```
head(gasoline_df)
# date gasoline
#1 2013-01-01 4.67
#2 2013-02-01 4.85
#3 2013-03-01 4.75
#4 2013-04-01 4.61
#5 2013-05-01 4.64
#6 2013-06-01 4.72
```

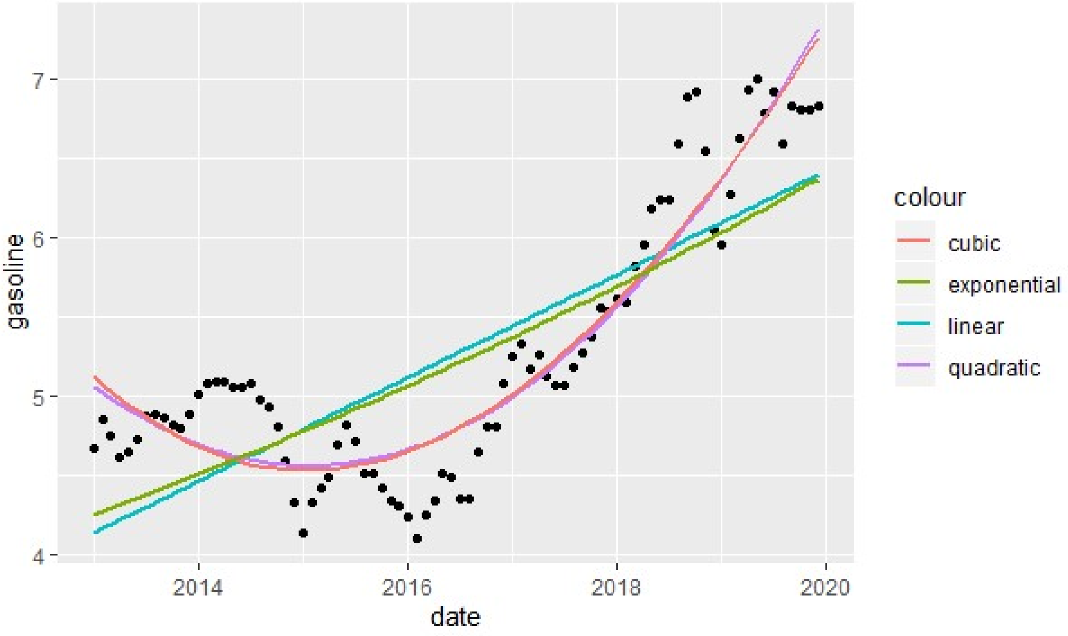
First of all, I wanted to see how gasoline prices went during the period and analyze the fitted trend lines.

```
#Trend lines
library(tidyverse)

ggplot(gasoline_df, aes(x = date, y = gasoline)) + geom_point() +
  stat_smooth(method = 'lm', aes(colour = 'linear'), se = FALSE) +
  stat_smooth(method = 'lm', formula = y ~ poly(x,2), aes(colour =
  stat_smooth(method = 'lm', formula = y ~ poly(x,3), aes(colour =
  stat_smooth(data=exp.model.df, method = 'loess',aes(x,y,colour =
```

Because the response variable is measured by logarithmic in the exponential model, we have to get the exponent of it and create a different data frame for the exponential trend line.

```
#exponential trend model
exp.model <- lm(log(gasoline)~date,data = gasoline_df)
exp.model.df <- data.frame(x=gasoline_df$date,
                           y=exp(fitted(exp.model)))
```



As we see above, the cubic and quadratic models almost overlap each other and they seem to be fit better to the data. However, we will analyze each model in detail.

The Forecasting Trend Models

The linear trend; y_t, the value of the series at given time, t, is described as:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

β_0 and β_1 are the coefficients.

```
model_linear <- lm(data = gasoline_df,gasoline~date)
```

Above, we created a model variable for the linear trend model. In order to compare the models, we have to extract the adjusted coefficients of determination, that is used to compare regression models with a different number of explanatory variables, from each trend models.

Adjusted $R^2 = 1 - (1 - R^2)(\frac{n-1}{n-k-1})$
n: sample size
k: number of variables

R^2: coefficient of determination which is the square of correlation of observation values with predicted values: $r^2_{y\hat{y}}$

```
adj_r_squared_linear <- summary(model_linear) %>%
  .$adj.r.squared %>%
  round(4)
```

The exponential trend; unlike the linear trend, allows the series to increase at an increasing rate in each period, is described as:

$\ln(y_t) = \beta_0 + \beta_1 t + \epsilon_t$. $\ln(y_t)$ is a natural logarithm of the response variable.

To make predictions on the fitted model, we use exponential function as $\hat{y}_t = exp(b_0 + b_1 t + s_e^2/2)$ because the dependent variable was transformed by a natural logarithmic function. In order for $\hat{y}_t$ not to be under the expected value of yt, we add half of the residual standard error's square. In order to find S_e, we execute the summary function as below. In addition, we can see the level of significance of the coefficients and the model. As we see below, because the p-value of the coefficients and the model are less than 0.05, they are significant at the %5 level of significance.

```
summary(model_exponential)

#Call:
#lm(formula = log(gasoline) ~ date, data = gasoline_df)

#Residuals:
#      Min        1Q    Median        3Q       Max
#-0.216180 -0.083077  0.009544  0.087953  0.151469

#Coefficients:
#              Estimate Std. Error t value Pr(<|t|)
#(Intercept) -1.0758581  0.2495848  -4.311  4.5e-05 ***
#date         0.0001606  0.0000147  10.928  < 2e-16 ***
#---
#Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

#Residual standard error: 0.09939 on 82 degrees of freedom
#Multiple R-squared:  0.5929,    Adjusted R-squared:  0.5879
#F-statistic: 119.4 on 1 and 82 DF,  p-value: < 2.2e-16
```

We create an exponential model variable.

```
model_exponential <- lm(data = gasoline_df,log(gasoline)~date)
```

To get adjusted R^2, we first transform the predicted variable.

```
library(purrr)

y_predicted <- model_exponential$fitted.values %>%
  map_dbl(~exp(.+0.09939^2/2))
```

And then, we execute the formula shown above to get adjusted R^2

```
r_squared_exponential <- (cor(y_predicted,gasoline_df$gasoline))^2

adj_r_squared_exponential <- 1-(1-r_squared_exponential)*
  ((nrow(gasoline_df)-1)/(nrow(gasoline_df)-1-1))
```

Sometimes a time-series changes the direction with respect to many reasons. This kind of target variable is fitted to **polynomial trend models**. If there is one change of direction we use **the quadratic trend model.**

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \epsilon$$

If there are two change of direction we use **the cubic trend models:**

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \epsilon$$

Then again, we execute the same process as we did in the linear model.

```
#Model variables
model_quadratic <- lm(data = df,gasoline~poly(date,2))
model_cubic <- lm(data = df,gasoline~poly(date,3))

#adjusted coefficients of determination
adj_r_squared_quadratic <- summary(model_quadratic) %>%
  .$adj.r.squared

adj_r_squared_cubic <- summary(model_cubic) %>%
  .$adj.r.squared
```

In order to compare all models, we create a variable that gathers all adjusted R^2.

```
adj_r_squared_all <- c(linear=round(adj_r_squred_linear,4),
                       exponential=round(adj_r_squared_exponential,4),
                       quadratic=round(adj_r_squared_quadratic,4),
                       cubic=round(adj_r_squared_cubic,4))
```

As we can see below, the results justify the graphic we built before; polynomial models are much better than the linear and exponential models. Despite polynomial models look almost the same, the quadratic model is slightly better than the cubic model.

```
adj_r_squared_all

      #linear exponential   quadratic       cubic
      #0.6064      0.6477      0.8660      0.8653
```

Seasonality

The seasonality component represents the repeats in a specific period of time. Time series with weekly monthly or quarterly observations tend to show seasonal variations that repeat every year. For example, the sale of retail goods increases every year in the Christmas period or the holiday tours increase in the summer. In order to detect seasonality, we use decomposition analysis.

Decomposition analysis: T, S, I, are trend, seasonality and random components of the series respectively. When seasonal variation increases as the time series increase, we'd use the multiplicative model.

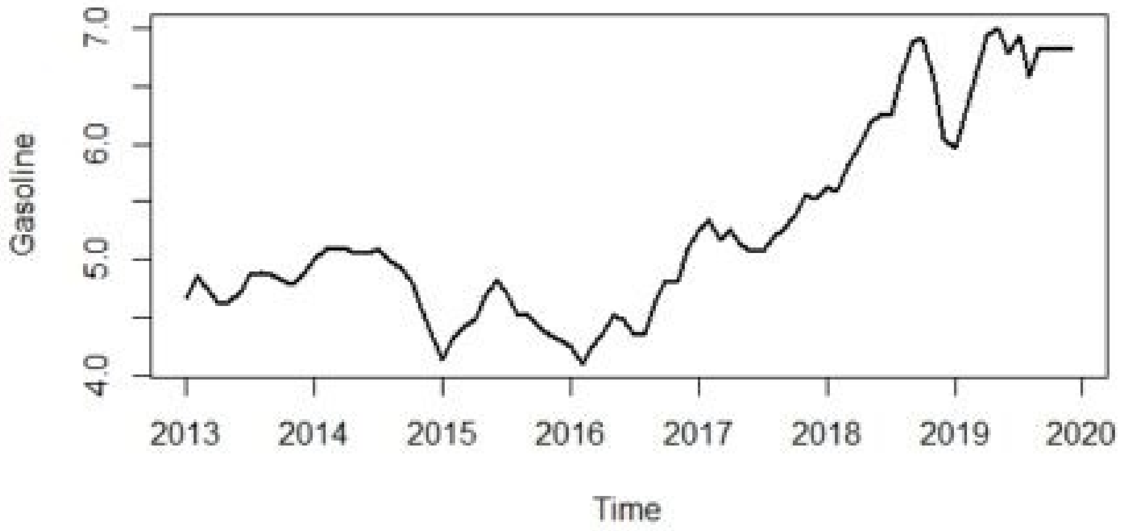
$$y_t = T_t \times S_t \times I_t$$

If the variation looks constant, we should use additive model.

$$y_t = T_t + S_t + I_t$$

To find which model is fit, we have to look at it on the graph.

```
#we create a time series variable for the plot
gasoline_ts <- ts(gasoline_df$gasoline,start = c(2013,1),end = c(2
#The plot have all the components of the series(T, S, I)
plot(gasoline_ts,lwd=2,ylab="Gasoline")
```



As we can see from the plot above, it seems that the multiplicative model would be more fit for the data; especially if we look at between 2016-2019. When we analyze the graph, we see some **conjuncture variations** caused by the expansion or contraction of the economy. Unlike seasonality, conjuncture fluctuations can be several months or years in length.

Additionally, the magnitude of conjuncture fluctuation can change in time because the fluctuations differ in magnitude and length so they are hard to reflect with historical data; because of that, we neglected the conjuncture component of the series.

Extracting the seasonality

Moving averages (**ma**) usually be used to separate the effect of a trend from seasonality.

$$m\ term\ moving\ averages = \frac{Average\ of\ the\ last\ m\ observations}{m}$$

We use the cumulative moving average (**CMA**), which is the average of two consecutive averages, to show the even-order moving average. For example; first two ma terms are $\bar{y}_{6.5}$ and $\bar{y}_{7.5}$ but there are no such terms in the original series; therefore we average the two terms to find CMA term matched up in the series:

$$\bar{y}_7 = \frac{\bar{y}_{6,5}+\bar{y}_{7,5}}{2}$$

The default value of the 'centre' argument of the ma function remains TRUE to get CMA value.

```
library(forecast)

gasoline_trend <- forecast::ma(gasoline_ts,12)
```

If noticed, $\bar{y}_t$ eliminates both seasonal and random variations; hence: $\bar{y}_t = T_t$

Since $y_t = T_t \times S_t \times I_t$ and $\bar{y}_t = T_t$, the detrend variable is found by: $\frac{\bar{y}_t}{y_t} = S_t \times I_t$; it is also called **ratio-to moving averages method**.

```r
gasoline_detrend <- gasoline_ts/gasoline_trend
```

Each month has multiple ratios, where each ratio coincides with a different year; in this sample, each month has seven different ratios. The arithmetic average is used to determine common value for each month; by doing this, we eliminate the random component and subtract the seasonality from the detrend variable. It is called the **unadjusted seasonal index.**

```r
unadjusted_seasonality <- sapply(1:12, function(x) mean(window(gas
c(2013,x), c(2019,12),deltat=1), na.rm = TRUE)) %>%
round(4)
```

Seasonal indexes for monthly data should be completed to 12, with an average of 1; in order to do that each unadjusted seasonal index is multiplied by 12 and divided by the sum of 12 unadjusted seasonal indexes.

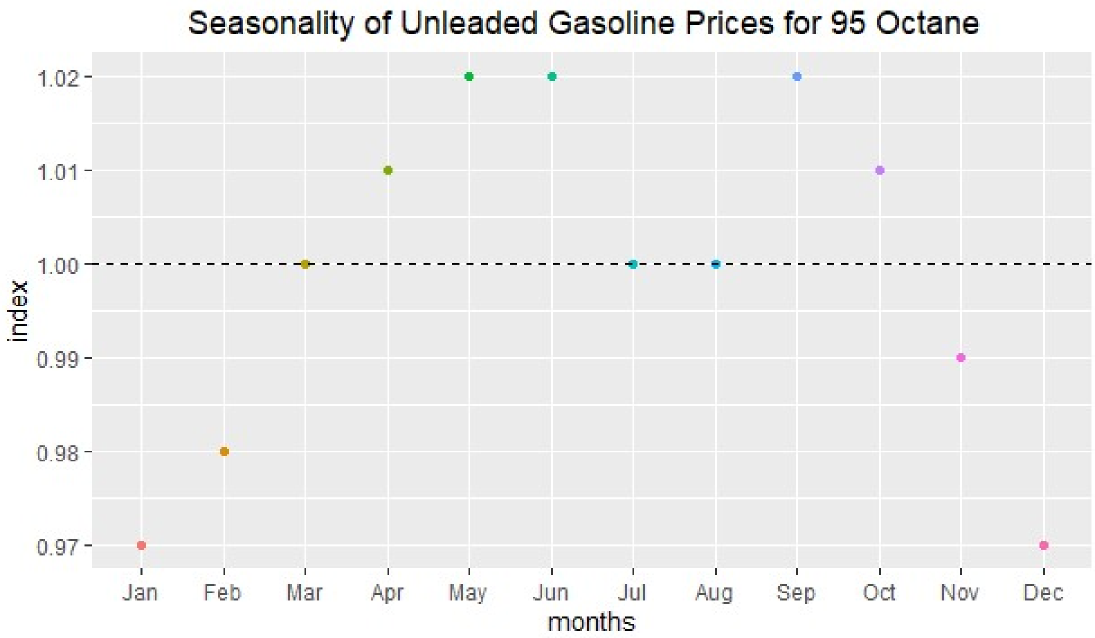
```r
adjusted_seasonality <- (unadjusted_seasonality*(12/sum(unadjustec
                        round(4)
```

The average of all adjusted seasonal indices is 1; if an index is equal to 1, that means there would be no seasonality. In order to plot seasonality:

```r
#building a data frame to plot in ggplot
adjusted_seasonality_df <- data_frame(
  months=month.abb,
  index=adjusted_seasonality)

#converting char month names to factor sequentially
adjusted_seasonality_df$months <- factor(adjusted_seasonality_df$

ggplot(data = adjusted_seasonality_df,mapping = aes(x=months,y=ir
 geom_point(aes(colour=months))+
 geom_hline(yintercept=1,linetype="dashed")+
 theme(legend.position = "none")+
 ggtitle("Seasonality of Unleaded Gasoline Prices for 95 Octane")
 theme(plot.title = element_text(h=0.5))
```

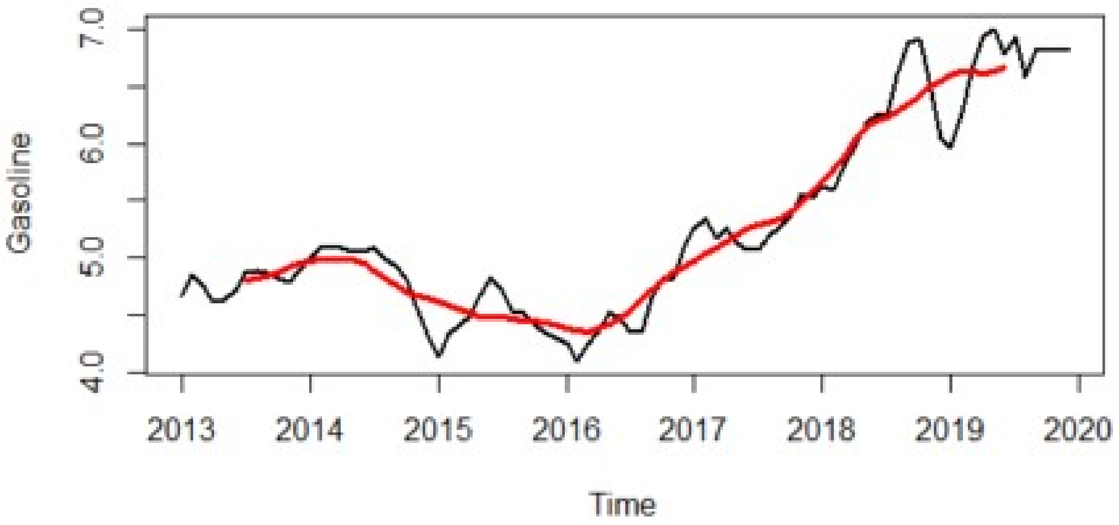


As we can see above, there is approximately a three-percent seasonal decrease in January and December; on the other hand, there is a two-percent seasonal increase in May and June. Seasonality is not seen in March, July, and August; because their index values are approximately equal to 1.

Decomposing the time series graphically

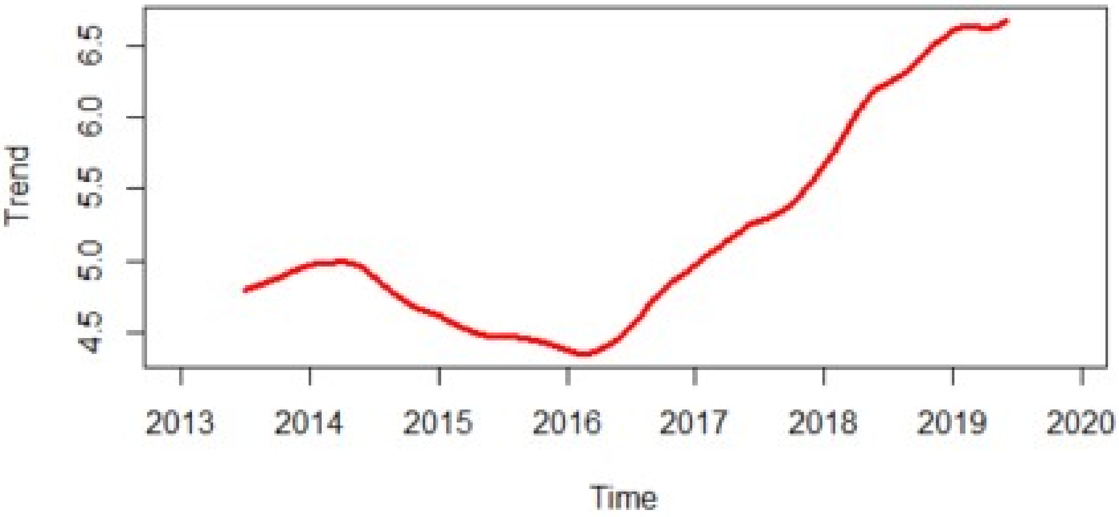
We will first show the trend line on the time series.

```
#Trend is shown by red line
plot(gasoline_ts,lwd=2,ylab="Gasoline")+
lines(gasoline_trend,col="red",lwd=3)
```



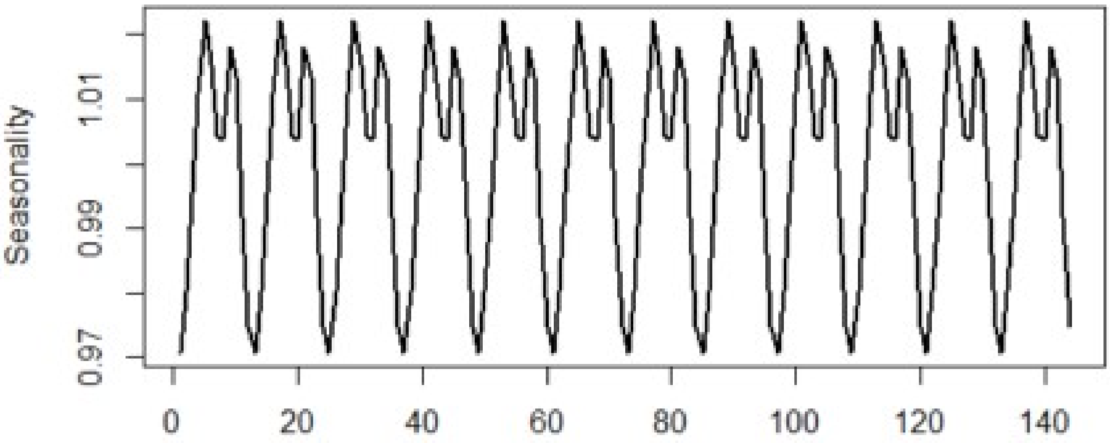
And will isolate the trend line from the plot:

```
plot(gasoline_trend,lwd=3,col="red",ylab="Trend")
```



Let’s see the seasonality line:

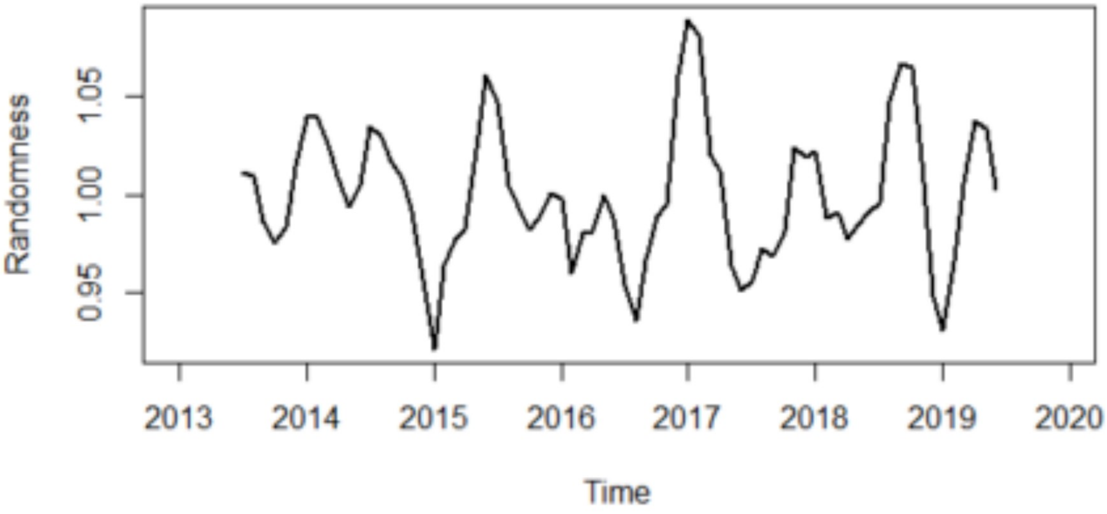
```
plot(as.ts(rep(unadjusted_seasonality,12)),lwd=2,ylab="Seasonality
```



And finally, we will show the randomness line; in order to do that:

$I_t = \frac{y_t}{S_t \times T_t}$

```r
randomness <- gasoline_ts/(gasoline_trend*unadjusted_seasonality)
plot(randomness,ylab="Randomness",lwd=2)
```


Randomness
Time