^s(x) \leq v_{n-1}^s(x)$.

Proof. Rewriting Eq. (7), it can be obtained that

$$\Delta v_n^s(x) = v_{n-1}^s(x) - v_n^s(x) = \frac{\kappa}{\kappa+\lambda} \cdot [v_{n-1}^s(x) - V(\Delta x_n)](n \geq 2) \tag{13}$$

where $V(\Delta x_n) \leq v_n^s(x) \leq v_{n-1}^s(x)$ makes sure that $\Delta v_{\mathrm{n}}^{\mathrm{s}}(x) \geq 0$. The content of Lemma 3 is depicted in Fig. 2.

Proposition 2. In FVD model, l_R cannot be less than ηL too much; otherwise, the second commuter will decelerate in the acceleration process.

Proof. It is supposed that the length of the road is long enough and the first commuter's location is x_1. If $l_R \ll \eta L$, there is a metastable state in which $\Delta x_2 = l_R$, and $v_2^s(x) > V(l_R)$ according to Lemma 3. The second commuter's velocity at the cruise process is approximate to that of the first commuter. Therefore, when $\Delta x_2 = l_R$, the second commuter is in the acceleration process.

When the first commuter arrives at the next location x_1', it is entirely possible that $l_R < \Delta x_2' < V^{-1}(v_2^s(x))$, the second commuter tends to slow down under the FVD model in the acceleration process. It does not square with the facts.

If there is an electric control unit calculating the speed in real time and control the vehicle based on FVD model, the condition in Proposition 2 will become $l_R \geq \eta L$.

Trip Cost and Equilibrium State

User equilibrium is a significant research content of departure time choice (Arnott et al. 1990; Newell 1988; De Palma and Arnott 2012; Li and Huang 2017) in the morning commute problem, which was first proposed by Wardrop (1952). No one has the incentive to change their departure time when the system has reached the UE state. If a commuter departs at time t, their trip cost consists of the travel time cost and the schedule delay cost, which can be formulated as follows (De Palma and Arnott 2012):

$$C(t) = \alpha_1 \tau(t) + \max\{\alpha_2[\bar{t} - t - \tau(t)], \alpha_3[t + \tau(t) - \bar{t}]\} \tag{14}$$

where $C(t)$ = trip cost when commuter departs at time t; $\tau(t)$ = travel time of the vehicle departing at time t; α_1 = per unit cost of travel time; α_2 = per unit of early arrival time; α_3 = per unit of late arrival time; and $\bar{t}$ = common preferred arrival time. The trip cost function can be rewritten in a discrete form (Tang et al. 2017, 2015) with respect to the commuter's number

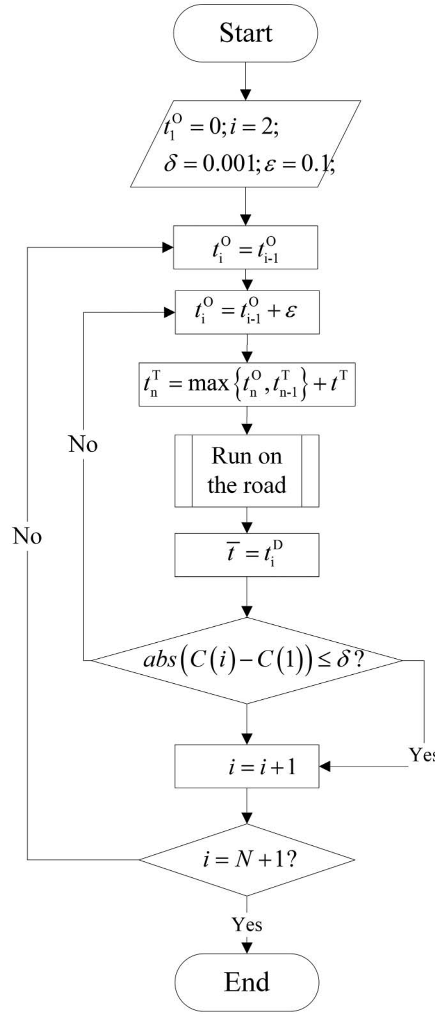

Fig. 3. Flow diagram for solving inflow rate under user equilibrium state.

$$C(n) = \alpha_1(t_n^D - t_n^O) + \max\{\alpha_2(\bar{t} - t_n^D), \alpha_3(t_n^D - \bar{t})\} \tag{15}$$

where $C(n)$ = nth commuter's trip cost; and other parameters have been defined in the "Notation" section. The following condition must be met when commuters reach the UE state:

$$C(1) = C(2) = \cdots = C(n) \tag{16}$$

This paper defines the conception of inflow rate and develops the solving method for inflow rate (Tang et al. 2017) under the UE state. The flow chart (Fig. 3) has provided the solving method for inflow rate under the UE state even though no commuter is late to work. The case considering the late arrival can be solved by a similar method. We will analyze the speed limit, the length of the road, and the effects of the toll station's service time on the time headway at the origin under the UE state in numerical tests.

Numerical Test

This section conducts some numerical tests to verify the previous analytic results and investigate how to obtain the UE pattern. The previous section proposed some basic results about the commute problem. However, there are two obstacles the study has to go over if presenting more detailed results. The first one is that the car-following model consists of numerous differential equations, which

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Table 1. Basic parameter settings in numerical tests

Parameter	Value
t^T (s)	5
κ	0.41
λ	0.5
Δt_n^o (s)	3
l_c (m)	5
α_1 (RMB$\cdot$s^{-1})	0.01
α_2 (RMB$\cdot$s^{-1})	0.008
L (m)	5,000
v^R (m$\cdot$s^{-1})	18

Note: RMB = Ren Min Bi.

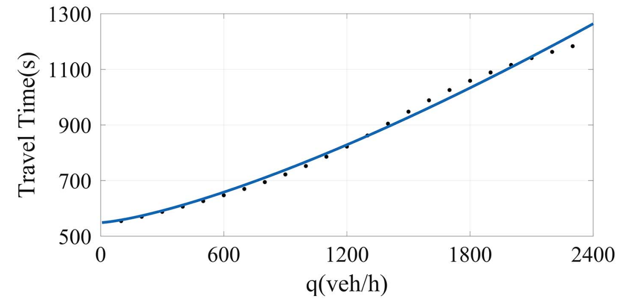

Fig. 5. Fitting curve of BPR function under FVD model.

is difficult to solve analytically. In addition, we have struggled to gain access to the boundary conditions (inflow rate) of the system in the UE state. Therefore, the numerical method is necessary to propose to overcome these problems. Furthermore, many factors including the length of the road, the service time of the toll station, and the limiting speed of the road are investigated in numerical tests combined with the inflow rate under the UE state.

Tang et al. (2017) applied the FVD model to study the commute problem and proposed the optimal speed formula as follows:

$$V(\Delta x) = 19.037 \exp\left(-18.94 \frac{1}{\Delta x + l_c}\right) \tag{17}$$

where l_c = length of the vehicle.

First, we verify the analytic results obtained from previous section in this section. Some parameters of numerical study are shown in Table 1. For simplicity, the time headway at the origin is 2 s, and the service rate of the toll station is 0 s in the first group numerical tests. It means that there is no toll station in the first group numerical test. The relationship between the convergence value and service rate is investigated in the following contents.

Fig. 4 shows that the commuters' optimal velocity and time headway at location $x = 4{,}000$ m. From the figure, it is evident that commuters' optimal velocity and time headway is convergent with commuters' number. The fitting curve of BPR function under FVD model is shown in Fig. 5, where the fitting function is given by

$$t(q) = 548.8 \times \left[1 + 0.8776 \times \left(\frac{q}{1810}\right)^{1.314}\right] \tag{18}$$

The correlation coefficients of the fit function are displayed in Table 2, where $t(q)$ is the average travel time for 1,000 m with road flow q. It is distinct that the road capacity under the FVD model is 1,810 veh/h, which can be transferred to the time headway based on the following function:

Table 2. Correlation coefficients of the fit function

Index	Value
SSE	1,841
R^2	0.9565
Adjusted R^2	0.9853
RMSE	28.2991

$$h_t = \frac{3600}{q} \tag{19}$$

where h_t = time headway. It can be easily found that commuters' time headways decrease and are convergent to a constant, which can be calculated by Eq. (19) when the flow is equal to the road capacity.

Additionally, the second commuter's speed curve with respect to the system time is plotted to verify Proposition 2. As shown in Fig. 6, when l_R is equal to 100 m, the second commuter will decrease their speed at around 50 s. the When the system time is equal to 50 s, the second commuter does not cruise even though the acceleration is small and has no need to decelerate in this case. A valley appears on the velocity curve of the second commuter. The reason for the appearance of the valley is proposed in Proposition 2. However, when l_R is equal to 300 m, the unreasonable phenomenon does not happen, even though ηL is about 330 m. The valley becomes flatter and flatter with the increase of l_R. The metastable state makes sure that the second vehicle can go through the road without the valley when l_R is close to ηL. If the vehicle changes its velocity in real time, it cannot avoid the valley unless $l_R \geq \eta L$.

This section investigates the influence of some factors of the road on the inflow rate under user equilibrium state. The time headway at origin will be calculated as the method proposed in Fig. 3. It is well known that there is a relationship between the inflow rate

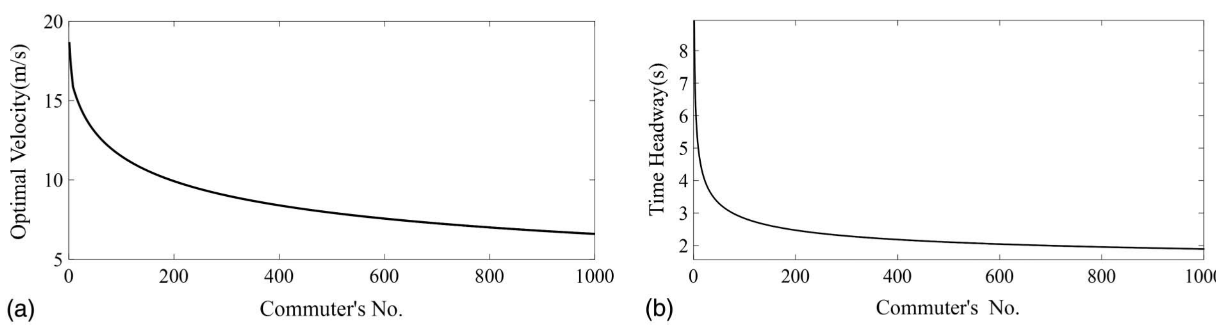

Fig. 4. Commuters' properties at $x = 4{,}000$ m: (a) commuters' optimal velocity at $x = 4{,}000$ m; and (b) commuters' time headway at $x = 4{,}000$ m.

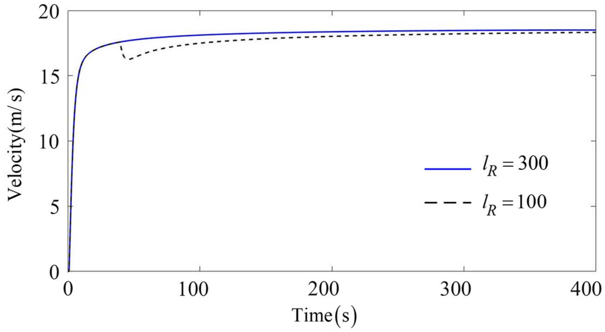

Fig. 6. Second commuter's velocity.

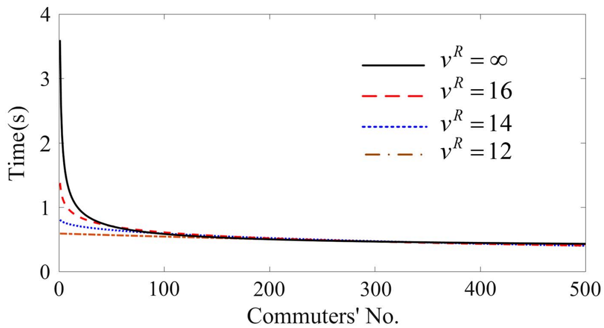

Fig. 7. Influence of limiting speed on the time headway at the origin.

and outflow rate under the UE state in the bottleneck model, which has been transferred to the discrete form (Tang et al. 2017). Thus, the time headway at the UE state can be obtained

$$\Delta t_n^O = \frac{\alpha_2}{\alpha_1 + \alpha_2} \Delta t_n^D \tag{20}$$

Eq. (20) reveals the relation of the time headways for those who are early to work. If considering the commuters who are late to work, the result has a similar form. Therefore, it is adequate to study the impacts of the factors on the time headway at the origin.

First of all, speed limit is investigated in Fig. 7 under the condition that the service time of the toll station is zero. The speed limit has a huge influence on some commuters in the front of the motorcade. However, it is not conclusive that the speed limit affects the convergence value of the time headway. In fact, the speed limit can reduce the convergence value as long as the speed limit is low enough to ensure that the capacity of the road is changeless.

In addition, the length of the road is investigated in Fig. 8. In this case, $t^T = 0$ without a speed limit. The length of the road can reduce the convergence speed of the time headway, and the effect decreases with the length of the road. From the perspective of commuters, commuters' time headway at the origin increase with the road's length and the effect is decreasing with commuters' number.

The influence of the service time of the toll station is shown in Fig. 9. In this case, the service time is 0, 3, and 5 s without a speed limit. The service time of the toll station has a slight influence on the time headway of some commuters who are in the front of the motorcade. These commuters' time headway from the origin is long enough, and the $(n+1)$th (n is a small positive integer) commuter almost has no need to wait at the toll station unless $t_{n-1}^T - t_n^O > 0$. Even though $t_{n-1}^T - t_n^O > 0$, the waiting time for those is short enough. However, the service time has a great influence on the convergence value of the time headway at the origin.

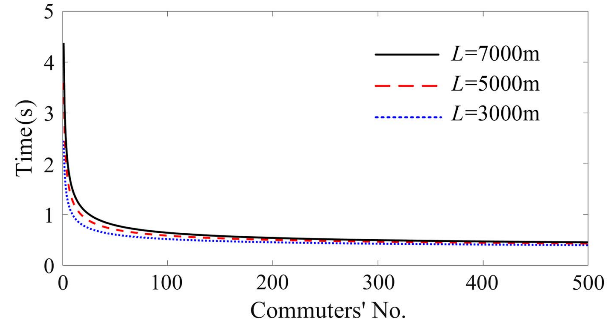

Fig. 8. Influence of the length of road on the time headway at the origin.

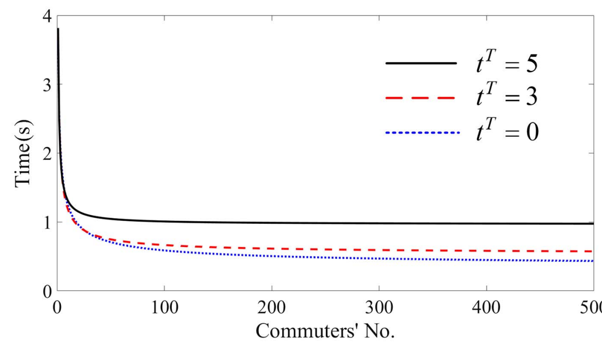

Fig. 9. Influence of service time of toll station on time headway at the origin.

Combined with the previous analyses, it can be concluded that the convergence value of the time headway is equal to the maximum of

$$\left(\frac{\alpha_2}{\alpha_1 + \alpha_2} \times \frac{3600}{c}\right) \quad \text{and} \quad \left(\frac{\alpha_2}{\alpha_1 + \alpha_2} \times t^T\right)$$

where c = road capacity.

Conclusions

Morning commute problem is a hot topic both in the transportation research field and the urban economic field. Many scholars have made numerous endeavors to investigate the inflow rate of the system under the UE state and multitudinous economic policies for relieving traffic congestion. In this paper, we propose a novel definition that is the metastable state, which is a reality communter's decisions based on the speed limit corridor with a toll station. Drivers make decisions and accelerate their cars to a metastable state and then the vehicles' condition remains unless drivers decide to accelerate with a relative acceleration. Some properties of the traffic flow can be analyzed based on our assumptions. Finally, this paper conducts some numerical tests to verify our analytic results and investigate the impacts of some factors on the inflow rate under the UE state, and revealed the convergence value of commuters' time headway. The results reveal how commuters' time headway and other properties form and explain the influence of the road's properties on the traffic flow under the UE state.

From the previous analysis, the properties of the commute problem in the traffic corridor, especially the convergence of the time headway at UE condition, is related to the bottleneck with minimum capacity. This paper indicates that the toll station (without electronic toll collection) should improve its service rate to close to the capacity of the most congested bottleneck in the traffic corridor. In addition, it is obvious that the speed limit can affect the commuters' departure rate, and therefore the administrator can set a time-varying speed limit curve in the road to induce commuters to depart with an economical departure rate, which will be investigated in our future research. From the perspective of the traffic flow model, this paper demonstrates that the FVD model will describe unreasonable phenomenon when the parameter of the optimal velocity is set improperly, which is insightful to the microscopic simulation of the traffic flow.

Unfortunately, this study has not explained analytically why these factors affect the inflow rate under the UE state and what the convergence value and mathematical formulas of velocity are. Some innovative models are supposed to be put forward for solving these gaps. Moreover, many factors (Ravishankar and Mathew 2011; Lam et al. 2013) in the traffic system have a significant influence on the traffic properties of traffic flow. Therefore, these factors will influence the properties of the commute problem, and we will investigate it further in our research. Furthermore, researchers (Khoury and Srour 2015) investigated congestion pricing in a high congested corridor by nonlinear optimization model and simulation, which gives fascinating insight into our future work and we will propose some effective strategies to alleviate the traffic congestion.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available from the corresponding author by request.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (71422001 and 71890971).

Notation

The following symbols are used in this paper:

l_R = threshold for λ;
t_n^D = time when the nth commuter arrives at the destination (s);
t_n^O = time when the nth commuter departs from the origin (s);
t^T = time when a car passes through the toll station (s);
t_n^T = time when the nth commuter leaves the toll station (s);
t_n^w = nth commuter's waiting time at the toll station (s);
$V(\cdot)$ = commuter's optimal velocity $(\mathrm{m\cdot s^{-1}})$;
$v_n(t)$ = nth commuter's velocity at time t $(\mathrm{m\cdot s^{-1}})$;
v^R = limiting speed of the road $(\mathrm{m\cdot s^{-1}})$;
$v_n^s(x)$ = nth commuter's speed at metastable state around location x $(\mathrm{m\cdot s^{-1}})$;
x_n^c = position when the nth commuter finishes acceleration process
$x_n(t)$ = nth commuter's spacing at time t (m);
Δt_n^D = $(t_n^D - t_{n-1}^D)$ (s);
Δt_n^O = $(t_n^O - t_{n-1}^O)$ (s);
$\Delta v_n(t)$ = nth commuter's relative spacing at time t $(\mathrm{m\cdot s^{-1}})$;
κ = commuter's reaction parameter on relative spacing; and
λ = commuter's reaction parameter on relative velocity.

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