Homework 8

1. Describe (in pseudo code) the algorithm of fast least squares method via SRHT projection and evaluate its relative error on the following inputs A.

Let $A \in \mathbb{R}^{mxn}$ be a real matrix where m >> n. We do the following steps:

a. Generate an *m x n* Gaussian random matrix M (where entries are i.i.d. $N(0,1)$.
b. Compute SVD of matrix $M = U_M \sum_M V_M^T$.
c. Generate a diagonal matrix S = diag(σ_1, ..., σ_n) where σi's are uniformly chosen in the range (1, 10^6). Change the value of σ_1 to 1 and σ_n to 10^6.
d. Compute matrix A as A = $U_M \ S_M \ V_M^T$
e. Generate an m-dimensional Gaussian random vector b.

The following is the pseudo code for the solving the over-strained least square approxiamtion problem via a SRHT projection. The algorithm will return a vector *x_output* that will satisfy the relative bounds stated in class.

Input: $A \in \mathbb{R}^{mxn}$ and $b \in \mathbb{R}^{m}$.
Output: x_output ∈ $\mathbb{R}^n$.

1. Let c be the sampling parameter, choose a c value that is more than $\frac{m}{2}$.
2. Let P be an empty matrix.
3. For i = 1 to c, select uniformly at random and with replacement an integer from = {1 to m}. Update the value of element at jth row and ith column of P randomly to either $\sqrt{\frac{m}{r}}$, $-\sqrt{\frac{m}{r}}$ or 0 with different probability. The probability of P_{ij} to be $\sqrt{\frac{m}{r}}$ or $-\sqrt{\frac{m}{r}}$ is the same. J is the number representing the cth row number from matrix A.
4. Let $H \in \mathbb{R}^{mxm}$ be a normalized Hadamard transform matrix.
5. Let $D \in \mathbb{R}^{mxm}$ be a diagonal matrix where $D_{ii} \in \{+1, -1\}$ each with probability $\frac{1}{2}$.
6. Compute and return $x\_output = (P^T HDA)^+ \ P^T HDb$.

Task: Take reasonable values of m, n, and sampling parameter c, compare the relative approximation error ‖*Ax_output* − b‖$_2$ / ‖*A_xopt* − b‖$_2$, where *x_output* is the solution produced by the randomized algorithm, and *x_opt* is the solution produced by a deterministic least square method.