Spatial and drones

An exciting target

- Exciting demo for spatial $\Rightarrow$ Answer to why spatial, and what can spatial do
- Intellectually stimulating $\Rightarrow$ Lots of research on the subject

Full of potential

Air is still very much an uncharted territory:

- Surveillance
- Search and rescue
- Logistic inside warehouses
- Transport of materials or documents
- Monitoring (crops, protection of species in danger)

A constrained problem

Energy bound

Improved efficiency $\Rightarrow$ Extended flight time

- Hovering rule of thumb: ~**150W/Kg**
- A drone like the AF450 from our lab ~**100W**
- His FMU (Flight Management Unit), a Pixhawk: ~**1-2W**
- Jetson TX2, the latest embedded CUDA board from NVIDIA consumes around: ~**8W**

Latency bound

SLAM (Software localization and mapping) is critical for **motion planning** and **motion control**.

A critical subproblem of **SLAM** is **POSE** (position estimation)

$$\hat{x}_t = f(x_{t-1}, O_t)$$

- x_t is the state of the drone (including position, attitude (orientation), velocity, etc ...) at time t
- $\hat{x}$ is the estimation of that state
- O_t is the observation of the universe by the drone at time t
- f is the SLAM prediction algorithm

Goal

Latency $= \Delta t = \text{Max}(f \text{ time}, O \text{ sample rate})$

Reduce f computation time closer to O sample rate.

Reduced latency result in more accurate $\hat{x}_t$:

- Smoother control $\Rightarrow$ Less jiggering + "agile" drone
- Better sync between planning and control
- Better collision avoidance $\Rightarrow$ safer for the drone and its surrounding.

Performance bound

Currently, heavy tasks are usually done:

- On a companion computer on the ground
- Sometimes, offline (after the flight) from the data gathered

Preferable or critical to do them **onboard** and **online**

The vision

Accelerating hardware!

(A plasticine in every drone)

- Efficient
- Low-latency
- Performant

Focus on **FPGA** and likely the **OcPoc** from aerotenna which include a **cyclone V**.

- Plasticine: An hardware architecture made by us for spatial
- FPGA: A common reprogrammable hardware achitecture targetable by spatial
- Spatial: the compiler from DSL to spatial hardware architecture program

Sensor fusion

Sensor fusion

Sensor fusion is the fusion of the data from different sensor to get accurate estimator of one state.

- Dual GPS
- accelerometer + gyroscope for attitude
- LIDAR + IMU

Sensor fusion can be achieved through the combination of filtered signals.

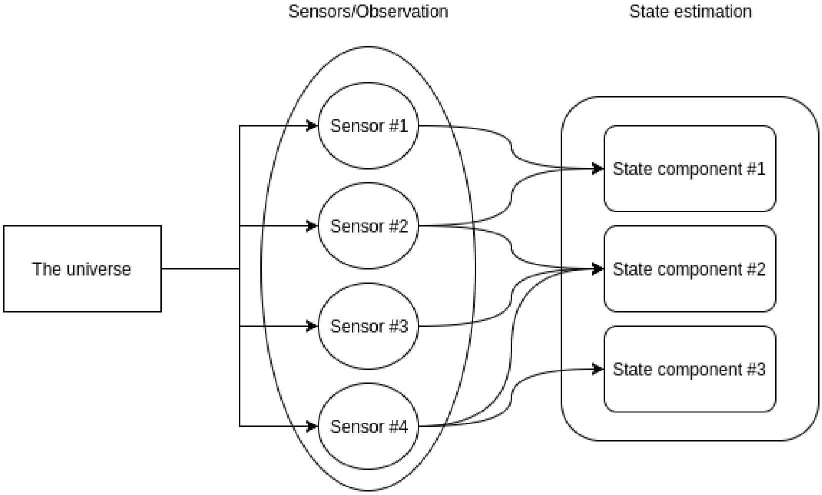


Figure 1: Sensor fusion

Sensor Filters

There is two main filters for POSE:

- Complementary filters
- Kalman filters

Complementary filters

Complementary filters come from the complentarity of a HPF and a LPF applied to different sensors

For instance, retrieving the attitude/orientation from the gyroscope + accelerometer

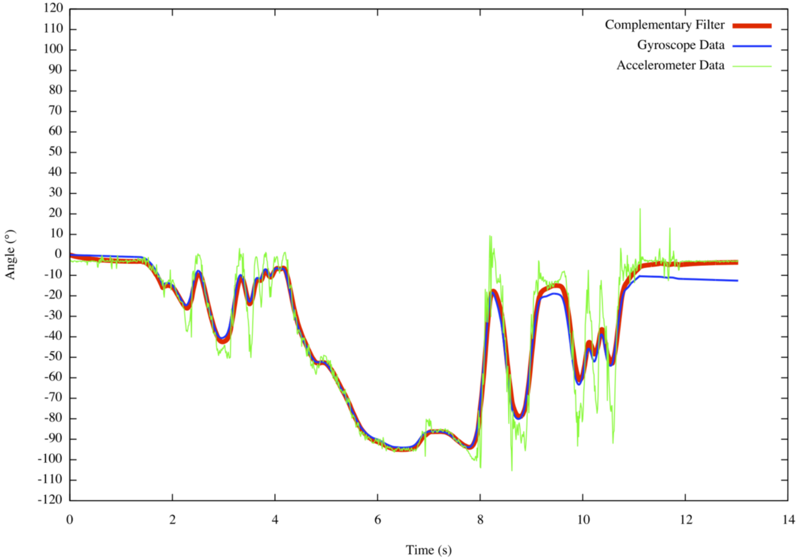
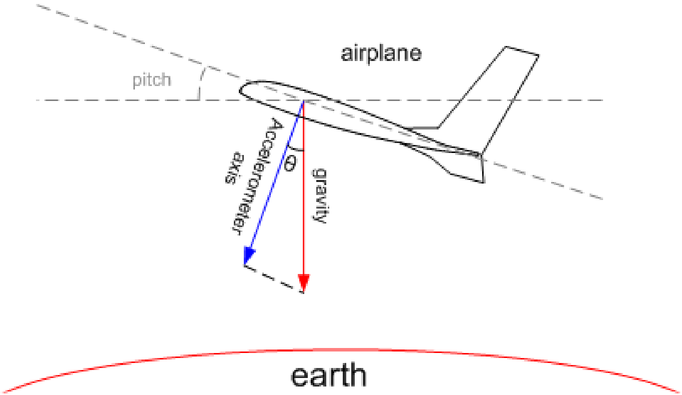


Figure 2: orientation

Accelerometer

accelerometer (through g acceleration) no drift but high-variance at high frequency (vibrations, other forces)

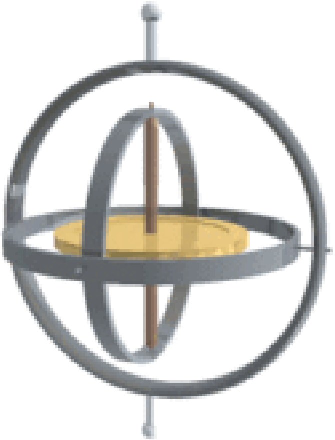
Accurate in the long-term: **Low-pass filter**



Gyroscope

gyroscope drift (because of integral over numerical error accumulate)

Accurate in the short term: **High-pass filter**



Drift

- Why does the gyro drift ? Because of the nature of an integration over a gaussian.
- Even if the noise (sensor noise + floating point error) has no bias, it accumulates errors over time.
- $$Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$$
- For better intuition, see Wiener process $Var(W_t) = t$

Kalman Filters

Also called linear quadratic estimation (LQE)

Estimate the joint state random variables (like position) conditonned on a a series of noisy observation (from the sensors).

Basic principle, true state x_t is a linear noisy process:

$$\mathbf{x}_t = \mathbf{F}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_t + \mathbf{w}_t$$

- $\mathbf{F}_t$ the state transition model
- $\mathbf{B}_t$ the control-input model
- $\mathbf{u}_t$ the control vector
- $\mathbf{w}_t$ process noise drawn from $\mathbf{w}_t \sim N(0, \mathbf{Q}_k)$

The kalman filter keeps track of our estimation of the gaussian random variable X_t

($[]_{a|b}$ reads as at time a knowing all observations until and including b)

$$\mathbf{X}_{t|t-1} \sim N(\hat{\mathbf{x}}_{t|t-1}, \mathbf{P}_{t|t-1})$$

- $\mathbf{X}$ state gaussian random variable
- $\hat{\mathbf{x}}$ estimated state mean (best guess)
- $\mathbf{P}$ estimated covariance matrix

Kalman filter proceed in two steps, predict and update:

Predict:

- $\hat{\mathbf{x}}_{t|t-1} = \mathbf{F}_k \hat{\mathbf{x}}_{t-1|t-1} + \mathbf{B}_t \mathbf{u}_t$
- $\mathbf{P}_{t|t-1} = \mathbf{F}_t \mathbf{P}_{t-1|t-1} \mathbf{F}_t^{\mathrm{T}} + \mathbf{Q}_t$

A simple example

A robot position and velocity in 1D.

- $\mathbf{x}_t = (p_t, v_t)$
- $p_t = p_{t-1} + v_t \Delta t + \frac{1}{2} a \Delta t^2$
- $v_t = v_{t-1} + a \Delta t$
- $F_t = (1, \Delta t)^t$
- $B_t = (\frac{1}{2}\Delta t^2, \Delta t)^t$
- $u_t = a$

Our sensor data z_k which is also noisy. We get a likelihood gaussian distribution:

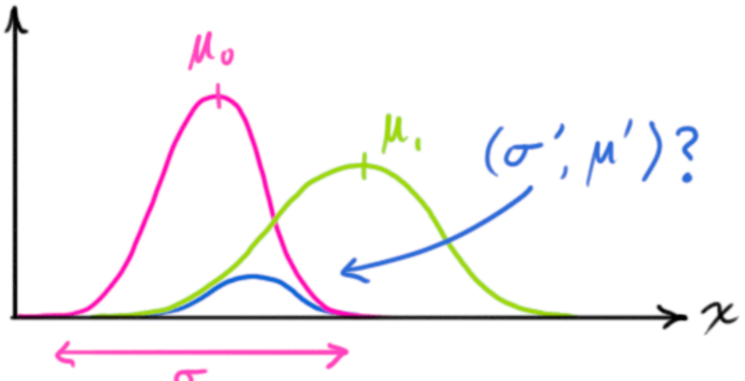
$$\mathbf{Z}_t \sim N(\mathbf{z}_t, \mathbf{R}_t)$$

$$\mathbf{X}_{t|t-1} \sim N(\hat{\mathbf{x}_{t|t-1}}, \mathbf{P}_{t|t})$$

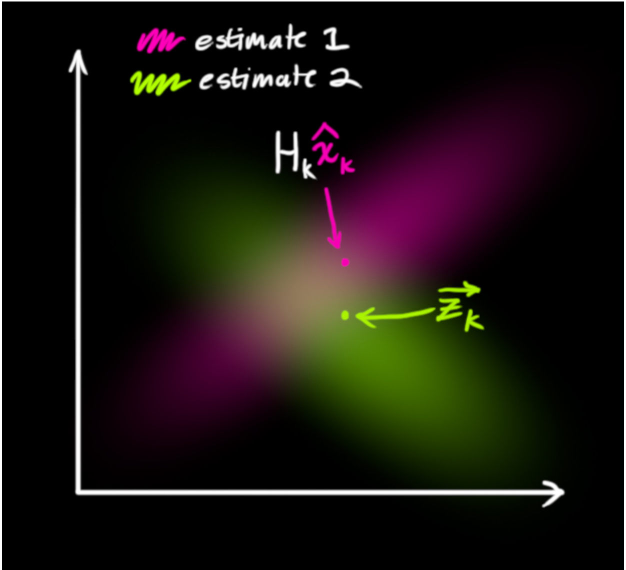
Now it suffices to combines them.

$$P(\mathbf{X}_{t|t}) \propto P(\mathbf{X}_{t|t-1}) \cdot P(\mathbf{Z}_t)$$

$$\mathbf{X}_{t|t-1} \cdot \mathbf{Z}_t \sim ?$$



- $\mu' = \mu_0 + \frac{\sigma_0^2(\mu_1 - \mu_0)}{\sigma_0^2+\sigma_1^2}$
- $\sigma'^2 = \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2+\sigma_1^2}$



Update

- $\mathbf{H}_t$ is the obs matrix (obs to state mapping)
- Innovation or measurement residual: $\tilde{\mathbf{y}}_t = \mathbf{z}_t - \mathbf{H}_t \hat{\mathbf{x}}_{t|t-1}$
- Innovation covariance: $\mathbf{S}_t = \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^{\mathrm{T}} + \mathbf{R}_t$
- Optimal Kalman gain: $\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^{\mathrm{T}} \mathbf{S}_t^{-1}$
- Updated (a posteriori) state estimate: $\hat{\mathbf{x}}_{t|t} = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t \tilde{\mathbf{y}}_t$
- Updated (a posteriori) estimate covariance $\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_{t|t-1} \ |\}$

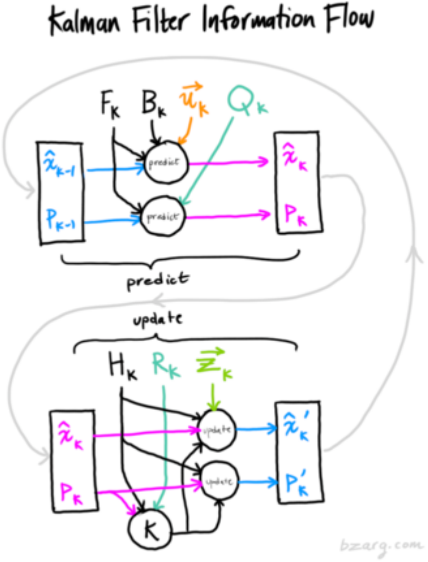


Figure 7: Kalman filter flow

The random variables we are interested in here are:

- **position** (x, y, z)
- **attitude** (orientation)
- **velocity**
- **angular velocity**
- **sensor biases**
- Earth magnetic field components

(In bold the ones I will focus on for this project)

The observations can come under many form:

- **motion capture systems like Vicon** (output relative position from 6 cameras around the lab tracking some markers)
- **accelerometer** (for linear velocity)
- **gyroscope** (for angular velocity)
- magnetometer
- GPS
- Optical flow (camera with some points as referentials)
- LIDAR for altitude or cloudpoints

(In bold the ones I will focus on for this project)

Non-linearity

Rotations are non-linear operations so we cannot just apply vanilla KF.

Because $Cov(f(X))$ for an arbitrary f has no closed form solution.

Differentiation to the rescue!

Extended Kalman Filter

Extended Kalman filter are an extension of kalman filters for **non linear systems**.

F and H are linearized by an approximation of the first order using Jacobians:

- $\hat{\boldsymbol{x}}_{k|k-1} = f(\hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{u}_{k-1})$
- $\tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - h(\hat{\boldsymbol{x}}_{k|k-1})$
- $\boldsymbol{F}_{k-1} = \left.\frac{\partial f}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{u}_{k-1}}$
- $\boldsymbol{H}_k = \left.\frac{\partial h}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k|k-1}}$

Quaternions (optional)

Quaternions are an extensions of complex numbers but with 2 extra dimensions

$$i^2 = j^2 = j^2 = ijk = -1$$

Unit quaternions, also known as versors, can be used to represent orientations and rotations in 3D.

Compared to Euler Angles, they are easier to compose and avoid gimbal lock.

Extensions

Parallelizable, Pipelinable ?

Matrixes involved are small and some known in advance.

Unrolling and parallelizing potential to shorten latency time.

Investigate theorotical implication of pipelining by using

$$\mathbf{x}_{t|t-k}$$

with k the length of the pipeline.

Other applications

- VR headsets also include an IMU whose reactivity is crucial for immersion
- Not only drones but the whole field of robotic use kalman filter for various planning tasks.