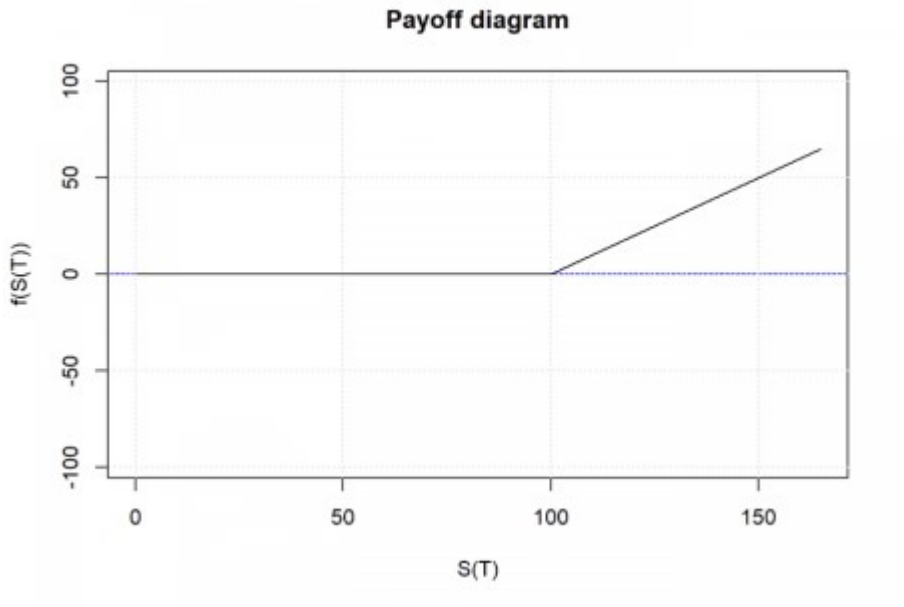
First, we need a way to define the *payoff function*: for each kink we provide two values, one for the *underlying* which goes from 0 to infinity and one for the payoff we want to replicate. We will use the names used in the paper for all the needed variables for clarity. Let us start by defining a *plain vanilla call*:

```
payoff <- data.frame(pi = c(0, 100, 110, Inf), f_pi = c(0, 0, 10, Inf))
payoff
##    pi f_pi
## 1   0    0
## 2 100    0
## 3 110   10
## 4 Inf  Inf
```

The last value of the payoff must be either equal to the penultimate value (= payoff staying flat at the given value) or must be (minus) infinity for a linear continuation in the given direction. Next we want to plot this payoff:

```
plot_payoff <- function(payoff, xtrpol = 1.5) {
  k <- nrow(payoff) - 1
  payoff_prt <- payoff
  payoff_prt$pi[k+1] <- payoff$pi[k] * xtrpol
  # linear extrapolation of last kink
  slope <- diff(c(payoff$f_pi[k-1], payoff$f_pi[k])) /
diff(c(payoff$pi[k-1], payoff$pi[k]))
  payoff_prt$f_pi[k+1] <- ifelse(payoff$f_pi[k] == payoff$f_pi[k+1],
payoff$f_pi[k+1], payoff$f_pi[k] + payoff$pi[k] * (xtrpol - 1) * slope)
  plot(payoff_prt, ylim = c(-max(abs(payoff_prt$f_pi) * xtrpol),
max(abs(payoff_prt$f_pi) * xtrpol)), main = "Payoff diagram", xlab =
"S(T)", ylab = "f(S(T))", type = "l")
  abline(h = 0, col = "blue")
  grid()
  lines(payoff_prt, type = "l")
  invisible(payoff_prt)
}

plot_payoff(payoff)
```



Now comes the actual replication. We need to functions for that: a helper function to calculate some parameters…

```
calculate_params <- function(payoff) {
  params <- payoff
  k <- nrow(params) - 1

  # add additional columns s_f_pi, lambda and s_lambda
  params$s_f_pi <- ifelse(params$f_pi < 0, -1, 1)
  # linear extrapolation of last kink
  slope <- diff(c(params$f_pi[k-1], params$f_pi[k])) /
diff(c(params$pi[k-1], params$pi[k]))
  f_pi_k <- ifelse(params$f_pi[k] == params$f_pi[k+1],
params$f_pi[k+1], slope)
  params$lambda <- c(diff(params$f_pi) / diff(params$pi), f_pi_k)
  params$s_lambda <- ifelse(params$lambda < 0, -1, 1)
  # consolidate
  params[k, ] <- c(params[k, 1:3], params[(k+1), 4:5])
  params <- params[1:k, ]
  params
}
```

…and the main function with the replication algorithm:

```
replicate_payoff <- function(payoff) {
  params <- calculate_params(payoff)
  suppressMessages(attach(params))
  k <- nrow(params)

  portfolios <- as.data.frame(matrix("", nrow = k, ncol = 6))
  colnames(portfolios) <- c("zerobonds", "nominal", "calls",
"call_strike", "puts", "put_strike")

  # step 0 (initialization)
  i <- 1
```

```
  i_r <- 1
  i_l <- 1

  while (i <= k) {

    # step 1 (leveling)
    if (f_pi[i] != 0) {
      portfolios[i, "zerobonds"] <- s_f_pi[i]
      portfolios[i, "nominal"] <- abs(f_pi[i])
    }

    # step 2 (replication to the right)
    while (i_r <= k) {
      if (i_r == i) {
        if (lambda[i] != 0) {
          portfolios[i, "calls"] <- paste(portfolios[i, "calls"],
lambda[i])
          portfolios[i, "call_strike"] <- paste(portfolios[i,
"call_strike"], pi[i])
        }
        i_r <- i_r + 1
        next
      }
      if ((lambda[i_r] - lambda[i_r-1]) != 0) {
        portfolios[i, "calls"] <- paste(portfolios[i, "calls"],
(lambda[i_r] - lambda[i_r-1]))
        portfolios[i, "call_strike"] <- paste(portfolios[i,
"call_strike"], pi[i_r])
      }
      i_r <- i_r + 1
    }

    # step 3 (replication to the left)
    while (i_l != 1) {
      if (i_l == i) {
        if (-lambda[i_l-1] != 0) {
          portfolios[i, "puts"] <- paste(portfolios[i, "puts"],
-lambda[i_l-1])
          portfolios[i, "put_strike"] <- paste(portfolios[i,
"put_strike"], pi[i_l])
        }
      } else {
        if ((lambda[i_l] - lambda[i_l-1]) != 0) {
          portfolios[i, "puts"] <- paste(portfolios[i, "puts"],
(lambda[i_l] - lambda[i_l-1]))
          portfolios[i, "put_strike"] <- paste(portfolios[i,
"put_strike"], pi[i_l])
        }
      }
      i_l <- i_l - 1
    }
```

```
    # step 4
    i <- i + 1
    i_r <- i
    i_l <- i
  }

  # remove duplicate portfolios
  portfolios <- unique(portfolios)
  # renumber rows after removal
  row.names(portfolios) <- 1:nrow(portfolios)
  portfolios
}
```

Let us test our function for the plain vanilla call:

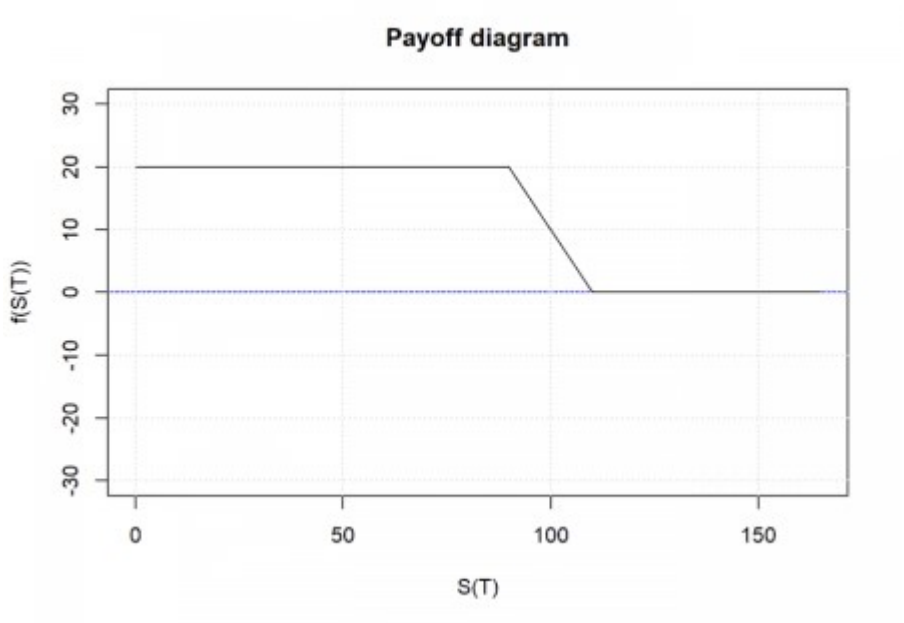
```
replicate_payoff(payoff)
##   zerobonds nominal calls call_strike  puts put_strike
## 1                         1         100
## 2         1      10     1         110  -1 1    110 100
```

There are always several possibilities for replication. In this case, the first is just our call with a strike of 100. Another possibility is buying a zerobond with a nominal of 10, going long a call with strike 110 and simultaneously going short a put with strike 110 and long another put with strike 100.

Let us try a more complicated payoff, a classic *bear spread* (which is also the example given in the paper):

```
payoff <- data.frame(pi = c(0, 90, 110, Inf), f_pi = c(20, 20, 0, 0))
payoff
##    pi f_pi
## 1   0   20
## 2  90   20
## 3 110    0
## 4 Inf    0

plot_payoff(payoff)
```

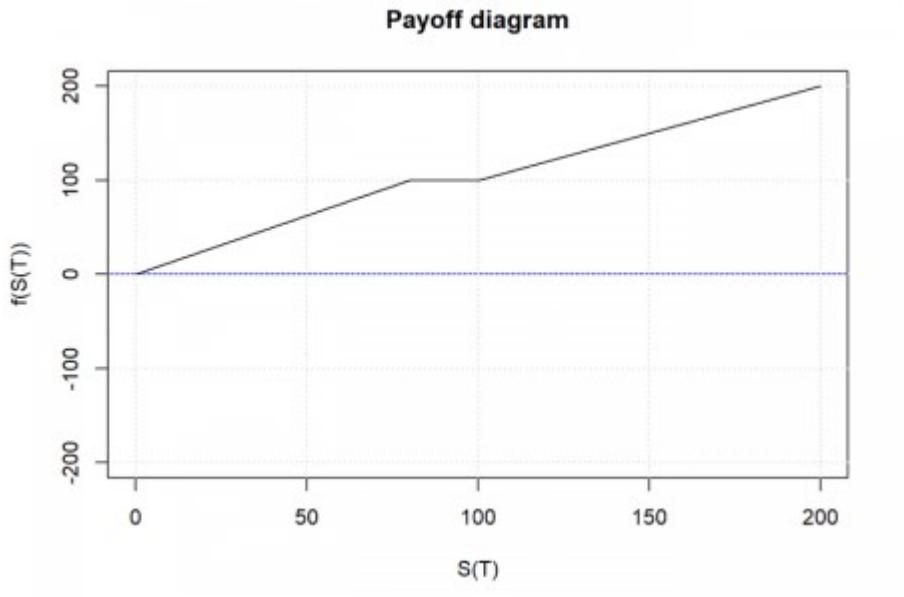



```
replicate_payoff(payoff)
##   zerobonds nominal calls call_strike  puts put_strike
## 1         1      20  -1 1      90 110
## 2                                    1 -1     110 90
```

Or for a so-called *airbag note*:

```
payoff <- data.frame(pi = c(0, 80, 100, 200, Inf), f_pi = c(0, 100,
100, 200, Inf))
payoff
##    pi f_pi
## 1   0    0
## 2  80  100
## 3 100  100
## 4 200  200
## 5 Inf  Inf

plot_payoff(payoff, xtrpol = 1)
```

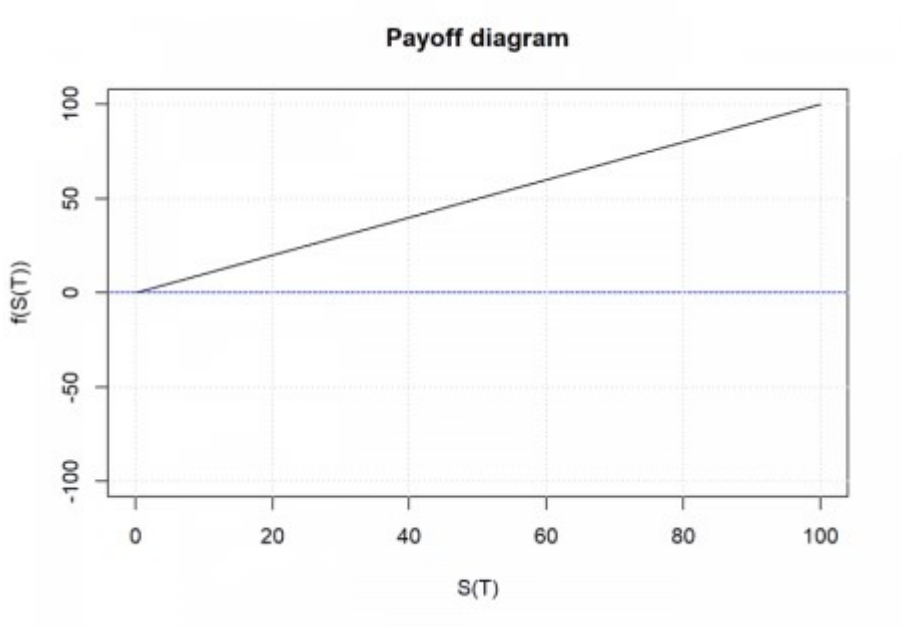


```
replicate_payoff(payoff)
##   zerobonds nominal         calls call_strike        puts
put_strike
## 1                    1.25 -1.25 1    0 80 100
## 2         1     100             1         100       -1.25
80
## 3         1     200             1         200  -1 1 -1.25  200 100
80
```

As a final example: how to replicate the underlying itself? Let's see:

```
payoff <- data.frame(pi = c(0, 100, Inf), f_pi = c(0, 100, Inf))
payoff
##    pi f_pi
## 1   0    0
## 2 100  100
## 3 Inf  Inf

plot_payoff(payoff, 1)
```




```
replicate_payoff(payoff)
##   zerobonds nominal calls call_strike puts put_strike
## 1                         1           0
## 2         1     100     1         100   -1        100
```

The first solution correctly gives us what is called a *zero-strike call*, i.e. a call with the strike of zero!

I hope you find this helpful! If you have any questions or comments, please leave them below.

I am even thinking that it might be worthwhile to turn this into a package and put it on CRAN, yet I don't have the time to do that at the moment… if you are interested in cooperating on that please leave a note in the comments too. Thank you!