Introduction

The aim of this series of blog is to predict monthly admissions to Singapore public acute adult hospitals. The dataset starts from Jan 2016 and ends in Feb 2021.

```
library(tidyverse)
library(timetk)
library(fpp3)

# cleaned up dataset downloaded from my github. Clean up of OG dataset done in 1st post
raw<- read_csv("https://raw.githubusercontent.com/notast/hierarchical-forecasting/main/
stat_sg_CLEAN.csv")

# dataset w national total; recalculate total hospital admissions
df<-raw %>% group_by(Date) %>% summarise(Admission=sum(Admission, na.rm
= T)) %>% mutate(Cluster= "National", Hospital= "National") %>%
bind rows(raw)
```

EDA for trend, seasonality and anomalies were explored in the last post. This post will complete EDA with lags and correlation of features of the time series.

1. Lags

Lags can be used to screen for seasonality. The values of lags can be used to calculate autocorrelation (ACF) and partial autocorrelation (PACF) There are two approaches for using lags for forecasting:

Short lags (lag length < forecast horizon)

When the lag length (e.g. 1 month) is less than the forecast horizon (e.g. 3 months), missing values are generated in future data which may pose a problem for some forecasting models.

```
df %>% group by (Hospital) %>%
  future frame(.date var= Date, .length out = "3 months", .bind data =
T) %>% ungroup() %>%
  mutate(Lag1month= lag vec(Admission, lag=1)) %>%
  tail()
## # A tibble: 6 x 5
   Hospital Date
                       Admission Cluster Lag1month
##
##
   <chr> <date> <dbl> <chr> <dbl>
## 1 TTSH 2020-12-01
## 2 TTSH 2021-01-01
                             4719 NHG
                                               4210
                             4502 NHG
                                               4719
## 3 TTSH 2021-02-01
## 4 TTSH 2021-03-01
## 5 TTSH 2021-04-01
                             4235 NHG
                                               4502
                               NA <NA>
                                               4235
                              NA <NA>
                                                  NA
## 6 TTSH
            2021-05-01
                               NA <NA>
                                                  NA
```

Recursive predictions are iterated to artificially generate lags (e.g. lag of 1 month again and again). Recursive prediction is used in ARIMA.

2. Long term lags (lag length >= forecast horizon)

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Long-term lags can be used to create lagged rolling features, which have proven predictive properties in the M5 competition. A disadvantage of using long term lags is the absent of short term lags which may have predictive properties.

1.1 Auto-correlation (ACF)

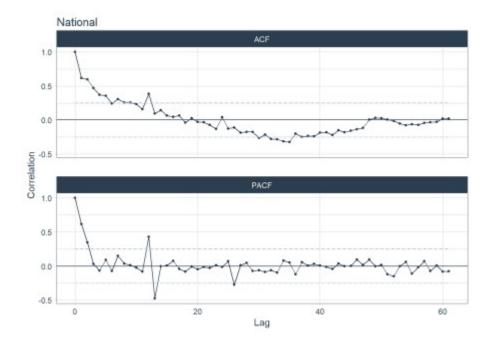
- ACF measures the linear relationship between lagged values of a time series. It
 measures how much the most recent value of the series is correlated with past values of
 the series, there is a tendency for values in a time series to be correlated with past
 observations in that time series.
- [If the time series has a trend, the ACF for small lags tend to be positive and large because observations nearby in time are also nearby in size.
- If the time series has seasonality, the ACF will have spikes. The autocorrelations will be larger at multiples of seasonal frequency than other lags.](https://otexts.com/fpp3/acf. html) e.g. for a daily dataset, if there are a spikes at day 7 and day 14, there is weekly seasonality. Nonetheless, if the spike at day 14 is larger than day 7, something is happening at the week 2 interval.
- ACF can also provide candidate periods for Fourier terms. A regression model containing
 Fourier terms (aka harmonic regression) is better for time series with longer seasonal
 periods (even seasonal ARIMA and seasonal ETS have ceiling seasonality periods). The
 longer seasonal pattern is modelled using Fourier terms while the short term dynamics
 can be modelled with ARIMA or ETS.

1.2 Partial Autocorrelation (PACF)

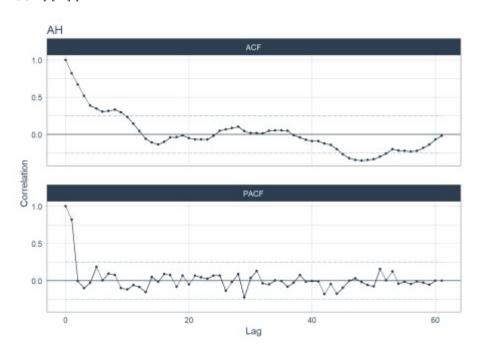
PACF measures the linear relationship between the correlations of the residuals (while ACF measures the linear relationship between lagged values). Using the correlation of residuals, PACF removes the dependence of lags on other lags. e.g. it looks at the direct effect between Jan and Mar and omits indirect effect between Jan to Feb and Feb to Mar. ACF tells us which lags are important while PACF tells us which lags are different and have functional implications.

- Based on the seasonality year strength values, CGH has the highest yearly seasonality.
 Spikes can be seen for CGH at lag 12 and 24 for CGH's ACF plot and spikes can be seen at lag 12 for CGH's PACF plot (CGH is plot 2).
- AH and SKH are hospitals with the lowest seasonality year strength values. Their ACF and PACF plots have no spikes (AH is plot 1 and SKH is plot 6).
- The time series are unlikely white noises as one or more large spikes are outside these bound

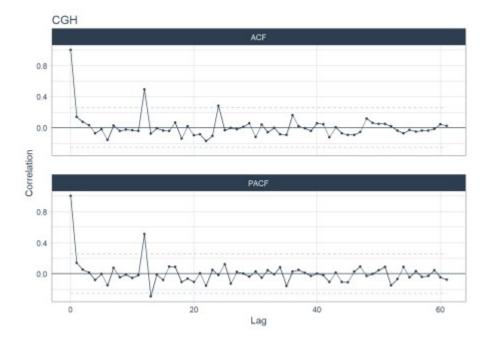
```
## Max lag exceeds data available. Using max lag: 61
## Max lag exceeds data available. Using max lag: 61
## Max lag exceeds data available. Using max lag: 61
## Max lag exceeds data available. Using max lag: 61
## Max lag exceeds data available. Using max lag: 61
plt_acf$p
## [[1]]
```



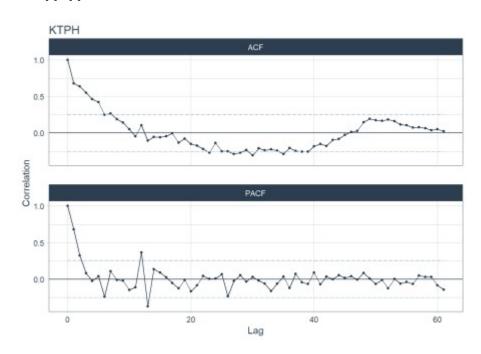
[[2]]



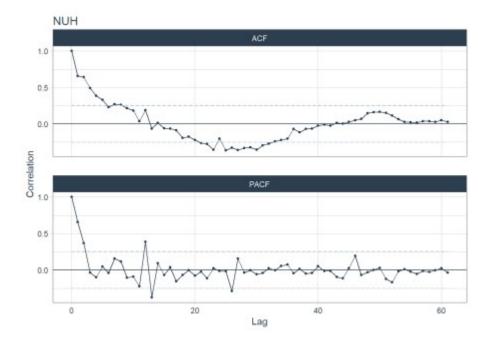
[[3]]



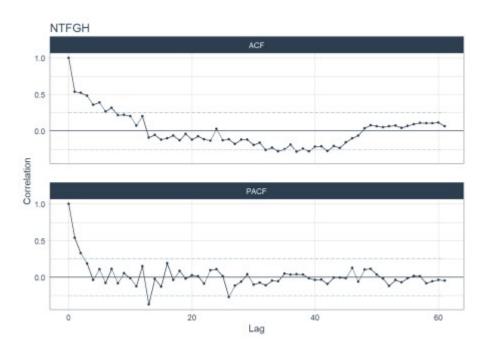
[[4]]



[[5]]



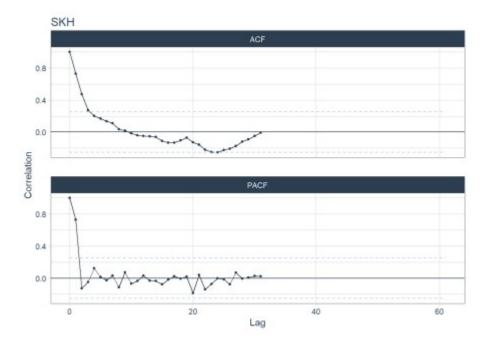
[[6]]



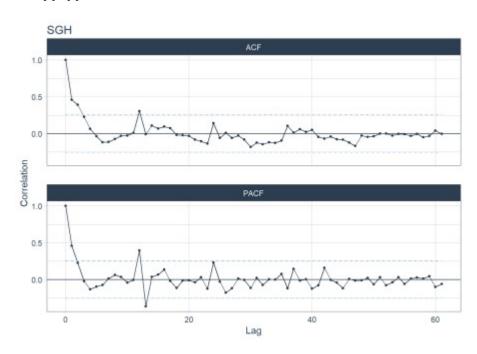
[[7]]

Warning: Removed 30 row(s) containing missing values (geom_path).

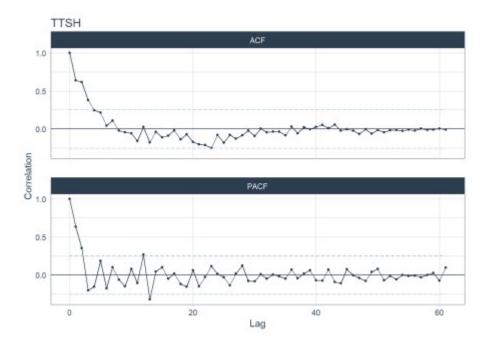
Warning: Removed 60 rows containing missing values (geom_point).



[[8]]



[[9]]



2. Correlation of features

Features and statistical values can be extracted from a time series. The fpp3 meta-package has a library feasts which provide provides up to 48 features. The features are tagged to 22 relevant concepts. 48 features is a relatively large number of variables to analyze, the correlation can help determine the associative relationship between the features.

The desired features are established with fabletools::feature_set and the features are extracted with fabeltools::features.

Every country has an organisation order to its public hospitals. In Singapore, there are 3 levels:

National level

|- Cluster level (Clusters are a network of hospitals based based on geographical regions. There are 3 health clusters in Singapore.)

|- Hospital level (There are 8 public acute adult hospitals.)

Features for each hierarchical level will be created.

```
feat hos<- df%>%
  filter(Hospital!= "National") %>%
 mutate(Date= yearmonth(as.character(Date))) %>%
  as tsibble(key= Hospital , index= Date) %>%
  features (Admission, feature_set(pkgs="feasts")) %>%
 mutate(Level="Hospital id") %>% rename(Name=Hospital)
## Warning: `n flat spots()` was deprecated in feasts 0.1.5.
## Please use `longest_flat_spot()` instead.
## Warning: 1 error encountered for feature 3
## [1] missing values in object
## Warning: 1 error encountered for feature 4
## [1] missing value where TRUE/FALSE needed
## Warning: 1 error encountered for feature 5
## [1] missing value where TRUE/FALSE needed
## Warning: 1 error encountered for feature 9
## [1] series is not periodic or has less than two periods
```

```
## Warning: 1 error encountered for feature 20
## [1] invalid time series parameters specified
feat clu<- df%>%
 filter(Cluster!= "National") %>%
 group by (Cluster, Date) %>% summarise (Admission= sum (Admission,
na.rm = T), .groups= "drop") %>%
 mutate(Date= yearmonth(as.character(Date))) %>%
 as tsibble(key= Cluster , index= Date) %>%
 features (Admission, feature set(pkgs="feasts")) %>%
 mutate(Level="Cluster id") %>% rename(Name=Cluster)
feat sg<- df %>% filter(Hospital=="National") %>%
  mutate(Date= yearmonth(as.character(Date))) %>%
 as tsibble(key= Hospital , index= Date) %>%
 features(Admission, feature set(pkgs="feasts")) %>%
 mutate(Level="National_id") %>% rename(Name=Hospital)
feat_all<- bind_rows(feat_hos, feat_clu, feat_sg)</pre>
glimpse(feat all)
## Rows: 12
## Columns: 50
## $ Name
                         <chr> "AH", "CGH", "KTPH", "NTFGH", "NUH",
"SGH", "SK~
## $ trend strength <dbl> 0.8008467, 0.2753427, 0.7806859,
0.8074394, 0.7~
## $ seasonal strength year <dbl> 0.2054133, 0.6550050, 0.4371513,
0.5688091, 0.5~
## $ seasonal_trough_year <dbl> 2, 2, 2, 2, 2, 2, 8, 2, 2, 2, 2
                         <dbl> 34308.47, 157238.54, 191874.46,
## $ spikiness
56234.36, 17971~
                         <dbl> 1131.4782, 347.8845, -512.1808,
## $ linearity
-759.2038, 753.~
## $ curvature
                         <dbl> -97.4100, -313.7077, -1564.4875,
-1325.6676, -2~
## $ stl e acf1
                        <dbl> 0.49046893, 0.17755315, 0.48299062,
-0.01179575~
                        <dbl> 0.8845025, 0.3302167, 0.6178947,
## $ stl e acf10
0.1922015, 0.7~
                         <dbl> 0.8206216, 0.1402620, 0.6816803,
## $ acf1
0.5401742, 0.6~
                        <dbl> 2.11098809, 0.06089474, 1.76735644,
## $ acf10
1.39798436,~
                        <dbl> -0.1443261, -0.4596242, -0.4699489,
## $ diff1 acf1
-0.5438977,~
## $ diff1 acf10
                        <dbl> 0.1936206, 0.2893855, 0.4387209,
0.5788380, 0.4~
## $ diff2 acf1
                        <dbl> -0.5765913, -0.6587037, -0.6950116,
-0.7213507,~
```

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```
## $ diff2 acf10
                        <dbl> 0.5990664, 0.6023470, 0.8644087,
0.9993514, 0.9~
                    <dbl> 0.04584300, 0.49355004, 0.10840987,
## $ season acf1
0.20553676,~
                        <dbl> 0.7185930, 0.0294867, 0.5769965,
## $ pacf5
0.4474341, 0.5~
                        <dbl> 0.2053477, 0.7647219, 0.6325854,
## $ diff1 pacf5
0.6161773, 0.6~
                 <dbl> 0.6957611, 1.1845925, 1.2335587,
## $ diff2 pacf5
1.3390279, 1.2~
## $ season pacf
                        <dbl> -0.0848245, 0.5090732, 0.3632837,
0.1520572, 0.~
## $ zero_run mean
                       <dbl> 0, 0, 0, 0, 0, 0, NA, 0, 0, 0, 0
                        <dbl> 0.131243894, 0.002854840,
## $ nonzero_squared_cv
0.007221997, 0.006827~
                       <dbl> 0, 0, 0, 0, 0, NA, 0, 0, 0, 0
## $ zero_start_prop
                        <dbl> 0, 0, 0, 0, 0, NA, 0, 0, 0, 0
## $ zero end prop
## $ lambda_guerrero <dbl> 1.2755867, 0.4304620, 1.9999268,
1.1297130, 1.9~
                       <dbl> 1.0798491, 0.2729302, 0.4187294,
## $ kpss stat
0.6144483, 0.3~
0.02132288,~
                        <dbl> -2.764480, -6.575707, -3.028578,
## $ pp stat
-3.247063, -3.~
                        <dbl> 0.07280602, 0.01000000, 0.04242816,
## $ pp pvalue
0.02856648,~
                        <int> 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 1, 1
## $ ndiffs
                        <int> 0, 1, 0, 0, 0, 0, NA, 0, 0, 0, 0
## $ nsdiffs
                        <dbl> 41.752026, 1.219753, 28.810659,
## $ bp stat
18.090865, 26.9~
## $ bp pvalue
                        <dbl> 1.036148e-10, 2.694091e-01,
7.981068e-08, 2.106~
                        <dbl> 43.805404, 1.279741, 30.227576,
## $ lb stat
18.980579, 28.2~
                        <dbl> 3.627021e-11, 2.579472e-01,
## $ lb_pvalue
3.842076e-08, 1.320~
## $ var tiled var
                       <dbl> 0.019075484, 0.390044129,
0.029382931, 0.007222~
                        <dbl> 0.7902109, 0.1644186, 0.7186914,
## $ var tiled mean
1.5074989, 0.7~
## $ shift level max <dbl> 290.2500, 222.6667, 488.4167,
406.0833, 776.583~
## $ shift_level_index <dbl> 14, 3, 50, 7, 50, 50, 33, 50, 50, 14,
34, 50
                        <dbl> 22794.73, 66035.73, 111015.11,
## $ shift var max
42842.55, 368944~
## $ shift_var_index <dbl> 21, 44, 43, 43, 43, 43, 38, 43, 43,
43, 26, 43
                        <dbl> 3.1278495, 0.9981751, 1.7897819,
## $ shift kl max
2.0810018, 1.2~
## $ shift kl index
                      <dbl> 14, 2, 49, 2, 49, 2, 33, 49, 49, 14,
```

Missing values need to be dropped before the correlation can be calculated. 11/48 features have missing values

```
(feat_ms<-feat_all %>% select_if(function(x) any(is.na(x))) %>%
colnames())
## [1] "pacf5" "diff1_pacf5" "diff2_pacf5"
## [4] "season_pacf" "zero_run_mean" "nonzero_squared_cv"
## [7] "zero_start_prop" "zero_end_prop" "lambda_guerrero"
## [10] "nsdiffs" "coef_hurst"
```

The missing values are from SKH likely due to nil admissions before it was open in Jul 2018. We are unable to drop observations with NA values as that will mean removing the SKH so we removed variables with missing values. The time series features is reduced from 48 to 37.

```
feat all %>% filter all(any vars(is.na(.)))
## # A tibble: 1 x 50
## Name trend strength seasonal strength y~ seasonal peak ye~
seasonal_trough_y~
   <dbl>
                                                         <dbl>
##
<dbl>
## 1 SKH
                   0.838
                                        0.284
                                                            10
## # ... with 45 more variables: spikiness <dbl>, linearity <dbl>,
## # curvature <dbl>, stl e acf1 <dbl>, stl e acf10 <dbl>, acf1
<dbl>,
     acf10 <dbl>, diff1 acf1 <dbl>, diff1 acf10 <dbl>, diff2 acf1
####
<dbl>,
## #
     diff2 acf10 <dbl>, season acf1 <dbl>, pacf5 <dbl>, diff1 pacf5
<dbl>,
## # diff2 pacf5 <dbl>, season pacf <dbl>, zero run mean <dbl>,
## # nonzero squared cv <dbl>, zero_start_prop <dbl>, zero_end_prop
<dbl>,
####
     lambda guerrero <dbl>, kpss stat <dbl>, kpss pvalue <dbl>,
pp stat <dbl>,
     pp pvalue <dbl>, ndiffs <int>, nsdiffs <int>, bp stat <dbl>,
####
## #
     bp_pvalue <dbl>, lb_stat <dbl>, lb_pvalue <dbl>, var_tiled_var
<dbl>,
## #
     var tiled mean <dbl>, shift level max <dbl>, shift level index
<dbl>,
## #
      shift var max <dbl>, shift var index <dbl>, shift kl max <dbl>,
```

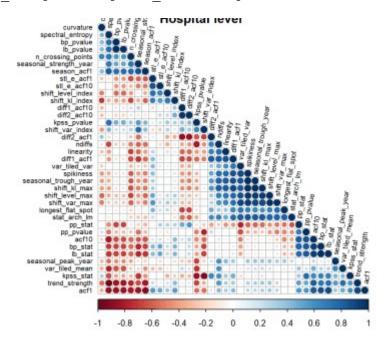
```
## # shift_kl_index <dbl>, spectral_entropy <dbl>, n_crossing_points
<int>,
## # longest_flat_spot <int>, coef_hurst <dbl>, stat_arch_lm <dbl>,
Level <chr>
# drop na columns
feat_all<- feat_all %>% select(-all_of(feat_ms))
```

The correlation for each level is visualized below.

Hospital level

```
fun_corrplt<- function(L,T) {
  feat_all %>%
  filter(Level==L) %>%
  # cor only takes numeric values
  select(-c(Level, Name)) %>% cor() %>%
  corrplot::corrplot(type="lower", order="hclust", tl.col="black",
tl.srt=80, tl.cex=0.65, title = T)}
```

fun corrplt("Hospital id", "Hospital level")



Cluster level

```
fun_corrplt("Cluster_id", "Cluster level")
## Warning in cor(.): the standard deviation is zero
## Error in hclust(as.dist(1 - corr), method = hclust.method):
NA/NaN/Inf in foreign function call (arg 10)
```

National level

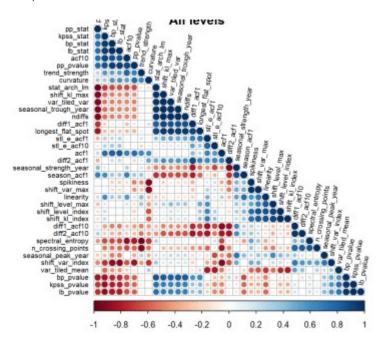
```
fun_corrplt("National_id", "National level")
## Warning in min(corr, na.rm = TRUE): no non-missing arguments to min;
returning
## Inf
## Warning in max(corr, na.rm = TRUE): no non-missing arguments to max;
returning
```

```
## -Inf
## Error in hclust(as.dist(1 - corr), method = hclust.method):
NA/NaN/Inf in foreign function call (arg 10)
```

All levels

Correlation at all levels was collectively calculated as admissions at a superordinate levels would have some correlation with admissions at the subordinate level.

```
fun_corrplt(c("Hospital_id", "Cluster_id", "National_id"), "All
levels")
```



Correlation findings

- Correlation could not be calculated at cluster level and national level, likely due to limited to observations.
- There are more features which are correlated at the collective level than at the hospital level
- The spread in the correlations revealed the heterogenity of the features, the features identify a variety of time series traits.

2.1 PCA of features

PCA was conducted as most features have moderate correlation with each other and to condense the information.

```
library(recipes)

(feat_pca_rec<-feat_all %>% recipe(~.) %>%
    update_role(Name, new_role = "id") %>% update_role(Level, new_role =
"id") %>%
    step_normalize(all_numeric_predictors()) %>%
    step_pca(all_numeric_predictors()))
## Data Recipe
##
## Inputs:
```

```
##
##
      role #variables
##
       id 2
## predictor
             37
##
## Operations:
##
## Centering and scaling for all_numeric_predictors()
## No PCA components were extracted.
feat pca rec %>% tidy()
## # A tibble: 2 x 6
## number operation type trained skip id ## <int> <chr> <chr> <lgl> <lgl> <chr>
## 1 1 step
                 normalize FALSE FALSE normalize YKTCL
## 2 2 step pca FALSE pca_h4nPa
feat pca rec %>% summary()
## # A tibble: 39 x 4
## variable
                       type role source
   <chr>
                        <chr> <chr>
                                       <chr>
## 1 Name
                       nominal id
                                      original
## 3 seasonal strength year numeric predictor original
## 4 seasonal peak year numeric predictor original
## 5 seasonal_trough_year numeric predictor original
## 6 spikiness
                      numeric predictor original
                       numeric predictor original
## 7 linearity
## 8 curvature
                       numeric predictor original
## 9 stl e acf1
                       numeric predictor original
## 10 stl_e_acf10 numeric predictor original
## # ... with 29 more rows
```

2.1 Variation captured

Here we examined the variation captured for each principal component.

```
(feat pca prep<- feat pca rec %>% prep())
## Data Recipe
##
## Inputs:
##
##
       role #variables
##
         id
## predictor
                37
##
## Training data contained 12 data points and no missing data.
##
## Operations:
##
## Centering and scaling for trend strength, ... [trained]
## PCA extraction with trend_strength, ... [trained]
```

The time series features were transformed into principal components and the first 5 principal

components were retained by default for step pca

The value of each time series feature from each principal component was tabulated.

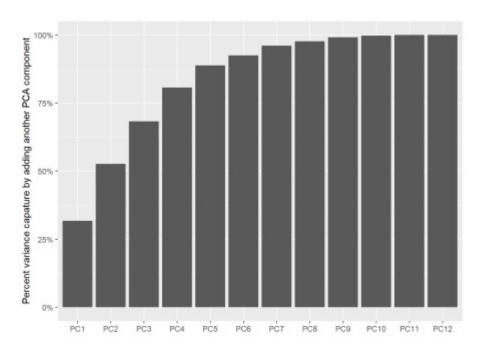
The percentage variance captured for each new principal component was plotted.

- The first 5 principal component captured 88% of the variance.
- There was no need to increase the number of components in step_pca as adequate variation was summarised in the first 5 components.
- # https://juliasilge.com/blog/best-hip-hop/

```
sdev<-feat_pca_prep$steps[[2]]$res$sdev
percent_variation <- sdev^2 / sum(sdev^2)

tibble(
  component = unique(feat_pca_PC$component),</pre>
```

```
percent_var = cumsum(percent_variation)
) %>%
  mutate(component = fct_inorder(component)) %>%
  ggplot(aes(component, percent_var)) +
  geom_col() +
  scale_y_continuous(labels = scales::percent_format()) +
  labs(x = NULL, y = "Percent variance capature by adding another PCA
component")
```



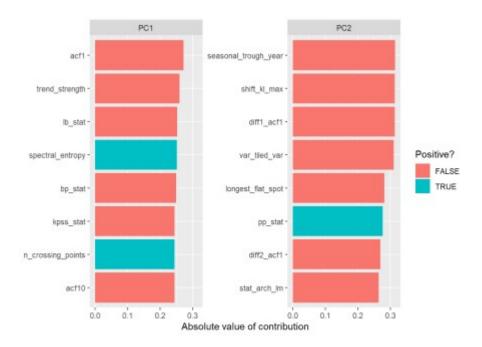
2.2 Time series features contributing to PC1

The first principal component captures 33% of the variance. PC1 is about

- trend & seasonality: acf1 (first ACF), acf10 (sum of squared of first 10 ACF), trend strength, spectral entropy.
- white noise: lb_stat (Ljung-Box test), bp_stat (box_pierce), spectral_entropy

```
# https://juliasilge.com/blog/best-hip-hop/
library(tidytext)
feat pca PC %>%
  filter(component %in% paste0("PC", 1:2)) %>%
  group_by(component) %>%
  top n(8, abs(value)) %>%
  ungroup() %>%
  mutate(terms = tidytext::reorder_within(terms, abs(value),
component)) %>%
  ggplot(aes(abs(value), terms, fill = value > 0)) +
  geom col() +
  facet wrap(~component, scales = "free y") +
  tidytext::scale_y_reordered() +
  labs(
    x = "Absolute value of contribution",
    y = NULL, fill = "Positive?"
```

)

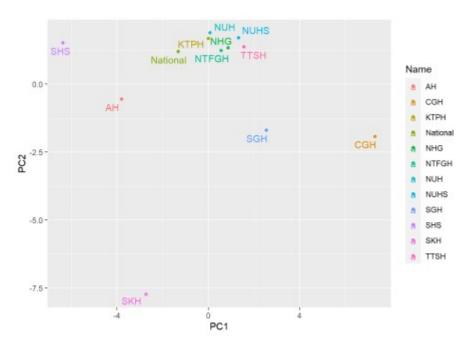


2.3 Distribution of hosptials in PC plane

```
feat_pca_df<-feat_pca_prep %>% bake(new_data=NULL)

feat_pca_df %>%

ggplot(aes(PC1, PC2, label = Name, colour=Name)) +
   geom_point() +
   ggrepel::geom_text_repel(force=.3)
```



Conclusion

- 1. There was an upward trend of admissions till the first half of 2020 which was flagged up as anomaly observations. => A dummy variable for this heightened COVID19 period would be considered when forecasting.
- 2. Seasonality varied though a dipped in admissions during Feb was common => Seasonality should be included when forecasting.
- 3. Lags varied (as sesaonlity varied too) => Lags should be considered when forecasting.

4. Features of time series are heterogeneous and most of them moderately correlated => Include these features when forecasting especially with machine learning approach. Principal components of these features can also be used instead.

```
# add prefix for easier selection
feat_all<-feat_all %>% rename_with(~paste0("F_",.x), !c(Level, Name))
save(feat_all, file = "feat_all.RData")
```

Errors

Errors encountered during scripting.

```
1 Unable to use {{}} with as_tsibble

fun_ft<- function(k) {
  mutate(Date= yearmonth(as.character(Date))) %>%
  as_tsibble(key= {{k}}, index= Date) %>%
  features(Admission, feature_set(pkgs="feasts"))
}

raw %>% fun_ft(Hospital)
## Error in fun ft(., Hospital): unused argument (Hospital)
```