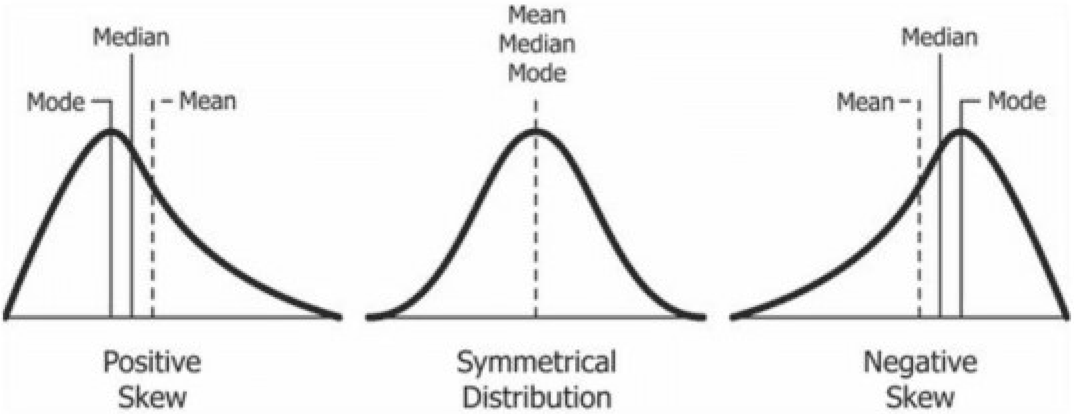
Skewness

The skewness is a measure of the asymmetry of the probability distribution assuming a unimodal distribution and is given by the third standardized moment.

We can say that the skewness indicates how much our underlying distribution deviates from the normal distribution since the normal distribution has skewness 0. Generally, we have three types of skewness.

- **Symmetrical**: When the skewness is close to 0 and the **mean** is almost the same as the **median**
- **Negative skew**: When the left tail of the histogram of the distribution is longer and the majority of the observations are concentrated on the right tail. In this case, we can use also the term “left-skewed” or “left-tailed”. and the median is greater than the mean.
- **Positive skew**: When the right tail of the histogram of the distribution is longer and the majority of the observations are concentrated on the left tail. In this case, we can use also the term “right-skewed” or “right-tailed”. and the median is less than the mean.

The graph below describes the three cases of skewness. Focus on the Mean and Median.



Wikipedia

Skewness formula

The skewness can be calculated from the following formula:

$skewness=\frac{\sum_{i=1}^{N}(x_i-\bar{x})^3}{(N-1)s^3}$

where:

- σ is the standard deviation
- $\bar{x }$ is the mean of the distribution
- N is the number of observations of the sample

Skewness values and interpretation

There are many different approaches to the interpretation of the skewness values. A rule of thumb states that:

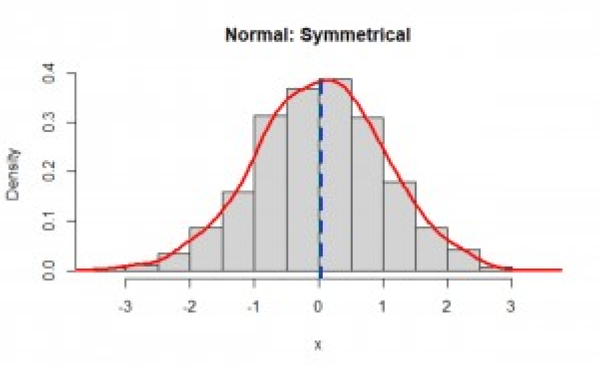
- **Symmetric**: Values between -0.5 to 0.5
- **Moderated Skewed data**: Values between **-1 and -0.5** or between **0.5 and 1**
- **Highly Skewed data**: Values **less than -1** or **greater than 1**

Skewness in Practice

Let’s calculate the skewness of three distribution. We will show three cases, such as a symmetrical one, and one positive and negative skew respectively.

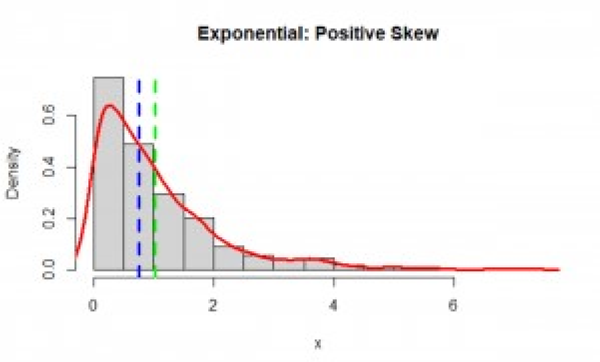
We know that the **normal** distribution is symmetrical.

```
set.seed(5)

# normal
x = rnorm(1000, 0,1)
hist(x, main="Normal: Symmetrical", freq=FALSE)
lines(density(x), col='red', lwd=3)
abline(v = c(mean(x),median(x)),  col=c("green", "blue"), lty=c(2,2), lwd=c(3
3))
```



The **exponential** distribution is positive skew:

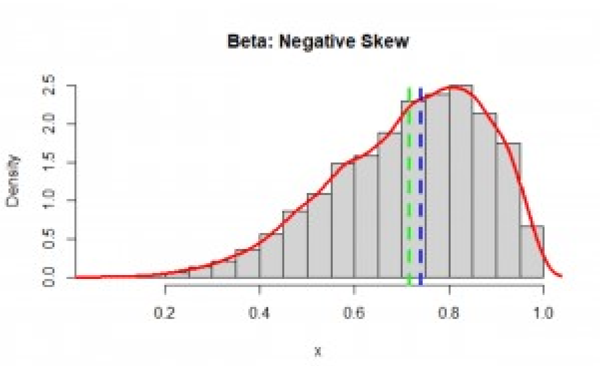
```
set.seed(5)
# exponential
x = rexp(1000,1)
hist(x, main="Exponential: Positive Skew", freq=FALSE)
lines(density(x), col='red', lwd=3)
abline(v = c(mean(x),median(x)),  col=c("green", "blue"), lty=c(2,2), lwd=c(3
3))
```



The **beta** distribution with hyper-parameters α=5 and β=2

```
set.seed(5)
# beta
```

```
x= rbeta(10000,5,2)
hist(x, main="Beta: Negative Skew", freq=FALSE)
lines(density(x), col='red', lwd=3)
abline(v = c(mean(x),median(x)),  col=c("green", "blue"), lty=c(2,2), lwd=c(3
3))
```



Notice that the **green** vertical line is the **mean** and the **blue** one is the **median**.

Let's see how we can calculate the skewness by applying the formula:

```
set.seed(5)
x= rbeta(10000,5,2)

sum((x-mean(x))^3)/((length(x)-1)*sd(x)^3)
```

We get:

```
3.085474
```

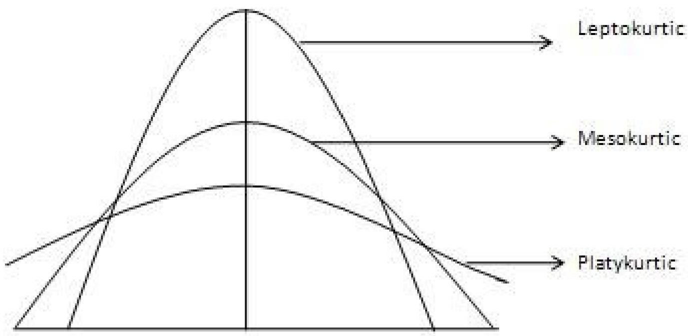
Notice that you can also calculate the skewness with the following packages:

```
library(moments)
moments::skewness(x)

# OR

library(e1071)
e1071::skewness(x)
```

There are some rounding differences between those two packages. Also at the e1071 the formula is without subtracting the 1from the (N-1).

Kurtosis

In statistics, we use the kurtosis measure to describe the "tailedness" of the distribution as it describes th shape of it. It is also a measure of the "peakedness" of the distribution. A high kurtosis distribution has a sharper peak and longer fatter tails, while a low kurtosis distribution has a more rounded pean and shorte thinner tails.



Tutorials Point

Let’s see the main three types of kurtosis.

- **Mesokurtic**: This is the normal distribution
- **Leptokurtic**: This distribution has fatter tails and a sharper peak. The kurtosis is “positive” with a value greater than 3
- **Platykurtic**: The distribution has a lower and wider peak and thinner tails. The kurtosis is “negative with a value greater than 3

Notice that we define the **excess kurtosis as kurtosis minus 3**

Kurtosis formula

The kurtosis can be derived from the following formula:

$kurtosis=\frac{\sum_{i=1}^{N}(x_i-\bar{x})^4}{(N-1)s^4}$

where:

- σ is the standard deviation
- $\bar{x}$ is the mean of the distribution
- N is the number of observations of the sample

Kurtosis interpretation

Kurtosis is the average of the standardized data raised to the fourth power. Any standardized values that are less than 1 (i.e., data within one standard deviation of the mean, where the “peak” would be), contribute virtually nothing to kurtosis, since raising a number that is less than 1 to the fourth power make it closer to zero. The only data values (observed or observable) that contribute to kurtosis in any meaningful way are those outside the region of the peak; i.e., the outliers. **Therefore, kurtosis measure outliers only; it measures nothing about the “peak”**.

Kurtosis in Practice

Let’s try to calculate the kurtosis of some cases:

Normal Distribution

```
set.seed(5)
# normal
x = rnorm(1000, 0,1)
sum((x-mean(x))^4)/((length(x)-1)*sd(x)^4)
```

```
[1] 3.058924
```

As expected we got a value close to 3!

Exponential distribution

```
set.seed(5)
# exponential
x = rexp(1000)
sum((x-mean(x))^4)/((length(x)-1)*sd(x)^4)
```

```
[1] 10.13425
```

As expected we get a positive excess kurtosis (i.e. greater than 3) since the distribution has a sharper peak.

Beta distribution

```
set.seed(5)
# beta
x = rbeta(1000,5,5)
sum((x-mean(x))^4)/((length(x)-1)*sd(x)^4)
```

```
[1] 2.634339
```

As expected we get a negative excess kurtosis (i.e. less than 3) since the distribution has a lower peak.

Notice that you can also calculate the kurtosis with the following packages:

```
library(moments)
moments::kurtosis(x)

# OR

library(e1071)
e1071::kurtosis(x)
```

Conclusion

We provided a brief explanation about two very important measures in statistics and we showed how we can calculate them in R.