Computation of distributed alternating currents

A numerical (PEEC) approach using the Java programming language.

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31-12-2011

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Chapter 1 General formulation of the mutual inductance between conductors with rectangular cross sections

In Figure 1 currents I_1 and I_1 are running perpendicular to the x and y plane and are carried through two conductors with rectangular cross sections. The rectangular areas are bound by the sides A, B, C and D respectively. The conductor axes are located at points (p, q) and (r, s). The conductors are characterized by a coefficient of mutual inductance M, and may be considered to consist of filament conductors with partial mutual inductance m between individual filaments from one conductor to the other. As an example two coupled filaments are outlined at points (x_1, y_1) and (x_2, y_2) in the figure.

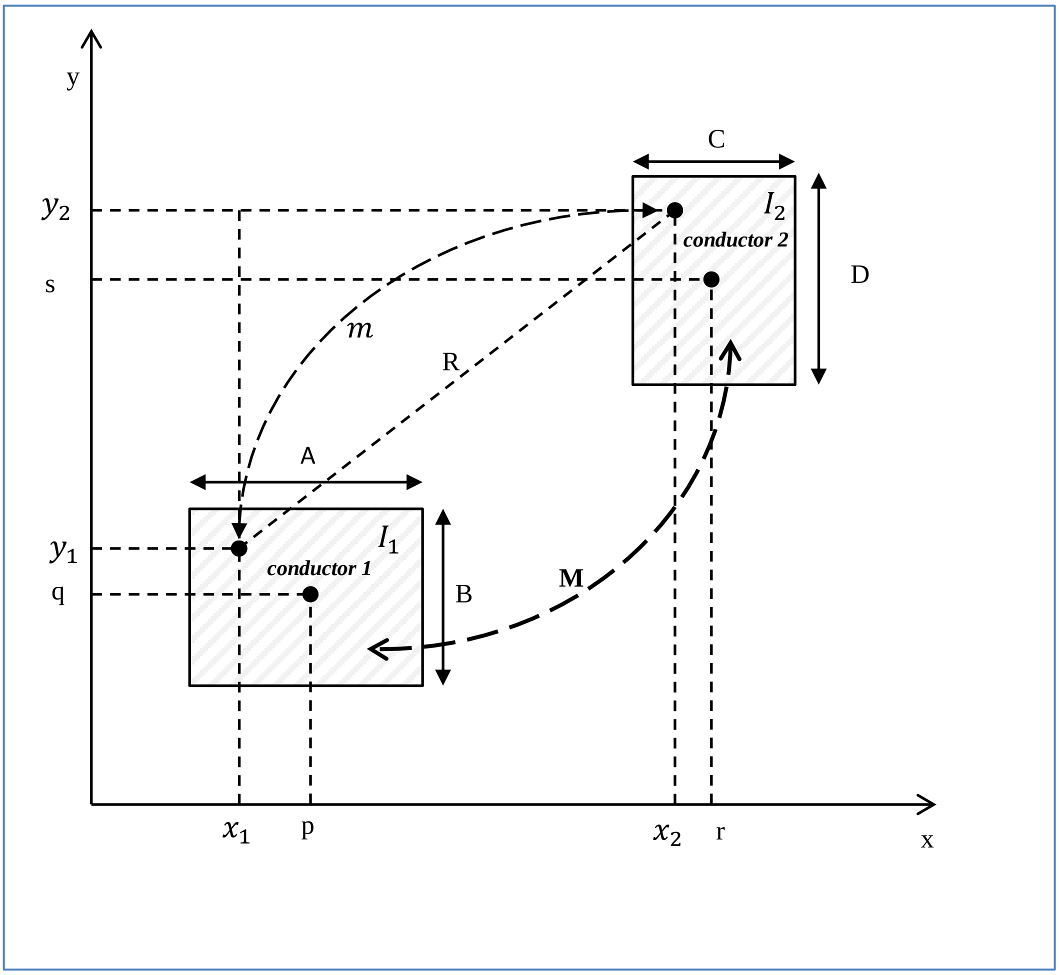

Figure 1 Mutual inductance M between two conductors carrying currents I_1 and I_2 with rectangular cross sections

The current ΔI flowing through one filament is contained within an area with sides Δx and Δy. Therefore the filement currents in the conductors can be expressed as:

$$\Delta I_1 = \frac{\Delta x_1 \, \Delta y_1}{A \, B} I_1$$

and

$$\Delta I_2 = \frac{\Delta x_2 \, \Delta y_2}{C \, D} I_2 \qquad (1)$$

The magnetic flux $\varphi(R)$ at one filament caused by the current from one other filament, where $m(R)$ is the mutual inductance between the filaments at a distance R, is given by:

$$\varphi(R) = m(R) \, \Delta I \qquad (2)$$

The total flux contribution from all filaments in conductor 1 to one filament in conductor 2 can be written as:

$$\varphi_1 = \sum_{I_1} m(R) \, \Delta I_1 \qquad (3)$$

and the partial mutual inductance $m_{12,p}$ between current I_1 and one filament in conductor 2:

$$m_{12,p} = \frac{\varphi_1}{I_1} = \frac{1}{I_1} \sum_{I_1} m(R) \, \Delta I_1 \qquad (4)$$

Substituting for ΔI_1 from formula (1):

$$m_{12,p} = \frac{1}{I_1} \sum_{A\,B} m(R) \frac{\Delta x_1\, \Delta y_1}{A\ B} I_1 \tag{5}$$

Which leads to:

$$m_{12,p} = \frac{1}{A\ B} \sum_{A\,B} m(R)\, \Delta x_1\, \Delta y_1 \tag{6}$$

Because of the reciprocity of mutual inductance

$$m_{12,p} = m_{21,p}$$

equation (6) also represents the mutual inductance between 1 filament in conductor 2 and conductor 1 as a whole. Therefore the flux contribution to conductor 1 from all filaments in conductor 2 can be written as:

$$\varphi_2 = \sum_{I_2} m_{12,p}\, \Delta I_2 \tag{7}$$

and hence the mutual inductance between all the filaments in conductor 2 and conductor 1:

$$M = \frac{\varphi_2}{I_2} = \frac{1}{I_2} \sum_{I_2} m_{12,p}\, \Delta I_2 \tag{8}$$

Combining equation (6) and (8):

$$M = \frac{1}{I_2} \sum_{I_2} \left\{ \frac{1}{A\,B} \sum_{A\,B} m(R)\,\Delta x_1\,\Delta y_1 \right\} \Delta I_2 \tag{9}$$

Substituting for ΔI_2 from formula (1):

$$M = \frac{1}{C\,D} \sum_{C\,D} \left\{ \frac{1}{A\,B} \sum_{A\,B} m(R)\,\Delta x_1\,\Delta y_1 \right\} \Delta x_2\,\Delta y_2 \tag{10}$$

Rearranging, converting to integral form, splitting the surface integrals and setting integral limits according to Figure 1:

$$M(p,q,r,s) = \frac{1}{A\,B\,C\,D} \int_{s-\frac{1}{2}D}^{s+\frac{1}{2}D} \int_{r-\frac{1}{2}C}^{r+\frac{1}{2}C} \int_{q-\frac{1}{2}B}^{q+\frac{1}{2}B} \int_{p-\frac{1}{2}A}^{p+\frac{1}{2}A} m(R(x_1,y_1,x_2,y_2))\; dx_1 dy_1 dx_2 dy_2 \tag{11}$$

Where it should be noted that R is a function of x_1, y_1, x_2 and y_2 :

$$R(x_1,y_1,x_2,y_2) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \tag{12}$$

Chapter 2 General formulation of the self inductance of a conductor with rectangular cross section

Self induction is a special case in the calculation of mutual induction. Substituting

$$A = C$$

$$B = D$$

$$p = r = \tfrac{1}{2}A$$

$$q = s = \tfrac{1}{2}B$$

in equation (11) and (12) and still referring at Figure 1 leads to the co-efficient of self-induction of a conductor with rectangular cross section and sides A and B:

$$L(A,B) = \frac{1}{(A\,B)^2}\int_0^B\int_0^A\int_0^B\int_0^A m(R(x_1,y_1,x_2,y_2))\; dx_1 dy_1 dx_2 dy_2 \tag{13}$$

Equation (12) remains unchanged for the value of R in the integrations:

$$R(x_1,y_1,x_2,y_2) = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} \tag{14}$$

Chapter 3 Specific Formulation of the mutual inductance between parallel filaments with shifted positions

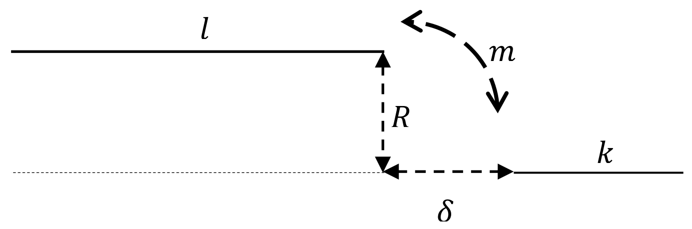

Figure 2 Mutual inductance *m* between two parallel filaments *l* and *k*

Grover, Chapter 6, formula 28:

$$m(R) = \frac{\mu_0 \mu_r}{\pi}\Bigg[\alpha \sinh^{-1}\frac{\alpha}{R} - \beta \sinh^{-1}\frac{\beta}{R} - \gamma \sinh^{-1}\frac{\gamma}{R} + \delta \sinh^{-1}\frac{\delta}{R}$$

$$- \sqrt{\alpha^2 + R^2} + \sqrt{\beta^2 + R^2} + \sqrt{\gamma^2 + R^2} + \sqrt{\delta^2 + R^2}\ \Bigg] \tag{15}$$

where

$$\alpha = l + m + \delta, \quad \beta = l + \delta, \quad \gamma = m + \delta$$

All lenghts in meters, inductance in Henry