…The workhorse of Machine Learning is *Gradient Descent*. If you want to understand how and why it works and, along the way, want to learn how to plot and animate 3D-functions in R read on!

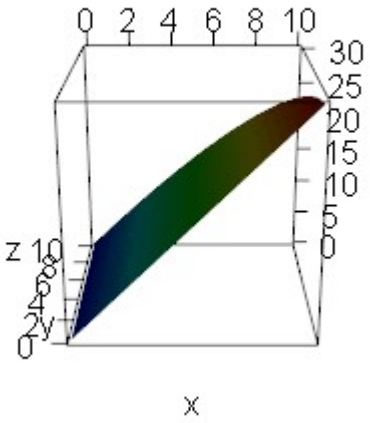
Gradient Descent is a mathematical algorithm to optimize functions, i.e. finding their minima or maxima. In Machine Learning it is used to *minimize* the *cost function* of many learning algorithms, e.g. artificial neural networks a.k.a. deep learning. The cost function simply is the function that measures how good a set of predictions is compared to the actual values (e.g. in regression problems).

The gradient (technically the *negative* gradient) is the direction of steepest descent. Just imagine a skier standing on top of a hill: the direction which points into the direction of steepest descent is the gradient!

Mathematically the gradient "stores" all the partial derivative information of a multivariable function, basically the slopes with respect to the directions of all axes. So, the Gradient Descent-algorithm always moves in the direction of the gradient to reach the minimum of the function (NB: in some cases unfortunately only a *local minimum*!), like our skier always taking the steepest route to reach the valley as fast as possible!

The question is: why does a bundle of partial derivatives point in the direction of the steepest descent? I answered this question some time ago here: Math.SE: Why is gradient the direction of steepest ascent?

> To give some intuition why the gradient (technically the negative gradient) has to point in the direction of steepest descent I created the following animation.
>
> It shows all of the points that can be reached by a vector of a given length and two variables x and y that are multiplied by a constant and summed up to give a very simple linear function (which give very simple directional derivatives).
>
> I then vary the constants relative to each other: when the constant of x goes up (down) the constant of y goes down (up). The red area equals the highest point which means that you have the steepest descent from there.
>
> As can be seen, this point varies smoothly with the proportion of the constants which represent the derivatives in each direction!
>
> Only when one constant equals zero do we have a corner solution, when both constants are the same the red area is exactly in the middle. There is no good reason why the red area (= steepest descent) should jump around between those points.
>
> This means that the gradient will always point in the direction of the steepest descent (nb: which is of course not a proof but a hand-waving indication of its behaviour to give some intuition only!)



I created the animation with the `rgl` package (on CRAN) and *ImageMagick* (see aso this post: Creating a Movie with Data from Outer Space in R).

The following fully documented code can be taken as a template to create plots and animations of 3D-functions with constraints (or inequalities). `f` is defined as follows:

$$f(x,y) = (3-n) * x + n * y \; with \; \sqrt{x^2+y^2} < 10,$$

where n is varied between 0 and 3.

```
library(rgl)
# create many x and y pairs as input of function
x <- y <- seq(0, 10, 0.01)

# define 3D-function with constraint
f <- function(x, y) {
  ifelse(sqrt(x^2 + y^2) < 10, (3-n)*x + n*y, NA)
}

# for rainbow colouring the function
nbcol <- 100
color <- rev(rainbow(nbcol, start = 0/6, end = 4/6))
zcol  <- cut(z, nbcol)

# loop for 3D-plots of function with n from 0 to 3 and back
olddir <- setwd("anim") # change path accordingly
nlim <- 3
step <- 0.1
niter <- c(seq(0, nlim, step), seq(nlim-step, step, -step))
for (i in 1:length(niter)) {
  n <- niter[i]
  z <- outer(x, y, f)
  zcol <- cut(z, nbcol)
  persp3d(x, y, z, col = color[zcol])
  filename <- paste0("pic", ifelse((i-1) < 10, paste0("0", (i-1)),
(i-1)), ".png")
  rgl.snapshot(filename, fmt = "png", top = TRUE)
}
```

```
# make animated gif with ImageMagick
system("cmd.exe", input = "convert -delay 20 -loop 0 pic*.png
gradient.gif")
setwd(olddir)
```

I am always fascinated by how versatile R is!