

Starting from zero, a random walk is produced by choosing moves between ± 1 and ± 2 at each step. If the choice between both is made towards maximising the probability of ending up positive after 100 steps, what is this probability?

Although the optimal path is not necessarily made of moves that optimise the probability of ending up positive after the remaining steps, I chose to follow a dynamic programming approach by picking between ± 1 and ± 2 at each step based on that probability:

```
bs=matrix(0,405,101) #best strategy with value i-203 at time j-1
bs[204:405,101]=1
for (t in 100:1){
  tt=2*t
  bs[203+(-tt:tt),t]=.5*apply(cbind(
    bs[204+(-tt:tt),t+1]+bs[202+(-tt:tt),t+1],
    bs[201+(-tt:tt),t+1]+bs[205+(-tt:tt),t+1]),1,max) }
```

resulting in the probability

```
> bs[203,1]
[1] 0.6403174
```

Just checking that a simple strategy of picking ± 1 above zero and ± 2 below leads to the same value

```
ga=rep(0,T)
for(v in 1:100) ga=ga+(1+(ga<1))*sample(c(-1,1),T,rep=TRUE)
```

or sort of

```
> mean(ga>0)
[1] 0.6403494
```

With highly similar probabilities when switching at $ga < 2$

```
> mean(ga>0)
[1] 0.6403183
```

or $ga < 0$

```
> mean(ga>0)
[1] 0.6403008
```

and too little difference to spot a significant improvement between the three boundaries.