…Bond portfolio strategies are too many to enumerate. But there are benchmark bond strategies which are frequently introduced to textbook. Among them are bullet, barbell, ladder and buy-and-hold portfolios. For these bond portfolios, we are going to calculate monthly and cumulative returns.

We assume the case of zero coupon bond portfolio for empirical exercises. Without target duration constraints, differences between zero coupon bond and coupon bearing bond are not large. It is well known that the coupon bond is the portfolio of zero coupon bonds.

1. Bullet : maintain one bond with its maturity fixed
2. Barbell : maintain two bonds with each maturities fixed and equal weights(1/2)
3. Ladder : maintain all relevant bonds with each maturity fixed and equal weights(1/n)
4. Buy and Hold : buy and hold one bond until its maturity and maintain this trade periodically

Return Calculation

At first, we need to calculate monthly returns of each bond. After getting these monthly returns, we can construct monthly returns of 4 portfolios.

The price of a pure discount (zero coupon) bond with maturity τ at time t is the discounted value of 1 receivable τ periods later. $$\begin{align} P_t^{[\tau]} =exp(-\tau \times s_t^{[\tau]}) \end{align}$$ Here, $s_t^{[\tau]}$ is the spot rate.

Using the log-return expression, the monthly holding period return is as follows.

$$\begin{align} r_t^{[\tau]}& = \log {P_t^{\left[\tau-\frac{1}{12}\right]}} - \log {P_{t-1}^{\left[\tau\right]}} \\ &= \left(\tau-\frac{1}{12}\right) \times s_t^{\left[\tau-\frac{1}{12}\right]} - \tau \times s_{t-1}^{\left[\tau\right]} \end{align}$$
However, yield curve is reported for some major relevant maturities such as 1-year or 3-year, and so on. In most cases, $s_t^{\left[\tau-\frac{1}{12}\right]}$ are not observed. Therefore, we need to interpolate these unobserved spot rates using observed spot rates. If $s_t^{\left[\tau-\frac{1}{12}\right]}$ are interpolated using spline or linear interpolation or Nelson-Siegel model, we can apply the above log-return expression to these interpolated spot rates. For our study, we use the spline interpolation using **splinefun** R function.

Now it is ready to use the time $t-1$ spot rates with maturities 1, 3, 5, 7, and 10 and the time t spot rates with maturities $1-\frac{1}{12}$, $3-\frac{1}{12}$, $5-\frac{1}{12}$, $7-\frac{1}{12}$, and $10-\frac{1}{12}$ to calculate monthly returns for each selected maturity.
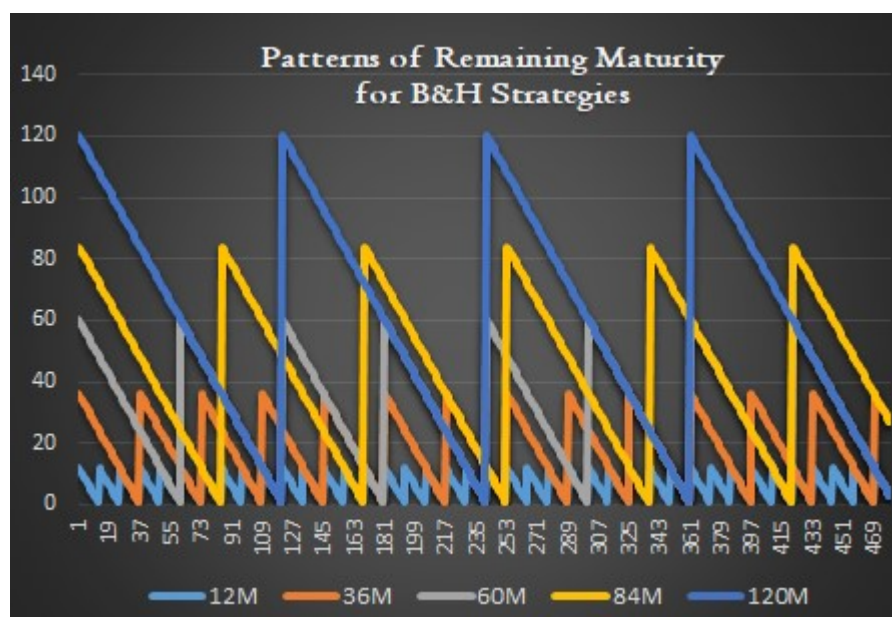
Monthly Returns of Portfolio

Given time series of monthly spot rate for each maturity, monthly returns of each bullet portfolio is the same as the previous monthly returns of each column respectively. It is only to read each column which corresponds each maturity respectively.

It is also easy to calculate monthly returns for barbell, and ladder. For barbell, 1- and 10-year returns are averaged. For ladder, returns of all 5 maturities are averaged. But for Buy-and-Hold, some caution is needed.

Buy-and-Hold (BH) strategy 1) buy a bond with issuance maturity and 2) hold it until maturity. Therefore, as soon as redemption is made, new BH strategy starts with a full issuance maturity. This means that remaining maturity shows periodic behavior. For example, time variation of remaining maturity of 3-year BH portfolio is as follows.

$$\begin{align} 2 &\rightarrow 1.92 \rightarrow 1.83 \rightarrow… \\ &\rightarrow 0.17 \rightarrow 0.08 \rightarrow 2 \rightarrow 1.92 \rightarrow … \end{align}$$

The following figure shows the pattern of remaining maturity of BH portfolio with selected maturities.

Hence, monthly return series for BH portfolios for each maturity are defined in in full interpolation monthly return grid. As time passes, position of current spot rates are moved to the left by $\frac{1}{12}$ in full interpolated spot rate grid until its maturity approaches zero. Of course, after redemption take places, position of current spot rate goes back to the original position which corresponds to the issuance maturity.

R code for Bond Portfolio

The following R code demonstrates the calculation of monthly and cumulative returns of 4 benchmark bond portfolio using Diebold, Rudebusch, and Aruoba (2006) data,

```
1   #========================================================
2   =============#
3   # Financial Econometrics & Derivatives, ML/DL using R, Python, Tensorflow
4   # by Sang-Heon Lee
5   #
6   # https://kiandlee.blogspot.com
7   #————————————————————————————————-#
8   # Benchmark Bond Portfolio Return Calculation
```

```r
 9   #=========================================================
10   =============#
11
12   library(readxl)
13   library(xlsx)
14   library(RColorBrewer)
15
16   graphics.off()  # clear all graphs
17   rm(list = ls()) # remove all files from your workspace
18
19   setwd("D:/SHLEE/a_blog_ki_and_Lee/bond_portfolio")
20
21   #————————————————————————————
22   # 0) Read DRA (2006) spot yield curve data
23   #————————————————————————————
24
25      fname <– "dra_spot_cc_data.xlsx"
26
27      # maturity, date, spot rate
28      vmatm   <– read_excel(fname, "spotcc","B1:R1",   col_names = FALSE)
29      df.ym   <– read_excel(fname, "spotcc","A2:A480", col_names = FALSE)
30      df.spot <– read_excel(fname, "spotcc","B2:R480", col_names = FALSE)
31
32      # maturity as decimal, spot as y
33      vmatm <– as.numeric(vmatm); vmaty <– vmatm/12
34      y     <– as.matrix(df.spot/100);
35      colnames(y) <– vmatm
36
37      # counting
38      nmat <– length(vmaty);
39      ny <– nrow(y); nr <– ny – 1; # number of yields and returns
40
41   #————————————————————————————
42   # 1) Interpolate monthly spot rates using cubic spline
43   #————————————————————————————
44
45      # .ip : interpolated
46
47      # use 0 for 1-month return temporarily
48      matm.ip     <– (0:max(vmatm))
49      max.matm.ip <– max(matm.ip)
50
51      # use apply() for row-wise interpolation
52      # output => collection of column vector => so transpose
53      y.ip <– t(apply(y, 1,
54            function (x) {
55              # make interpolation function
56              fs<–splinefun(vmatm,x);
57              # apply fs function to each row
58              fs(matm.ip)
59            }
60          ))
61
62   #————————————————————————————
63   # 2) Calculate 1-month returns
64   #————————————————————————————
65
66      # interpolated spot rate
67      r.ip <– matrix(0, ny–1, length(matm.ip)–1)
68
```

```r
69      # calculate monthly returns
70      for (t in 2:ny) {
71
72        # spot rates and maturities at t and (t-1)
73        spot.t   <- y.ip[t,1:max.matm.ip]
74        spot.t_1 <- y.ip[t-1,2:(max.matm.ip+1)]
75        mat.t    <- matm.ip[1:max.matm.ip]/12
76        mat.t_1  <- matm.ip[2:(max.matm.ip+1)]/12
77
78        # log-return expression
79        r.ip[t-1,] = log(exp(-spot.t*mat.t)/exp(-spot.t_1*mat.t_1))
80      }
81
82      # draw a graph for monthly returns
83      x11(width=6, height=4);
84      par(mar =  c(5, 4, 4, 6) + 0.1)
85      matplot(r.ip[,c(120, 60, 36, 12, 3)], type = "l", lty = 1,
86            col = c(brewer.pal(4, "Paired"),"black"),
87            ylab = "return", xlab = "date", lwd = 2,
88            main = "Monthly Zero Coupon Bond Returns")
89      legend("right", inset = c(-0.3,0),
90            legend = paste(c(120, 60, 36, 12, 3),"M"), xpd = TRUE,
91            horiz = FALSE, col = c(brewer.pal(4, "Paired"),"black"),
92            lty = 1, bty = "n", lwd = 2)
93
94   #———————————————————————————
95   # 3) Construct benchmark bond portfolios
96   #    – barbell, bullet, ladder, Buy-and-Hold
97   #———————————————————————————
98
99      # selected maturity
100     maty_bm <- c(1,3,5,7,10)
101     matm_bm <- maty_bm*12
102     nc <- length(maty_bm)     # number of selected maturities
103
104     # 1. Barbell (1, 10 year)
105     maty1 <- c(1,10)
106     col1.ret <- rowMeans(r.ip[1:nr, maty1*12])
107     col1.dur <- matrix(1/2, nr,2)%*%maty1
108
109     # 2. Bullet
110     col2.ret <- r.ip[1:nr, matm_bm]
111     colnames(col2.ret) <- paste0("Bullet(", maty_bm, ")")
112     col2.dur <- matrix(1,nr,nc)%*%diag(maty_bm);
113     colnames(col2.dur) <- colnames(col2.ret)
114
115     # 3. Ladder (equal weights for all maturities)
116     col3.ret <- rowMeans(r.ip[1:nr, matm_bm])
117     col3.dur <- matrix(1/nc, nr, nc)%*%maty_bm
118
119     # 4. Buy-and-Hold
120     bah.ret <- bah.maty <- bah.matm <- matrix(0,nr, nc)
121     for (t in 1:nr) { for(j in 1:nc) {
122
123        # As t move forward, remaining maturity decreases.
124        # When remaining maturity is zero,
125        # a new bond is purchased
126        # the remaining maturity is set to the issuance maturity.
127        matm <- matm_bm[j]-(t-1)%%matm_bm[j]
128
```

```
129        bah.matm[t,j] <- matm
130        bah.maty[t,j] <- matm/12
131        bah.ret[t,j] <- r.ip[t, matm]
132    }}
133    col4.ret <- bah.ret; col4.dur <- bah.maty;
       colnames(col4.ret) <- paste0("BH(", maty_bm,")")
134    colnames(col4.dur) <- colnames(col4.ret)
135
136    # collect portfolio returns
137    port.ret <- cbind(col1.ret,col2.ret,col3.ret,col4.ret)
138    colnames(port.ret)[1] <- "Barbell(1,10)"
139    colnames(port.ret)[7] <- "Ladder"
140
141  #————————————————————————
142  # 4) Cumulative Returns
143  #————————————————————————
144
145    port.cum.ret <- port.ret*0;
146
147    # cumulative return
148    for(i in 1:ncol(port.ret)) {
149        port.cum.ret[,i] <- cumprod(1+port.ret[,i])-1
150    }
151
152  #————————————————————————
153  # 5) Performance Statistics
154  #————————————————————————
155
156    # Use data.frame not matrix when using sapply
157    out.port <- sapply(as.data.frame(port.ret),
158        function(x) {
159            # average, stdev, Sharpe
160            col1 <- mean(x)*100*12
161            col2 <- sd(x)*100*sqrt(12)
162            col3 <- col1/col2
163            return(c(avg=col1, stdev = col2, Sharpe = col3))})
164
165    # print out
166    round(out.port[,1:3],2)
167    round(out.port[,4:6],2)
168    round(out.port[,7:12],2)
```

Colored by Color Scripter

The following figure shows the monthly returns of 5-maturity bond which is, indeed, that of bullet portfolio. We can find that the longer the maturity of a bond, the more sensitive is its return to a change in interest rates. Therefore, it is not easy to forecast future return of a long-term bond.
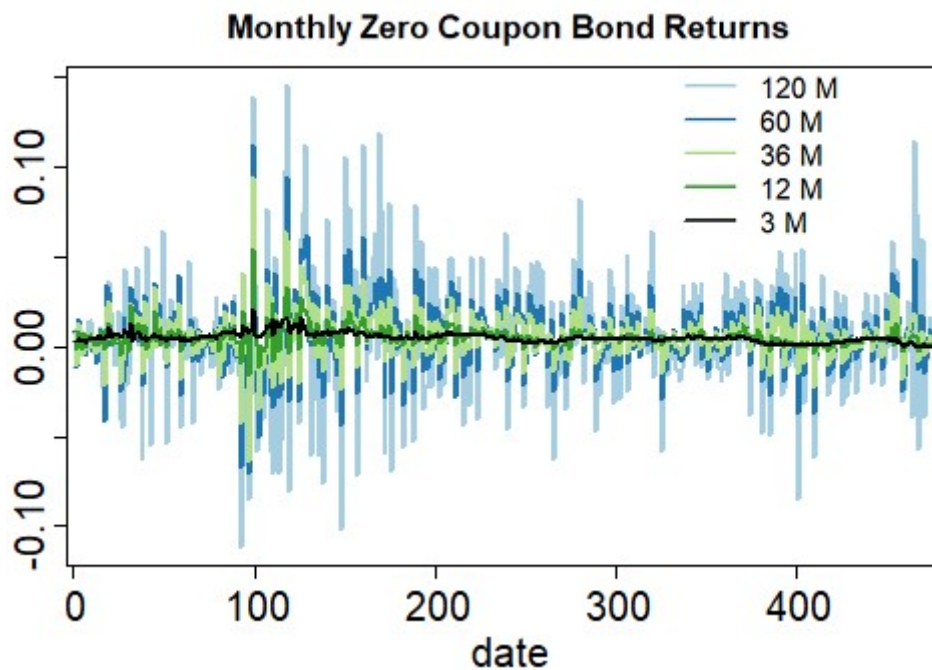
To investigate the performance of bond portfolio strategy, it is useful to calculate the cumulative returns as follows. We can find a stylized fact that long term bond shows the higher returns and higher volatility.
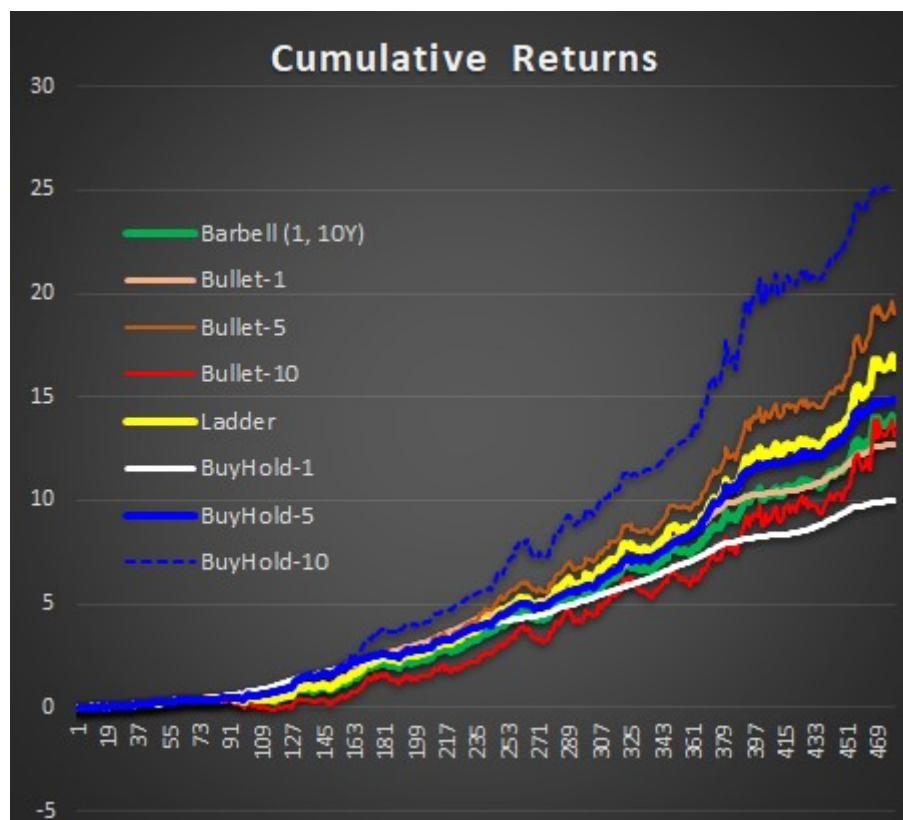
Using mean and standard deviation, we can calculate the Sharpe ratio which is the risk-adjusted return as follows. (In fact, it is typical to use the excess return when calculating the Sharpe ratio and therefore this is the same as we assume 0% risk-free rate)

```
>       round(out.port[,1:3],2)
        Barbell(1,10) Bullet(1) Bullet(3)
avg                6.97         6.61       7.36
stdev              6.52         1.93       4.49
Sharpe             1.07         3.43       1.64
>       round(out.port[,4:6],2)
        Bullet(5) Bullet(7) Bullet(10)
avg          7.77         7.94        7.34
stdev        6.65         8.98       11.72
Sharpe       1.17         0.88        0.63
>       round(out.port[,7:12],2)
        Ladder BH(1) BH(3) BH(5) BH(7) BH(10)
avg       7.40  6.04  6.40  7.03  7.03   8.44
stdev     6.42  1.41  2.80  3.91  5.93   6.69
Sharpe    1.15  4.27  2.28  1.80  1.19   1.26
> |
```

From this post, we have constructed some benchmark bond portfolio and calculate its monthly returns and cumulative performances.

This analysis can be also applied to coupon bond portfolio. For more information, refer to Deguest, Fabozzi, Martellini, and Milhau (2018).

Reference

Deguest, R., F. Fabozzi, L. Martellini, and V Milhau (2018), "Bond Portfolio Optimization in the Presence of Duration Constraints," The Journal of Fixed Income 28, 6-26

Diebold, F. X., G. D. Rudebusch, and S. B. Aruoba (2006), "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach," Journal of Econometrics 131, 309-338. $\blacksquare$