**Elasticity**

**Stress and strain**

The stress on a body acted on by a deforming force is equal to F/A, the magnitude of the force divided by the cross-sectional area A over which it acts. The unit of stress in SI units is newtons per square meter, which is known as the pascal (Pa). In the British system it is customary to use pounds per square inch. The three categories of stress – tension, compression, and share – are illustrated in the figures bellow. The unstressed shape is shown by the dashed lines, and the stressed shape by the solid lines.

No stress Compression Tension Shear

The relative change in the size of a body due to applied stress is called strain. Strain is a dimensionless quantity, for instance, the longitudinal strain that tension produces in a body is its change in length divided by its original length , which is a pure number.

**Elasticity**

The elastic limit of a material is the maximum stress that can be applied to a body without causing a permanent deformation. For stresses below the elastic limit, the material exhibits elastic behavior: when the stress is removed, the body returns to its original size and shape.

Below elastic limit, strain is found to be proportional to stress. This relationship is known as Hooke’s Law. In the case of tension, for example, doubling the applied force on a body will double the amount by which the body stretches. The modulus of elasticity of a material subjected to a particular kind of stress below its elastic limit is defined by the relationship

The ultimate strength of a material is the greatest stress it can withstand without rapture. In many materials the ultimate strength considerably exceeds the elastic limit. When a stress greater than its elastic limit but less than its ultimate strength is applied to such a material, the result is permanent deformation. Bending a piece of metal is an example.

Example 1

A nylon rope 24 mm in diameter has a breaking strength of 120 kN. Find the breaking strengths of similar ropes (a) 12 mm and (b) 48 mm in diameter.

1

**Young’s Modulus**

When a tension or compression force F acts on an object of length and cross-sectional area A, the result is a change in length . Below the elastic limit, the ratio between stress and strain in this situation is called

The value of Young’s modulus depends only on the composition of the object, not on its size or shape. The usual units of are newtons per square meter or pounds per square inch.

Example 2

An aluminum wire 3 mm in diameter and 4 m long is used to support a mass of 50 kg. What is the elongation of the wire? Young’s modulus for aluminum is .

Example 3

The elastic limit of aluminum is . What is the maximum mass that the wire of example 2 can support without exceeding its elastic limit?

Example 4

A wire 8 ft long with a cross-sectional area of 0.01 stretches by 0.05 in. when a weight of 100 lb is suspended from it. Find the stress on the wire, the resulting strain, and the value of Young’s modulus for the wire’s material.

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Example 5

A steel pipe 3.6 m long is placed vertically under a sagging floor to support it. The inside diameter of the pipe is 80 mm, its outside diameter is 100 mm, Pa. A sensitive strain gauge indicates that the pipe’s length decreases by 0.1 mm. What is the magnitude of the load the pipe supports?

Example 6

By how much can a steel wire 3 m long and 2 mm in diameter be stretched before the elastic limit exceeded? Young’s modulus for the wire is Pa. and its elastic limit is Pa.

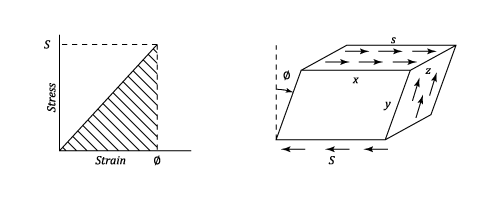
Example 7

A steel cable whose cross-sectional area is 1 in.2 is used to support an elevator cab weighing 5000 lb. If the stress in the cable is not to exceed 20 percent of the cable’s elastic limit of 40,000 lb/in2, find the maximum permissible upward acceleration.

3

**Shear Modulus**

A shear stress changes the shape of an object, not its volume. The figure below shows a rectangular block acted by shear forces F. the shearing stress is equal to F/A, and its shearing strain is equal to the angle of shear , express in radians. Because is always small, it is very nearly the same as the ratio between the displacement of the block’s faces and the distance between these faces. Below the elastic limit, then there are two equivalent expressions for the shear modulus or (modulus of rigidity):

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Example 8

The shearing strength of a certain steel alloy is Pa. Two 5 mm-diameter bolts of this alloy are used to fasten a bracket to a wall. What is the maximum load the bracket can support with shearing off the bolts?

Example

How much force is required to punch a hole ½ in. in diameter in a steel sheet in thick whose shearing strength is ?

4

**Bulk Modulus**

When compressive forces act over the entire surface of a body, its volume decreases. If the compressive force per unit area is uniform, the is given by

The minus sign is included because an increase in the volume stress leads to a decrease in the volume.

Volume stresses occur when objects are immersed in liquids, since liquid exerts a uniform force perpendicular to any surface in its interior. The stress exerted by a liquid is called

Example 10

The pressure at a depth of 300 m in the ocean exceeds sea-level atmospheric pressure by kPa. By how much does the volume of a 0.2 m3 aluminum object contract when lowered to this depth in the ocean? The bulk modulus of aluminum is Pa.

Example 11

The reciprocal of the bulk modulus of a liquid is called its , so . The bulk modulus of water is Pa. (a) Find its compressibility per atmosphere of pressure, where Pa is the pressure exerted by the earth’s atmosphere at sea level. (b) How much pressure in atmospheres is needed to compress a sample of water by 0.1 percent?

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Exercises

1. A wire 5 m long and 4 mm in diameter supports a load of 80 kg. If the wire stretches by 2.6 mm, find the value of Young’s modulus for its material.
2. A steel wire 6 ft long and 0.002 in cross section supports a load of 15 lb. (a) What is its elongation? (b) What would its elongation be if the load were be doubled to 30 lb?
3. A lead cube 20 mm on each edge is held in the jaws of a vise with a force of 3 kN. By how much is the cube compressed? Young’s modulus for lead is Pa.
4. The ultimate strength in tension of a certain type of steel is Pa. What is the maximum tension a rod made of this steel and 25 mm in diameter can withstand?
5. An elevator cab weighing 3000 lb is designed for a maximum upward acceleration of 12 ft/s2. If the stress in its cable is not to exceed 6000 lb/in2, what should the cable be?
6. A steel cable of cross-sectional area 2.5 cm2 supports a 1000-kg elevator. The elastic limit of the cable is Pa. If the stress in the cable is not to exceed 20 percent of the elastic limit, find the maximum upward acceleration of the elevator.
7. Two steel beams are riveted together to form a single longer beam. Eight rivets 10 mm in diameter are used. If a tension force of 55 kN is applied to the new beam, what is the shearing stress on the rivets? How does this compare with their shearing strength Pa?
8. Find the force needed to punch a hole 1 in square in a steel sheet 0.05 in thick whose shearing strength is lb/in2.
9. A punch press that exerts a force of 20 kN is used to punch holes 1 cm square in sheet aluminum. If the shear strength of aluminum is 70 MPa, what is the maximum thickness of aluminum sheet that can be used with the press?
10. When a pressure of 300 lb/in2 is applied to a mercury sample, it contracts by 0.008 percent. Find the bulk modulus of mercury.

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