Heuristic analysis:

Custom heuristic functions taken into consideration for isolation game are following:

1. ***eval_param_score_fn(game, player,x,y,z)***
2. ***eval_normalize_score_fn(game, player)***
3. ***eval_reduce_opp_score_fn(game, player)***

Details:

1. ***eval_param_score_fn(game, player,x,y,z)*** : Parametrized evaluation function. Here x,y,z are tuning parameter. Evaluation score is derived from following equation.

Code : `return float(x*own_moves - y*opp_moves +z)`

Own_move = Move count avaliable for active player
Opp_move = Move count avaliable for opponent player

The matches are played with different variable x,y, z
Function fn$x$

1 fn1(x=1,y=0.5,z=0)
2. fn2(x=1,y=0.5,z=1)
3. fn3(x=1,y=0.25,z=1)
4. fn4(x=0.5,y=1,z=1)
5. fn5(x=0.75,y=0.25,z=1)
6. fn6(x=0.5,y=0.75,z=2)
7. fn7(x=1,y=0.5,z=0.5)

Match chart:

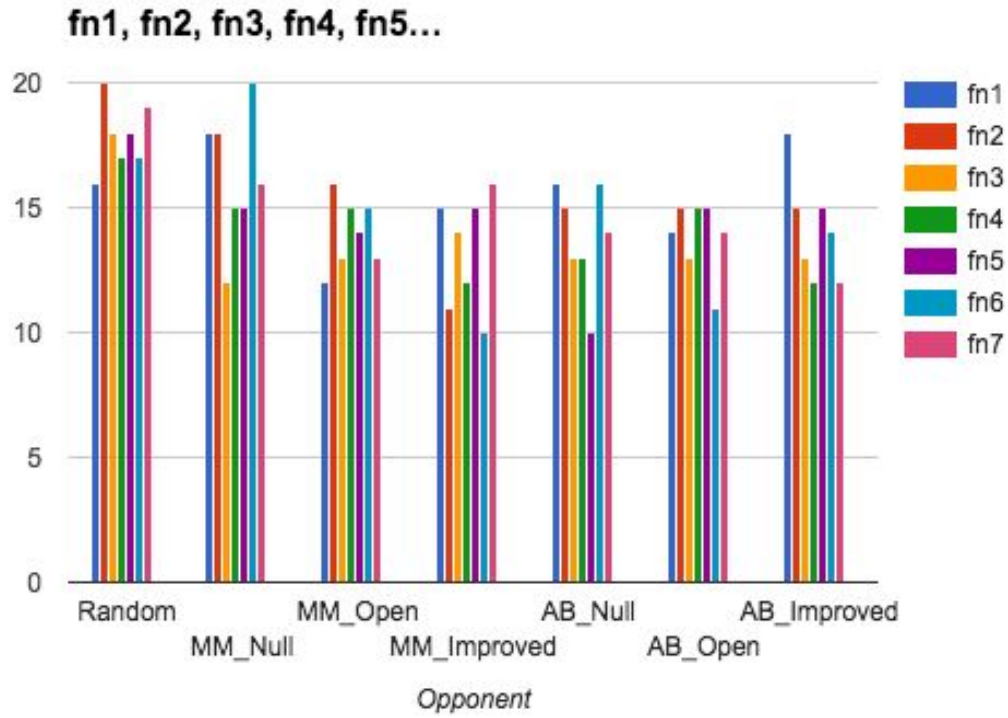
Table 1 - Evaluation of ID_Improved.
Notation - fn$x$ - i
where i = match iteration . Ex: fn2-3 indicates fn2 evaluation function is used and it's 3rd the iteration

Eval: ID_Improved		Win Count out of 20											
SN	Opponent	fn1-1	fn2-1	fn3-1	fn4-1	fn5-1	fn6-1	fn7-1	fn2-2	fn2-3	fn2-4	fn2-5	fn2-Mean
1	Random	20	18	19	18	18	17	19	19	20	17	19	19
2	MM_Null	18	16	19	18	18	15	9	16	13	17	18	16
3	MM_Open	12	11	13	14	15	13	17	15	13	16	15	14
4	MM_Improved	14	13	14	10	9	14	11	10	14	12	13	12
5	AB_Null	15	16	16	15	14	9	13	16	16	15	16	16
6	AB_Open	8	9	13	12	11	10	13	15	14	16	13	13
7	AB_Improved	14	14	10	13	11	11	12	11	13	12	12	12
	Winning %	**69.29**	**69.29**	**74.29**	**74.29**	**68.57**	**63.57**	**67.14**	**72.86**	**73.57**	**75**	**75.71**	**72.86**

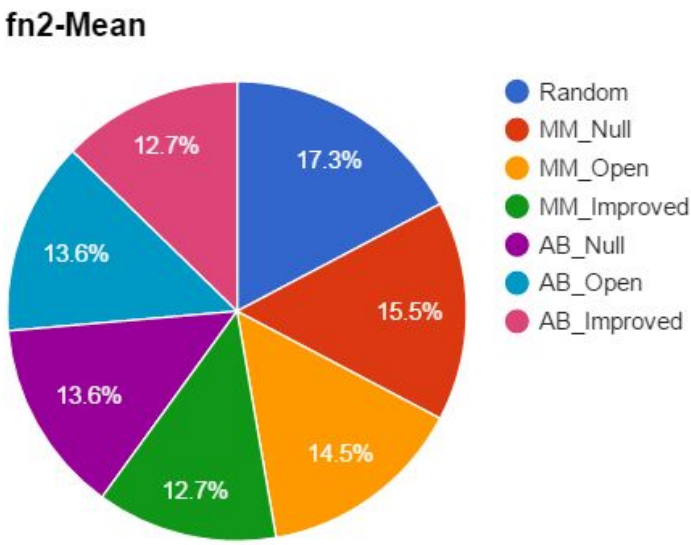
Table 2: ***Evaluation of Student.***

Eval: Student		Win Count out of 20											
SN	Opponent	fn1-1	fn2-1	fn3-1	fn4-1	fn5-1	fn6-1	fn7-1	fn2-2	fn2-3	fn2-4	fn2-5	fn2-Mean
1	Random	16	20	18	17	18	17	19	18	20	17	18	19
2	MM_Null	18	18	12	15	15	20	16	16	18	17	16	17
3	MM_Open	12	16	13	15	14	15	13	15	18	16	14	16
4	MM_Improved	15	11	14	12	15	10	16	15	11	17	15	14
5	AB_Null	16	15	13	13	10	16	14	15	11	18	16	15
6	AB_Open	14	15	13	15	15	11	14	15	15	13	15	15
7	AB_Improved	18	15	13	12	15	14	12	12	14	13	14	14
	Winning %	**77.86**	**78.57**	**68.57**	**70.71**	**72.86**	**73.57**	**74.28**	**77.14**	**80**	**79.29**	**77.14**	**78.57**

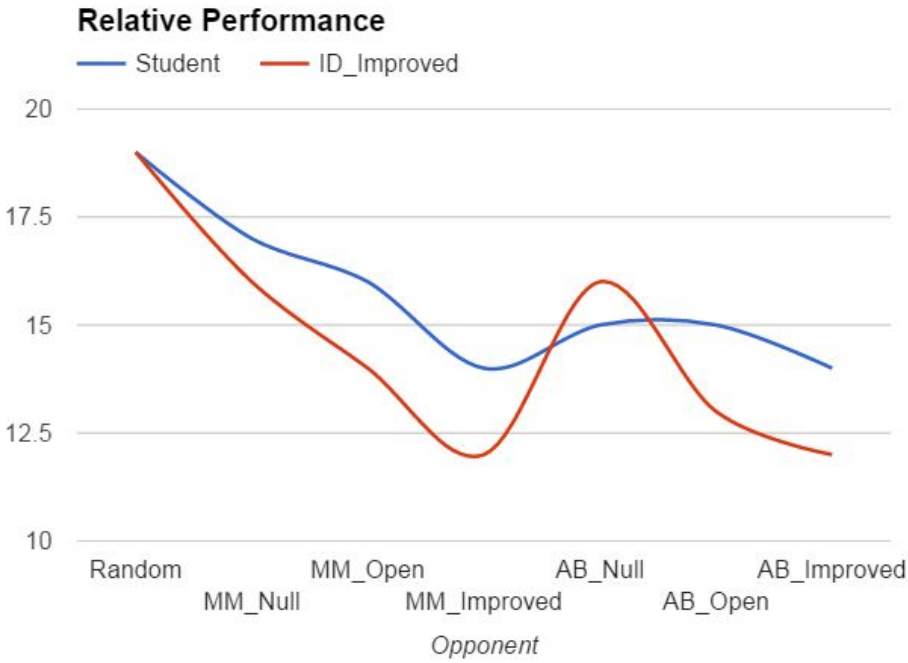
Based on Table 2 data,relative performance of fn_x is shown below



After multiple tries, fn2 shows better winning ratio. Fn2 is iterated multiple time to calculate the fn2-Mean. Mean winning percentage of fn2= 78.57%



Bench match comparison: Student vs ID_Improved
Mean winning % of D_Improved :**72.86%**
Mean winning % of Student :**78.57%**



2. ***eval_normalize_score_fn(game, player)*** : The function evaluates based on the number of occupied space. When board positions are less than half occupied, agent weighs to reduce opponent's legal move. (Aggressive play)

Code:

```
if len(game.get_blank_spaces())>25:
    return  float(0.5*own_moves-opp_moves)
else:
    return float(own_moves - 0.5*opp_moves)
```

len(game.get_blank_spaces()) = Remaining number of board position

Match chart:

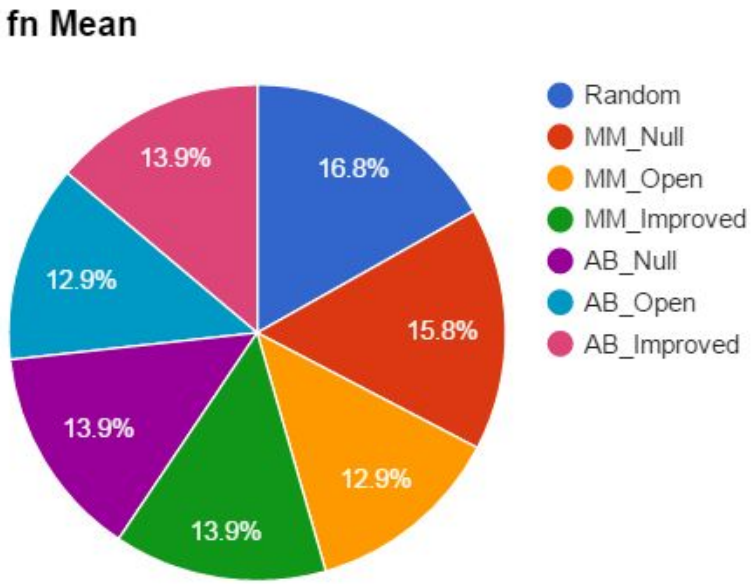
Table 1 - Evaluation of ID_Improved.
Notation - fn-i
where i = match iteration

Eval: ID_Improved		Win Count out of 20							
SNo.	Opponent	fn-1	fn-2	fn-3	fn-4	fn-5	fn-6	fn-7	fn-Mean
1	Random	18	17	19	15	18	11	19	17
2	MM_Null	17	15	19	11	17	19	18	17
3	MM_Open	16	11	15	10	15	10	8	12
4	MM_Improved	14	12	14	11	16	11	13	13
5	AB_Null	15	16	13	15	18	18	13	15
6	AB_Open	15	14	14	11	18	12	9	13
7	AB_Improved	13	12	10	15	13	15	12	13
	Winning %	**77.14**	**69.29**	**74.29**	**62.86**	**77.86**	**68.57**	**64.29**	**71.42**

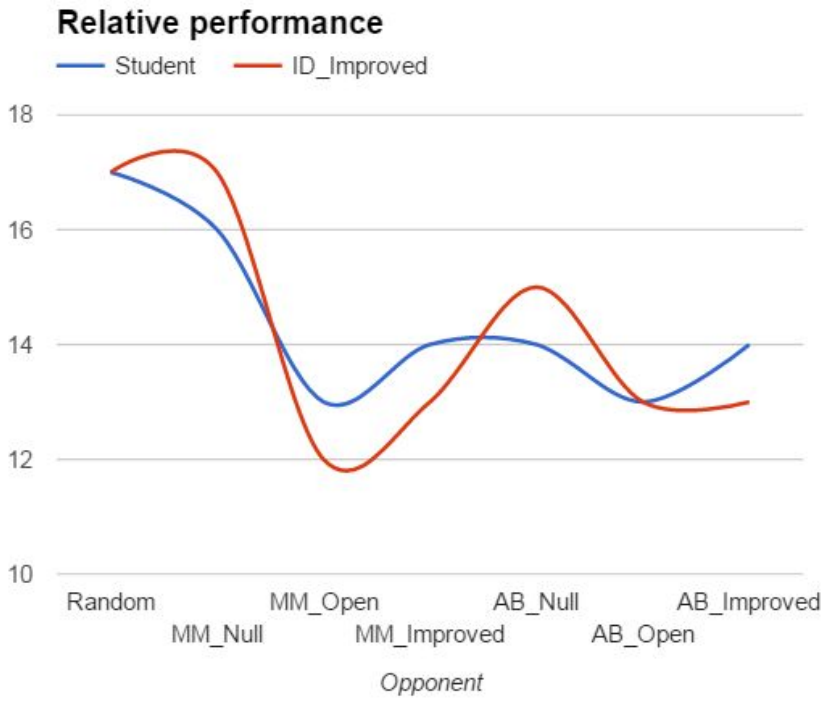
Table 2: ***Evaluation of Student.***

Eval: Student		Win Count out of 20							
SNo.	Opponent	fn-1	fn-2	fn-3	fn-4	fn-5	fn-6	fn-7	fn-Mean
1	Random	18	15	17	19	18	14	17	17
2	MM_Null	18	15	17	16	18	13	18	16
3	MM_Open	12	12	15	12	15	14	13	13
4	MM_Improved	14	12	15	15	15	13	13	14
5	AB_Null	18	12	15	12	17	13	12	14
6	AB_Open	13	13	12	14	12	15	12	13
7	AB_Improved	12	13	17	14	14	15	11	14
	Winning %	**75**	**65.71**	**77.14**	**72.87**	**77.86**	**69.29**	**68.57**	**72.14**

Mean winning percentage of **72.14**%



Bench match comparison: Student vs ID_Improved
Mean winning % of D_Improved **71.42%**
Mean winning % of Student :**72.14%**



3. ***eval_reduce_opp_score_fn(game, player)*:** This evaluation function solely based on opponent's number of legal moves.

Code:

```
return float(-opp_moves)
```

Opp_move = Move count avaliable for opponent player

Match chart:

Table 1 - Evaluation of ID_Improved.

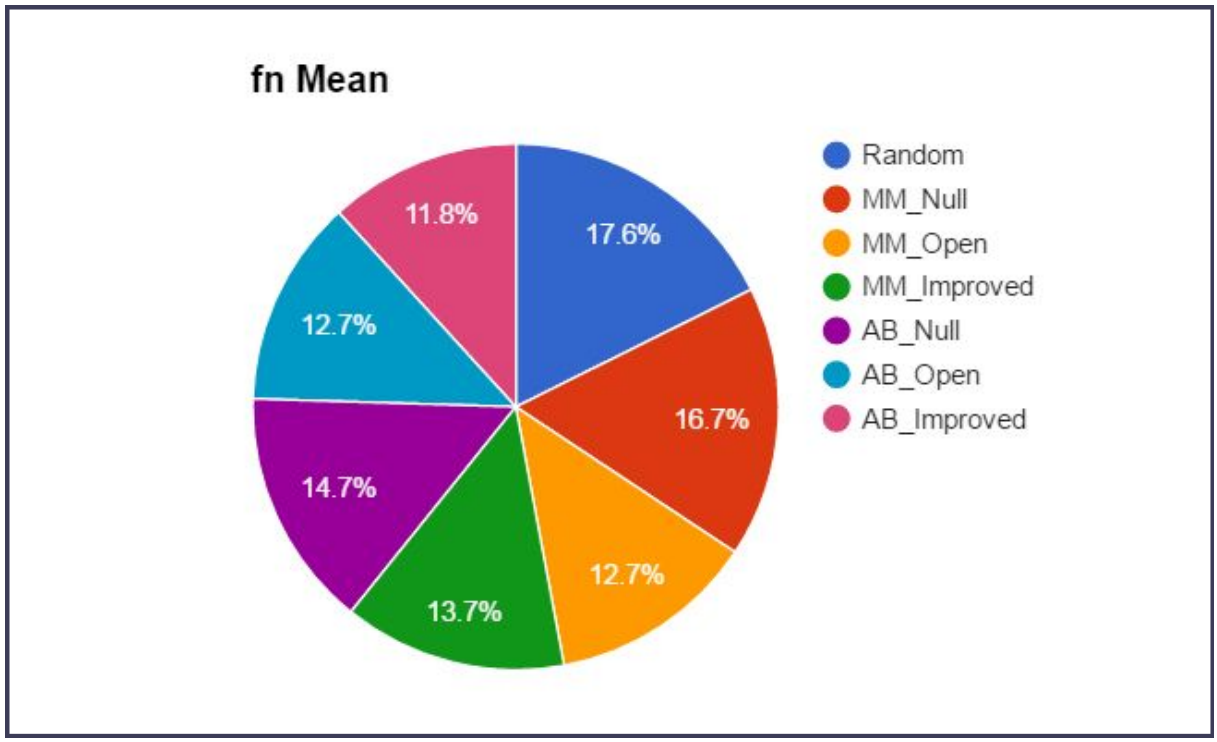
Notation - fn-i

where i = match iteration

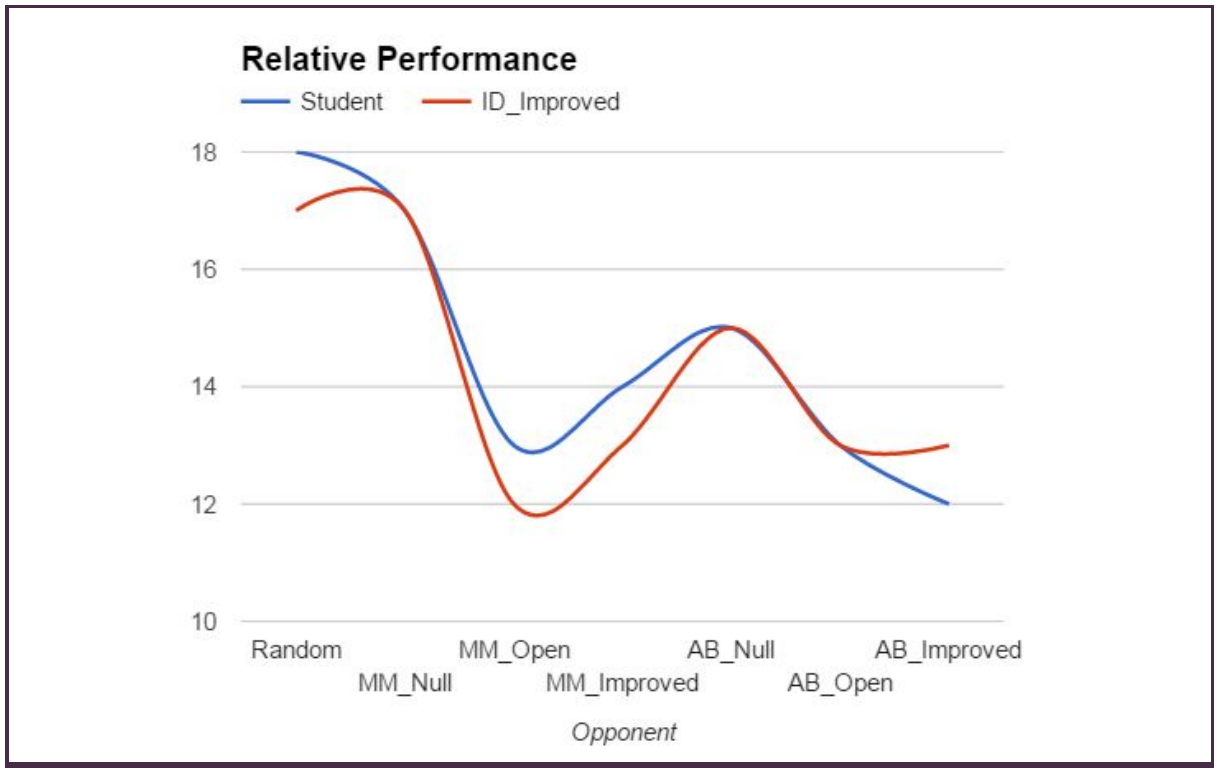
Eval: ID_Improved		Win Count out of 20							
SNo.	Opponent	fn-1	fn-2	fn-3	fn-4	fn-5	fn-6	fn-7	fn-Mean
1	Random	19	15	19	20	17	19	18	18
2	MM_Null	16	15	18	19	15	20	17	17
3	MM_Open	12	12	13	14	15	14	14	13
4	MM_Improved	11	16	11	14	12	13	13	13
5	AB_Null	15	14	12	16	13	16	15	14
6	AB_Open	15	12	13	8	13	10	15	12
7	AB_Improved	14	13	14	12	12	14	14	13
	Winning %	**72.86**	**69.28**	**71.43**	**74.29**	**69.28**	**75.71**	**75.71**	**71.42**

Table 2: ***Evaluation of Student.***

Eval: Student		Win Count out of 20							
SNo.	Opponent	fn-1	fn-2	fn-3	fn-4	fn-5	fn-6	fn-7	fn-Mean
1	Random	18	19	19	19	17	18	19	18
2	MM_Null	15	17	17	18	16	17	17	17
3	MM_Open	11	14	12	12	14	11	16	13
4	MM_Improved	13	12	18	14	14	12	12	14
5	AB_Null	12	13	15	15	18	16	18	15
6	AB_Open	13	12	15	13	14	12	14	13
7	AB_Improved	15	12	10	13	12	13	12	12
	Winning %	**69.28**	**70.71**	**75.71**	**74.29**	**75**	**70.71**	**77.14**	**72.86**



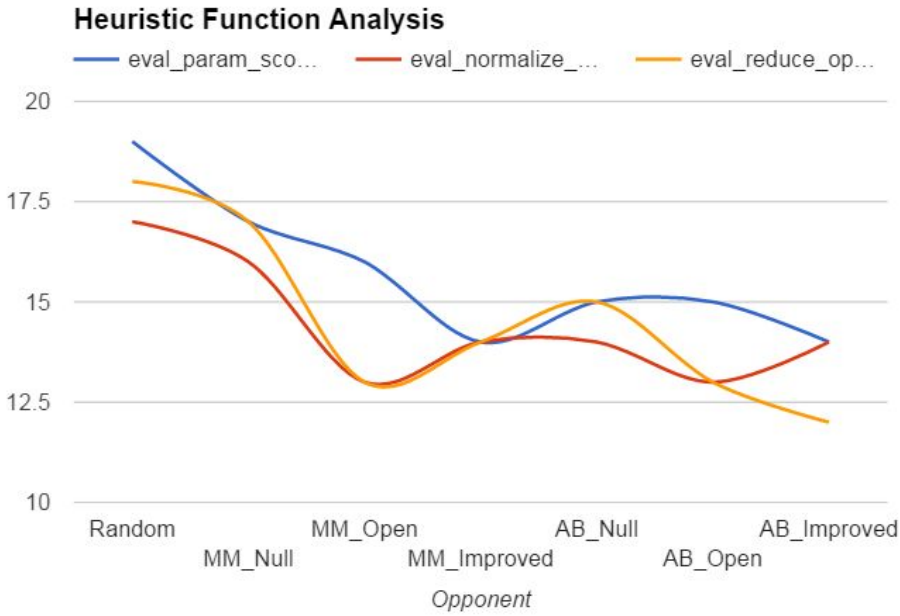
Bench match comparison: Student vs ID_Improved
Mean winning % of D_Improved **71.42%**
Mean winning % of Student :**72.86 %**



Conclusion:

Relative comparison of heuristic function result:

Mean winning % using ***eval_param_score_fn*** :**78.57%**
Mean winning % using ***eval_normalize_score_fn*** :**72.14%**
Mean winning % using ***eval_reduce_opp_score_fn*** :**72.86 %**



Based on above analysis results, ***eval_param_score_fn*** is the better choice compared to other two. It marginally performs better against all opponents. Adding further, since its parameterized model, by changing the parameter, performance can be further improved. By using knowledge base of game results, the parameter can be tuned for the optimal solution. So more games it play, better it gets. Also, collecting position data of every move can further tune the function for better results.