Completely randomized design

$y_1 x_1$ $y_7 x_7$ $y_5 x_5$
$y_2 x_2$
$y_4 x_4$
$y_3 x_3$ $y_6 x_6$

$x_i = 0$ if control
$x_i = 1$ if treatment

Assume continuous, real response,
~~i.e. change in height, + is larger, - is less. Treatment~~ is
Treatment & a control.
Write a full model and interpret coefficients.

$$g(\underline{\beta}, x_i) = \beta_0 + \beta_1 x_i$$

$$[\underline{\beta}, \sigma^2 | \underline{y}] = \prod_{i=1}^{n} \text{normal}(g(\underline{\beta}, x_i), \sigma^2)$$
$$\times \text{normal}(\beta_0 | 0, 10000)$$
$$\times \text{normal}(\beta_1 | 0, 1000)$$
$$\times \text{uniform}(\sigma^2 | 0, 200)$$

$x_i = 1$ if treated, $x_i = 0$ if not

β_0 is mean in absence of treatment

β_1 is additive effect of treatment.

Do Means model first

Now assume the data are counts. Write the model and interpret the coefficients

Change notation to: ~~μ_0 for~~
~~to~~

$\beta_0 = \mu_0$

$\beta_1 = \gamma$

$$g(\mu_0, \gamma, x_i) = e^{\mu_0 + \gamma x_i}$$

$$[\mu_0, \gamma | \underline{y}] = \prod_{i=1}^{n} \text{Poisson}(g(\mu_0, \gamma, x_i))$$
$$[\mu_0, \gamma]$$

What is e^{μ_0}? What is e^{γ}?

What is μ_0? What is γ?

effects model

Means model is

$$[\underline{\mu}, \sigma^2 | \underline{y}] = \prod_{i=1}^{n_j} \prod_{j=1}^{2} \text{normal}(y_{ij} | \mu_j, \sigma^2)$$

Completely random design

design matrix $= \underline{X}$

Observation (i)	X_1	X_2
1	1	0
2	0	1
3	1	1
4	1	0
5	~~etc~~	
⋮		
n		

vector $= x_i = (1,0)'$

Completely random design of ~~blocked design~~

each plot gets 1 combination of trt.

$x_i \Rightarrow Y_i$

$\mu_0 \quad \underline{\gamma} \quad \sigma^2$

$$g(\mu_0, \underline{\gamma}) = \mu_0 + \gamma_1 x_{1,i} + \gamma_2 x_{2,i} + \gamma_3 x_{1,i} x_{2,i}$$

$x_1 = 1$ if rainout present, 0 otherwise
$x_2 = 1$ if fence present, 0 otherwise

Main effects & interactions

100 plots receive one of the 4 treatment combinations

Continuous, real valued response

$$[\mu_0, \gamma_1, \gamma_2, \gamma_3, \sigma^2 \mid \underline{y}] \propto \prod_{i=1}^{100} \text{normal}(y_i \mid \mu_0 + \underbrace{\gamma_1 x_{1,i}}_{\text{effect of rain}} + \underbrace{\gamma_2 x_{2,i}}_{\text{effect of removing herbivory}} + \underbrace{\gamma_3 x_{1,i} x_{2,i}}_{\text{interaction}}) \times [\mu_0, \gamma_1, \gamma_2, \gamma_3, \sigma^2]$$

$x_2 = 1$ if fence present
$x_1 = 1$ if rainout present

Means

1. No rainout, no fence $= \mu_0$
2. No rainout, fence present $\mu_2 = \mu_0 + \gamma_2$
3. Rainout present, fence absent $\mu_3 = \mu_0 + \gamma_1$
4. rainout present, fence present $\mu_{..} = \mu_0 + \sum x$

Means model

$j=1$ control
$j=2$ fence only
$j=3$ rainout only
$j=4$ fence + rainout

$$[\underline{\mu}, \sigma^2 \mid \underline{Y}] = \prod_{i=1}^{n_j} \prod_{j=1}^{4} \text{normal}(y_{ij} \mid \mu_j, \sigma^2)$$

$$[\mu, \sigma^2]$$

How do you get effects?

Effect of rainout:

$\gamma_1 = \mu_2 - \mu_1$

$\gamma_2 = \mu_3 - \mu_1$

Interaction: $\gamma_3 = \mu_4 - (\mu_3 + \mu_2) - \mu_1$

Could also include both likelihoods in code, but not as efficient

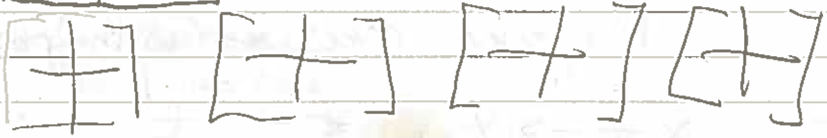
Design matrix for blocked design

i = observation j = group x_1 x_2

	no fence	fence
no rainout	1	2
rainout	3	4

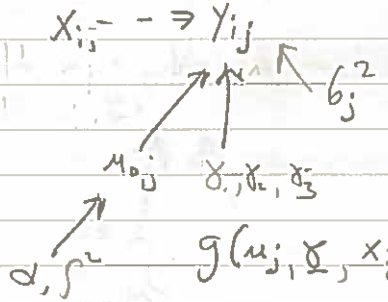
$x_1 = 1$ if rainout present, 0 otherwise
$x_2 = 1$ if fence present, 0 otherwise

4 sites, replications within site



Effects model with group level effects for site – each site has its own mean for control

$$[\underline{\mu_0}, \gamma_1, \gamma_2, \gamma_3 \mid \underline{Y}]$$



$g(\mu_j, \underline{\gamma}, \underline{x}_{ij}) = \mu_0 + \gamma_1 x_{1,ij} + \gamma_2 x_{2,ij} + \gamma_3 x_{1,ij} x_{2,ij}$

$$[\underline{\mu_0}, \underline{\gamma}, \underline{\sigma}^2, \alpha, \varsigma^2 \mid \underline{Y}] \propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} \text{normal}(y_{ij} \mid \text{normal}(g(\mu_j, \underline{\gamma}, \underline{x}_{ij}))$$

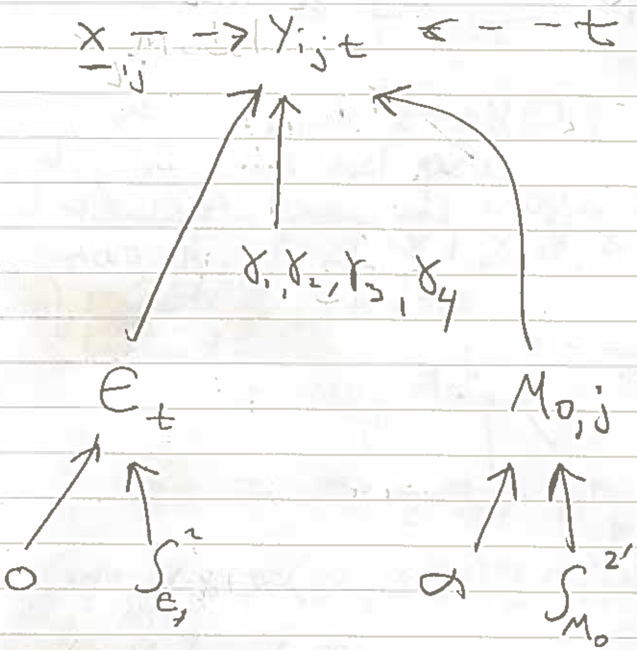
$$\times \text{normal}(\mu_j \mid \alpha, \varsigma^2)$$

$$\times [\underline{\gamma}, \underline{\sigma}^2]$$

Means grand mean z α

$$g(\mu_j, \underline{\gamma}, \underline{x}_{ij}, t) = \mu_0 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_1 x_2 + \gamma_4 t + \epsilon_t$$

Repeated measures with "random effect" for time



$$[\underline{\gamma}, \underline{\mu_0}, \underline{\epsilon}, \alpha, \sigma^2_{\mu_0}, \sigma^2_{\epsilon} \mid \underline{Y}] \propto$$

$$\prod_{i=1}^{n_j} \prod_{j=1}^{J} \prod_{t=1}^{1} \text{normal}\left(g(\mu_j, \underline{\gamma}, x_{ij}, t), \sigma^2\right)$$

$$\times \text{ normal}(\mu_j \mid \alpha, \sigma^2_{\mu_0})$$

$$\times \text{ normal}(\epsilon_t \mid 0, \sigma^2_{\epsilon})$$