

HARAMAYA UNIVERSITY
College of Natural and Computational Sciences
Department of Mathematics

FINAL EXAM⁴
MATHEMATICS FOR NATURAL SCIENCE

Date: August 29, 2022

Time allowed: 3 hours

Total Weight: 50%

Name: _____ ID: _____
Section: _____ Instructor/s Name: _____

General Instructions:

- ☞ Write your Name, ID, No & other information's on the space provided!
 - ☞ Please check that the exam booklet has 7 pages and contains 24 questions!
 - ☞ Read the instructions carefully and attempt all questions!
 - ☞ Make your handwriting orderly, clear and readable!
 - ☞ Any type of cheating during the examination will invalidate your result!
 - ☞ Mobile Phones or any related electronic materials are **not allowed** to be with you during the examination!!!
 - ☞ All other exam rules & regulations of the University are also binding!
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FOR INSTRUCTORS' USE ONLY:

Part No & Weightage	Part I [7%]	Part II [15%]	Part III [14%]	Part IV [14%]	Total [50%]
Mark earned					

'Department Seal'

Part I: Write "True" if the statement is correct and "False" if incorrect (7 points)

1. Any two distinct vertical lines in a plane are parallel to each other.
2. Every rational function has a vertical asymptote.
3. The range and codomain of any onto function are equal.
4. The domain of $f(x) = \coth x$ is the set of all real numbers.
5. For a and b two nonzero real numbers, the equation of the line that passes through the points $(a, 0)$ and $(0, b)$ can be expressed as $\frac{x}{a} + \frac{y}{b} = 1$.
6. Every polynomial function is rational function.
7. If the distance between the center of a circle and a line l equals to its radius, then the line must be a secant line to the circle.

Part II: Choose the best answer from the given alternatives

(15 points)

1. One of the following is **not true** about the complex number $1 + \sqrt{3}i$.
A. $\text{Arg}(z) = \frac{\pi}{3}$ **B.** $z = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$ **C.** $z = 2e^{\frac{\pi i}{3}}$ **D.** $z = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$
2. Which one of the following is **not true** about the function $f(x) = a^x$, $a > 0$, $a \neq 1$?
A. The domain is the set of all real numbers.
B. The range is the set of all positive real numbers.
C. The graph of $f(x)$ passes through the point $(0, 1)$.
D. The x -intercept of the graph of $f(x)$ is $(1, 0)$.
3. What is the distance from the point $(0, 0)$ to the line $l: y + x - 1 = 0$?
A. 1 **B.** $\frac{1}{2}$ **C.** $\sqrt{2}$ **D.** $\frac{\sqrt{2}}{2}$
4. The equation of the line that is perpendicular to $3x + 4y = 6$ and that passes through the point $(2, -3)$ is
A. $4x + 3y = -17$ **B.** $4x - 3y = 17$ **C.** $3x - 4y = 17$ **D.** $3x + 4y = -17$

5. What is $f(x)$ if $(f \circ g)(x) = 4x^2 + 4x + 9$ and $g(x) = 2x + 3$?
A. $x^2 - 3x + 3$ B. $2x^2 - 4x + 3$ C. $x^2 - 4x + 12$ D. $3x^2 - x + 2$

6. Which one of the following equations can represent a circle?
A. $x^2 + y^2 - 2x + 4 = 0$ B. $(x - 1)^2 + (y + 1)^2 = -10$
C. $x^2 + y^2 - 8x - 4y = -16$ D. $3x^2 + 3y^2 - 6x + 12y = -18$

7. Which one of the following is **true** about the function $p(x) = 6x^5 - 2x^4 + 4x - 8$?
A. $p(x)$ have at most 5 turning point. B. The leading coefficient is -2 .
C. $x - 1$ is a factor of $p(x)$. D. -1 is a zero of $p(x)$.

8. Which of the following is **not true** about the function $f(x) = 1 + \log_5(x - 2)$?
A. The vertical asymptote is $x = 0$.
B. The domain is $(2, \infty)$.
C. The x-intercept is $\left(\frac{7}{3}, 0\right)$.
D. It is an increasing function.

9. Which one of the following is **true** about the cosine function $f(x) = \cos x$?
A. Its period is π .
B. Its domain is $[-1, 1]$.
C. It is an odd function.
D. Its range is $[-1, 1]$.

10. Let C be a circle with radius 5 and centered at $(1, -2)$. Which one of the following is **true** about the point $P(3, 2)$?
A. The point P is on the circle C .
B. The point P is inside the circle C .
C. The point P is outside the circle C .
D. The point P is one of the end points of the diameter of the circle C .

Part III: Short Answer (Write the most simplified answer on the space provided) (14 points)

1. Given a complex number $z = -4 - 4\sqrt{3}i$, find

a) the principal argument of z .

Answer: _____

b) the polar form of z

Answer: _____

2. If $f(x) = -2 \cos 3x$, then find

a) the period of $f(x)$.

Answer: _____

b) the frequency of $f(x)$.

Answer: _____

3. Find the value of $3\cosh(\ln 3)$.

Answer: _____

4. If $f(x) = 4x + 1$ and $g(x) = 3x + k$, then find the value of k for which $(fog)(x) = (gof)(x)$.

Answer: _____

5. Let $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$. Then, find

a) the domain of $(gof)(x)$.

Answer: _____

b) the value of $(fog)(0)$.

Answer: _____

6. Find the inverse of the function $f(x) = e^{x+1}$.

Answer: _____

7. Find the polynomial function $p(x)$ whose zeros are $3, -2$, and -1 with multiplicity $1, 3$, and 4 respectively such that $p(0) = 48$.

Answer: _____

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Part IV. Work out (Show all the necessary steps clearly and neatly)

(14 points)

1. Find the equation of the circle that passes through the point $(2, 5)$ and is tangent to the line $y = 3x - 2$ at the point $(1, 1)$. (4 points)

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2. Given the rational function $f(x) = \frac{2x^2}{x^2 - 1}$. Then, find the domain, intercepts, asymptotes and sketch the graph of $f(x)$.

(6 points)

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ID:

3. Suppose that $1 - i$ is the root of the polynomial function $f(x) = x^3 - 3x^2 + ax + b$, where $a, b \in \mathbb{R}$. Then, find the remaining two roots of $f(x)$ and the values of a and b . (4 points)