The Wilcoxon test (also referred as the Mann-Withney-Wilcoxon test) is a non-parametric test, meaning that it does not rely on data belonging to any particular parametric family of probability distributions. Non-parametric tests have the same objective as their parametric counterparts. However, they have an advantage over parametric tests: they **do not** require the assumption of normality of distributions. A Student's t-test for instance is only applicable if the data are Gaussian or if the sample size is large enough (usually \(n \ge 30\)). A non-parametric should be used in other cases.

One may wonder why we would not always use a non-parametric test so we do not have to bother about testing for normality. The reason is that non-parametric tests are usually less powerful than corresponding parametric tests when the normality assumption holds. Therefore, all else being equal, with a non-parametric test you are less likely to reject the null hypothesis when it is false if the data follows a normal distribution. It is thus preferred to use the parametric version of a statistical test when the assumptions are met.

In the remaining of the article, we present the two scenarios of the Wilcoxon test and how to perform them in R through two examples.

2 different scenarios

As for the Student's t-test, the Wilcoxon test is used to compare two groups and see whether they are significantly different from each other. The 2 groups to be compared are either:

1. independent, or
2. paired (i.e., dependent)

Independent samples

For the Wilcoxon test with independent samples, suppose that we want to test whether grades at the statistics exam differ between female and male students.

We have collected grades for 24 students (12 girls and 12 boys):

```
dat <- data.frame(
  Sex = as.factor(c(rep("Girl", 12), rep("Boy", 12))),
  Grade = c(
    19, 18, 9, 17, 8, 7, 16, 19, 20, 9, 11, 18,
    16, 5, 15, 2, 14, 15, 4, 7, 15, 6, 7, 14
  )
)
```

```
dat
```

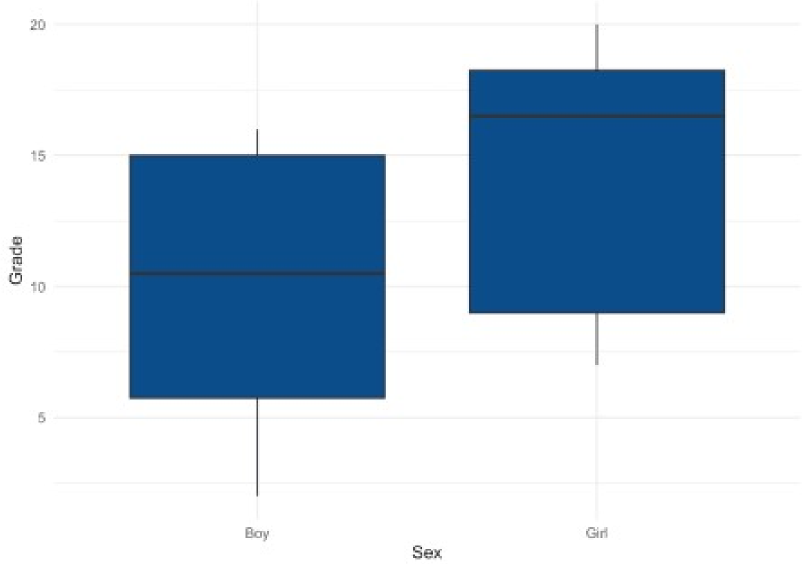
```
##      Sex Grade
## 1  Girl    19
## 2  Girl    18
## 3  Girl     9
## 4  Girl    17
## 5  Girl     8
## 6  Girl     7
## 7  Girl    16
## 8  Girl    19
## 9  Girl    20
## 10 Girl     9
## 11 Girl    11
## 12 Girl    18
## 13  Boy    16
## 14  Boy     5
## 15  Boy    15
## 16  Boy     2
```

```
## 17   Boy     14
## 18   Boy     15
## 19   Boy      4
## 20   Boy      7
## 21   Boy     15
## 22   Boy      6
## 23   Boy      7
## 24   Boy     14
```

Here are the distributions of the grades by sex:

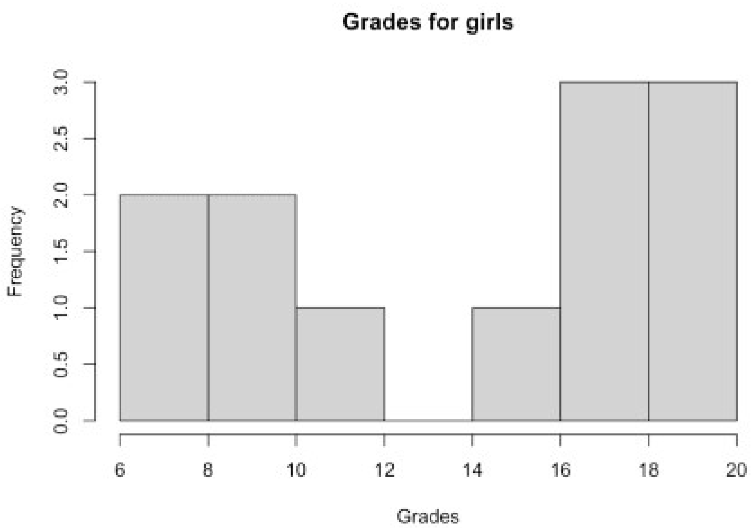
```
library(ggplot2)

ggplot(dat) +
  aes(x = Sex, y = Grade) +
  geom_boxplot(fill = "#0c4c8a") +
  theme_minimal()
```

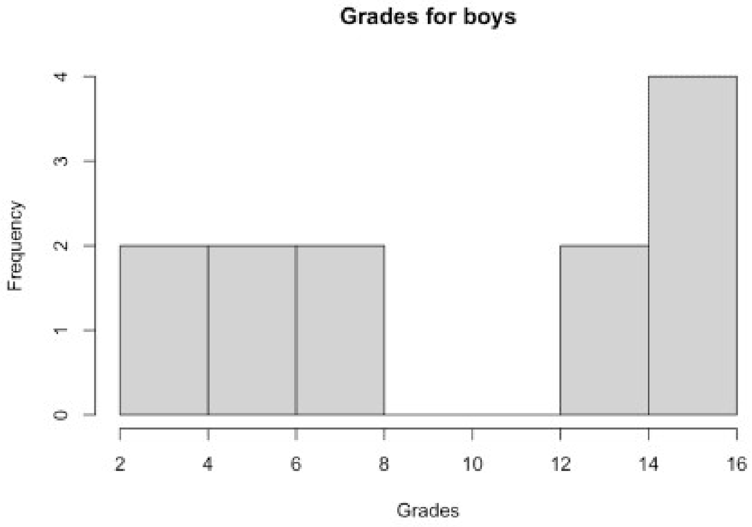


We first check whether the 2 samples follow a normal distribution via a histogram and the Shapiro-Wilk test:

```
hist(subset(dat, Sex == "Girl")$Grade,
  main = "Grades for girls",
  xlab = "Grades"
)
```



```
hist(subset(dat, Sex == "Boy")$Grade,
  main = "Grades for boys",
  xlab = "Grades"
)
```




```
shapiro.test(subset(dat, Sex == "Girl")$Grade)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  subset(dat, Sex == "Girl")$Grade
## W = 0.84548, p-value = 0.0323
```

```
shapiro.test(subset(dat, Sex == "Boy")$Grade)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  subset(dat, Sex == "Boy")$Grade
## W = 0.84313, p-value = 0.03023
```

The histograms show that both distributions do not seem to follow a normal distribution and the *p*-values of the Shapiro-Wilk tests confirm it (since we reject the null hypothesis of normality for both distributions at the 5% significance level).

We just showed that normality assumption is violated for both groups so it is now time to see how to perform the Wilcoxon test in R.[2] Remember that the null and alternative hypothesis of the Wilcoxon test are as follows:

- \(H_0\): the 2 groups are similar
- \(H_1\): the 2 groups are different

```
test <- wilcox.test(dat$Grade ~ dat$Sex)
test
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  dat$Grade by dat$Sex
## W = 31.5, p-value = 0.02056
## alternative hypothesis: true location shift is not equal to 0
```

We obtain the test statistic, the *p*-value and a reminder of the hypothesis tested.[3]

The *p*-value is 0.021. Therefore, at the 5% significance level, we reject the null hypothesis and we conclude that grades are significantly different between girls and boys.

Given the boxplot presented above showing the grades by sex, one may see that girls seem to perform better than boys. This can be tested formally by adding the `alternative = "less"` argument to the `wilcox.test()` function:[4]

```
test <- wilcox.test(dat$Grade ~ dat$Sex,
  alternative = "less"
)
test
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  dat$Grade by dat$Sex
## W = 31.5, p-value = 0.01028
## alternative hypothesis: true location shift is less than 0
```

The *p*-value is 0.01. Therefore, at the 5% significance level, we reject the null hypothesis and we conclude that boys performed significantly worse than girls (which is equivalent than concluding that girls performed significantly better than boys).

Paired samples

For this second scenario, consider that we administered a math test in a class of 12 students at the beginning of a semester, and that we administered a similar test at the end of the semester to the exact same students. We have the following data:

```
dat <- data.frame(
  Beginning = c(16, 5, 15, 2, 14, 15, 4, 7, 15, 6, 7, 14),
  End = c(19, 18, 9, 17, 8, 7, 16, 19, 20, 9, 11, 18)
)

dat
```

```
##    Beginning End
## 1         16  19
## 2          5  18
## 3         15   9
## 4          2  17
## 5         14   8
## 6         15   7
## 7          4  16
## 8          7  19
## 9         15  20
## 10         6   9
## 11         7  11
## 12        14  18
```

We transform the dataset to have it in a tidy format:

```
dat2 <- data.frame(
  Time = c(rep("Before", 12), rep("After", 12)),
  Grade = c(dat$Beginning, dat$End)
)
dat2
```

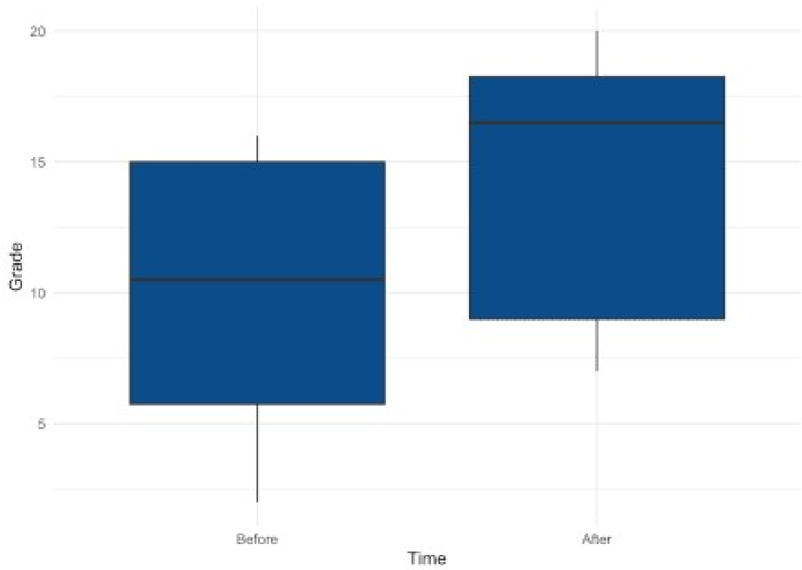
```
##      Time Grade
## 1  Before    16
```

```
## 2  Before     5
## 3  Before    15
## 4  Before     2
## 5  Before    14
## 6  Before    15
## 7  Before     4
## 8  Before     7
## 9  Before    15
## 10 Before     6
## 11 Before     7
## 12 Before    14
## 13  After    19
## 14  After    18
## 15  After     9
## 16  After    17
## 17  After     8
## 18  After     7
## 19  After    16
## 20  After    19
## 21  After    20
## 22  After     9
## 23  After    11
## 24  After    18
```

The distribution of the grades at the beginning and after the semester:

```
# Reordering dat2$Time
dat2$Time <- factor(dat2$Time,
  levels = c("Before", "After")
)

ggplot(dat2) +
  aes(x = Time, y = Grade) +
  geom_boxplot(fill = "#0c4c8a") +
  theme_minimal()
```



(See the `{esquisse}` and `{questionr}` addins to help you reorder levels of a factor variable and to easily draw plots with the `{ggplot2}` package.)

In this example, it is clear that the two samples are not independent since the same 12 students took the exam before and after the semester. Supposing also that the normality assumption is violated, we thus use the Wilcoxon test for **paired samples**.

The R code for this test is similar than for independent samples, except that we add the `paired = TRUE` argument to the `wilcox.test()` function to take into consideration the dependency between the 2 samples:

```
test <- wilcox.test(dat2$Grade ~ dat2$Time,
  paired = TRUE
)
test
```

```
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  dat2$Grade by dat2$Time
## V = 21, p-value = 0.1692
## alternative hypothesis: true location shift is not equal to 0
```

We obtain the test statistic, the *p*-value and a reminder of the hypothesis tested.

The *p*-value is 0.169. Therefore, at the 5% significance level, we do not reject the null hypothesis that the grades are similar before and after the semester.