…When I first saw the *Computer Algebra System Mathematica* in the nineties I was instantly fascinated by it: you could not just calculate things with it but solve equations, simplify, differentiate and integrate expressions and even solve simple differential equations… not just numerically but *symbolically*! It helped me a lot during my studies at the university back then. Normally you cannot do this kind of stuff with R but fear not, there is, of course, a package for that!

There are many so-called *Computer Algebra Systems (CAS)* out there, commercial but also open-source. One very mature one is called *YACAS* (for *Yet Another Computer Algebra System*). You find the documentation here: Yacas Documentation (many of the following examples are taken from there).

You can use the full power of it in R by installing the `Ryacas` package from CRAN. You can use Yacas commands directly by invoking the `yac_str` function, the `as_r` function converts the output to R. Let us first *simplify a mathematical expression*:

```
library(Ryacas)
##
## Attaching package: 'Ryacas'
## The following object is masked from 'package:stats':
##
##     deriv
## The following objects are masked from 'package:base':
##
##     %*%, determinant, diag, diag<-, I, lower.tri, upper.tri

# simplify expressions
as_r(yac_str("Simplify(a*b*a^2/b-a^3)"))
## [1] 0
```

Or *solve an equation*:

```
as_r(yac_str("Solve(a+x*y==z,x)"))
## [1] "x==-(a-z)/y"
```

And you can do all kinds of tedious stuff that is quite error-prone when done differently, e.g. *expanding* expressions like $(x-2)^{10}$ by using the binomial theorem:

```
as_r(yac_str("Expand((x-2)^20)"))
## expression(x^20 - 40 * x^19 + 760 * x^18 - 9120 * x^17 + 77520 *
##     x^16 - 496128 * x^15 + 2480640 * x^14 - 9922560 * x^13 +
##     32248320 * x^12 - 85995520 * x^11 + 189190144 * x^10 - 343982080 *
##     x^9 + 515973120 * x^8 - 635043840 * x^7 + 635043840 * x^6 -
##     508035072 * x^5 + 317521920 * x^4 - 149422080 * x^3 + 49807360 *
##     x^2 - 10485760 * x + 1048576)
```

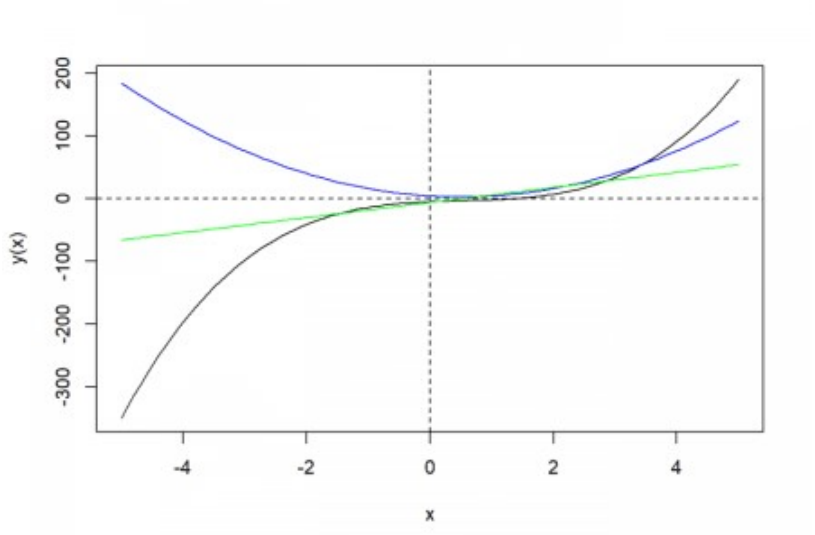
To demonstrate how easily the results can be integrated into R let us do some *curve sketching* on a function. First, we define two helper function for converting an expression into a function (which can then be used to plot it) and for determining the *derivative of order n* of some function (we redefine the `D` function for that):

```
as_function <- function(expr) {
  as.function(alist(x =, eval(parse(text = expr))))
}

# redefine D function
D <- function(eq, order = 1) {
  yac_str(paste("D(x,", order, ")", eq))
}
```

Now, we define the function (in this case a simple *polynomial* $2x^3 - 3x^2 + 4x - 5$), determine the first

and second derivatives symbolically and plot everything:

```
xmin <- -5
xmax <- 5

eq <- "2*x^3 - 3*x^2 + 4*x - 5"
eq_f <- as_function(eq)
curve(eq_f, xmin, xmax, ylab = "y(x)")
abline(h = 0, lty = 2)
abline(v = 0, lty = 2)

D_eq <- D(eq)
D_eq
## [1] "6*x^2-6*x+4"

D_eq_f <- as_function(D_eq)
curve(D_eq_f, xmin, xmax, add = TRUE, col = "blue")

D2_eq <- D(eq, 2)
D2_eq
## [1] "12*x-6"

D2_eq_f <- as_function(D2_eq)
curve(D2_eq_f, xmin, xmax, add = TRUE, col = "green")
```



Impressive, isn't it! Yacas can also determine *limits* and *integrals*:

```
# determine limits
yac_str("Limit(x,0) 1/x")
## [1] "Undefined"

yac_str("Limit(x,0,Left) 1/x")
## [1] "-Infinity"

yac_str("Limit(x,0,Right) 1/x")
## [1] "Infinity"

# integration
yac_str("Integrate(x) Cos(x)")
## [1] "Sin(x)"
```

```
yac_str("Integrate(x,a,b) Cos(x)")
## [1] "Sin(b)-Sin(a)"
```

As an example, we can prove in no-time that the famous approximation $\pi \approx \frac{22}{7}$ is actually too big (more details can be found here: Proof that 22/7 exceeds π):

$$0 < \int_0^1 \frac{x^4 (1-x)^4}{1+x^2} dx = \frac{22}{7} - \pi$$

```
yac_str("Integrate(x,0,1) x^4*(1-x)^4/(1+x^2)")
## [1] "22/7-Pi"
```

And, as the grand finale of this post, Yacas is even able to solve *ordinary differential equations* symbolically! Let us first take the simplest of them all:

```
as_r(yac_str("OdeSolve(y' == y)"))
## expression(C115 * exp(x))
```

It correctly returns the *exponential function* (The `C`-term is just an arbitrary *constant*). And finally a more complex, higher-order one:

```
as_r(yac_str("OdeSolve(y'' - 4*y == 0)"))
## expression(C154 * exp(2 * x) + C158 * exp(-2 * x))
```

I still find CAS amazing and extremely useful… and an especially powerful one can be used from within R!