Fundamentally, there are three basic steps to optimizing a goal-based portfolio:

1. Determine your goal variables: time horizon, amount of wealth dedicated to the goal today, and future required wealth value.
2. Develop capital market expectations for your investment universe: correlations, return expectations, and volatility.
3. Run a standard optimizer with a goal-based utility function.

This post is all about how to optimize a goal based portfolio in R.

First, we need to understand the goal, what is it you want to do with the money? To keep things simple, let's say you need $1,000 in 10 years, and you have $750 dedicated to it today. We organize this into a goal vector with the goal's value, the required funding value ($1,000), and the time horizon (10 years). Note that the goal's value is only relevant when optimizing your current wealth across your goals, which this post does not cover.

```
pool <- 750 # Total amount dedicated to this goal
goal_vector <- c(1, 1000, 10) # c(Goal value, goal funding requirement, time
horizon)
```

Ok. Step 1 is done.

Second, we need to develop capital market expectations for our investment universe. This topic is so big numerous books have been written on it because it is a very important step. Better forecasts yield better results. Since this post isn't about building CMEs, let's just input something simple.

- Stocks: 9% average return with 15% volatility
- Bonds: 4.5% average return with 5% volatility
- Gamble: -1% average return with 80% volatility
- Cash: 0.5% average return with 0.01% volatility

```
# Asset names
assets <- c('Stocks', 'Bonds', 'Gamble', 'Cash')
# Capital Market expectations - each is a vector in order of assets
cme <- data.frame( 'Return_Forecast' = c(0.09, 0.045, -0.01, 0.005),
                   'Volatility Forecast' = c(0.15, 0.05, 0.80, 0.001) )
# Correlation matrix, in order of assets both sideways and vertically
correlations <- matrix( c(1.00, 0.10, 0.00, 0.00,
                          0.10, 1.00, 0.00, 0.00,
                          0.00, 0.00, 1.00, 0.00,
                          0.00, 0.00, 0.00, 1.00),
                        nrow = length(assets), ncol = length(assets), byrow=T )
```

Note the "gamble" asset–we are going to have fun with that in a minute! Step 2 is complete.

Finally, now that we have our human-based inputs, let's proceed with the algorithm. Load our required libraries.

```
library(tidyverse)
library(Rsolnp) # this is the optimizer solnp()
```

And build the functions we will use.

```
# Required Functions
# This function converts the covariance table and weight vector into a
# portfolio standard deviation.
sd.f = function(weight_vector, covar_table){
  covar_vector = 0
  for(z in 1:length(weight_vector)){
```

```
    covar_vector[z] = sum(weight_vector * covar_table[,z])
  }
  return( sqrt( sum( weight_vector * covar_vector) ) )
}
# This function will return the expected portfolio return, given the
# forecasted returns and proposed portfolio weights
mean.f = function(weight_vector, return_vector){
  return( sum( weight_vector * return_vector ) )
}
# This function will return the probability of goal achievement, given
# the goal variables, allocation to the goal, expected return of the
# portfolio, and expected volatiltiy of the portfolio
phi.f = function(goal_vector, goal_allocation, pool, mean, sd){
  required_return = (goal_vector[2]/(pool * goal_allocation))^(1/goal_vector[3])
- 1

  if( goal_allocation * pool >= goal_vector[2]){

    return(1)

  } else {

    return( 1 - pnorm( required_return, mean, sd, lower.tail=TRUE ) )

  }
}
# For use in the optimization function later, this is failure probability,
# which we want to minimize.
optim_function = function(weights){

  1 - phi.f(goal_vector, allocation, pool,
            mean.f(weights, return_vector),
            sd.f(weights, covar_table) )
}
# For use in the optimization function later, this allows the portfolio
# weights to sum to 1.
constraint_function = function(weights){
  sum(weights)
}
```

Since we input correlations and volatilities, we need to build a covariance table. This uses the

$$\sigma_{i,j}^2 = \rho_{i,j}\sigma_i\sigma_j$$

form (covariance of asset *i* to *j* equals the correlation of *i* and *j* times the volatility of *i* times the volatility of *j*).

```
# Convert correlations to covariances
covariances <- matrix( nrow = length(assets), ncol = length(assets) )
for(i in 1:length(assets)){
  for(j in 1:length(assets)){
    covariances[j,i] <- cme[i,2] * cme[j,2] * correlations[i,j]
  }
}
```

All that is left is to do the optimization

```
# Optimization
return_vector <- cme$Return_Forecast
covar_table <- covariances
```

```
allocation <- 1
starting_weights <- rep(0.25, length(assets)) # start with 25% weights
result <- solnp( starting_weights, # Initialize weights
                 optim_function, # The function to minimize
                 eqfun = constraint_function, # The constraint function (
sum(weights) )
                 eqB = 1, # Constraint function must equal 1
                 LB = rep(0, length(assets)), # Lower bound of constraint,
weight >= 0
                 UB = rep(1, length(assets)) ) # Upper bound of constraint,
weight <= 1
# Results
optimal_weights <- data.frame( 'Assets' = assets,
                               'Optimal Weights' = round(result$pars, 2) )
```

The solnp function from the Rsolnp package is quite powerful. Plus, when you are running it on complicated problems, the output makes me feel like a hacker, which is always a plus!

And we find that our optimal weights are 25% stocks, 75% bonds.

```
> optimal_weights
  Assets Optimal.Weights
1 Stocks            0.25
2  Bonds            0.75
3 Gamble            0.00
4   Cash            0.00
```

So What's Different About Goals-Based Investing?

Now that you've got the basics of goals-based portfolio optimization, we may ask what is so different about GBI? Well, let's find out!

To illustrate, let's build allocations for various levels of starting wealth.

```
pool_seq <- seq(50, 1000, 50)
```

And empty lists to hold the various allocation results

```
# Empty lists of allocation to hold results
stock_allocation <- 0
bond_allocation <- 0
gamble_allocation <- 0
cash_allocation <- 0
```

Then iterate through each starting wealth value and determine the optimal investment allocation (this code assumes you've run the code in the previous section).

```
# Loop through each level of starting wealth to determine optimal allocation
for(i in 1:length(pool_seq)){
  pool <- pool_seq[i]
  result <- solnp( starting_weights,
                   optim_function,
                   eqfun = constraint_function,
                   eqB = 1,
                   LB = rep(0, length(assets)),
                   UB = rep(1, length(assets)) )

  # Store results for this iteration
  stock_allocation[i] <- result$pars[1] %>% round(digits = 2)
  bond_allocation[i] <- result$pars[2] %>% round(digits = 2)
```

```
  gamble_allocation[i] <- result$pars[3] %>% round(digits = 2)
  cash_allocation[i] <- result$pars[4] %>% round(digits = 2)
}
```

Since I am using ggplot, I'll now need to store the results in a long form data frame.

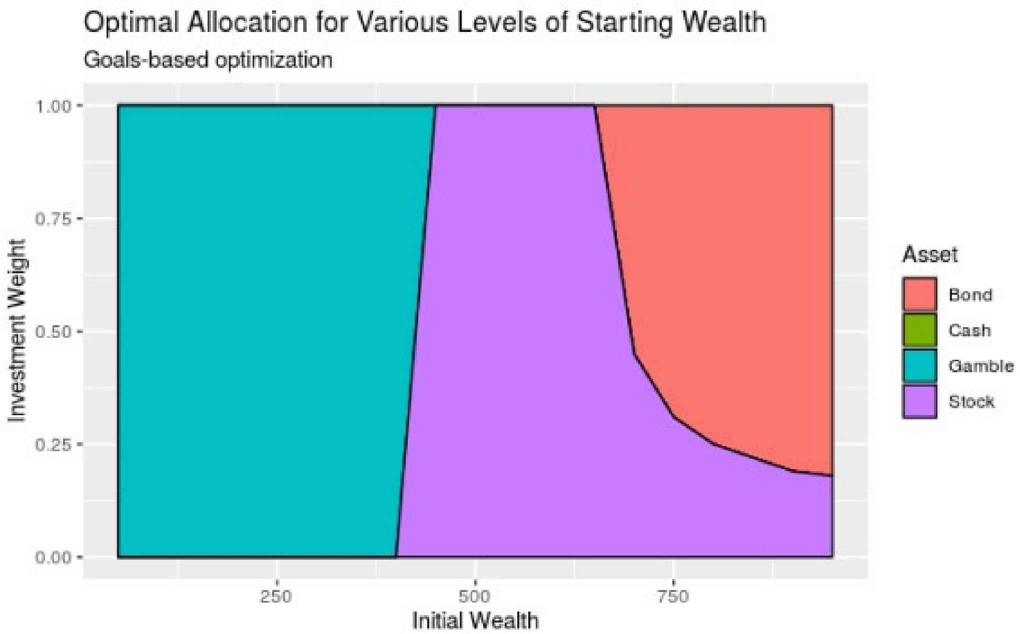
```
seq_results <- data.frame( 'Weight' = c(stock_allocation,
                                        bond_allocation,
                                        gamble_allocation,
                                        cash_allocation ),

                           'Asset' = c( rep('Stock', length(stock_allocation)),
                                        rep('Bond', length(bond_allocation)),
                                        rep('Gamble',
length(gamble_allocation)),
                                        rep('Cash', length(cash_allocation))),

                           'Wealth' = rep(pool_seq, length(assets)) )
```

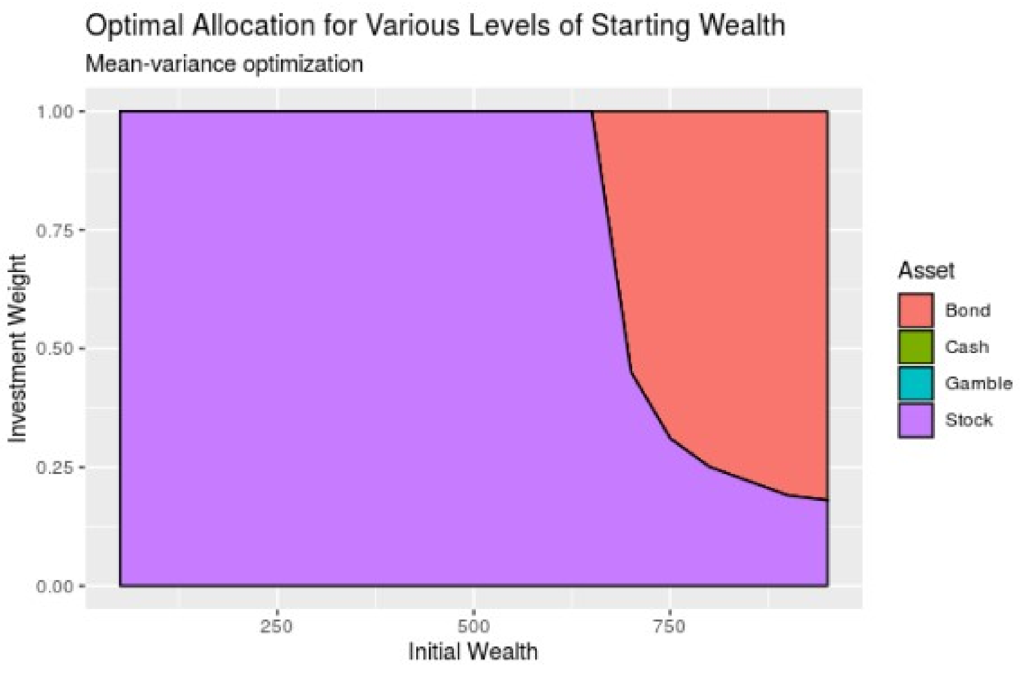
And, finally, visualize the results.

```
ggplot( seq_results, aes(x = Wealth, y = Weight, fill = Asset))+
  geom_area( linetype=1, size=0.5, color='black')+
  xlab('Initial Wealth')+
  ylab('Investment Weight')+
  labs(fill = 'Asset',
       title = 'Optimal Allocation for Various Levels of Starting Wealth',
       subtitle = 'Goals-based optimization')
```



As you can see, goals-based portfolios will lean on high-variance, low return investments when your starting wealth is small enough. Technically speaking, GBI portfolios begin allocating to lottery-like investments whenever the return required to hit the goal is greater than the return offered by the mean-variance efficient frontier. In traditional mean-variance optimization, the optimizer will maintain exposure to the endpoint of the frontier, or 100% stock allocation, in our example.

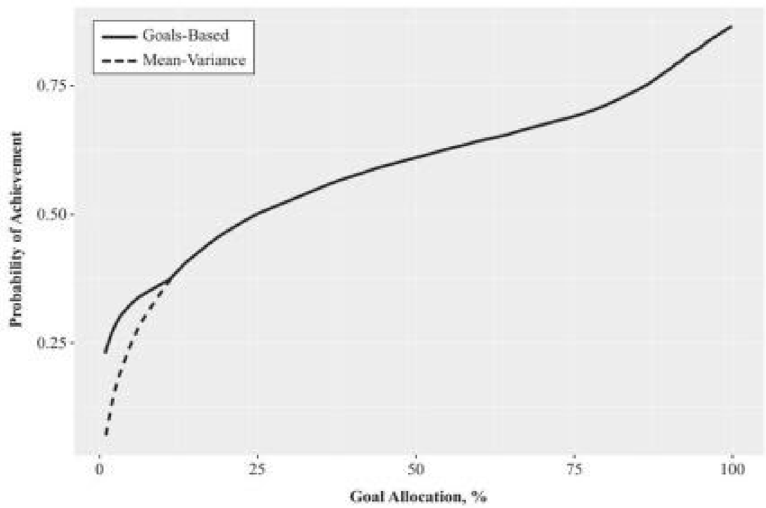
As a comparison, here is the mean-variance optimizer result. As you can see, the "gamble" asset is eliminated from consideration.



Because of this, goal-based portfolios yield higher probabilities of goal achievement than mean-variance portfolios (mean-variance portfolios are stochastically dominated by goals-based portfolios). This was Exhibit 5 in my recent paper on this subject.

EXHIBIT 5

Mean-Variance Portfolios are Stochastically Dominated by Goals-Based Portfolios



Notes: For higher across-goal allocations, mean-variance portfolios and goals-based portfolios are the same. As the wealth allocation drops, goals-based portfolios begin to deliver more achievement probability than those that are mean-variance constrained. The point of departure between the two is the point at which solutions become infeasible under current goals-based paradigms. This mean-variance adapted form, although inferior to strict goals-based optimization, has the benefit of feasible solutions for investors who are mean-variance constrained.

For all of these reasons (and more), if you have goals to achieve then you should be using goals-based portfolio theory. I hope this post helped you understand how to implement the basic framework!