Course: Algorithm
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Homework: Lab 4

I referenced the **Lecture 3 – Probability.pdf, page 13 – 14**
p is probability of the success
q is probability of the failure
Expected number of trials to get a success when trying *m* times is:
$S = 1.p + 2.q.p + 3.q^2.p + ... + m.q^{m-1}.p$
⇔ $S.q = \quad q.p + 2.q2.p + ... + (m - 1).q^{m-1}.p + m.q^m.p$
⇔ $S - S.q = p + p.q + q^2.p + ... + q^{m-1}.p - m.q^m.p$
⇔ $S(1 - q) = p(1 + q + q^2 + ... + q^{m-1} - m.q^m)$
⇔ $S.p = p(\frac{1-q^m}{1-q} - m.q^m)$
⇔ $S = \frac{1-q^m}{p} - m.q^m$ **(1)**

1. **Question 1**
   a. *What is the average number of array locations to inspect to find a D*

      We have 4 letter A, B, C, D. Let say, D is a success, [A, B, C] is a failure.
      So, P(Success) = ¼, P(Failure) = ¾
      Expected number of array locations to inspect to find a D – success is:
      By using formula **(1)**, in this case we have m = 10
      $S = \frac{1-(3/4)^{10}}{\frac{1}{4}} - 10.(\frac{3}{4})^{10} = 4 - 4.(3/4)^{10} - 10.(3/4)^{10} = 4 - 14.(3/4)^{10} \approx 3.2116$

   b. *Calculate expected value of Z – random variable that indicate the index of D(success) in the array of 10 elements*

      Z can be any of [0,1,2,3,4,5,6,7,8,9,10],
      $P_{(Z=0)} = ¾$ (the failure)
      We have 10 positions for success = ¼, so each $P_{(Z!=0)} = (¼).(1/10) = 1/40$
      $E(Z) = 0.(3/4) + 1.(1/40) + 2.(1/40) + 3.(1/40) + \dots + 10.(1/40)$
      $= (1/40).(1 + 2 + 3 + \dots + 10) = (1/40) . \frac{(1+10).10}{2} = 11/8 \approx 1.375$

2. **Question 2**
   a. *Average number of array locations to inspect to find 10 Ds*

      By using formula **(1)**, in this question m = 100. Expected number of array locations for 1 D is
      $S = \frac{1-(3/4)^{100}}{\frac{1}{4}} - 100.(3/4)^{100} = 4 - 104.(3/4)^{100} \approx 4$
      In order to get k = 10 D = 10.4 = 40

b. *The average number of array locations to inspect to find k Ds*

$(\frac{1-q^m}{p} - m.q^m).k$

Size of the array is 100, so $(\frac{1-q^m}{p} - m.q^m).k = (\frac{1-q^{100}}{p} - 100.q^{100}).k \approx \frac{k}{p}$

(because q^{100} is approximate 0)

c. *Average time complexity to find k D in an array*

We have the average number of array locations to inspect to find k D is $\frac{k}{p}$

(size of array is 100, so q^{100} is approximate zero)

$T_{avg} = \frac{k}{p} = \frac{k}{\frac{1}{4}} = 4k$

3. **Question 3 -** Prove: $1 + ½ + 1/3 + … + 1/n = O(\log n)$

n = 7

1 + (1/2 + 1/3) + (1/4 + 1/5 + 1/6 + 1/7) < 1 + (1/2 + ½) + (1/4 + ¼ + ¼ + 1/4) = 3 = log(7 + 1) ➔ Holds

So, 1 + (½ + ½) + (¼ + ¼ + ¼ + ¼) + … = log(n+1)

S = 1 + (1/2 + 1/3) + (1/4 + 1/5 + 1/6 + 1/7) + … + 1/n + 1/n+1

As we can see (1/2 + ½) > (1/2 + 1/3); (1/4 + ¼ + ¼ + ¼) > (1/4 + 1/5 + 1/6 + 1/7), so (1/k + … + 1/k) > (…+ 1/n + 1/n + 1)

So S < 1 + (1/2 + ½) + (1/4 + ¼ + ¼ + ¼) + … = log(n + 1)

4. **Question 4 –** Find the sum: ½ + 2/4 + 3/8 + 4/16 + 5/32 + …

S = ½ + 2/4 + 3/8 + 4/16 + 5/32 + …

S/2 = ¼ + 2/8 + 3/16 + 4/32 + …

S – S/2 = ½ + ¼ + 1/8 + 1/16 + 1/32 + …

⇔ $S(1 – ½) = ½ [\frac{1 - (½)^n}{1-1/2}]$

⇔ $S = \frac{1-(\frac{1}{2})^n}{\frac{1}{2}}$

⇔ $S = 2(1 - (\frac{1}{2})^n)$

When n goes to big, S = 2