An even number of teams play one another once a week with no tie allowed and have played all other teams. Four weeks into the tournament, A has won all its games, B,C, and D have won three games, the other teams have won at least one games. What is the minimum number of teams? Show an instance.

By sheer random search

```
tnmt=function(K=10,gamz=4){
 t1=t0=matrix(1,K,K)
tnmt=function(K=10,gamz=4){
 tnmt=t0=matrix(0,K,K)
 while (!prod(apply(tnmt^2,1,sum)==4)){
   tnmt=t0
   for (i in 1:(K-2)){
     if((a<-gamz-sum(tnmt[i,]^2))> K-i-1) break()
     if(a>0){
      j=sample((i+1):K,a)
      tnmt[i,j]=sample(c(-1,1),a,rep=TRUE)
      tnmt[j,i]=-tnmt[i,j]}}}
 tnmt}
chck=function(1,gamz=4){
    sumz=apply(tnmt,1,sum)
    max(sumz)==gamz&
    sum(sumz==2)>2&
    min(sumz)>-gamz}
```

I found that 8 teams were not producing an acceptable game out of 10^6 tries. Here is a solution for 9 teams:

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
 [1,]             -1   -1         1              -1
 [2,]             -1         1         -1        -1
 [3,]    1    1                   1              -1
 [4,]    1                   1          1   -1
 [5,]        -1        -1                    1   -1
 [6,]   -1        -1                   -1    1
 [7,]         1        -1         1          1
 [8,]                   1   -1   -1    -1
 [9,]    1    1    1         1
```

where team 9 wins all four games, 7,4 and 3, win three games, and the other 4 teams win one game. Which makes sense since this is a zero sum game, with a value of 10 over the four top teams and 2(N-4)=10 if no team has two wins (adding an even number of such teams does not change the value of the game).