

7.	40%	Misalkan $S=\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ basis untuk $\mathbb{R}^3$ , dengan $\mathbf{u}_1=[1, a, 1]$ , $\mathbf{u}_2=[1, 1, -1]$ , dan $\mathbf{u}_3=[-1, 1, 0]$ . Carilah basis ortonormal $B=\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ untuk $\mathbb{R}^3$
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$$\mathbf{u}_1 = \begin{bmatrix} 1 & 5 & 1 \end{bmatrix}$$

$$\mathbf{u}_2 = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

$$\mathbf{u}_3 = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{v}_1 = \frac{\mathbf{u}_1}{|\mathbf{u}_1|} = \frac{\begin{bmatrix} 1 & 5 & 1 \end{bmatrix}}{\sqrt{27}} = \begin{bmatrix} \frac{1}{\sqrt{27}} & \frac{5}{\sqrt{27}} & \frac{1}{\sqrt{27}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{9} & \frac{5\sqrt{3}}{9} & \frac{\sqrt{3}}{9} \end{bmatrix}$$

$$\mathbf{v}_2 = \frac{\mathbf{x}_2}{|\mathbf{x}_2|}, \text{ dengan } \mathbf{x}_2 = \mathbf{u}_2 - [\mathbf{u}_2 * \mathbf{v}_1]\mathbf{v}_1$$

$$[\mathbf{u}_2 * \mathbf{v}_1] = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} * \begin{bmatrix} \frac{\sqrt{3}}{9} & \frac{5\sqrt{3}}{9} & \frac{\sqrt{3}}{9} \end{bmatrix}$$

$$[\mathbf{u}_2 * \mathbf{v}_1] = \frac{\sqrt{3}}{9} + \frac{5\sqrt{3}}{9} - \frac{\sqrt{3}}{9} = \frac{5\sqrt{3}}{9}$$

$$[\mathbf{u}_2 * \mathbf{v}_1]\mathbf{v}_1 = \frac{5\sqrt{3}}{9} * \begin{bmatrix} \frac{\sqrt{3}}{9} & \frac{5\sqrt{3}}{9} & \frac{\sqrt{3}}{9} \end{bmatrix} = \begin{bmatrix} \frac{5}{27} & \frac{25}{27} & \frac{5}{27} \end{bmatrix}$$

$$\mathbf{x}_2 = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} - \begin{bmatrix} \frac{5}{27} & \frac{25}{27} & \frac{5}{27} \end{bmatrix} = \begin{bmatrix} \frac{22}{27} & \frac{2}{27} & -\frac{32}{27} \end{bmatrix}$$

$$\mathbf{v}_2 = \frac{\begin{bmatrix} \frac{22}{27} & \frac{2}{27} & -\frac{32}{27} \end{bmatrix}}{\sqrt{\frac{56}{27}}} = \begin{bmatrix} \frac{11\sqrt{42}}{126} & \frac{\sqrt{42}}{126} & -\frac{8\sqrt{42}}{63} \end{bmatrix}$$

$$\mathbf{v}_3 = \frac{\mathbf{x}_3}{|\mathbf{x}_3|}, \text{ dengan } \mathbf{x}_3 = \mathbf{u}_3 - [\mathbf{u}_3 * \mathbf{v}_1]\mathbf{v}_1 - [\mathbf{u}_3 * \mathbf{v}_2]\mathbf{v}_2$$

$$[\mathbf{u}_3 * \mathbf{v}_1] = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} * \begin{bmatrix} \frac{\sqrt{3}}{9} & \frac{5\sqrt{3}}{9} & \frac{\sqrt{3}}{9} \end{bmatrix} = \frac{4\sqrt{3}}{9}$$

$$[\mathbf{u}_3 * \mathbf{v}_1]\mathbf{v}_1 = \frac{4\sqrt{3}}{9} * \begin{bmatrix} \frac{\sqrt{3}}{9} & \frac{5\sqrt{3}}{9} & \frac{\sqrt{3}}{9} \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{20}{27} & \frac{4}{27} \end{bmatrix}$$

$$[\mathbf{u}_3 * \mathbf{v}_2] = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} * \begin{bmatrix} \frac{11\sqrt{42}}{126} & \frac{\sqrt{42}}{126} & -\frac{8\sqrt{42}}{63} \end{bmatrix} = -\frac{5\sqrt{42}}{63}$$

$$[u_3 * v_2]v_2 = \left(-\frac{5\sqrt{42}}{63}\right) * \begin{bmatrix} \frac{11\sqrt{42}}{126} & \frac{\sqrt{42}}{126} & -\frac{8\sqrt{42}}{63} \end{bmatrix} = \begin{bmatrix} -\frac{55}{189} & -\frac{5}{189} & \frac{80}{189} \end{bmatrix}$$

$$x_3 = u_3 - [u_3 * v_1]v_1 - [u_3 * v_2]v_2$$

$$x_3 = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{27} & \frac{20}{27} & \frac{4}{27} \end{bmatrix} - \begin{bmatrix} -\frac{55}{189} & -\frac{5}{189} & \frac{80}{189} \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -\frac{6}{7} & \frac{2}{7} & -\frac{4}{7} \end{bmatrix}$$

$$v_3 = \frac{x_3}{|x_3|}$$

$$v_3 = \frac{\begin{bmatrix} -\frac{6}{7} & \frac{2}{7} & -\frac{4}{7} \end{bmatrix}}{\sqrt{\frac{8}{7}}} = \begin{bmatrix} -\frac{3\sqrt{14}}{14} & \frac{\sqrt{14}}{14} & -\frac{\sqrt{14}}{7} \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} \frac{\sqrt{3}}{9} \\ \frac{5\sqrt{3}}{9} \\ \frac{\sqrt{3}}{9} \end{bmatrix}, \begin{bmatrix} \frac{11\sqrt{42}}{126} \\ \frac{\sqrt{42}}{126} \\ -\frac{8\sqrt{42}}{63} \end{bmatrix}, \begin{bmatrix} -\frac{3\sqrt{14}}{14} \\ \frac{\sqrt{14}}{14} \\ -\frac{\sqrt{14}}{7} \end{bmatrix} \right\}$$