Deterministic Models in Ecology

ESS 575 Models for Ecological Data

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January 24, 2017

Graduate Degree Program in Ecology
Department of Ecosystem Science and Sustainability
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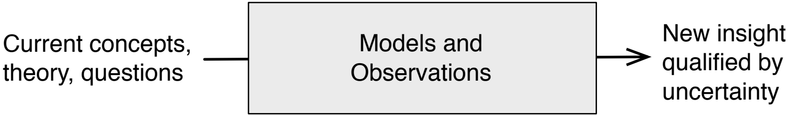
Goal of course



Linking models to data

Probability model

Observations (data)

Deterministic model

Idea!

What is the probability that I would observe the data if my model is a faithful representation of the processes that gave rise to the data?

Deterministic models

$$\mu_i = g(\boldsymbol{\theta}, \mathbf{x}_i) \tag{1}$$

$$\underbrace{\mu_i}_{\text{response}} = g(\underbrace{\boldsymbol{\theta}}_{\text{parameters}}, \overbrace{\mathbf{x}_i}^{\text{predictors}}) \tag{2}$$

Also range and domain, dependent variable and independent variable. Predictors also called predictor variables and covariates.

$g(\boldsymbol{\theta}, \mathbf{x}_i)$

Any type of process

- Effects of soil properties on nitrogen mineralization
- Effects of elephant poaching on recruitment
- Responses of carnivore to energy development
- Food web dynamics of streams
- Hydrologic regime and riparian plant dynamics
- Controls on plant invasion in sagebrush steppe

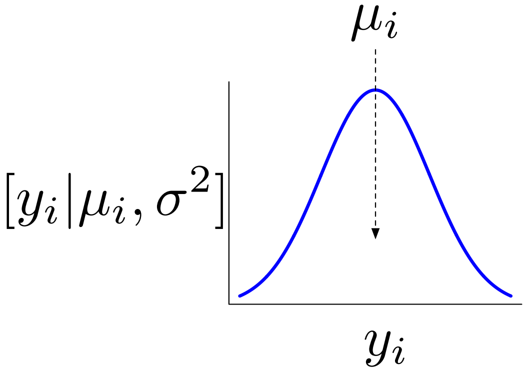
$g(\boldsymbol{\theta}, \mathbf{x}_i)$

Any type of mathematical function

- linear models
- non-linear models
- differential equations
- difference equations
- landscape transmission models

Linking models to data

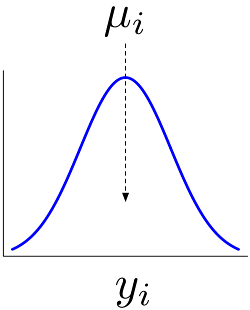
$$\mu_i = g(\boldsymbol{\theta}, x_i)$$



For example,

$$\boldsymbol{\theta} = (\beta_0, \beta_1)'$$
$$\mu_i = g(\boldsymbol{\theta}, x_i) = \beta_0 + \beta_1 x_i$$

$$\underbrace{[y_i | \mu_i, \sigma^2]}_{\text{normal}}$$



Layout of next few lectures

- Today: a toolbox for $g(\boldsymbol{\theta}, \mathbf{x})$
- Thursday: basic laws of probability
- Next week: probability distributions
- Week after: Likelihood and Bayes' Theorem

Objectives of lecture(s)

- Introduce a set of functional forms useful for composing deterministic models
- Illustrate mechanistic approach to deriving models of ecological processes.
- Cross cutting themes
  - A relatively small set of functions can be used to describe a broad array of ecological processes .
  - The same process can be represented by different functional forms.

Some functional forms for $g(\boldsymbol{\theta}, \mathbf{x})$

- Functions for additive effects
- Asymptotic processes
- Biological scaling and other power functions
- Change points
- Competing controls, interactions, composites of functions

What if response is > 0 and < 1 ?

- Proportion of plots with invasive species
- Nitrogen content of soil (gN / g OM)
- Proportion of landscape burned
- Survival probability of juveniles
- Prevalence of a disease in a population

Inverse logit function

Let p = variable that can take on values between 0 and 1.

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) \text{ converts } p \text{ to -inf } \rightarrow \text{ +inf}$$

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 .... + \beta_n x_n$$

$$p = \text{inverse logit}(\beta_0 + \beta_1 x_1 .... + \beta_n x_n)$$

$$p = \frac{e^{\beta_0 + \beta_1 x_1 .... + \beta_n x_n}}{1 + e^{\beta_0 + \beta_1 x_1 .... + \beta_n x_n}}$$

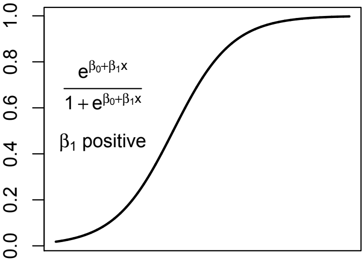
You will also see

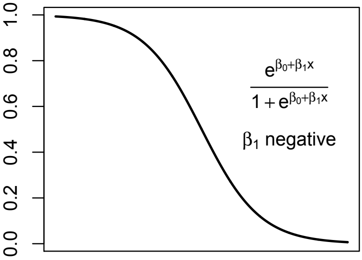
$$p = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 .... + \beta_n x_n)}}$$

BUT BE CAREFUL ABOUT THE MINUS

IN THE EXPONENT!!!

Inverse logit function



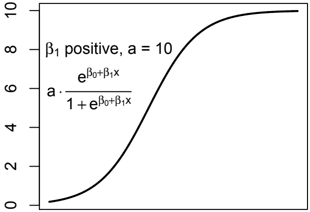


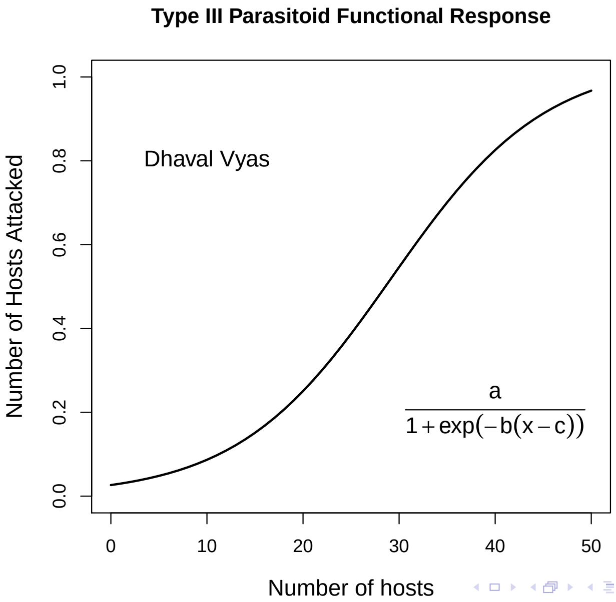
What if response is between 0 and a?

Multiply by a:

$$\frac{a\,e^{(\beta_0+\beta_1 x_i....+\beta_n x_n)}}{1+e^{(\beta_0+\beta_1 x_i....+\beta_n x_n)}} \tag{3}$$

Always non-negative and does not reach excessively large values




Type III Parasitoid Functional Response
Dhaval Vyas
$\frac{a}{1+\exp(-b(x-c))}$
Number of Hosts Attacked
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
Number of hosts

What if response must be ≥ 0 ?

What if the response must be ≥ 0?
For example we want to model λ_t as an additive function of covariates:

$$N_{t+1} = \lambda_t N_t \tag{4}$$
$$\lambda_t = g(\boldsymbol{\beta}, \mathbf{x}_t) \tag{5}$$

Other example responses that must be non-negative:

- biomass
- energy expenditure
- population density
- species richness

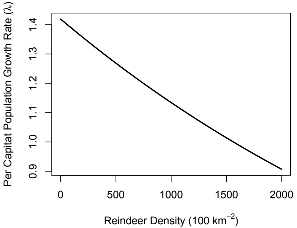
Exponential model

$$\mu_i = \exp(\beta_0 + \beta_1 x_{1,i}, ..., +\beta_n x_{n,i}),$$

which is also written as

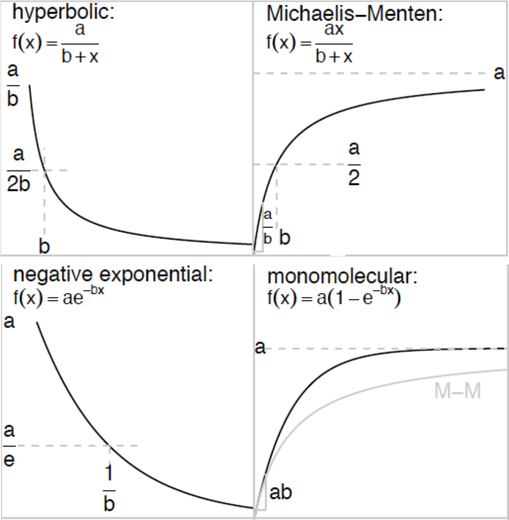
$$\log(\mu_i) = \beta_0 + \beta_1 x_{1,i}, ..., +\beta_n x_{n,i}$$

$$\lambda_{i,t} = e^{(B_0+B_1 D_{i,t-1}+B_2 L_{i,t}+B_3 W_{i,t}+B_4 G_i+B_5 O_t)\Delta t}$$

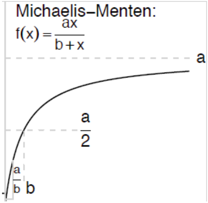


Hobbs, N. T., H. Andren, J. Persson, M. Aronsson, and G. Chapron. 2012. Native predators reduce harvest of reindeer by Sami pastoralists. Ecological Applications 22:1640-1654.

Asymptotic functions

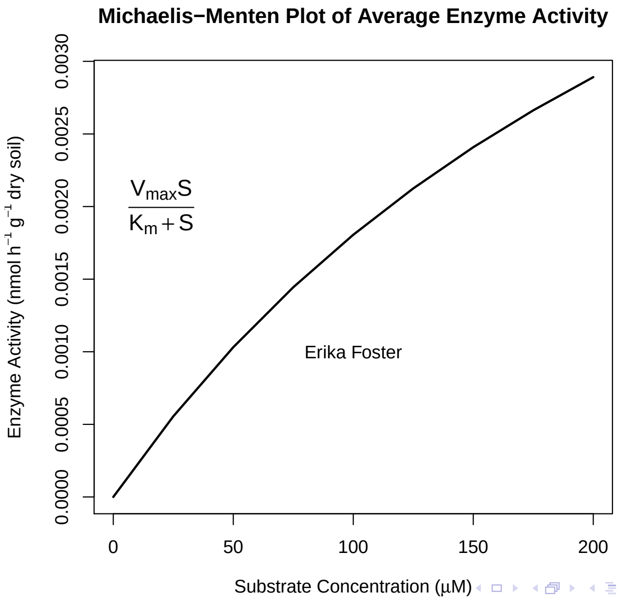


Figures courtesy of Bolker, B. 2008. Ecological Models and Data in R. Princeton University Press, Princeton, N. J USA.



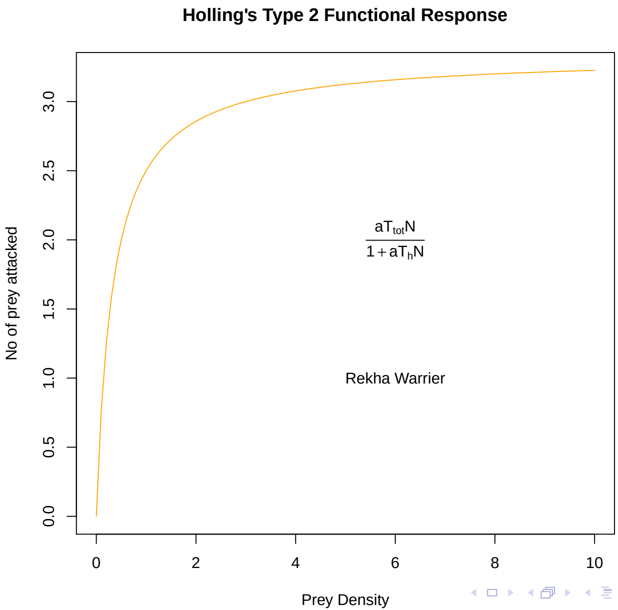
Also written as $\frac{ax}{\frac{a}{b}+x}$

- Enzyme kinetics
- As Beverton-Holt equation, describes dynamics of populations, particularly fish.
- As Holling's disc equation and Spalinger-Hobbs model, portrays functional response
- As the "Monad equation" used to represent the limitation of soil nutrients on plant growth.


Michaelis−Menten Plot of Average Enzyme Activity
Enzyme Activity (nmol h^{-1} g^{-1} dry soil)
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
$\frac{V_{max}S}{K_m + S}$
Erika Foster
0
50
100
150
200
Substrate Concentration (µM)

You will also see functions that look like:

$$
\begin{aligned}
&\mu = \frac{\alpha x}{1+\alpha\gamma x} \\
&\text{let a} = \frac{1}{\gamma},\ b = \frac{1}{\alpha\gamma} \\
&\\
&\text{multiply numerator and denominator by } \frac{1}{\alpha\gamma} \\
&\mu = \frac{\frac{\alpha x}{\alpha\gamma}}{\frac{1}{\alpha\gamma}+\frac{\alpha\gamma x}{\alpha\gamma}} \\
&\\
&\text{simplify and substitute:} \\
&\mu = \frac{ax}{b+x}
\end{aligned}
\tag{6}
$$


Holling's Type 2 Functional Response
$\frac{aT_{tot}N}{1+aT_hN}$
Rekha Warrier
No of prey attacked
Prey Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
2
4
6
8
10

Some useful rescaling tricks

Applied to Michaelis-Menton but applicable to any function
$\mu = \frac{x}{1+x}$,simpliest form of MM model
asymptote at 1, half - maximum at 1

substitute $\frac{x}{b}\text{for}x$ multiply numerator and denominator by b
$\mu = \frac{x}{b+x}$
asymptote at 1, half - maximum at b

multiply rhs by a
$\mu = \frac{ax}{b+x}$
asymptote at $a, \text{half} - \text{maximumat} b$

More rescaling tricks

We can shift curve to right or left by adding or subtracting a constant from x. Useful when y= 0 is reached at positive x to represent lower threshold (see light limitation of trees example)

$$\mu = \frac{a\,(x-c)}{b+(x-c)} \tag{7}$$

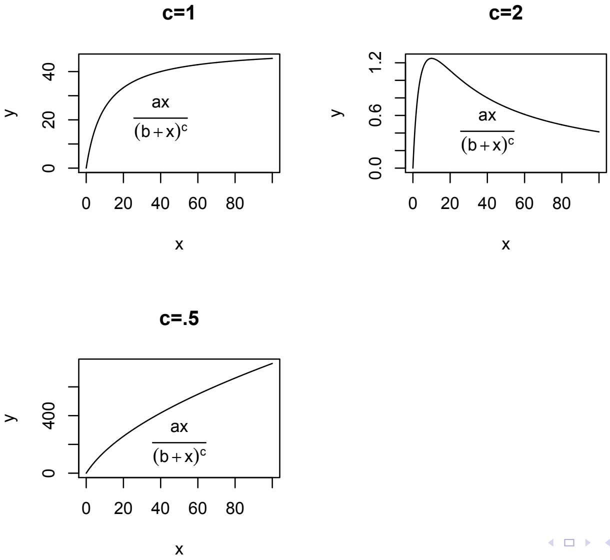
We can shift the curve up or down by adding or subtracting a constant to the right hand side. If we add d, the curve starts a y = d instead of 0 when x = c. What happens to the asymptote?

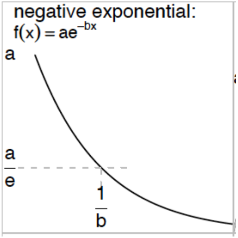
$$\mu = \frac{a\,(x-c)}{b+(x-c)} + d \tag{8}$$

If it makes sense, we can give biological meaning to b by defining it as the ratio of a divided by dy/dx when x is “small” , i.e

$$b = \frac{a}{f}, f = \frac{dy}{dx}\,\text{when}\,x = 0\,\text{or}\,x = c \tag{9}$$

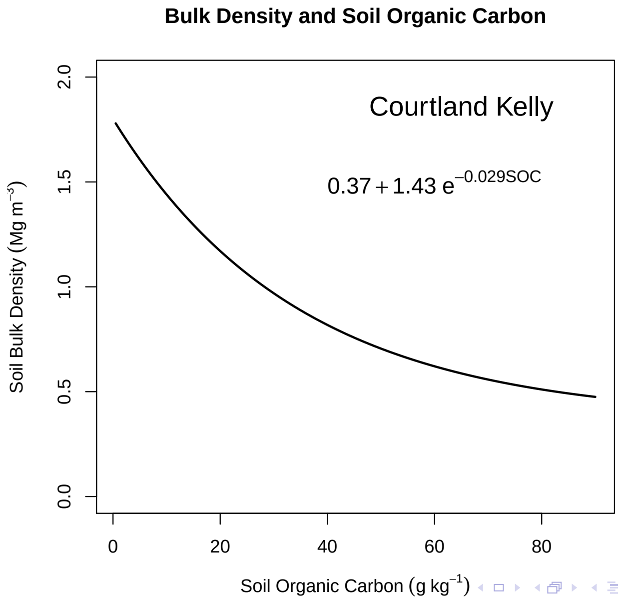
Using exponents

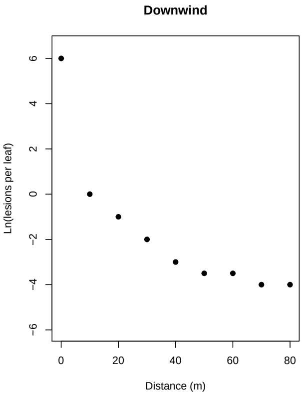




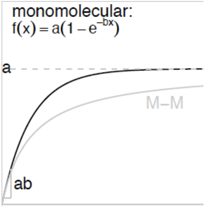
Solution to differential equation $\frac{dy}{dx} = -bx$

- Many applications in population ecology
- Detection functions in distance sampling
- Beer's law for light attenuation through plant canopy


Bulk Density and Soil Organic Carbon
Courtland Kelly
$0.37 + 1.43\,e^{-0.029SOC}$
Soil Bulk Density (Mg m^{-3})
0.0
0.5
1.0
1.5
2.0
0
20
40
60
80
Soil Organic Carbon (g kg^{-1})

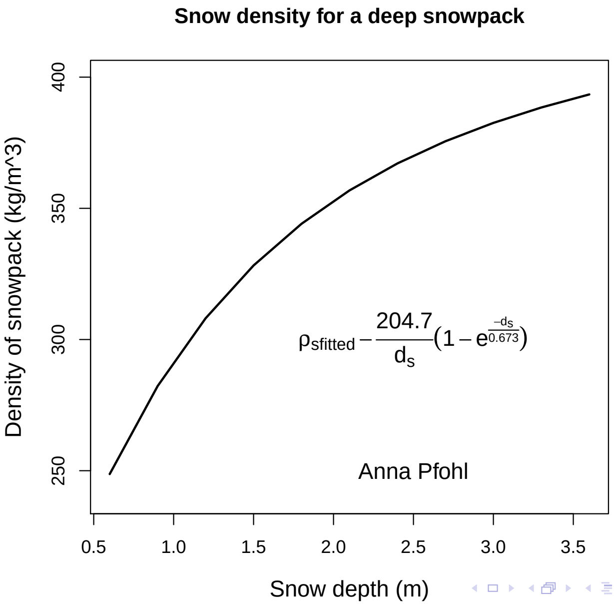

Downwind
Ln(lesions per leaf)
6
4
2
0
−2
−4
−6
0
20
40
60
80
Distance (m)

Examples of Mono-molecular



- the *catalytic curve* in infectious disease epidemiology, where it represents the change over time in the fraction of a cohort that has been exposed to disease (Anderson and May, 1991);
- the simplest form of the *von Bertalanffy* growth curve in organismal biology and fisheries, where it arises from the competing effects of changes in catabolic and metabolic rates with changes in size (Essington et al., 2001);
- the *Skellam model* in population ecology, giving the number of offspring in the next year as a function of the adult population size this year when competition has a particularly simple form (Skellam, 1951; Brännström and Sumpter, 2005).

Bolker, B. 2008. Ecological Models and Data in R. Princeton University Press, Princeton, N. J USA.


Snow density for a deep snowpack
Density of snowpack (kg/m^3)
250
300
350
400
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Snow depth (m)
$\rho_{sfitted} - \frac{204.7}{d_s}\left(1 - e^{\frac{-d_s}{0.673}}\right)$
Anna Pfohl

Droop Model

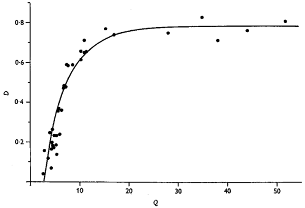
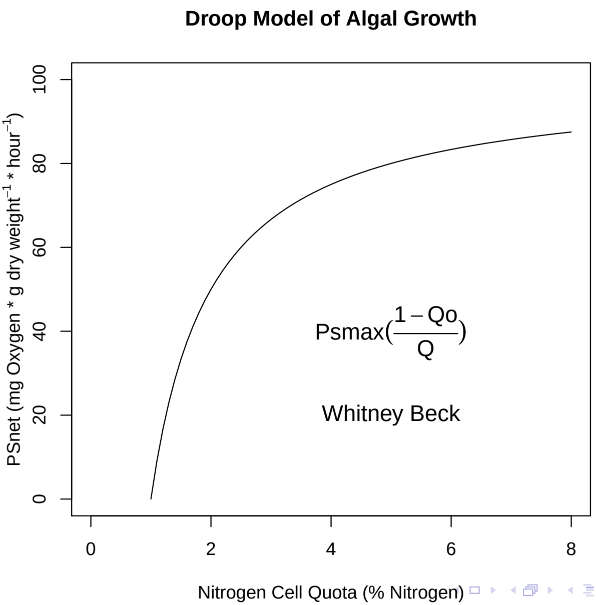


Figure 3. Chemostat steady states. Relation between dilution rate (*D*) and cell quota (*Q*). Reproduced from Droop 1968 with permission of the Journal of the Marine Biological Association UK.

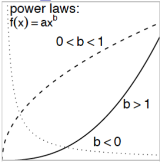
$$\mu = \mu_{\text{max}} \left(1 - \frac{k_q}{x}\right)$$

- μ = realized growth rate
- μ_{max} = maximum growth rate
- k_q= intercept (abscissa)
- x = organism or population size



Droop Model of Algal Growth
PSnet (mg Oxygen * g dry weight^{-1} * hour^{-1})
0
20
40
60
80
100
0
2
4
6
8
Nitrogen Cell Quota (% Nitrogen)
$\mathrm{Psmax}(\frac{1-\mathrm{Qo}}{\mathrm{Q}})$
Whitney Beck

Scaling models, power functions



- Species area curves
- Scaling physiological and ecological parameters to body mass
- Adjusting measurements taken at different spatial scales

Can be log transformed to linear form:

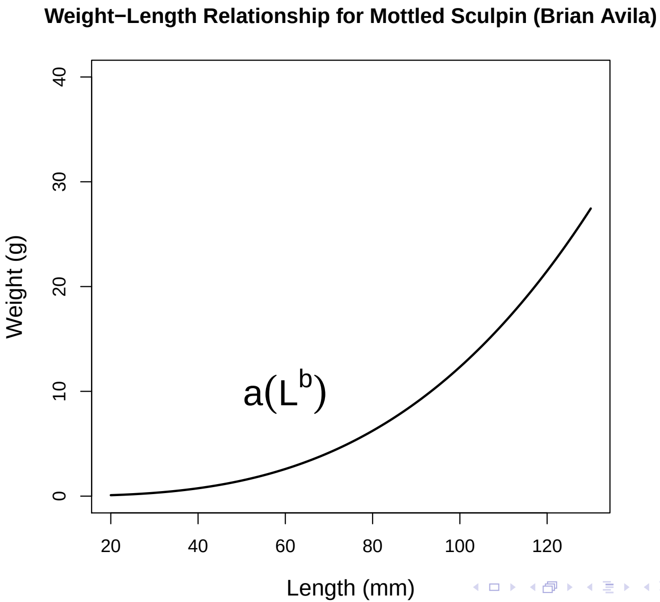
$$y = ax^b \tag{10}$$

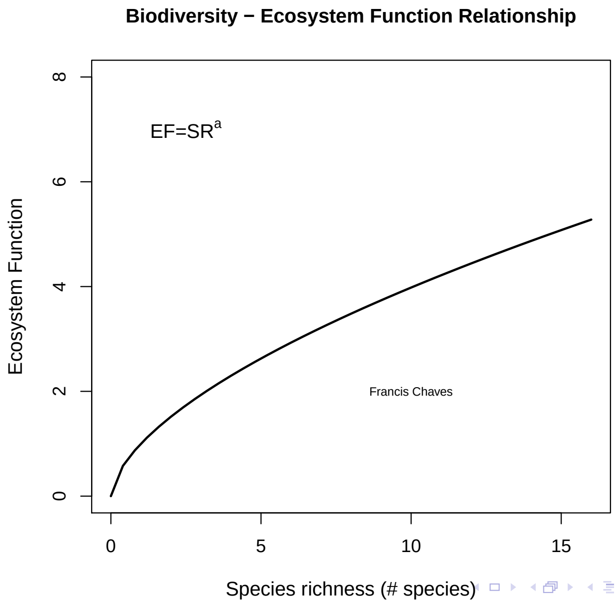
$$\log(y) = \log(a) + b\log(x) \tag{11}$$

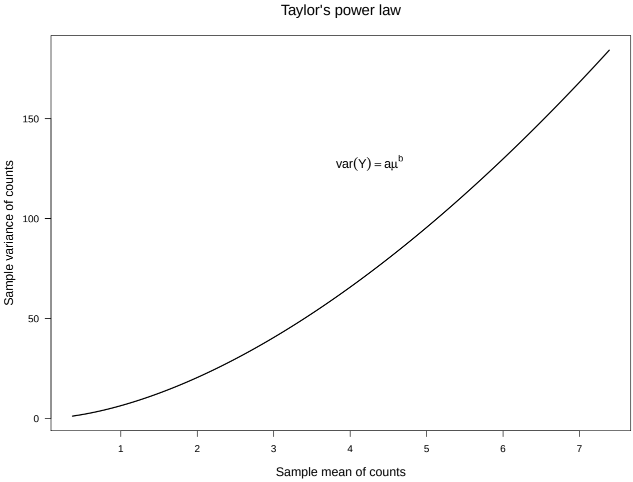
$$\beta_0 = \log(a) \tag{12}$$

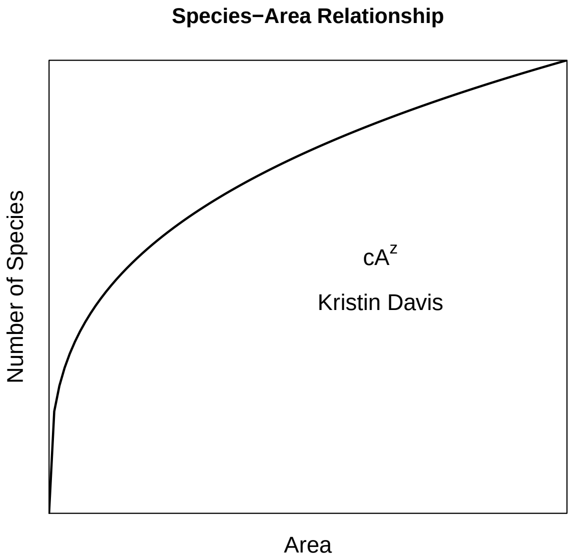
$$\beta_1 = b \tag{13}$$

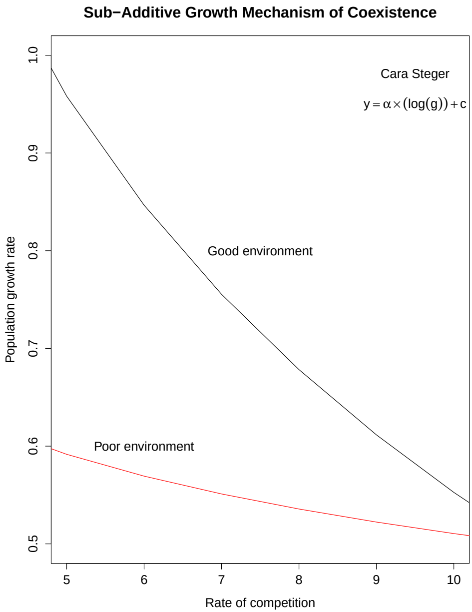
$$\log(y) = \beta_0 + \beta_1 \log(x) \tag{14}$$


Weight−Length Relationship for Mottled Sculpin (Brian Avila)
Weight (g)
0
10
20
30
40
20
40
60
80
100
120
Length (mm)
a(L^b)


Biodiversity − Ecosystem Function Relationship
EF=SR^a
Francis Chaves
Ecosystem Function
0
2
4
6
8
0
5
10
15
Species richness (# species)


Taylor's power law
var(Y) = aμ^b
Sample variance of counts
0
50
100
150
1
2
3
4
5
6
7
Sample mean of counts


Species−Area Relationship
Number of Species
Area
cA^z
Kristin Davis


Sub−Additive Growth Mechanism of Coexistence
Cara Steger
y = α × (log(g)) + c
Good environment
Poor environment
Population growth rate
Rate of competition
0.5
0.6
0.7
0.8
0.9
1.0
5
6
7
8
9
10

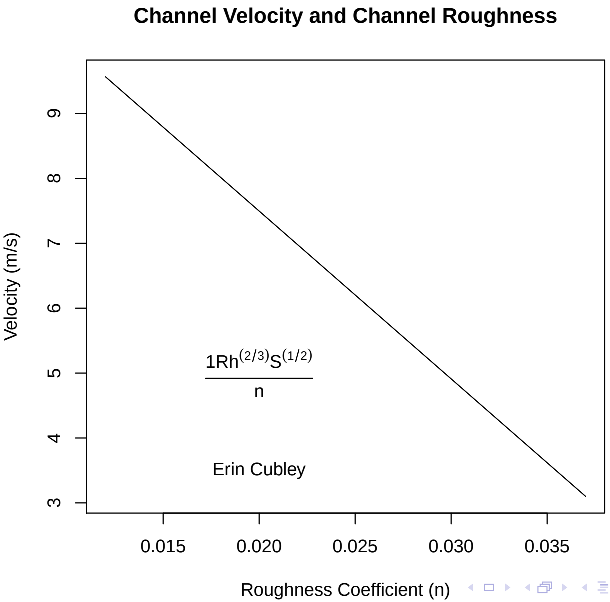
Scaling models, power functions

Multiple predictor variables can be accommodated using

$$g(\boldsymbol{\beta},\mathbf{x}) = \beta_0 x_1^{\beta_1} x_2^{\beta_2}, \ldots, x_n^{\beta_n}$$

which you will see rewritten as

$$\log(y) = \log(\beta_0) + \beta_1 \log(x_1) + \beta_2 \log(x_2), + \ldots + \beta \log(x_n).$$


Channel Velocity and Channel Roughness
Velocity (m/s)
3
4
5
6
7
8
9
1Rh(2/3)S(1/2)
n
Erin Cubley
0.015
0.020
0.025
0.030
0.035
Roughness Coefficient (n)

Some change point possibilities

Bolker, B. 2008. Ecological Models and Data in R. Princeton University Press, Princeton, N. J USA.

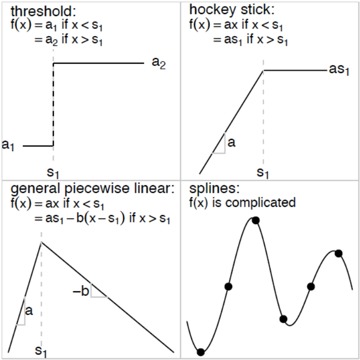
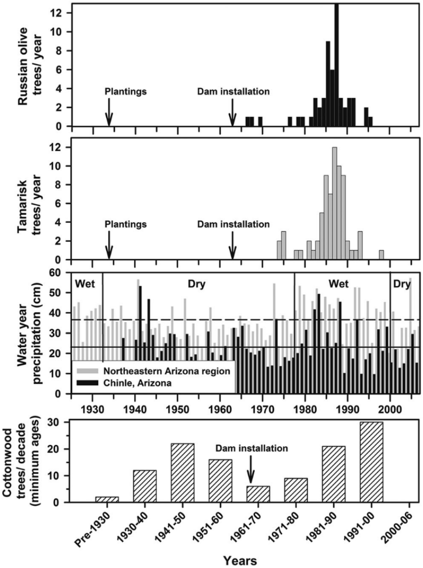


Figure 3.7 Piecewise polynomial functions: the first three (threshold, hockey stick, general piecewise linear) are all piecewise linear. Splines are piecewise cubic; equations are complicated and usually handled by software (see `?spline` `?smooth.spline`).



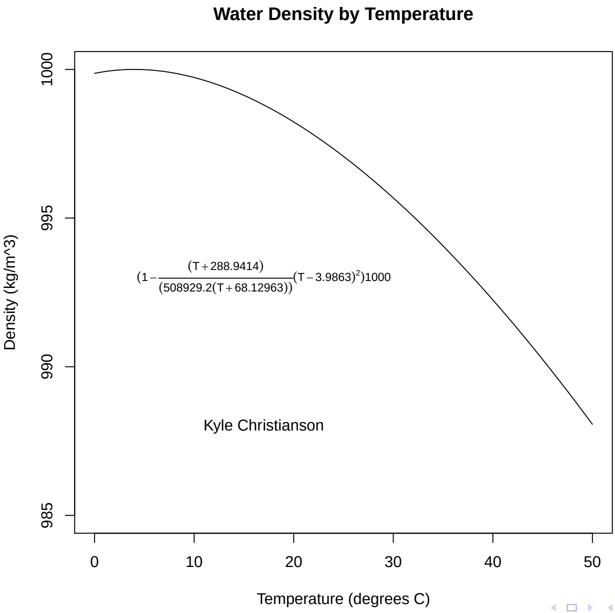
Reynolds, L. V., D. J. Cooper, and N. T. Hobbs. 2014. Drivers of riparian tree invasion on a desert stream. River Research and Applications 30:60-70.

Combined Feedbacks

- Combination of positive and negative feedback, e.g., Allee effect, photoinhibition, plant water relations.
- Example:

$$\text{rate of photosynthesis} = \underbrace{P_{max}}_{\substack{\text{maximum} \\ \text{rate g/time}}} \overbrace{\left[\underbrace{(aI)}_{\text{facilitation}}\underbrace{\left(e^{1-aI}\right)}_{\text{inhibition}}\right]}^{\text{Between 0-1}}$$

I=light intensity, a = unitless shape parameter


Water Density by Temperature
(1 − (T + 288.9414) / (508929.2(T + 68.12963)) (T − 3.9863)^2)1000
Kyle Christianson
Density (kg/m^3)
1000
995
990
985
0
10
20
30
40
50
Temperature (degrees C)

Linking models to data

Probability model

Observations (data)

Deterministic model

Idea!

What is the probability that I would observe the data if my model is a faithful representation of the processes that gave rise to the data?