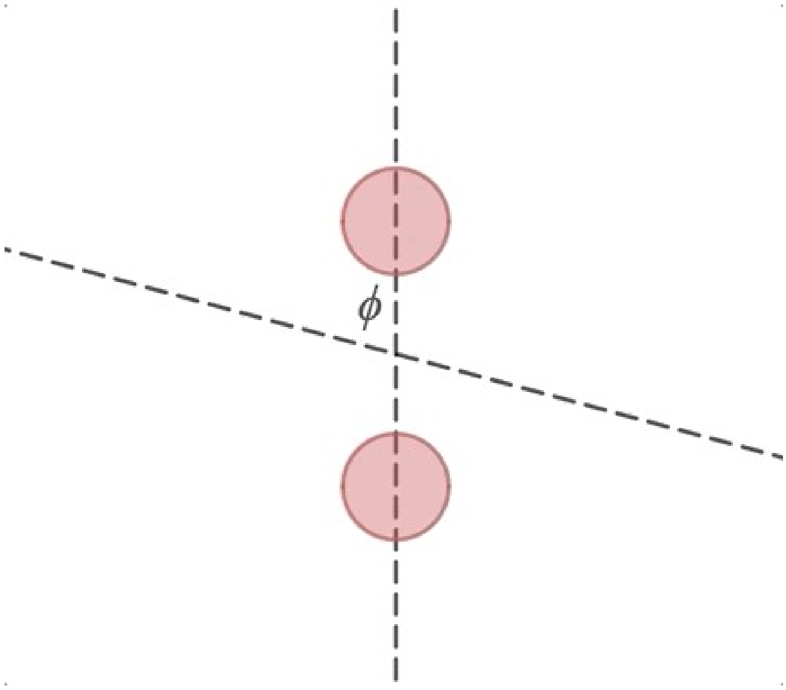
…:



What is the minimum value of phi that will allow your bowl to split your opponents' bowls without hitting them?

Plan

I will approximate the solution to this puzzle by simulating the game from many different angles. Thankfully, because the game is vertically and horizontally symmetric, I only need to simulate the green ball reaching the middle point between the red balls and only need to see if it collides with the top red ball.

Setup

```
knitr::opts_chunk$set(echo = TRUE, comment = "#>", cache = TRUE)
library(glue)
library(clisymbols)
library(ggforce)
library(gganimate)
library(tidyverse)
theme_set(theme_minimal())
# Some standard colors used throughout
green <- "#54c761"
red <- "#c75454"
purple <- "#a06bdb"
light_grey <- "grey70"
grey <- "grey40"
set.seed(0)
```

Simulate a single pass

I split the code into two pieces. The first simulates a bowl with a given angle, and the second decides on the angle to narrow down the approximation. The following functions take care of the first part: simulating a bowl.

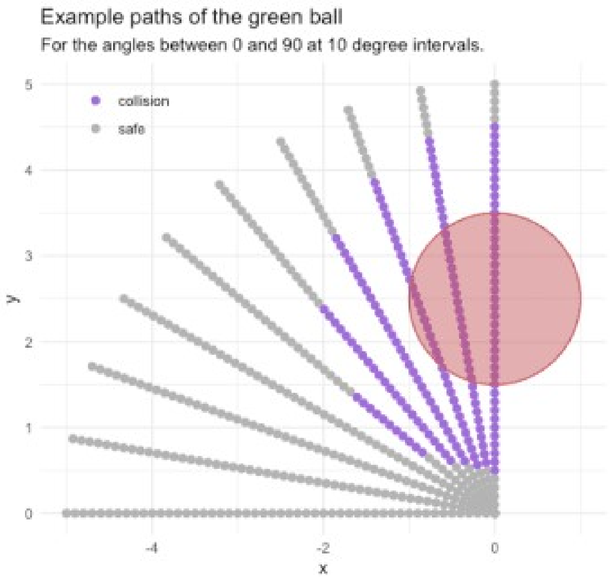
A single simulation can be run by calling `run_bowl_simulation()` with an angle (in degrees). The function works by changing the hypotenuse, starting with `h_start = 5` and decreasing it to 0 by `step_size` steps (the steps are held in the numeric vector `h_vals`). The actual position of the ball is calculated from the length of the hypotenuse and angle with a bit of trigonometry in `make_green_ball()`. For each hypotenuse value, the green ball is positioned and then tested to see if it collides with the red ball (set at ((x,y) = (0,2.5)) as per the riddle) using the function `did_balls_collide()`. This information is recorded by building a single data frame with the data for each step of the simulation. The data frame is returned at the end of the simulation.

```
# Run a simulation of the bowling game.
run_bowl_simulation <- function(angle,
step_size = 0.1,
red_ball_loc = list(x = 0, y = 2.5)) {
h_start <- 5
h_vals <- seq(h_start, 0, by = -step_size)
angle <- angle * (pi / 180)
all_ball_pos <- NULL
for (h in h_vals) {
green_ball <- make_green_ball(h, angle)
collision <- did_balls_collide(green_ball, red_ball_loc, radius = 1)
all_ball_pos <- bind_rows(
all_ball_pos,
tibble(h = h,
x = green_ball$x,
y = green_ball$y,
collision = collision)
)
}
return(all_ball_pos)
}

# Make a green ball location from the x-position and angle.
make_green_ball <- function(h, angle) {
x <- -1 * h * cos(pi/2 - angle)
y <- h * sin(pi/2 - angle)
list(x = x, y = y)
}

# Decide wether the two balls of radius `r` collided.
did_balls_collide <- function(ball1, ball2, radius) {
d <- sqrt((ball1$x - ball2$x)^2 + (ball1$y - ball2$y)^2)
return(d <= 2*radius)
}
```

Below are the results from running the simulation at angles between 90 degrees (horizontal) and 0 degrees (vertical) at 10 degree increments. Each line is an individual simulation, and each point is a round of the simulation. A red ball is positioned as per the riddle, and the purple points indicate where the green ball would collide with the red ball. These example simulations show that the `run_bowl_simulation()` function is working as expected.

```
map(seq(90, 0, -10), run_bowl_simulation, step_size = 0.1) %>%
map2(seq(90, 0, -10), ~ .x %>% add_column(angle = .y)) %>%
bind_rows() %>%
mutate(collision = ifelse(collision, "collision", "safe")) %>%
```

```
ggplot() +
geom_point(aes(x, y, color = collision), size = 2) +
geom_circle(aes(x0 = x0, y0 = y0, r = r),
data = tibble(x0 = 0, y0 = 2.5, r = 1),
color = red, fill = red, alpha = 0.5) +
scale_color_manual(values = c(purple, light_grey)) +
coord_fixed() +
theme(
legend.position = c(0.15, 0.9),
legend.title = element_blank()
) +
labs(x = "x", y = "y",
title = "Example paths of the green ball",
subtitle = "For the angles between 0 and 90 at 10 degree intervals.")
```



Find the smallest angle

The second part of the code is to find the smallest (narrowest) angle at which there is no collision. Instead of trying every angle between 90 degrees and 0 degrees at some very small increment, I approach this problem a bit more efficiently. I built an algorithm than starts at 90 degrees and takes large steps until there is an angle that causes a collision. It then takes a step back an tries again with a progressively smaller step, until it no longer collides. This continues with the step size getting smaller and smaller. The algorithm stops when the step size is small enough for a good approximation and the angle does not cause a collision. The code chunk below carries out this process, printing the information for each pass.

The purpose of the `angle` and `previous_angle` parameters are fairly obvious. The `angle_delta` parameter is the value by which the angle is reduced at each step. `epsilon` is used to reduce `angle_delta` when there are collisions at an angle. Finally, `min_angle_delta` is one of the stopping criteria: when `angle_delta` gets below this value, the algorithm is sufficiently close to the correct answer and it stops trying new angles. *Thus, this parameter determines the precision of the algorithm.* It is set relatively high for now, because this first pass is just a demonstration and prints out the results of each iteration.

For efficiency, the while loop uses a memoised version of `run_bowl_simulation()` because when the balls collide, the previous step is tried again. Therefore, memoising the function saves some time instead of running the simulation from the same angle multiple times.

```
# The starting angle.
angle <- 90
previous_angle <- angle
# The "learning rate" paramerters.
angle_delta <- 10
epsilon <- 0.5
min_angle_delta <- 0.01
# Start with TRUE, though it doesn't matter.
collision <- TRUE
memo_bowl_sim <- memoise::memoise(run_bowl_simulation)
while (angle_delta >= min_angle_delta | collision) {
# Run the bowling simulation with the current angle.
sim_res <- memo_bowl_sim(angle = angle, step_size = 0.1)
# Were there any collisions?
collision <- any(sim_res$collision)
# Print results
msg <- "collision: {ifelse(collision, symbol$cross, symbol$tick)}" %>%
paste("{collision},") %>%
paste("angle: {round(angle, 4)},") %>%
paste("angle_delta: {round(angle_delta, 4)}")
print(glue(msg))
if (!collision) {
# Reduce the angle if there is no collision.
previous_angle <- angle
angle <- angle - angle_delta
} else {
# Revert to the previous angle and reduce delta if there is a collision.
angle_delta <- epsilon * angle_delta
angle <- previous_angle
}
}
```

```
#> collision: ✓ FALSE, angle: 90, angle_delta: 10
#> collision: ✓ FALSE, angle: 80, angle_delta: 10
#> collision: ✓ FALSE, angle: 70, angle_delta: 10
#> collision: ✓ FALSE, angle: 60, angle_delta: 10
#> collision: ✕ TRUE, angle: 50, angle_delta: 10
#> collision: ✓ FALSE, angle: 60, angle_delta: 5
#> collision: ✓ FALSE, angle: 55, angle_delta: 5
#> collision: ✕ TRUE, angle: 50, angle_delta: 5
#> collision: ✓ FALSE, angle: 55, angle_delta: 2.5
#> collision: ✕ TRUE, angle: 52.5, angle_delta: 2.5
#> collision: ✓ FALSE, angle: 55, angle_delta: 1.25
#> collision: ✓ FALSE, angle: 53.75, angle_delta: 1.25
#> collision: ✕ TRUE, angle: 52.5, angle_delta: 1.25
#> collision: ✓ FALSE, angle: 53.75, angle_delta: 0.625
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.625
#> collision: ✓ FALSE, angle: 53.75, angle_delta: 0.3125
#> collision: ✓ FALSE, angle: 53.4375, angle_delta: 0.3125
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.3125
#> collision: ✓ FALSE, angle: 53.4375, angle_delta: 0.1562
#> collision: ✓ FALSE, angle: 53.2812, angle_delta: 0.1562
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.1562
#> collision: ✓ FALSE, angle: 53.2812, angle_delta: 0.0781
```

```
#> collision: ✓ FALSE, angle: 53.2031, angle_delta: 0.0781
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.0781
#> collision: ✓ FALSE, angle: 53.2031, angle_delta: 0.0391
#> collision: ✓ FALSE, angle: 53.1641, angle_delta: 0.0391
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.0391
#> collision: ✓ FALSE, angle: 53.1641, angle_delta: 0.0195
#> collision: ✓ FALSE, angle: 53.1445, angle_delta: 0.0195
#> collision: ✕ TRUE, angle: 53.125, angle_delta: 0.0195
#> collision: ✓ FALSE, angle: 53.1445, angle_delta: 0.0098
```

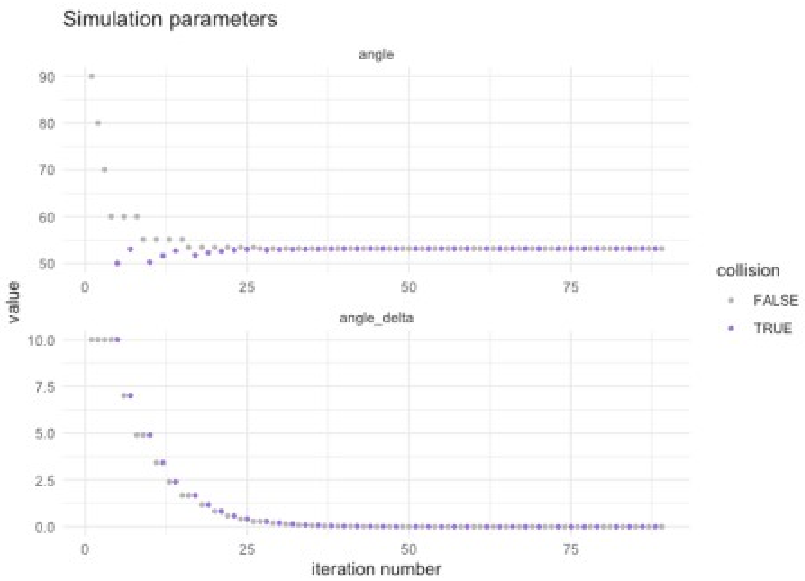
From the print-out above, we can see how the algorithm jumps back an forth, narrowing in on a solution around 53 degrees.

With that successful proof-of-concept, the following code runs the algorithm with a smaller `min_angle_delta = 1e-5` to achieve greater precision. Instead of printing out the results of each iteration, the simulation results and parameters are saved to `sim_results_tracker` and `sim_parameters_tracker`, respectively, and are inspected below.

```
angle <- 90
previous_angle <- angle
angle_delta <- 10
epsilon <- 0.7
min_angle_delta <- 1e-5
collision <- TRUE
sim_results_tracker <- tibble()
sim_parameters_tracker <- tibble()
memo_bowl_sim <- memoise::memoise(run_bowl_simulation)
while (angle_delta >= min_angle_delta | collision) {
sim_res <- memo_bowl_sim(angle = angle, step_size = 0.01)
collision <- any(sim_res$collision)
sim_results_tracker <- bind_rows(sim_results_tracker,
sim_res %>% add_column(angle = angle))
sim_parameters_tracker <- bind_rows(sim_parameters_tracker,
tibble(angle, angle_delta,
collision, epsilon))
if (!collision) {
previous_angle <- angle
angle <- angle - angle_delta
} else {
angle_delta <- epsilon * angle_delta
angle <- previous_angle
}
}
```

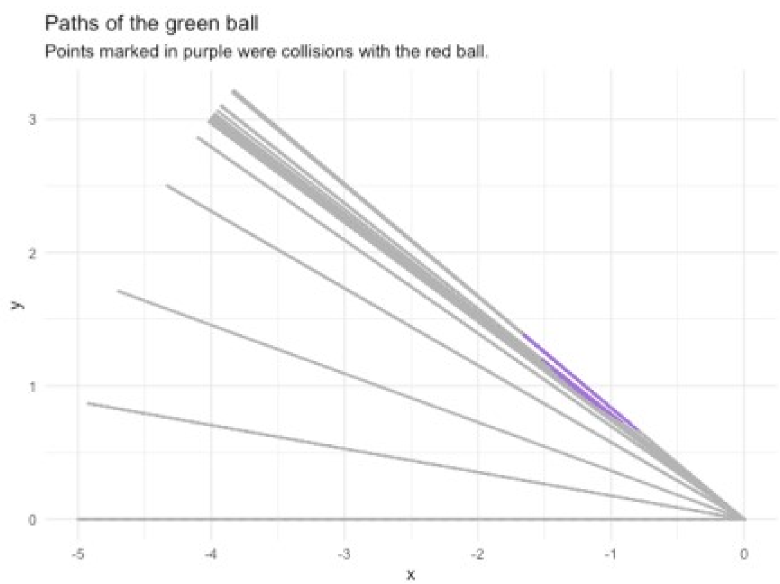
The simulation took 89 steps. The plot below shows the angle and `angle_delta` at each step, colored by whether there was a collision or not.

```
sim_parameters_tracker %>%
mutate(row_idx = row_number()) %>%
pivot_longer(-c(row_idx, epsilon, collision)) %>%
ggplot(aes(x = row_idx, y = value)) +
facet_wrap(~ name, nrow = 2, scales = "free") +
geom_point(aes(color = collision), size = 0.9) +
scale_color_manual(values = c(light_grey, purple)) +
labs(x = "iteration number",
y = "value",
title = "Simulation parameters")
```



The following plot shows each of the paths tried, again, coloring the locations of collisions in purple.

```
sim_results_tracker %>%
mutate(collision = ifelse(collision, "collision", "safe")) %>%
ggplot() +
geom_point(aes(x = x, y = y, color = collision),
size = 0.1) +
scale_color_manual(values = c(collision = purple,
safe = light_grey)) +
coord_fixed() +
theme(legend.position = "none") +
labs(x = "x",
y = "y",
title = "Paths of the green ball",
subtitle = "Points marked in purple were collisions with the red ball.")
```



Finally, we can find the approximated angle by taking the smallest angle tried in the rounds of simulation that did not have any collisions.

```
smallest_angle <- sim_parameters_tracker %>%
filter(collision == FALSE) %>%
top_n(1, wt = -angle) %>%
```

```
pull(angle) %>%
unique()
```

The algorithm approximates the solution to be: 53.1301 degrees (0.9273 in radians).

The simulation with this angle is shown in an animated plot below.

```
final_result <- sim_results_tracker %>%
filter(angle == smallest_angle) %>%
mutate(row_idx = row_number()) %>%
filter(row_idx == 1)
bind_rows(
final_result,
final_result %>%
mutate(x = -1 * x, y = -1 * y)
) %>%
mutate(row_idx = row_number()) %>%
ggplot() +
geom_point(aes(x = x, y = y),
color = green, size = 2) +
geom_circle(aes(x0 = x, y0 = y, r = 1),
fill = green, alpha = 0.2, size = 0) +
geom_point(aes(x, y),
data = tibble(x = 0, y = 2.5),
color = red, size = 2) +
geom_circle(aes(x0 = x, y0 = y, r = r),
data = tibble(x = 0, y = 2.5, r = 1),
fill = red, alpha = 0.2, size = 0) +
geom_point(aes(x, y),
data = tibble(x = 0, y = -2.5),
color = red, size = 2) +
geom_circle(aes(x0 = x, y0 = y, r = r),
data = tibble(x = 0, y = -2.5, r = 1),
fill = red, alpha = 0.2, size = 0) +
coord_fixed() +
labs(
x = "x",
y = "y",
title = glue(
"The tightest angle of the perfect bowl: {round(smallest_angle, 3)} deg."
)) +
transition_states(row_idx, transition_length = 2,
state_length = 0, wrap = FALSE) +
ease_aes("sine-in-out")
```

The tightest angle of the perfect bowl: 53.13 deg.