```
# In R console
suppressPackageStartupMessages(library(ESGtoolkit))
```

G2++ Model input **parameters**:

```
# Observed maturities
u <- 1:30

# Yield to maturities
txZC <- c(0.01422,0.01309,0.01380,0.01549,0.01747,0.01940,
          0.02104,0.02236,0.02348, 0.02446,0.02535,0.02614,
          0.02679,0.02727,0.02760,0.02779,0.02787,0.02786,
          0.02776,0.02762,0.02745,0.02727,0.02707,0.02686,
          0.02663,0.02640,0.02618,0.02597,0.02578,0.02563)

# Zero-coupon prices = 'Observed' market prices
p <- c(0.9859794,0.9744879,0.9602458,0.9416551,0.9196671,
       0.8957363,0.8716268,0.8482628,0.8255457,0.8034710,
       0.7819525,0.7612204,0.7416912,0.7237042,0.7072136
       ,0.6922140,0.6785227,0.6660095,0.6546902,0.6441639,
       0.6343366,0.6250234,0.6162910,0.6080358,0.6003302,
       0.5929791,0.5858711,0.5789852,0.5722068,0.5653231)
```

G2++ **simulation function** (HCSPL stands for Hermite Cubic Spline interpolation of the Yield Curve):

```
# Function of the number of scenarios
simG2plus <- function(n, methodyc = "HCSPL", seed=13435,
                      b_opt=NULL, rho_opt=NULL, eta_opt=NULL,
                      randomize_params=FALSE)
{
    set.seed(seed)

    # Horizon, number of simulations, frequency
    horizon <- 20
    freq <- "semi-annual"
    delta_t <- 1/2

    # Parameters found for the G2++
    a_opt <- 0.50000000 + ifelse(randomize_params, 0.5*runif(1), 0)
    if(is.null(b_opt))
      b_opt <- 0.35412030 + ifelse(randomize_params, 0.5*runif(1), 0)
    sigma_opt <- 0.09416266
    if(is.null(rho_opt))
      rho_opt <- -0.99855687
    if(is.null(eta_opt))
      eta_opt <- 0.08439934

    print(paste("a:", a_opt))
    print(paste("b:", b_opt))
    print(paste("sigma:", sigma_opt))
    print(paste("rho:", rho_opt))
    print(paste("eta:", eta_opt))

    # Simulation of gaussian correlated shocks
    eps <- ESGtoolkit::simshocks(n = n, horizon = horizon,
                     frequency = "semi-annual",
                     family = 1, par = rho_opt)
```

```
    # Simulation of the factor x
    x <- ESGtoolkit::simdiff(n = n, horizon = horizon,
                 frequency = freq,
                 model = "OU",
                 x0 = 0, theta1 = 0, theta2 = a_opt, theta3 = sigma_opt,
                 eps = eps[[1]])

    # Simulation of the factor y
    y <- ESGtoolkit::simdiff(n = n, horizon = horizon,
                 frequency = freq,
                 model = "OU",
                 x0 = 0, theta1 = 0, theta2 = b_opt, theta3 = eta_opt,
                 eps = eps[[2]])

    # Instantaneous forward rates, with spline interpolation
    methodyc <- match.arg(methodyc)
    fwdrates <- ESGtoolkit::esgfwdrates(n = n, horizon = horizon,
    out.frequency = freq, in.maturities = u,
    in.zerorates = txZC, method = methodyc)
    fwdrates <- window(fwdrates, end = horizon)

    # phi
    t.out <- seq(from = 0, to = horizon,
                 by = delta_t)
    param.phi <- 0.5*(sigma_opt^2)*(1 - exp(-a_opt*t.out))^2/(a_opt^2) +
    0.5*(eta_opt^2)*(1 - exp(-b_opt*t.out))^2/(b_opt^2) +
      (rho_opt*sigma_opt*eta_opt)*(1 - exp(-a_opt*t.out))*
      (1 - exp(-b_opt*t.out))/(a_opt*b_opt)
    param.phi <- ts(replicate(n, param.phi),
                    start = start(x), deltat = deltat(x))
    phi <- fwdrates + param.phi
    colnames(phi) <- c(paste0("Series ", 1:n))

    # The short rates
    r <- x + y + phi
    colnames(r) <- c(paste0("Series ", 1:n))

    return(r)
}
```

Simulations of G2++ for **4 types of parameters' sets**:

```
r.HCSPL <- simG2plus(n = 10000, methodyc = "HCSPL", seed=123)

r.HCSPL2 <- simG2plus(n = 10000, methodyc = "HCSPL", seed=2020)

r.HCSPL3 <- simG2plus(n = 10000, methodyc = "HCSPL", seed=123,
                     randomize_params=TRUE)

r.HCSPL4 <- simG2plus(n = 10000, methodyc = "HCSPL", seed=123,
                      b_opt=1, rho_opt=0, eta_opt=0,
                      randomize_params=FALSE)
```

Stochastic **discount factors** derived from short rates simulations:

```
deltat_r <- deltat(r.HCSPL)

Dt.HCSPL <- ESGtoolkit::esgdiscountfactor(r = r.HCSPL, X = 1)
```

```
Dt.HCSPL <- window(Dt.HCSPL, start = deltat_r, deltat = 2*deltat_r)

Dt.HCSPL2 <- ESGtoolkit::esgdiscountfactor(r = r.HCSPL2, X = 1)
Dt.HCSPL2 <- window(Dt.HCSPL2, start = deltat_r, deltat = 2*deltat_r)

Dt.HCSPL3 <- ESGtoolkit::esgdiscountfactor(r = r.HCSPL3, X = 1)
Dt.HCSPL3 <- window(Dt.HCSPL3, start = deltat_r, deltat = 2*deltat_r)

Dt.HCSPL4 <- ESGtoolkit::esgdiscountfactor(r = r.HCSPL4, X = 1)
Dt.HCSPL4 <- window(Dt.HCSPL4, start = deltat_r, deltat = 2*deltat_r)
```

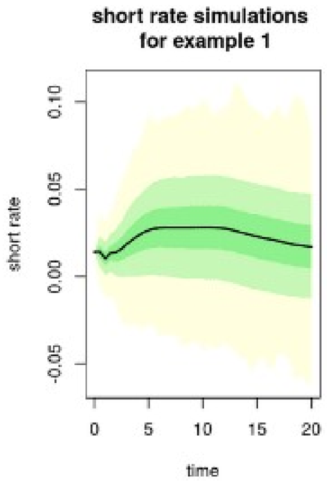
Prices (*observed* vs Monte Carlo for previous 4 examples):

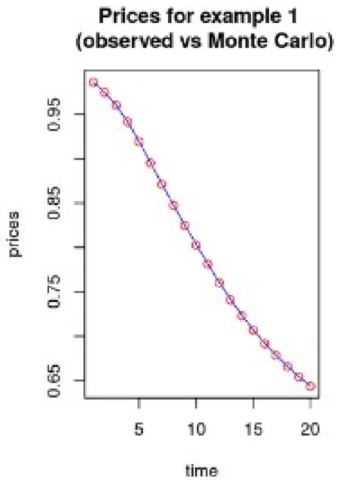
```
# Observed market prices
horizon <- 20
marketprices <- p[1:horizon]

# Monte Carlo prices
## Example 1
montecarloprices.HCSPL <- rowMeans(Dt.HCSPL)
## Example 2
montecarloprices.HCSPL2 <- rowMeans(Dt.HCSPL2)
## Example 3
montecarloprices.HCSPL3 <- rowMeans(Dt.HCSPL3)
## Example 4
montecarloprices.HCSPL4 <- rowMeans(Dt.HCSPL4)
```

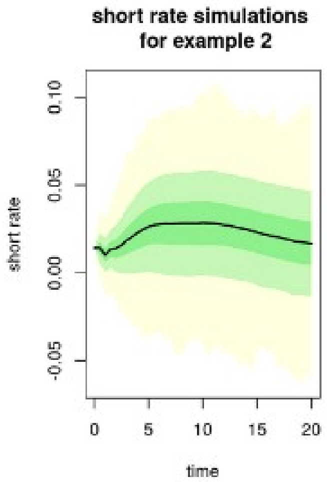
Plots *observed* prices vs Monte Carlo prices:

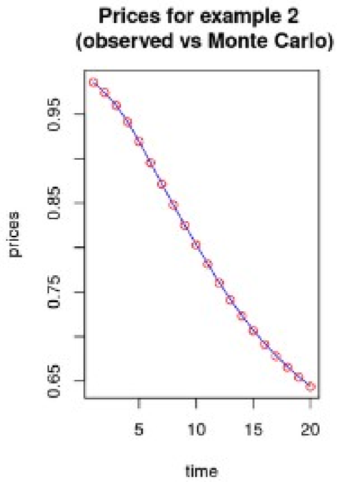
```
par(mfrow=c(4, 2))

ESGtoolkit::esgplotbands(r.HCSPL, xlab = 'time', ylab = 'short rate',
                         main="short rate simulations \n for example 1")
plot(marketprices, col = "blue", type = 'l',
     xlab = "time", ylab = "prices", main = "Prices for example 1 \n (observed
vs Monte Carlo)")
points(montecarloprices.HCSPL, col = "red")

ESGtoolkit::esgplotbands(r.HCSPL2, xlab = 'time', ylab = 'short rate',
                         main="short rate simulations \n for example 2")
plot(marketprices, col = "blue", type = 'l',
     xlab = "time", ylab = "prices", main = "Prices for example 2 \n (observed
vs Monte Carlo)")
points(montecarloprices.HCSPL2, col = "red")

ESGtoolkit::esgplotbands(r.HCSPL3, xlab = 'time', ylab = 'short rate',
                         main="short rate simulations \n for example 3")
plot(marketprices, col = "blue", type = 'l',
     xlab = "time", ylab = "prices", main = "Prices for example 3 \n (observed
vs Monte Carlo)")
points(montecarloprices.HCSPL3, col = "red")

ESGtoolkit::esgplotbands(r.HCSPL4, xlab = 'time', ylab = 'short rate',
                         main="short rate simulations \n for example 4")
plot(marketprices, col = "blue", type = 'l',
     xlab = "time", ylab = "prices", main = "Prices for example 4 \n (observed
vs Monte Carlo)")
```

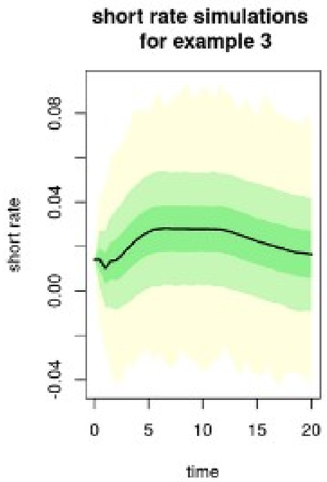
```
points(montecarloprices.HCSPL4, col = "red")
```

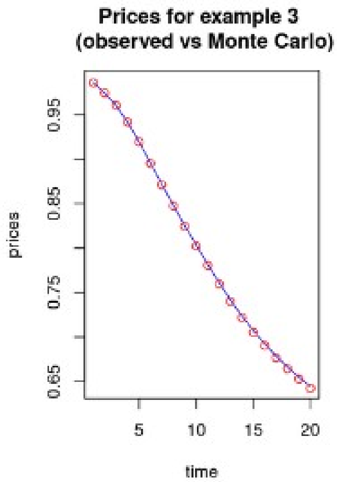

short rate simulations
for example 1
short rate
0.10
0.05
0.00
-0.05
0
5
10
15
20
time

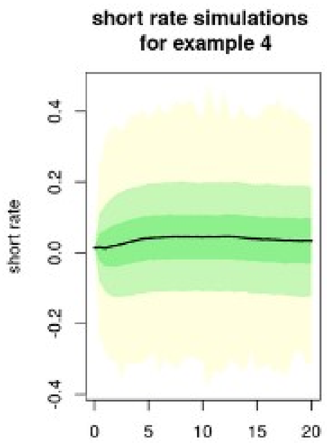

Prices for example 1
(observed vs Monte Carlo)
prices
0.95
0.85
0.75
0.65
5
10
15
20
time

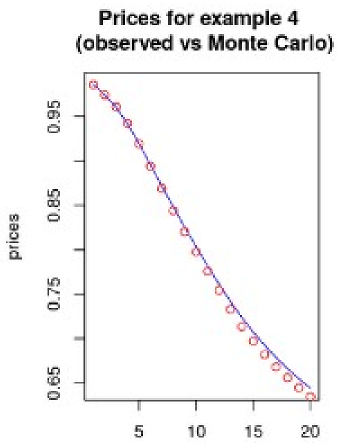

short rate simulations
for example 2
short rate
0.10
0.05
0.00
-0.05
0
5
10
15
20
time


Prices for example 2
(observed vs Monte Carlo)
prices
0.95
0.85
0.75
0.65
5
10
15
20
time


short rate simulations
for example 3
short rate
0.08
0.04
0.00
-0.04
0
5
10
15
20
time


Prices for example 3
(observed vs Monte Carlo)
prices
0.95
0.85
0.75
0.65
5
10
15
20
time


short rate simulations
for example 4
short rate
0.4
0.2
0.0
-0.2
-0.4
0
5
10
15
20


Prices for example 4
(observed vs Monte Carlo)
prices
0.95
0.85
0.75
0.65
5
10
15
20

time time

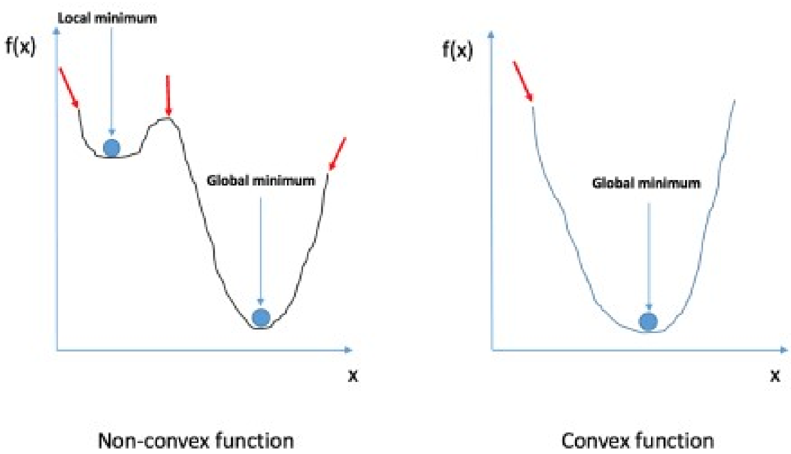
What do we **observe** on these graphs, both on simulations and prices? What will happen if we add a **third factor** to this model, meaning, three more parameters; a G3++/any other *hydra*?

Optimization in Deep learning neural networks

On a different type of question/problem, but still on the subject of model specification, identification, degrees of freedom and regularization: **Deep learning neural networks**. Some people suggest that if you keep adding parameters (degrees of freedom?) to these models, you'll **still obtain a good generalization.** Well, there's this picture that I like a lot:

Source: https://towardsdatascience.com/optimizers-be-deeps-appetizers-511f3706aa67

When we optimize the loss function in Deep learning neural networks models, we are most likely using gradient descent, which is **fast and scalable.** Still, no matter how sophisticated the gradient descent procedure we're using, we will likely get stuck into a local minimum – because the loss function is rarely convex.



Stuck is a rather unfortunate term here, because it's not an actual problem, but instead, an indirect way to avoid overtraining. Also, in our gradient descent procedure, we tune the number of epochs (number of iterations in the descent/ascent), the learning rate (how fast we roll in the descent/ascent), in addition to the dropout (randomly dropping out some nodes in networks' layers), etc. These are also ways to **avoid learning too much, to stop the optimization relatively early, and preserve the model's ability to generalize**. They regularize the model, whereas the millions of network nodes serve as degrees of freedom. This is a different problem than the first one we examined, with different objectives, but… still on the subject of model specification, identification, degrees of freedom and regularization.