A Fermat-like riddle from the Riddler (with enough room to code on the margin)

> *An arbitrary positive integer N is to be written as a difference of two distinct positive integers. What are the impossible cases and else can you provide a list of all distinct representations?*

Since the problem amounts to finding a>b>0 such that

[redacted]

both *(a+b)* and *(a-b)* are products of some of the prime factors in the decomposition of *N* and both terms must have the same parity for the average *a* to be an integer. This eliminates decompositions with a single prime factor 2 (and N=1). For other cases, the following R code (which I could not deposit on tio.run because of the packages R.utils!) returns a list

```
library(R.utils)
library(numbers)
bitz<-function(i,m) #int2bits
  c(rev(as.binary(i)),rep(0,m))[1:m]
ridl=function(n){
a=primeFactors(n)
if((n==1)|(sum(a==2)==1)){
  print("impossible")}else{
  m=length(a);g=NULL
  for(i in 1:2^m){
    b=bitz(i,m)
    if(((d<-prod(a[!!b]))%%2==(e<-prod(a[!b]))%%2)&(d
```

For instance,

```
> ridl(1456)
     [,1] [,2]
[1,]  365  363
[2,]  184  180
[3,]   95   87
[4,]   59   45
[5,]   40   12
[6,]   41   15
```

Checking for the most prolific N, up to 10^6, I found that $N=6720=2^6\cdot3\cdot5\cdot7$ produces 20 different decompositions. And that $N=887{,}040=2^8\cdot3^2\cdot5\cdot7\cdot11$ leads to 84 distinct differences of squares.