Random Processes- Project 2

Binoy Thomas, North Carolina State University **12/07/2018**

Introduction

Echo SONAR is used for the detection of fishes in various water bodies. The transducer sends bursts of high frequency sounds which are recorded by the receiver. The signals received from all the fishes are combined and the position of the fishes are detected.

If there are N fishes, the received signal is written as

$$X(t) = \sum_{i=1}^{N} A_i \cos\left(2\pi f_0 t + \phi_i\right)$$

Where A_i and ϕ_i depend on the fish's position, orientation and motion and since they are not known they are taken to be random variables.

Question 1

Rewrite the equation of the received waveform

$$X(t) = \sum_{i=1}^{N} A_i \cos\left(2\pi f_0 t + \phi_i\right)$$

$$X(t) = \sum_{i=1}^{N} A_i \cos(2\pi f_0 t)\cos(\phi_i) - A_i \sin(2\pi f_0 t)\sin(\phi_i)$$

Replacing $U_i = A_i cos\ \phi$ and $V_i = A_i sin\ \phi$

$$X(t) = \sum_{i=1}^{N} U_i \cos(2\pi f_0 t) - V_i \sin(2\pi f_0 t)$$

Question 2

$\sum_{i=1}^{N} U_i\ and \sum_{i=1}^{N} V_i$ are sums of independent random variables. They are also identically distributed because all the fishes are considered to be of the same size. Therefore, $\sum_{i=1}^{N} U_i\ and \sum_{i=1}^{N} V_i$ are identically distributed independent random variables. Hence, if we consider a considerably large sample space then we can invoke the central limit theorem and say that the distribution is approximated to a normal distribution and postulate a Gaussian distribution for U and V

Question 3

The mean of a Rayleigh distribution for U and V – N(0, Nσ^2) can be given as

$$E(A) = \sqrt{\frac{\pi}{2}N\sigma^2}$$

From this equation we can infer that the total number of fishes is approximately

$$N = \frac{2}{\pi\sigma^2}E(A)^2$$

Question 4

If we measure the mth envelope of A_m after transmitting the P pulses for the P fishes the expected value can be calculated as

$$E(A) = (\frac{1}{P}\sum_{m=1}^{P}\hat{A}_m)^2$$

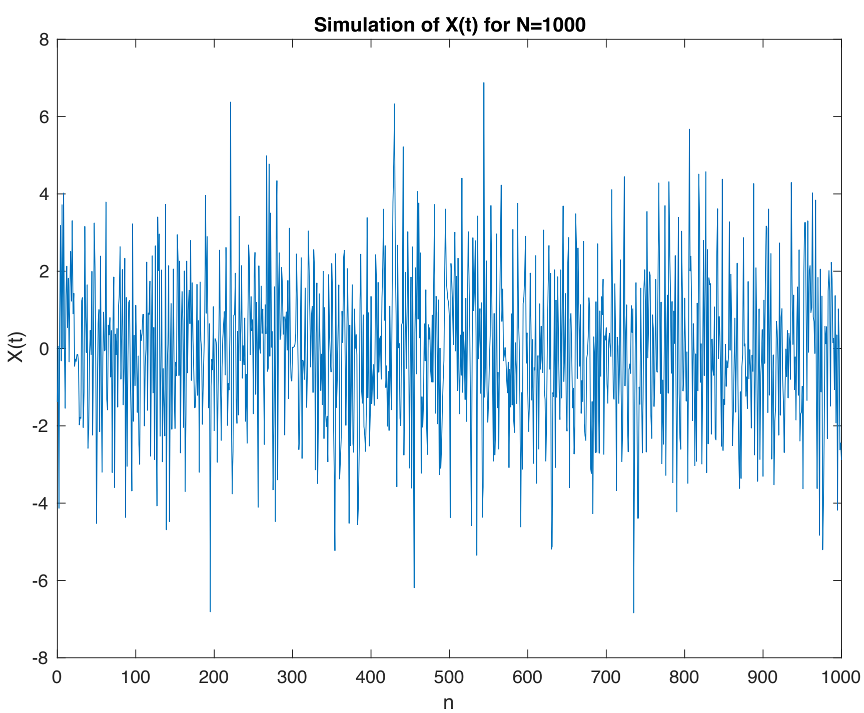
$$\widehat{N} = \frac{2}{\pi\sigma^2}(\frac{1}{P}\sum_{m=1}^{P}\sqrt{{U_m}^2 + {V_m}^2})^2$$

Since $\hat{A}_m = \sqrt{{U_m}^2 + {V_m}^2}$

Question 5

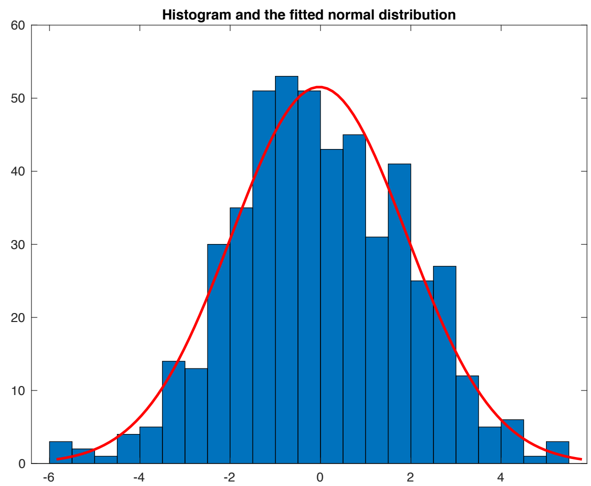
If we consider a very high value for the number of fishes, we obtain the simulation for X(t) as follows

X(t) for N=1000



Question 6

If we consider P=500, then we observe that the histogram of X(t) approaches a Gaussian curve. When we fit the Gaussian distribution on the histogram it is observed that the mean is equal to -0.0279 which is approximately 0. We asserted that the values obtained for X(t) would be a normally distributed and the observed results are in accordance to this assertion



Question 7

U and V are considered to be normally distributed curves and hence when we take the root of their squares, we observe a Rayleigh distribution. This can be proved by the results obtained. The shaping coefficient of the Rayleigh curve was equal to 0.9934