```
knitr::opts_chunk$set(echo = TRUE, comment = "#>")
library(glue)
library(tidygraph)
library(ggraph)
library(tidyverse)
theme_set(theme_minimal())
```

Simulation method

The first method I tried was to use a simulation to find the path from the blank space to the final space.

I abstracted the chessboard as a matrix with 0 as empty space, 1 as a taken space, and 2 as the knight.

```
chessboard <- matrix(c(
1, 1, 1, 0, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 1, 1, 1, 1, 1, 1,
1, 2, 1, 1, 1, 1, 1, 1
), nrow = 8, byrow = TRUE)
chessboard
```

```
#> [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#> [1,] 1 1 1 0 1 1 1 1
#> [2,] 1 1 1 1 1 1 1 1
#> [3,] 0 0 0 0 0 0 0 0
#> [4,] 0 0 0 0 0 0 0 0
#> [5,] 0 0 0 0 0 0 0 0
#> [6,] 0 0 0 0 0 0 0 0
#> [7,] 1 1 1 1 1 1 1 1
#> [8,] 1 2 1 1 1 1 1 1
```

I then created a bunch of functions that take care of different parts of the algorithm.

```
# Return the current location of the knight on the chessboard `mat`.
get_knight_location <- function(mat) {
knight_row <- which(apply(mat, 1, function(x) any(x == 2)))
knight_col <- which(apply(mat, 2, function(x) any(x == 2)))
return(list(x = knight_col, y = knight_row))
}
get_knight_location(chessboard)
```

```
#> $x
#> [1] 2
#>
#> $y
#> [1] 8
```

```
# A helper for visiualizing the chessboard.
print_chessboard <- function(mat) {
new_mat <- mat
new_mat[new_mat == "0"] <- " "
```

```
new_mat[new_mat == "1"] <- "+"
new_mat[new_mat == "2"] <- "H"
new_mat[1, 4] <- "o"
print(new_mat)
invisible(NULL)
}
print(chessboard)
```

```
#> [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#> [1,] 1 1 1 0 1 1 1 1
#> [2,] 1 1 1 1 1 1 1 1
#> [3,] 0 0 0 0 0 0 0 0
#> [4,] 0 0 0 0 0 0 0 0
#> [5,] 0 0 0 0 0 0 0 0
#> [6,] 0 0 0 0 0 0 0 0
#> [7,] 1 1 1 1 1 1 1 1
#> [8,] 1 2 1 1 1 1 1 1
```

```
# Movement: (horizontal movement, vertical movement)
possible_knight_movements <- rbind(
expand.grid(c(1, -1), c(2, -2)),
expand.grid(c(2, -2), c(1, -1))
) %>%
as_tibble() %>%
set_names(c("x", "y"))
possible_knight_movements
```

```
#> # A tibble: 8 x 2
#> x y
#>
#> 1 1 2
#> 2 -1 2
#> 3 1 -2
#> 4 -1 -2
#> 5 2 1
#> 6 -2 1
#> 7 2 -1
#> 8 -2 -1
```

One optimization I added to the simulation to help it preform better than a purely random walk was to prevent it from retracing its steps. This was achieved by adding a check in `is_available_move()` to prevent it from returning to the previous step. (As we see with the final solution, this wasn't really necessary.)

```
# Select a random move for the knight.
get_random_movement <- function() {
sample_n(possible_knight_movements, 1)
}
# Get the new location of the knight after a move.
get_new_location <- function(movement, current_loc) {
new_x_loc <- movement$x + current_loc$x
new_y_loc <- movement$y + current_loc$y
return(list(x = new_x_loc, y = new_y_loc))
}
# Move the knight on the board.
move_knight_to_new_location <- function(movement, mat) {
current_loc <- get_knight_location(mat)
new_loc <- get_new_location(movement, current_loc)
```

```
new_mat <- mat
new_mat[current_loc$y, current_loc$x] <- 0
new_mat[new_loc$y, new_loc$x] <- 2
return(new_mat)
}
previous_location <- get_knight_location(chessboard)
# Is the new position on a board possible or available.
# i.e. can the knight make the `movement` on the `mat`.
# This function "remembers" the previous location and will not let the knight
# move backwards. Because this is reset at the beginning, the knight won't get
# trapped in a corder forever, just one round.
is_available_move <- function(movement, mat) {
current_loc <- get_knight_location(mat)
new_loc <- get_new_location(movement, current_loc)
# Check that the piece stays on the board.
if (new_loc$x < 1 | new_loc$x > ncol(chessboard)) {
return(FALSE)
} else if (new_loc$y < 1 | new_loc$y > nrow(chessboard)) {
return(FALSE)
}
# Check if the new location would be the same as the previous location.
if (new_loc$x == previous_location$x & new_loc$y == previous_location$y) {
return(FALSE)
}
# Check the new space is not already taken.
if (mat[new_loc$y, new_loc$x] == 1) {
return(FALSE)
}
previous_location <<- current_loc
TRUE
}
# Move the knight one time randomly, but legally.
move_knight <- function(mat) {
old_loc <- get_knight_location(mat)
movement <- get_random_movement()
while(!is_available_move(movement, mat)) {
movement <- get_random_movement()
}
move_knight_to_new_location(movement, mat)
}
```

The function to play a round just calls `move_knight()` 8 times on the same chessboard. It returns a tibble with the locations of the knight during the process.

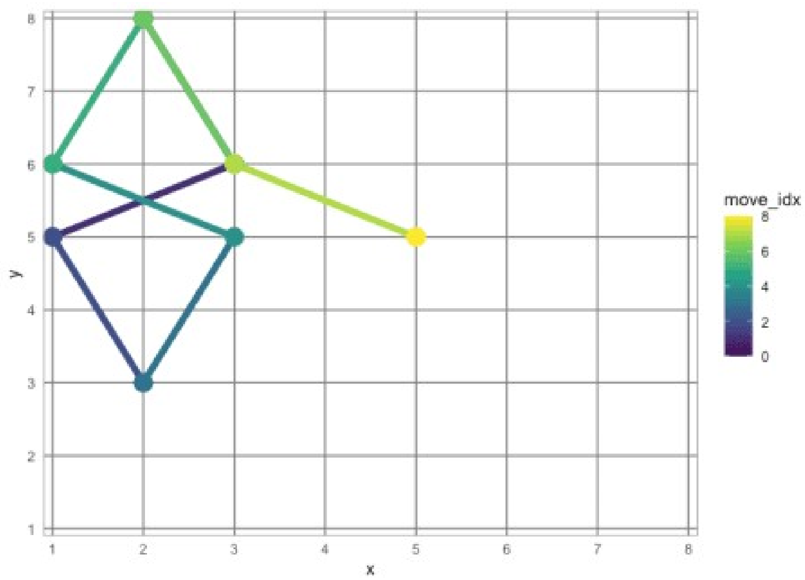
```
# Return a tidy tibble of the knights locations.
knight_location_tidy <- function(l) {
enframe(l, name = "move_idx") %>%
mutate(x = map_dbl(value, ~ .x[[1]]),
y = map_dbl(value, ~ .x[[2]])) %>%
select(move_idx, x, y) %>%
mutate(move_idx = move_idx - 1)
}
# Play a round of the simulation.
play_round <- function(num_moves = 8) {
gameboard <- chessboard
knight_locs <- rep(NA, num_moves + 1)
knight_locs[1] <- list(get_knight_location(gameboard))
```

```
for (i in seq(1, num_moves)) {
gameboard <- move_knight(gameboard)
knight_locs[i + 1] <- list(get_knight_location(gameboard))
}
return(knight_location_tidy(knight_locs))
}
play_round()
```

```
#> # A tibble: 9 x 3
#> move_idx x y
#>
#> 1 0 2 8
#> 2 1 3 6
#> 3 2 1 5
#> 4 3 3 4
#> 5 4 4 6
#> 6 5 5 4
#> 7 6 3 3
#> 8 7 2 5
#> 9 8 1 3
```

I also added a simple function to plot the path of the knight. Each step is labeled with its place in the sequence.

```
# A visualization tool for the path of the knight.
plot_knight_locations <- function(df) {
df %>%
ggplot(aes(x = x, y = y, color = move_idx)) +
geom_path(aes(group = game_idx), size = 2) +
geom_point(size = 5) +
scale_x_continuous(limits = c(1, 8),
expand = expansion(add = c(0.1, 0.1)),
breaks = 1:8) +
scale_y_continuous(limits = c(1, 8),
expand = expansion(add = c(0.1, 0.1)),
breaks = 1:8) +
scale_color_viridis_c(breaks = seq(0, 8, 2)) +
theme(
panel.grid.major = element_line(color = "grey50", size = 0.5),
panel.grid.minor = element_blank(),
panel.border = element_rect(fill = NA, color = "grey50")
)
}
play_round() %>%
add_column(game_idx = 1) %>%
plot_knight_locations()
```



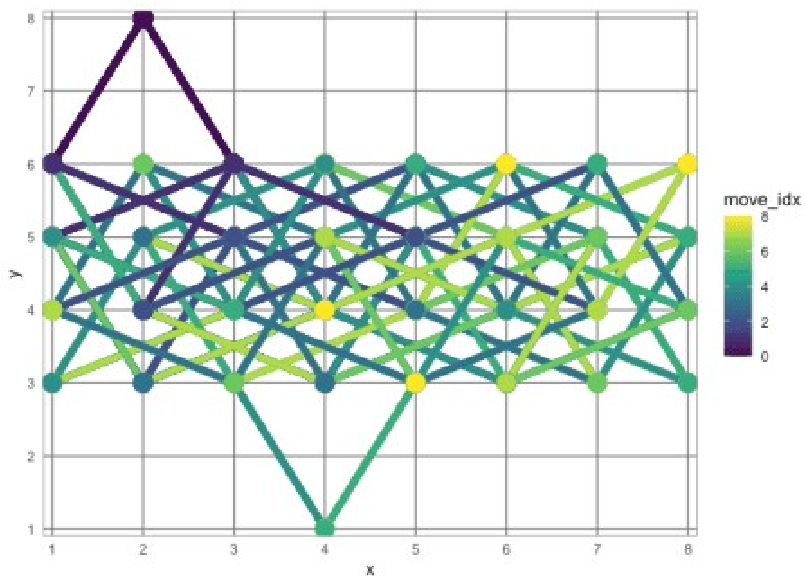
Finally, we can play the game many times until a solution is found.

```
# `TRUE` is returned if the riddle was solved.
finished_riddle <- function(df) {
last_loc <- df %>% slice(nrow(df))
if (last_loc$x == 4 & last_loc$y == 1) {
return(TRUE)
} else {
return(FALSE)
}
}

set.seed(0)
n_max <- 5e2
all_games <- rep(NA, n_max)
for (i in seq(1, n_max)) {
moves <- play_round()
all_games[i] <- list(moves)
if (finished_riddle(moves)) {
print("RIDDLE SOLVED!")
break
}
}
```

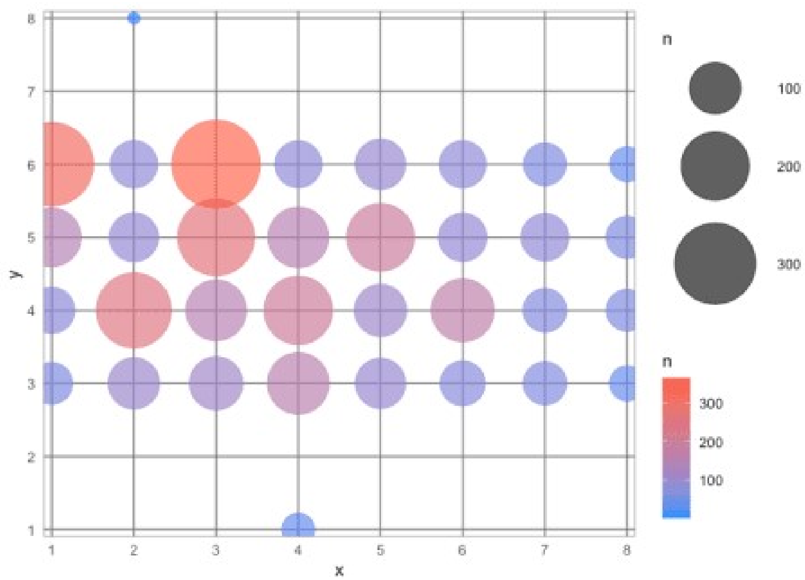
Interestingly, the desired end point, `(4, 1)`, was reached, just not at the end of the path.

```
bind_rows(all_games, .id = "game_idx") %>%
plot_knight_locations()
```



We can look at the most visited locations (ignoring the starting location).

```
bind_rows(all_games, .id = "game_idx") %>%
count(x, y) %>%
mutate(n = ifelse(x == 2 & y == 8, 1, n)) %>%
ggplot(aes(x = x, y = y, color = n)) +
geom_point(aes(size = n), alpha = 0.7) +
scale_x_continuous(limits = c(1, 8),
expand = expansion(add = c(0.1, 0.1)),
breaks = 1:8) +
scale_y_continuous(limits = c(1, 8),
expand = expansion(add = c(0.1, 0.1)),
breaks = 1:8) +
scale_color_gradient(low = "dodgerblue", high = "tomato") +
scale_size_continuous(range = c(3, 25)) +
theme(
panel.grid.major = element_line(color = "grey50", size = 0.5),
panel.grid.minor = element_blank(),
panel.border = element_rect(fill = NA, color = "grey50")
)
```



After a lot of simulations (only 500 shown above, but I also tried

10,000), no solution was found. I was inspired by the visualization to try a graph-based approach.

Graph method

I can build a graph of all the possible paths of the knight given the state of the board and then find the path between the start and end that is 8 steps long.

The graph building process is a bit complicated, but it follows the basic algorithm outlined below:

1. Start from a seed location (`(2, 8)` at the beginning).
2. Find all possible next locations for the knight.
3. Of these locations, add the new ones to a record of visted locations (`position_table`).
4. Add to the edge list (`edge_list`) a link between the parent `(x,y)` to these next positions.
5. For the nodes that have not yet been visited, repeat this algorithm for each.

```
# A table to track where the algorithm has been already.
position_table <- tibble(x = 2, y = 8)
# An edge list for the graph.
edge_list <- tibble()
# A tibble with the possible x and y changes of position for the knight.
possible_knight_changes <- possible_knight_movements %>%
set_names(c("change_x", "change_y"))
# Is the position allowed on the chessboard?
position_is_allowed <- function(x, y) {
if (x > 8 | x < 1 | y > 8 | y < 1) {
return(FALSE)
} else if (chessboard[y, x] != 0) {
return(FALSE)
}
TRUE
}
# A tibble of the next possible locations for the knight.
possible_next_positions <- function(x, y) {
rep(list(tibble(x = x, y = y)),
nrow(possible_knight_changes)) %>%
bind_rows() %>%
bind_cols(possible_knight_changes) %>%
mutate(x = x + change_x,
y = y + change_y,
is_legal = map2_lgl(x, y, position_is_allowed)) %>%
filter(is_legal) %>%
select(x, y)
}
# Build the graphs starting from a seed x and y position.
get_knight_edges <- function(x, y) {
df <- possible_next_positions(x, y)
# Add the new edges to the edge list.
edge_list <<- bind_rows(
edge_list,
tibble(from = paste0(x, ",", y),
to = paste0(df$x, ",", df$y))
)
```

```
# Remove positions already recorded.
df <- df %>% anti_join(position_table, b = c("x", "y"))
if (nrow(df) != 0) {
position_table <<- bind_rows(position_table, df)
for (i in 1:nrow(df)) {
get_knight_edges(df$x[[i]], df$y[[i]])
}
}
invisible(NULL)
}
get_knight_edges(2, 8)
edge_list
```

```
#> # A tibble: 134 x 2
#> from to
#>
#> 1 2,8 3,6
#> 2 2,8 1,6
#> 3 3,6 4,4
#> 4 3,6 2,4
#> 5 3,6 5,5
#> 6 3,6 1,5
#> 7 4,4 5,6
#> 8 4,4 3,6
#> 9 4,4 6,5
#> 10 4,4 2,5
#> # … with 124 more rows
```

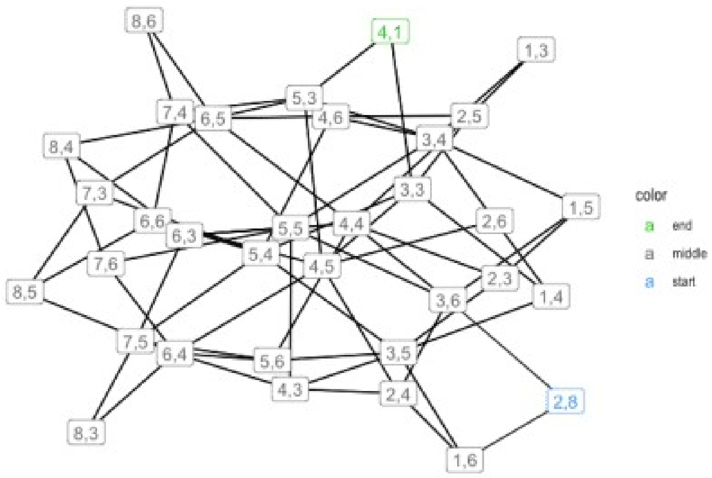
The edge list can be turned into a `tidygraph` from the 'tidygraph' library.

```
knight_graph <- as_tbl_graph(edge_list, directed = FALSE)
knight_graph
```

```
#> # A tbl_graph: 34 nodes and 134 edges
#> #
#> # An undirected multigraph with 1 component
#> #
#> # Node Data: 34 x 1 (active)
#> name
#>
#> 1 2,8
#> 2 3,6
#> 3 4,4
#> 4 5,6
#> 5 6,4
#> 6 7,6
#> # … with 28 more rows
#> #
#> # Edge Data: 134 x 2
#> from to
#>
#> 1 1 2
#> 2 1 34
#> 3 2 3
#> # … with 131 more rows
```

Here is a simple visualization of the graph.

```
knight_graph %N>%
mutate(color = case_when(name == "2,8" ~ "start",
name == "4,1" ~ "end",
TRUE ~ "middle")) %>%
ggraph(layout = "stress") +
geom_edge_link() +
geom_node_label(aes(label = name, color = color)) +
scale_color_manual(values = c("green3", "grey40", "dodgerblue")) +
theme_graph()
```



One possible way to find the path of 8 steps between the "start" and "end" would be to elucidate all the possible paths and then find those of length 8. This takes way too long, though, so I instead used a random walk method. However, I was still unable to find a solution after 1,000 random walks.

```
n_max <- 1e3
set.seed(0)
for (i in seq(1, n_max)) {
path <-igraph::random_walk(knight_graph,
start = "2,8",
steps = 9,
mode = "all")
if (names(path)[[9]] == "4,1") {
print("RIDDLE SOLVED!")
break
}
}
```

That means both of my methods have failed to find a solution to this Riddler…

Problem with my solving methods

The knight cannot travel from the original blank square to the final position. This is true because every time the knight moves, it goes from a black to a white square or a white to a black square. Thus, it is not possible for the knight in the bottom-left to travel from a white square to a black square in 8 moves. In 8 moves, it will always be on a white square again.

Solution

Thus the knight that killed the queen must have come from the bottom-right and the bottom-left knight took its place. We can solve the puzzle by just changing the original chessboard and re-running the simulations and graph search.

Simulation

If we change the chessboard and re-try the simulation method, it finds a solution easily.

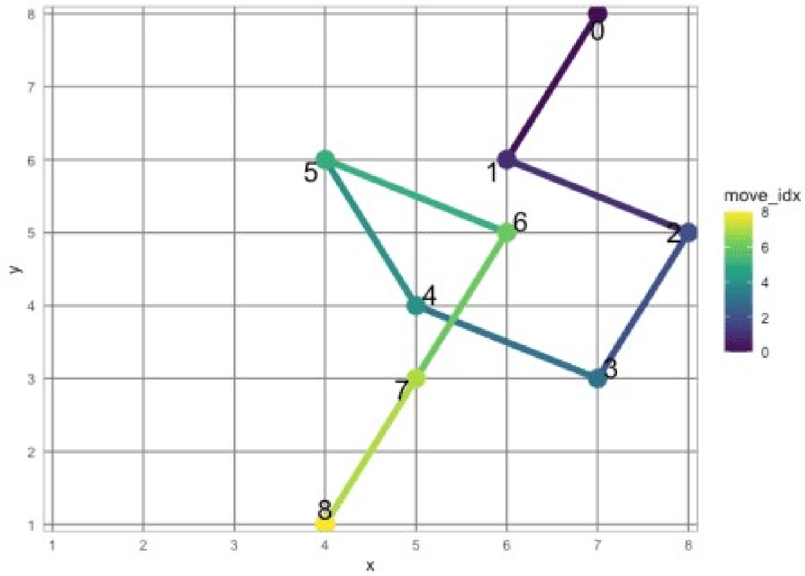
```
chessboard <- matrix(c(
1, 1, 1, 0, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 2, 1
), nrow = 8, byrow = TRUE)
chessboard
```

```
#> [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
#> [1,] 1 1 1 0 1 1 1 1
#> [2,] 1 1 1 1 1 1 1 1
#> [3,] 0 0 0 0 0 0 0 0
#> [4,] 0 0 0 0 0 0 0 0
#> [5,] 0 0 0 0 0 0 0 0
#> [6,] 0 0 0 0 0 0 0 0
#> [7,] 1 1 1 1 1 1 1 1
#> [8,] 1 1 1 1 1 1 2 1
```

```
previous_location <- get_knight_location(chessboard)
set.seed(0)
n_max <- 1e2
all_games <- rep(NA, n_max)
for (i in seq(1, n_max)) {
moves <- play_round()
all_games[i] <- list(moves)
if (finished_riddle(moves)) {
print("RIDDLE SOLVED!")
break
}
}
```

```
#> [1] "RIDDLE SOLVED!"
```

```
all_games <- all_games[!is.na(all_games)]
successful_game <- all_games[length(all_games)][[1]]
successful_game <- successful_game %>%
mutate(game_idx = 1)
p <- plot_knight_locations(successful_game)
p +
ggrepel::geom_text_repel(aes(label = move_idx),
color = "black", size = 6)
```



Graph

We can try the graph-based method again, too, and this time it finds a solution.

```
# A table to track where the algorithm has been already.
position_table <- tibble(x = 7, y = 8)
# An edge list for the graph.
edge_list <- tibble()
get_knight_edges(7, 8)
new_knight_graph <- as_tbl_graph(edge_list, directed = FALSE)
new_knight_graph
```

```
#> # A tbl_graph: 34 nodes and 134 edges
#> #
#> # An undirected multigraph with 1 component
#> #
#> # Node Data: 34 x 1 (active)
#> name
#>
#> 1 7,8
#> 2 8,6
#> 3 7,4
#> 4 5,5
#> 5 6,3
#> 6 7,5
#> # … with 28 more rows
#> #
#> # Edge Data: 134 x 2
#> from to
#>
#> 1 1 2
#> 2 1 34
#> 3 2 3
#> # … with 131 more rows
```

```
n_max <- 1e2
set.seed(0)
for (i in seq(1, n_max)) {
path <-igraph::random_walk(new_knight_graph,
start = "7,8",
```

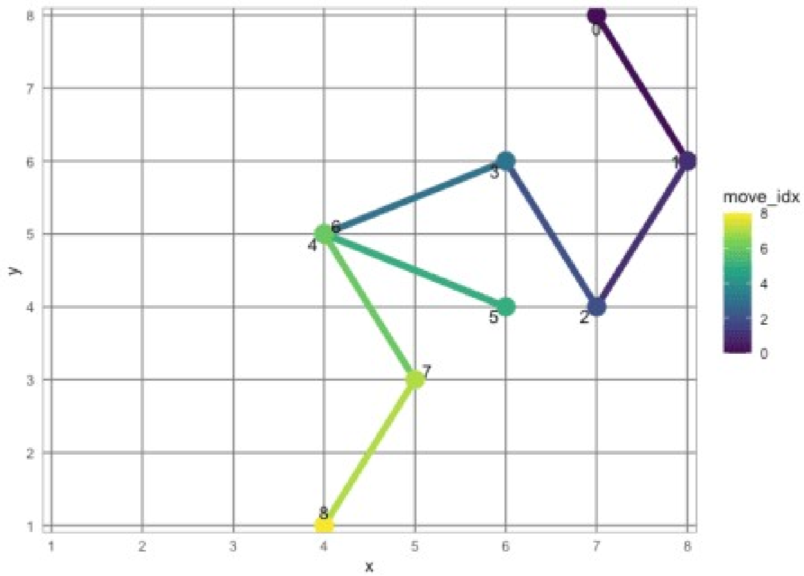
```
steps = 9,
mode = "all")
if (names(path)[[9]] == "4,1") {
print("RIDDLE SOLVED!")
break
}
}
```

```
#> [1] "RIDDLE SOLVED!"
```

```
print(path)
```

```
#> + 9/34 vertices, named, from d355afd:
#> [1] 7,8 8,6 7,4 6,6 4,5 6,4 4,5 5,3 4,1
```

```
p <- tibble(node = names(path)) %>%
mutate(x = as.numeric(str_extract(node, "^[:digit:]")),
y = as.numeric(str_extract(node, "[:digit:]$"))) %>%
mutate(move_idx = 1:n() - 1,
game_idx = 1) %>%
plot_knight_locations()
p +
ggrepel::geom_text_repel(aes(label = move_idx), color = "black")
```



It seems like there are actually a few different solutions. 34 different paths were found in 10,000 trials.

```
n_max <- 1e4
set.seed(0)
successful_paths <- c()
for (i in seq(1, n_max)) {
path <-igraph::random_walk(new_knight_graph,
start = "7,8",
steps = 9,
mode = "all")
if (names(path)[[9]] == "4,1") {
successful_paths <- c(successful_paths, path)
}
}
length(unique(successful_paths))
```

```
#> [1] 34
```