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Online Appendix: "A Model of Tournament Incentives with Corruption"

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Contents

A Patterns of Corruption Charges

The data of corruption charges are collected from the website of the Central Committee for Discipline Inspection of the CCP, which is the Party organ in charge of fighting corruption.[1] This website publishes the official announcements of disciplined cadres. A typical announcement looks as follows:

> Recently, approved by the CCP Central Committee, the CCP Central Commission for Discipline Inspection reviewed the case of WU Tianjun, the former member of the Provincial Standing Committee of the Henan Province and the former secretary of the Political and Legal Committee.
>
> During the course of the investigation, Wu Tianjun violated political discipline, resisted the investigation. He is found to have committed a breach of anti-graft discipline by **taking advantage of his office to seek benefits for his relatives' business** and **requesting his subordinate units to pay for his personal expenses**. He is also found to have committed a breach of work discipline by using fraud when receiving visitors from the masses. He is found to **use his job to seek benefits for others and obtain huge sums of money**. He is currently on suspicion of the criminal offense of **bribery**.
>
> Wu Tianjun, as a senior leading cadre of the Party, lost his faith and beliefs, seriously violated the Party's discipline, and caused serious consequences. Moreover, he did not modify his behaviors after the 18th National Congress of the Party. According to the relevant provisions of the *Communist Party of China Disciplinary Regulations* and other relevant provisions, after the discussion of the Standing Committee of the Central Commission for Discipline Inspection and upon the approval of the Central Committee, it is decided that Wu Tianjun should be expelled from the Party and the public office, with all his unlawful income confiscated, and be transfers to the judiciary with the evidence of the crime. *(Boldfaced letters added by the authors)*

The announcement enumerates the type of breach of discipline that the cadre committed. Our analysis is based on the information in the boldfaced phrases. We collect the announcements of 119 cases where the cadre in question is directly supervised by the Central Organization Committee published in the period from Feb. 8, 2013 to Feb. 23, 2017.[2]

Using a web crawler, we crawled the 119 announcements and summarized the nature of the accusation. When an announcement contains phrases such as or similar to "subordinate units to pay for his personal expenses," "travel fees payed by the public funds," "consumption payed by the public funds," "embezzlement," and "possess public belongings," we count that case towards an instance of embezzlement. When an announcement contains phrases such as or similar to "seek personal interests," "obtain colossal interests," "help relatives' business," "help others to get promoted," and "utilize power to seek interests," we count that case towards an instance of the conflict of interest. Lastly, we also count cases where bribery is present by looking for phrases such as "take colossal bribes," "take cash gift," "take gifts," and "take bribes."

Table A-1 presents our findings. Of the 119 cases, about 21% involve embezzlement while as much as 95% involve some kind of conflict of interest. Bribery is also quite common, which occurs 90.7% of all cases. Note that this is not a breakdown of the cases, it is perfectly possible for a single case to display all three "symptoms." The findings are consistent with a recent article by Guo (2013) that shows embezzlement has

[1] The website of the Central Commission for Discipline Inspection (CCDI) is http://www.ccdi.gov.cn/. The example is taken from the webpage http://www.ccdi.gov.cn/xwtt/201701/t20170123_93321.html (accessed on March 15, 2017).

[2] The announcements are categorized by the organization of the Party: the announcements covering the cadres who are supervised by Central Organization Committee (*zhong guan gan bu*), the announcements covering the cadres who are supervised by Provincial Organization Committee (*sheng guan gan bu*) and the announcements covering the cadres who work in the state-owned enterprises and institutions. We focus on the first group.

given way to the conflict of interest type of corruption as the economy develops, possibly due to better auditing measures in the government.

Table A-1: Patterns of Corruption Charges

Embezzlement	Conflict of Interest	Bribery
21%	95%	90.7%

Notes: The table shows the frequency of phrases describing each type of corruption appearing in the announcements of corruption cases against cadres managed by the Central Organization Committee published by the Central Commission for Discipline Inspection from February 8th, 2013 to February 23rd, 2017.

B The Model: Second-Order Conditions

Here in this section we prove that the lower-level officials' problems and the problem of the optimal set of incentives are concave under some conditions in the baseline model.

The Provincial Governor's Problem The second order condition for concavity of the provincial governor's problem is:

$$\frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}(W_0 + r_1 r_2 K + I(0,0) - W_1 - r_2 K) - \kappa < 0 \tag{B-1}$$

Evaluated at x_1:

$$T_1(W_0 - W_1 + (r_1 - 1) r_2 K + I(0,0)) \equiv T_1 \Delta W_1 < \kappa \tag{B-2}$$

where

$$T_1 = \frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}|_{x_j = x_1} = \int_{\mathbf{R}} (n_1 - 1)\left[(n_1 - 2) G(\eta_j)^{n_1 - 3} g(\eta_j) + G(\eta_j)^{n_1 - 2} g'(\eta_j)\right] g(\eta_j) d\eta_j. \tag{B-3}$$

The Prefectural Mayor's Problem The second order condition for concavity of the prefectural Mayor's problem is:

$$\frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}\left[\frac{1}{n_1}(W_0 - W_1 + (r_1 - 1) r_2 K + I(0,0)) + W_1 + r_2 K - \frac{1}{2}\kappa x_1^2 - W_2 - K\right] - \kappa < 0 \tag{B-4}$$

Evaluated at x_2:

$$\begin{aligned} & T_2\left[\frac{1}{n_1}(W_0 - W_1 + (r_1 - 1) r_2 K + I(0,0)) + W_1 - W_2 + (r_1 - 1) K - \frac{1}{2}\kappa x_1^2\right] \\ \equiv & T_2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2\right) < \kappa \end{aligned} \tag{B-5}$$

where

$$T_2 = \frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}|_{x_{ij} = x_2} = \int_{\mathbf{R}} (n_2 - 1)\left[(n_2 - 2) F(\varepsilon_{ij})^{n_2 - 3} f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2 - 2} f‘(\varepsilon_{ij})\right] f(\varepsilon_{ij}) d\varepsilon_{ij}. \tag{B-6}$$

So to guarantee the SOCs of the agents' problems, we need κ sufficiently large:

$$\kappa > \max\left\{T_1 \Delta W_1, T_2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2\right)\right\}.$$

We verify in the simulation that this condition is satisfied at the optimum.

The Optimal Set of Incentives The second order condition for concavity of the problem is that the Hessian matrix is negative definite:

$$H(x_1, x_2) = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \tag{B-7}$$

where $L_{ij} = \frac{\partial^2 L}{\partial x_i \partial x_j}$ and L is the Lagrangian function of the problem.

We have:

$$L_{11} = n_1 n_2 \mathcal{A}\alpha(\alpha - 1)x_1^{\alpha-2}x_2^{\beta} - n_1\kappa \tag{B-8}$$

$$L_{12} = L_{21} = n_1 n_2 \mathcal{A}\alpha\beta x_1^{\alpha-1}x_2^{\beta-1} \tag{B-9}$$

$$L_{22} = n_1 n_2 \mathcal{A}\beta(\beta - 1)x_1^{\alpha}x_2^{\beta-2} \tag{B-10}$$

It is obvious that $L_{11} < 0$ because $\alpha < 1$.

And the determinant of the Hessian matrix is:

$$\begin{aligned}
|H| =& L_{11}L_{22} - L_{21}L_{12} \\
=& \left(n_1 n_2 \mathcal{A}\alpha(\alpha - 1)x_1^{\alpha-2}x_2^{\beta} - n_1\kappa\right) n_1 n_2 \mathcal{A}\beta(\beta - 1)x_1^{\alpha}x_2^{\beta-2} - \left(n_1 n_2 \mathcal{A}\alpha\beta x_1^{\alpha-1}x_2^{\beta-1}\right)^2 \\
=& (n_1 n_2 \mathcal{A})^2\alpha\beta(\alpha - 1)(\beta - 1)x_1^{2\alpha-2}x_2^{2\beta-2} - n_1\kappa(n_1 n_2 \mathcal{A})\beta(\beta - 1)x_1^{\alpha}x_2^{\beta-2} - (n_1 n_2 \mathcal{A})^2\alpha^2\beta^2 x_1^{2\alpha-2}x_2^{2\beta-2} \\
=& (n_1 n_2 \mathcal{A})^2\alpha\beta(1 - \alpha - \beta)x_1^{2\alpha-2}x_2^{2\beta-2} + n_1\kappa(n_1 n_2 \mathcal{A})\beta(1 - \beta)x_1^{\alpha}x_2^{\beta-2} \\
>& 0
\end{aligned}$$

So the Hessian matrix is negative definite and hence the problem of the optimal set of incentives is concave globally.

C The Simulation: The Extended Model with a Four-Level Government

C.1 The Data

The county-level output data are from the Financial and Economic Statistics at the City and County Level Sub-database of the EPS China Fiscal and Taxation Database. This is a proprietary database, which consolidates data from the Ministry of Finance of the People's Republic of China and the State Administration of Taxation. Obtain the output data for 2,976 counties in China from 1997 to 2007. We also obtain provincial consumer price index (CPI) from the CEIC China Database, also proprietary, to deflate the output variables.

The sample selection proceeds in several steps. Firstly, after converting the nominal output into real output (normalizing the price level in Anhui in 2007 to 100) and computing the real GDP growth rates, we first trim the top and bottom 5% of the annual growth rates to exclude outliers. The 5th percentile of growth rates corresponds to a gross annual growth rate of 0.8 or a drop of output by 20%, while the 95th percentile of growth rates corresponds to a gross annual growth rate of 1.68 or an increase of output by 68%. Secondly, we drop four municipalities that are under direct control of the central government: Beijing, Tianjin, Shanghai, and Chongqing. Thirdly, we compute the average growth rates for each county over the sample period and drop prefectures under which we have fewer than five county observations. Fourthly, we drop provinces under which we have fewer than five prefectures. The last two steps ensure that each prefecture and each province has sufficiently large numbers of subordinate administrative regions so that it makes sense to compute the variances. We document the change in the number of counties remaining in the sample in Table C-1.

The resulting analysis sample consists of 2,613 counties in 274 prefectures of 25 provinces.

Table C-1: Sample Selection: County-Level Output Data

Operation	Number of counties
(Initial sample)	2,976
Trim top and bottom 5% of county growth rates	2,882
Drop four municipalities under central control	2,840
Drop prefectures with fewer than 5 counties	2,646
Drop provinces with fewer than 5 prefectures	2,613

Notes: The table shows the sample size (i.e. the number of counties) at each operation in the sample selection process.

C.2 The Extended Model with Exogenous Rates of Return

Here we extend the two-stage tournament model in the Model section of the paper to a three-stage tournament with a provincial, a prefectural and a county stage from top to down.

C.2.1 Enforcement of Disciplinary Inspection

For a losing county level official, he is indifferent between corrupt and not corrupt if:

$$W_3 + (1 - \pi_3^*) r_3 K = W_3 + K$$

$$\Rightarrow \quad \pi_3^* = 1 - \frac{1}{r_3}$$

For a losing prefecture level official, he is indifferent between corrupt and not corrupt if:

$$W_2 + (1 - \pi_2^*) r_2 r_3 K = W_2 + r_3 K$$
$$\Rightarrow \quad \pi_2^* = 1 - \frac{1}{r_2}$$

For a losing province level official, he is indifferent between corrupt and not corrupt if:

$$W_1 + (1 - \pi_1^*) r_1 r_2 r_3 K = W_1 + r_2 r_3 K$$
$$\Rightarrow \quad \pi_1^* = 1 - \frac{1}{r_1}$$

C.2.2 Production Decisions Made by the Lower-Level Officials

Provincial Governor's Problem. For the provincial governor of province j:

$$\begin{aligned} V_1 &= \max_{x_j} P(x_j; x_1)\left(W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)\right) + \left(1 - P(x_j; x_1)\right)\left(W_1 + (1 - \pi_1) r_1 r_2 r_3 K\right) - c(x_j) \\ &= \max_{x_j} P(x_j; x_1)\left(W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)\right) + \left(1 - P(x_j; x_1)\right)\left(W_1 + r_2 r_3 K\right) - c(x_j), \end{aligned}$$

where the probability of winning is:

$$P(x_j; x_1) = \int_{\mathbf{R}} G(x_j - x_1 + \eta_j)^{n_1 - 1} g(\eta_j) d\eta_j, \tag{C-1}$$

and $G(\cdot)$ ($g(\cdot)$) is the cdf (pdf) of η_j. Then,

$$\frac{\partial P(x_j; x_1)}{\partial x_j} = \int_{\mathbf{R}} (n_1 - 1) G(x_j - x_1 + \eta_j)^{n_1 - 2} g(x_j - x_1 + \eta_j) g(\eta_j) d\eta_j.$$

$$I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3) = \delta[(\pi_1^* - \bar{\pi}_1)(n_1 - 1) r_1 r_2 r_3 K + (\pi_2^* - \bar{\pi}_2) n_1 (n_2 - 1) r_2 r_3 K + (\pi_3^* - \bar{\pi}_3) n_1 n_2 (n_3 - 1) r_3 K]$$

In particular,

$$I(0, 0, 0) = \delta[(n_1 - 1)(r_1 - 1) r_2 r_3 K + n_1 (n_2 - 1)(r_2 - 1) r_3 K + n_1 n_2 (n_3 - 1)(r_3 - 1) K]$$

The FOC is

$$\frac{\partial P(x_j; x_1)}{\partial x_j}\left(W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3) - W_1 - r_2 r_3 K\right) - c'(x_j) = 0.$$

Evaluated at x_1, it becomes:

$$\int_{\mathbf{R}} (n_1 - 1) G(\eta_j)^{n_1 - 2} g(\eta_j)^2 d\eta_j \left(W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3) - W_1 - r_2 r_3 K\right) = c'(x_1) = \kappa x_1.$$

Let $\Delta W_1 = W_0 - W_1 + (r_1 - 1) r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)$, we have:

$$\frac{t_1 \Delta W_1}{\kappa} = x_1. \tag{C-2}$$

or

$$\Delta W_1 = \frac{\kappa}{t_1} x_1$$

The value function is then

$$V_1 = \frac{1}{n_1}\Delta W_1 + (W_1 + r_2 r_3 K) - \frac{1}{2}\kappa x_1^2. \tag{C-3}$$

The SOC is:

$$\frac{\partial^2 P(x_j; x_1)}{\partial x_j^2} (W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3) - W_1 - r_2 r_3 K) - c''(x_j) < 0.$$

Evaluated at x_1, it becomes:

$$T_1 (W_0 + r_1 r_2 r_3 K + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3) - W_1 - r_2 r_3 K) < \kappa.$$

where

$$T_1 = \frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}|_{x_j = x_1} = \int_{\mathbf{R}} (n_1 - 1) \left[(n_1 - 2) G(\eta_j)^{n_1 - 3} g(\eta_j) + G(\eta_j)^{n_1 - 2} g'(\eta_j) \right] g(\eta_j) d\eta_j.$$

The Prefectural Mayor's Problem. For the prefectural mayor i in province j:

$$\begin{aligned} V_2 &= \max_{x_{ij}} P(x_{ij}; x_2) V_1 + (1 - P(x_{ij}; x_2)) (W_2 + (1 - \pi_2) r_2 r_3 K) - c(x_{ij}) \\ &= \max_{x_{ij}} P(x_{ij}; x_2) V_1 + (1 - P(x_{ij}; x_2)) (W_2 + r_3 K) - c(x_{ij}), \end{aligned}$$

where $P(x_{ij}; x_2)$ is the probability that he will beat all other mayors in province j:

$$P(x_{ij}; x_2) = \int_{\mathbf{R}} F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2 - 1} f(\varepsilon_{ij}) d\varepsilon_{ij}. \tag{C-4}$$

Here $F(\cdot)$ ($f(\cdot)$) is the cdf (pdf) of ε_{ij}. Then,

$$\frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} = \int_{\mathbf{R}} (n_2 - 1) F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2 - 2} f(x_{ij} - x_2 + \varepsilon_{ij}) f(\varepsilon_{ij}) d\varepsilon_{ij}.$$

The FOC is

$$\frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} [V_1 - W_2 - r_3 K] - c'(x_{ij}) = 0.$$

Evaluated at x_2, it becomes:

$$\int_{\mathbf{R}} (n_2 - 1) F(\varepsilon_{ij})^{n_2 - 2} f(\varepsilon_{ij})^2 d\varepsilon_{ij} (V_1 - W_2 - r_3 K) = c'(x_2),$$

Let $\Delta W_2 \equiv W_1 - W_2 + (r_2 - 1) r_3 K$ and we have

$$\frac{t_2}{\kappa} \left(\Delta W_2 + \frac{1}{n_1} \Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2 \right) = x_2, \tag{C-5}$$

or

$$\Delta W_2 = \frac{\kappa}{t_2} x_2 - \frac{1}{n_1} \frac{\kappa}{t_1} x_1 + \frac{1}{2} \kappa x_1^2.$$

The value function is then:

$$V_2 = \frac{1}{n_2}\left(\frac{1}{n_1}\Delta W_1 + \Delta W_2 - \frac{1}{2}\kappa x_1^2\right) + W_2 + r_3 K - \frac{1}{2}\kappa x_2^2. \tag{C-6}$$

The SOC is:

$$\frac{\partial^2 P(x_{ij};x_2)}{\partial x_{ij}^2}\left[V_1 - W_2 - r_3 K\right] < \kappa.$$

Evaluated at x_2, it becomes:

$$T_2\left[V_1 - W_2 - r_3 K\right] - c''(x_{ij}) < 0.$$

where

$$T_2 = \frac{\partial^2 P(x_{ij};x_2)}{\partial x_{ij}^2}|_{x_{ij}=x_2} = \int_{\mathbf{R}} (n_2-1)\left[(n_2-2)F(\varepsilon_{ij})^{n_2-3}f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2-2}f'(\varepsilon_{ij})\right] f(\varepsilon_{ij})d\varepsilon_{ij}.$$

The County Head's Problem. For the county head h in prefecture of i province j:

$$\begin{aligned} V_3 &= \max_{x_{ij}} P(x_{hij};x_3)V_2 + (1-P(x_{hij};x_3))\left(W_3 + (1-\pi_3)r_3 K\right) - c(x_{hij}) \\ &= \max_{x_{hij}} P(x_{hij};x_3)V_2 + (1-P(x_{hij};x_3))\left(W_3 + K\right) - c(x_{hij}), \end{aligned}$$

where $P(x_{hij};x_3)$ is the probability that he will beat all other county heads in prefecture i of province j:

$$P(x_{hij};x_3) = \int_{\mathbf{R}} H(x_{hij} - x_3 + \varepsilon_{hij})^{n_3-1}h(\zeta_{hij})d\zeta_{ij}. \tag{C-7}$$

Here $H(\cdot)$ $(h(\cdot))$ is the cdf (pdf) of ζ_{hij}. Then,

$$\frac{\partial P(x_{hij};x_3)}{\partial x_{hij}} = \int_{\mathbf{R}} (n_3-1)H(x_{hij} - x_3 + \zeta_{hij})^{n_3-2}h(x_{hij} - x_2 + \zeta_{hij})h(\zeta_{ij})d\zeta_{hij}.$$

The FOC is

$$\frac{\partial P(x_{hij};x_3)}{\partial x_{hij}}\left[V_2 - W_3 - K\right] - c'(x_{hij}) = 0.$$

Evaluated at x_3, it becomes:

$$\int_{\mathbf{R}} (n_3-1)F(\zeta_{hij})^{n_3-2}f(\zeta_{hij})^2 d\zeta_{hij}\left(V_2 - W_3 - K\right) = c'(x_2),$$

Let $\Delta W_3 \equiv W_2 - W_3 + (r_3-1)K$ and we have

$$\frac{t_3}{\kappa}\left[\frac{1}{n_2}\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right) + \Delta W_3 - \frac{1}{2\kappa}t_2^2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right)^2\right] = x_3, \tag{C-8}$$

or

$$\Delta W_3 = \frac{\kappa}{t_3}x_3 - \frac{1}{n_2}\frac{\kappa}{t_2}x_2 + \frac{1}{2}\kappa x_2^2.$$

The SOC is:

$$\frac{\partial^2 P(x_{hij}; x_3)}{\partial x_{hij}^2} [V_2 - W_3 - K] - c''(x_{hij}) < 0.$$

Evaluated at x_3, it becomes:

$$T_3 (V_2 - W_3 - K) < \kappa,$$

where

$$T_3 = \frac{\partial^2 P(x_{hij}; x_3)}{\partial x_{hij}^2}|_{x_{hij}=x_3} = \int_{\mathbf{R}} (n_3 - 1) \left[(n_3 - 2) H(\zeta_{hij})^{n_3-3} h(\zeta_{hij}) + H(\zeta_{hij})^{n_3-2} f'(\zeta_{hij})\right] h(\zeta_{hij}) d\zeta_{hij}.$$

C.2.3 The Optimal Set of Incentives

To find the wage scale $\{W_0, W_1, W_2, W_3\}$ and the minimal fractions of corruption cases $(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)$, we maximize the expected total final output net the cost of incentives:

$$\max_{W_0, W_1, W_2, W_3, \bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3} \quad E(n_1 n_2 n_3 \mathcal{A} q_1^{\alpha} q_2^{\beta} q_3^{\gamma}) - W_0 - (n_1 - 1) W_1 - (n_2 - 1) n_1 W_2 - (n_3 - 1) n_2 n_1 W_3 \tag{C-9}$$

$$\Leftrightarrow \max_{W_0, W_1, W_2, W_3, \bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3} \quad n_1 n_2 n_3 \mathcal{A} x_1^{\alpha} x_2^{\beta} x_3^{\gamma} - \Delta W_1 - n_1 \Delta W_2 - n_1 n_2 \Delta W_3 - n_1 n_2 n_3 W_3 + I(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)$$
$$+ (r_1 - 1) r_2 r_3 K + n_1 (r_2 - 1) r_3 K + n_1 n_2 (r_3 - 1) K, \tag{C-10}$$

subject to the incentive compatibility constraints of the provincial governors, prefectural mayors and county heads, (C-2), (C-5) and (C-8). This is a standard problem. The optimal W_3 is set to 0. The minimal fractions of corruptions cases are set to $\bar{\pi}_1 = \bar{\pi}_2 = \bar{\pi}_3 = 0$. Then the problem equals to the one which optimizes on x_1, x_2, x_3 after substituting $\Delta W_1, \Delta W_2, \Delta W_3$ for x_1, x_2, x_3.

$$\max_{x_1, x_2, x_3 \geq 0} \quad n_1 n_2 n_3 \mathcal{A} x_1^{\alpha} x_2^{\beta} x_3^{\gamma} - \frac{1}{2} n_1 \kappa x_1^2 - \frac{1}{2} n_1 n_2 \kappa x_2^2 - n_1 n_2 \frac{\kappa}{t_3} x_3 + (1 + \delta(n_1 - 1))(r_1 - 1) r_2 r_3 K \tag{C-11}$$
$$+ n_1 (1 + \delta(n_2 - 1))(r_2 - 1) r_3 K + n_1 n_2 (1 + \delta(n_3 - 1))(r_3 - 1) K.$$

The FOC with respect to x_1, x_2, x_3 are:

$$x_1 : n_1 n_2 n_3 \mathcal{A} \alpha x_1^{\alpha-1} x_2^{\beta} x_3^{\gamma} - n_1 \kappa x_1 = 0. \tag{C-12}$$
$$x_2 : n_1 n_2 n_3 \mathcal{A} \beta x_1^{\alpha} x_2^{\beta-1} x_3^{\gamma} - n_1 n_2 \kappa x_2 = 0. \tag{C-13}$$
$$x_1 : n_1 n_2 n_3 \mathcal{A} \gamma x_1^{\alpha} x_2^{\beta} x_3^{\gamma-1} - n_1 n_2 \frac{\kappa}{t_3} = 0. \tag{C-14}$$

The optimal efforts in three levels are:

$$x_1 = \left(\mathcal{A} \alpha^{1-\frac{\beta}{2}-\gamma} \beta^{\frac{\beta}{2}} \gamma^{\gamma} n_2^{1-\frac{\beta}{2}-\gamma} n_3 t_3^{\gamma} \kappa^{-1}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{C-15}$$

$$x_2 = \left(\mathcal{A} \alpha^{\frac{\alpha}{2}} \beta^{1-\frac{\alpha}{2}-\gamma} \gamma^{\gamma} n_2^{\frac{\alpha}{2}} n_3 t_3^{\gamma} \kappa^{-1}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{C-16}$$

$$x_3 = \left(\mathcal{A}^2 \alpha^{\alpha} \beta^{\beta} \gamma^{2-\alpha-\beta} n_2^{\alpha} n_3^2 t_3^{2-\alpha-\beta} \kappa^{-2}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{C-17}$$

C.3 The Extended Model with Endogenous Rates of Return

Here in this section, we characterize the solution of the tournament with endogenous rate of return on private wealth. Basically, we first nail the disciplinary probability of each level down and then obtain the optimal effort of managers of each level by solving the Bellman equations. Then we solve the optimal set of incentives by maximizing the total outputs net costs of incentives given the incentive compatibility constraints of the managers (i.e. their FOC conditions).

C.3.1 Enforcement of Disciplinary Inspection

For a losing county level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_3 + (1-\pi_3^*(\cdot))r_3(x_{hij})K = W_3 + K \\ \Rightarrow\quad & \pi_3^*(x_{hij}) = 1 - \frac{1}{r_3(x_{hij})} \end{aligned}$$

For a losing prefecture level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_2 + (1-\pi_2^*(\cdot))r_2(x_ij)r_3(x_{hij})K = W_2 + r_3(x_{hij})K \\ \Rightarrow\quad & \pi_2^*(x_{ij}) = 1 - \frac{1}{r_2(x_{ij})} \end{aligned}$$

For a losing province level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_1 + (1-\pi_1^*(\cdot))r_1(x_j)r_2(x_{ij})r_3(x_{hij})K = W_1 + r_2(x_{ij})r_3(x_{hij})K \\ \Rightarrow\quad & \pi_1^*(x_j) = 1 - \frac{1}{r_1(x_j)} \end{aligned}$$

C.3.2 Production Decisions Made by the Lower-Level Officials

We solve the lower-level officials' problems by backward induction of the Bellman equations. Importantly, at each level, the officials consider the rates of return of private wealth from the past when they were at lower levels as given, but the officials understand that their current choice of effort will impact future choices at higher levels through the endogenous return. Then officials' objective functions of each level are maximized over efforts.

Provincial Governor's Problem The governor of province j maximizes the expected payoff by exerting effort to compete.

$$\begin{aligned} V_1 = \max_{x_j} & P(x_j;x_1)\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K + I(\bar\pi_1,\bar\pi_2,\bar\pi_3)\right) \\ & + (1-P(x_j;x_1))\left(W_1 + (1-\pi_1(x_j))r_1(x_j)r_2(x_2)r_3(x_3)K\right) - c(x_j) \\ = \max_{x_j} & P(x_j;x_1)\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K + I(\bar\pi_1,\bar\pi_2,\bar\pi_3)\right) + (1-P(x_j;x_1))\left(W_1 + r_2(x_2)r_3(x_3)K\right) - c(x_j), \end{aligned} \tag{C-18}$$

where the probability of winning is:

$$P(x_j;x_1) = \int_{\mathbf{R}} G(x_j - x_1 + \eta_j)^{n_1-1} g(\eta_j)d\eta_j,$$

and $G(\cdot)$ ($g(\cdot)$) is the cdf (pdf) of η_j. Then,

$$\frac{\partial P(x_j;x_1)}{\partial x_j} = \int_{\mathbf{R}} (n_1-1)G(x_j-x_1+\eta_j)^{n_1-2} g(x_j-x_1+\eta_j)g(\eta_j)d\eta_j.$$

And

$$\begin{aligned} I(\overline{\pi}_1,\overline{\pi}_2,\overline{\pi}_3) = \delta[&(\pi_1^*-\overline{\pi}_1)(n_1-1)r_1(x_j)r_2(x_{ij})r_3(x_{hij})K + (\pi_2^*-\overline{\pi}_2)n_1(n_2-1)r_2(x_{ij})r_3(x_{hij})K \\ &+ (\pi_3^*-\overline{\pi}_3)n_1n_2(n_3-1)r_3(x_{hij})K]. \end{aligned}$$

Specifically $I(0,0,0)$, which will be optimally chosen for the same reason as in the benchmark case, is given by:

$$\begin{aligned} I(0,0,0) = \delta[&(n_1-1)(r_1(x_j)-1)r_2(x_{ij})r_3(x_{hij})K + n_1(n_2-1)(r_2(x_{ij})-1)r_3(x_{hij})K \\ &+ n_1n_2(n_3-1)(r_3(x_{hij})-1)K] \equiv I(x_j,x_{ij},x_{hij}). \end{aligned}$$

The FOC is:

$$\begin{aligned} &\frac{\partial P(x_j;x_1)}{\partial x_j}\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K + I(x_j,x_2,x_3) - W_1 - r_2(x_2)r_3(x_3)K\right) \\ &+ P(x_j;x_1)r_1'(x_j)r_2(x_2)r_3(x_3)K + P(x_j;x_1)I'_{x_j} - c'(x_j) = 0. \end{aligned}$$

Evaluated at x_1:

$$t_1\left(W_0 - W_1 + (r_1(x_1)-1)\,r_2(x_2)r_3(x_3)K + I(x_1,x_2,x_3)\right) + \frac{1}{n_1}I'_{x_1} + \frac{1}{n_1}r_1'(x_1)r_2(x_2)r_3(x_3)K = \kappa x_1 \tag{C-19}$$

where

$$t_1 = \int_{\mathbf{R}} (n_1-1)G(\eta_j)^{n_1-2}g(\eta_j)^2 d\eta_j \tag{C-20}$$

and

$$I'_{x_1} = \delta(n_1-1)r_1^{'}(x_1)r_2(x_2)r_3(x_3)K.$$

The value function is

$$V_1 = \frac{1}{n_1}\left(W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K + I(x_1,x_2,x_3)\right) + W_1 + r_2(x_2)r_3(x_3)K - \frac{1}{2}\kappa x_1^2. \tag{C-21}$$

The SOC is:

$$\begin{aligned} &\frac{\partial^2 P(x_j;x_1)}{\partial x_j^2}(W_0 - W_1 + (r_1(x_j)-1)r_2(x_2)r_3(x_3)K + I(x_j,x_2,x_3)) \\ &+2\frac{\partial P(x_j;x_1)}{\partial x_j}[r_1'(x_j)r_2(x_2)r_3(x_3)K + I^{'}_{x_j}] + P(x_j;x_1)[r_1''(x_j)r_2(x_2)r_3(x_3)K + I^{''}_{x_j,x_j}] - \kappa \end{aligned} \tag{C-22}$$

where

$$I^{''}_{x_j,x_j} = \delta(n_1-1)r_1^{''}(x_j)r_2(x_2)r_3(x_3)K.$$

Evaluated at x_1:

$$T_1(W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K + I(x_1,x_2,x_3))$$

$$+2t_1 r_1'(x_1) r_2(x_2) r_3(x_3) K + 2t_1 I_{x_1}^{'} + \frac{1}{n_1} r_1''(x_1) r_2(x_2) r_3(x_3) K + \frac{1}{n_1} I_{x_1,x_1}^{''} - \kappa < 0 \tag{C-23}$$

where

$$T_1 = \frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}|_{x_j = x_1} = \int_{\mathbf{R}} (n_1 - 1) \left[(n_1 - 2) G(\eta_j)^{n_1 - 3} g(\eta_j) + G(\eta_j)^{n_1 - 2} g'(\eta_j) \right] g(\eta_j) d\eta_j. \tag{C-24}$$

Prefectoral Mayor's Problem. The mayor of prefecture i in province j maximizes the expected payoff of the competition.

$$\begin{aligned} V_2 =& \max_{x_{ij}} P(x_{ij}; x_2) V_1 + (1 - P(x_{ij}; x_2)) \left(W_2 + (1 - \pi_2(x_{ij})) r_2(x_{ij}) r_3(x_3) K \right) - c(x_{ij}) \\ =& \max_{x_{ij}} P(x_{ij}; x_2) \{ \frac{1}{n_1} \left[W_0 - W_1 + (r_1(x_1(x_{ij})) - 1) r_2(x_{ij}) r_3(x_3) K + I(x_1, x_{ij}, x_3) \right] + W_1 \\ & + r_2(x_{ij}) r_3(x_3) K - c(x_1(x_{ij})) \big\} + (1 - P(x_{ij}; x_2)) \left(W_2 + r_3(x_3) K \right) - c(x_{ij}), \end{aligned} \tag{C-25}$$

where $P(x_{ij}; x_2)$ is the probability that ij will come out first in his group:

$$P(x_{ij}; x_2) = \int_{\mathbf{R}} F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2 - 1} f(\varepsilon_{ij}) d\varepsilon_{ij}.$$

Here $F(\cdot)$ ($f(\cdot)$) is the cdf (pdf) of ε_{ij}. Then,

$$\frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} = \int_{\mathbf{R}} (n_2 - 1) F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2 - 2} f(x_{ij} - x_2 + \varepsilon_{ij}) f(\varepsilon_{ij}) d\varepsilon_{ij}.$$

The FOC is

$$\begin{aligned} & \frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} \left\{ \frac{1}{n_1} \left[W_0 - W_1 + (r_1(x_1(x_{ij})) - 1) r_2(x_{ij}) r_3(x_3) K + I(x_1, x_{ij}, x_3) \right] + W_1 + r_2(x_{ij}) r_3(x_3) K \right. \\ & \left. - c(x_1(x_{ij})) - W_2 - r_3(x_3) K \right\} + P(x_{ij}; x_2) \left\{ \frac{1}{n_1} \left[r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'}(x_{ij}) r_2(x_{ij}) + (r_1(x_1) - 1) r_2^{'}(x_{ij}) \right] r_3(x_3) K \right. \\ & \left. + \frac{1}{n_1} \left[I_{x_j}' x_1^{'}(r_2) r_2^{'}(x_{ij}) + I_{x_{ij}}' \right] + r_2^{'}(x_{ij}) r_3(x_3) K - \kappa x_1 x_1^{'}(r_2) r_2^{'}(x_{ij}) \right\} = \kappa x_{ij} \end{aligned}$$

where

$$I_{x_{ij}}^{'} = \delta [(n_1 - 1)(r_1(x_1) - 1) r_2^{'}(x_{ij}) r_3 K + n_1 (n_2 - 1) r_2^{'}(x_{ij}) r_3 K].$$

Evaluated at x_2:

$$\begin{aligned} & t_2 \left\{ \frac{1}{n_1} \left[W_0 - W_1 + (r_1(x_1) - 1) r_2(x_2) r_3(x_3) K + I(x_1, x_2, x_3) \right] + W_1 - W_2 + (r_2(x_2) - 1) r_3(x_3) K - \frac{1}{2} \kappa x_1^2 \right\} \\ & + \frac{1}{n_2} \left\{ \frac{1}{n_1} \left[r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'}(x_2) r_2(x_2) + (r_1(x_1) - 1) r_2^{'}(x_2) \right] r_3(x_3) K + \frac{1}{n_1} \left[I_{x_1}' x_1^{'}(r_2) r_2^{'}(x_2) + I_{x_2}' \right] \right. \\ & \left. + r_2^{'}(x_2) r_3(x_3) K - \kappa x_1 x_1^{'}(r_2) r_2^{'}(x_2) \right\} = \kappa x_2 \end{aligned} \tag{C-26}$$

where

$$t_2 = \int_{\mathbf{R}} (n_2 - 1) F(\varepsilon_{ij})^{n_2 - 2} f(\varepsilon_{ij})^2 d\varepsilon_{ij}, \tag{C-27}$$

The value function is:

$$V_2 = \frac{1}{n_2}\left\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K + I(x_1,x_2,x_3)\right] + W_1 - W_2 + (r_2(x_2)-1)r_3(x_3)K - \frac{1}{2}\kappa x_1^2\right\} + W_2 + r_3(x_3)K - \frac{1}{2}\kappa x_2^2 \tag{C-28}$$

The SOC of the second level officials' problem is:

$$\begin{aligned}
&\frac{\partial^2 P(x_{ij};x_2)}{\partial x_{ij}^2}\left\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1(x_{ij}))-1)r_2(x_{ij})r_3(x_3)K + I(x_1,x_{ij},x_3)\right] + W_1\right. \\
&\left. + r_2(x_{ij})r_3(x_3)K - \frac{1}{2}x_1^2 - W_2 - r_3(x_3)K\right\} \\
&+2\frac{\partial P(x_{ij};x_2)}{\partial x_{ij}}\left\{\frac{1}{n_1}\left[r_1'(x_1)x_1'(r_2)r_2'(x_{ij})r_2(x_{ij}) + (r_1(x_1)-1)r_2'(x_{ij})\right]r_3(x_3)K\right. \\
&\left.+\frac{1}{n_1}\left[I'_{x_j}x_1'(r_2)r_2'(x_{ij}) + I'_{x_{ij}}\right] + r_2'(x_{ij})r_3(x_3)K - \kappa x_1 x_1'(r_2)r_2'(x_{ij})\right\} \\
&+P(x_{ij};x_2)\left\{\frac{1}{n_1}[r_1''(x_1)x_1'^2(r_2)r_2'^2(x_{ij})r_2(x_{ij}) + r_1'(x_1)x_1''(r_2)r_2'^2(x_{ij})r_2(x_{ij})\right. \\
&+r_1'(x_1)x_1'(r_2)r_2''(x_{ij})r_2(x_{ij}) + r_1'(x_1)x_1'(r_2)r_2'^2(x_{ij}) + r_1'(x_1)x_1'(r_2)r_2'^2(x_{ij}) \\
&+(r_1(x_1)-1)r_2''(x_{ij})]r_3(x_3)K \\
&+\frac{1}{n_1}\left[I''_{x_j,x_j}x_1'^2(r_2)r_2'^2(x_{ij}) + I''_{x_j,x_{ij}}x_1'(r_2)r_2'(x_{ij}) + I'_{x_j}x_1''(r_2)r_2'^2(x_{ij}) + I'_{x_j}x_1'(r_2)r_2''(x_{ij})\right] \\
&+\frac{1}{n_1}\left[I''_{x_{ij},x_j}x_1'(r_2)r_2'(x_{ij}) + I''_{x_{ij},x_{ij}}\right] \\
&+r_2''(x_{ij})r_3(x_3)K - \kappa x_1'^2(r_2)r_2'^2(x_{ij}) \\
&\left.-\kappa x_1 x_1''(r_2)r_2'^2(x_{ij}) - \kappa x_1 x_1'(r_2)r_2''(x_{ij})\right\} - \kappa \\
&<0
\end{aligned} \tag{C-29}$$

where

$$\begin{aligned}
I''_{x_j,x_{ij}} &= \delta(n_1-1)r_1'(x_j)r_2'(x_{ij})r_3(x_3)K \\
I''_{x_{ij},x_{ij}} &= \delta\left[(n_1-1)(r_1(x_1)-1)r_2''(x_{ij})r_3(x_3)K + n_1(n_2-1)r_2''(x_{ij})r_3K\right]
\end{aligned}$$

Evaluated at x_2:

$$\begin{aligned}
\kappa > &T_2\left\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K + I(x_1,x_2,x_3)\right]\right. \\
&\left.+ W_1 + r_2(x_2)r_3(x_3)K - \frac{1}{2}x_1^2 - W_2 - r_3(x_3)K\right\} \\
&+2t_2\left\{\frac{1}{n_1}\left[r_1'(x_1)x_1'(r_2)r_2'(x_2)r_2(x_2) + (r_1(x_1)-1)r_2'(x_2)\right]r_3(x_3)K\right.
\end{aligned} \tag{C-30}$$

$$
\begin{aligned}
&+ \frac{1}{n_1} I'_{x_1} x_1^{'}(r_2) r_2^{'}(x_2) + I'_{x_2} + r_2^{'}(x_2) r_3(x_3) K - \kappa x_1 x_1^{'}(r_2) r_2^{'}(x_2) \Big\} \\
&+ \frac{1}{n_2} \left\{ \frac{1}{n_1} [r_1^{''}(x_1) x_1^{'2}(r_2) r_2^{'2}(x_2) r_2(x_2) + r_1^{'}(x_1) x_1^{''}(r_2) r_2^{'2}(x_2) r_2(x_2) \right. \\
&+ r_1^{'}(x_1) x_1^{'}(r_2) r_2^{''}(x_2) r_2(x_2) + 2 r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'2}(x_2) \\
&+ (r_1(x_1) - 1) r_2^{''}(x_2)] r_3(x_3) K \\
&+ \frac{1}{n_1} \left[I^{''}_{x_1,x_1} x_1^{'2}(r_2) r_2^{'2}(x_2) + I^{''}_{x_1,x_2} x_1^{'}(r_2) r_2^{'}(x_2) + I^{'}_{x_1} x_1^{''}(r_2) r_2^{'2}(x_2) + I^{'}_{x_1} x_1^{'}(r_2) r_2^{''}(x_2) \right] \\
&+ \frac{1}{n_1} \left[I^{''}_{x_2,x_1} x_1^{'}(r_2) r_2^{'}(x_2) + I^{''}_{x_2,x_2} \right] \\
&+ r_2^{''}(x_2) r_3(x_3) K - \kappa x_1^{'2}(r_2) r_2^{'2}(x_2) \\
&\left. - \kappa x_1 x_1^{''}(r_2) r_2^{'2}(x_2) - \kappa x_1 x_1^{'}(r_2) r_2^{''}(x_2) \right\},
\end{aligned}
$$

where

$$
T_2 = \frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2} |_{x_{ij}=x_2} = \int_{\mathbf{R}} (n_2 - 1) \left[(n_2 - 2) F(\varepsilon_{ij})^{n_2-3} f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2-2} f`(\varepsilon_{ij}) \right] f(\varepsilon_{ij}) d\varepsilon_{ij}. \tag{C-31}
$$

County Head's Problem. The head of county h in prefecture i of province j maximizes his expected payoff by exerting effort in the competition.

$$
V_3 = \max_{x_{hij}} P(x_{hij}; x_3) V_2 + (1 - P(x_{hij}; x_3)) (W_3 + (1 - \pi_3(x_{hij})) r_3(x_{hij}) K) - c(x_{ij}) \tag{C-32}
$$

$$
= \max_{x_{iij}} P(x_{iij}; x_2) \left\{ \frac{1}{n_2} \left\{ \frac{1}{n_1} [W_0 - W_1 + (r_1(x_1) - 1) r_2(x_2) r_3(x_{hij}) K + I(x_1, x_2, x_3)] + W_1 - W_2 \right. \right. \tag{C-33}
$$

$$
\left. \left. + (r_2(x_2) - 1) r_3(x_{hij}) K - \frac{1}{2} \kappa x_1^2 \right\} + W_2 + r_3(x_{hij}) K - \frac{1}{2} \kappa x_2^2 \right\}
$$

$$
+ (1 - P(x_{hij}; x_3)) W_3 - \frac{1}{2} \kappa x_{hij}^2,
$$

where $P(x_{hij}; x_3)$ is the probability that county hij comes out first in his group:

$$
P(x_{hij}; x_3) = \int_{\mathbf{R}} H(x_{hij} - x_3 + \zeta_{hij})^{n_3-1} h(\zeta_{hij}) d\zeta_{hij}.
$$

Here $H(\cdot)$ ($h(\cdot)$) is the cdf (pdf) of ζ_{hij}. Then,

$$
\frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}} = \int_{\mathbf{R}} (n_3 - 1) H(x_{hij} - x_3 + \zeta_{ij})^{n_3-2} h(x_{hij} - x_3 + \zeta_{hij}) h(\zeta_{hij}) d\zeta_{hij}.
$$

The FOC is:

$$
\frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}} \left\{ \frac{1}{n_2} \left\{ \frac{1}{n_1} [W_0 - W_1 + (r_1(x_1) - 1) r_2(x_2) r_3(x_{hij}) K + I(x_1, x_2, x_{hij})] \right. \right. \tag{C-34}
$$

$$
\left. \left. + W_1 - W_2 + (r_2(x_2) - 1) r_3(x_{hij}) K - \frac{1}{2} \kappa x_1^2 \right\} + W_2 - W_3 + (r_3(x_{hij}) - 1) K - \frac{1}{2} \kappa x_2^2 \right\}
$$

$$
\begin{aligned}
&+P(x_{hij};x_3)\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_2(x_2)r_3(x_{hij})\right.\right.\right.\\
&\left.+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij})+(r_1(x_1)-1)r_2(x_2)r_3^{'}(x_{hij})\right]K\\
&+\frac{1}{n_1}\left[I_{x_j}^{'}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})+I_{x_{ij}}^{'}x_2^{'}(r_3)r_3^{'}(x_{hij})+I_{x_{hij}}^{'}\right]\\
&+r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij})K+(r_2(x_2)-1)r_3^{'}(x_{hij})K\\
&\left.\left.-\ \kappa x_1x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})\right\}+r_3^{'}(x_{hij})K-\kappa x_2x_2^{'}(r_3)r_3^{'}(x_{hij})\right\}\\
&-\kappa x_{hij}=0
\end{aligned}
$$

Evaluated at x_3:

$$
\begin{aligned}
&t_3\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[W_0-W_1+(r_1(x_1)-1)r_2(x_2)r_3(x_3)K+I(x_1,x_2,x_3)\right]+W_1-W_2+(r_2(x_2)-1)r_3(x_3)K\right.\right. \qquad \text{(C-35)}\\
&\left.\left.-\ \frac{1}{2}\kappa x_1^2\right\}+W_2-W_3+(r_3(x_3)-1)K-\frac{1}{2}\kappa x_2^2\right\}\\
&+\frac{1}{n_3}\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_2(x_2)r_3(x_3)\right.\right.\right.\\
&\left.+\ (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3)+(r_1(x_1)-1)r_2(x_2)r_3^{'}(x_3)\right]K\\
&+\frac{1}{n_1}\left[I_{x_1}^{'}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)+I_{x_2}^{'}x_2^{'}(r_3)r_3^{'}(x_3)+I_{x_3}^{'}\right]\\
&+r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3)K+(r_2(x_2)-1)r_3^{'}(x_3)K\\
&\left.\left.-\ \kappa x_1x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)\right\}+r_3^{'}(x_3)K-\kappa x_2x_2^{'}(r_3)r_3^{'}(x_3)\right\}\\
&-\kappa x_3=0,
\end{aligned}
$$

where

$$
t_3=\frac{\partial P(x_{hij};x_3)}{\partial x_{hij}}|_{x_{hij}=x_3}=\int_{\mathbf{R}}(n_3-1)H(\zeta_{hij})^{n_3-2}h(\zeta_{hij})^2d\zeta_{hij},
$$

and

$$
\begin{aligned}
I_{x_{hij}}^{'}=\delta&\left[(n_1-1)(r_1(x_1)-1)r_2(x_2)r_3^{'}(x_{hij})K+n_1(n_2-1)(r_2(x_2)-1)r_3^{'}(x_{hij})K\right.\\
&\left.+n_1n_2(n_3-1)r_3^{'}(x_{hij})K\right].
\end{aligned}
$$

The SOC is:

$$
\begin{aligned}
&\frac{\partial^2P(x_{hij};x_3)}{\partial x_{hij}^2}\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[W_0-W_1+(r_1(x_1)-1)r_2(x_2)r_3(x_{hij})K+I(x_1,x_2,x_{hij})\right]+W_1-W_2\right.\right. \qquad \text{(C-36)}\\
&\qquad\left.\left.+\ (r_2(x_2)-1)r_3(x_{hij})K-\frac{1}{2}\kappa x_1^2\right\}+W_2-W_3+(r_3(x_{hij})-1)K-\frac{1}{2}\kappa x_2^2\right\}\\
&\qquad+2\frac{\partial P(x_{hij};x_3)}{\partial x_{hij}}\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_2(x_2)r_3(x_{hij})\right.\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij}) + (r_1(x_1)-1)r_2(x_2)r_3^{'}(x_{hij})\Big] K \\
&+\frac{1}{n_1}\left[I_{x_j}^{'}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij}) + I_{x_{ij}}^{'}x_2^{'}(r_3)r_3^{'}(x_{hij}) + I_{x_{hij}}^{'}\right] \\
&+r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij})K + (r_2(x_2)-1)r_3^{'}(x_{hij})K \\
&- \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})\Big\} + r_3^{'}(x_{hij})K - \kappa x_2 x_2^{'}(r_3)r_3^{'}(x_{hij})\Big\} \\
&+P(x_{hij};x_3)\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_2(x_2)r_3(x_{hij})\right.\right.\right. \\
&+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_2(x_2)r_3^{'}(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_3(x_{hij}) \\
&+(r_1(x_1)-1)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_3(x_{hij}) \\
&+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_{hij})r_3(x_{hij}) \\
&+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_{hij})r_3(x_{hij}) \\
&+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_{hij}) \\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_{hij})r_2(x_2) \\
&+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_{hij}) \\
&+ (r_1(x_1)-1)r_2(x_2)r_3^{''}(x_{hij})\Big] K \\
&+extraTermCountySOC1 \\
&-\kappa x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij}) \\
&-\kappa x_1 x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij}) \\
&-\kappa x_1 x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij}) \\
&-\kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_{hij}) \\
&-\kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_{hij}) \\
&+r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_{hij})r_3(x_{hij})K + r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_{hij})r_3(x_{hij})K \\
&+r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_{hij})r_3(x_{hij})K + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_{hij})K \\
&+ r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_{hij})K + (r_2(x_2)-1)r_3^{''}(x_{hij})K\Big\} \\
&+ r_3^{''}(x_{hij})K - \kappa x_2^{'2}(r_3)r_3^{'2}(x_{hij}) - \kappa x_2 x_2^{''}(r_3)r_3^{'2}(x_{hij}) - \kappa x_2 x_2^{'}(r_3)r_3^{''}(x_{hij})\Big\} \\
&-\kappa < 0,
\end{aligned}
$$

where

$$extraTermCountySOC1$$

$$
\begin{aligned}
=&\frac{1}{n_1}\Big[I''_{x_j,x_j}x_1'^2(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_{hij})+I''_{x_j,x_{ij}}x_1'(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_{hij})\\
&+I''_{x_j,x_{hij}}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_{hij})+I'_{x_j}x_1''(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_{hij})\\
&+I'_{x_j}x_1'(r_2)r_2''(x_2)x_2'^2(r_3)r_3'^2(x_{hij})+I'_{x_j}x_1'(r_2)r_2'(x_2)x_2''(r_3)r_3'^2(x_{hij})+I'_{x_j}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3''(x_{hij})\\
&+I''_{x_{ij},x_j}x_1'(r_2)r_2'(x_2)x_2'^2(r_3)r_3'^2(x_{hij})+I''_{x_{ij},x_{ij}}x_2'^2(r_3)r_3'^2(x_{hij})+I''_{x_{ij},x_{hij}}x_2'(r_3)r_3'(x_{hij})\\
&+I'_{x_{ij}}x_2''(r_3)r_3'^2(x_{hij})+I'_{x_{ij}}x_2'(r_3)r_3''(x_{hij})\\
&+I''_{x_{hij},x_j}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_{hij})\\
&+I''_{x_{hij},x_{ij}}x_2'(r_3)r_3'(x_{hij})\\
&+I''_{x_{hij},x_{hij}}\Big].
\end{aligned}
$$

Evaluated at x_3:

$$
\begin{aligned}
\kappa >&T_3\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\left[W_0-W_1+(r_1(x_1)-1)r_2(x_2)r_3(x_3)K+I(x_1,x_2,x_3)\right]\right.\right. \qquad \text{(C-37)}\\
&\left.\left.+W_1-W_2+(r_2(x_2)-1)r_3(x_3)K-\frac{1}{2}\kappa x_1^2\right\}+W_2-W_3+(r_3(x_3)-1)K-\frac{1}{2}\kappa x_2^2\right\}\\
&+2t_3\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\Big[r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)r_2(x_2)r_3(x_3)\right.\right.\\
&+(r_1(x_1)-1)r_2'(x_2)x_2'(r_3)r_3'(x_3)r_3(x_3)+(r_1(x_1)-1)r_2(x_2)r_3'(x_3)\Big]K\\
&+\frac{1}{n_1}\Big[I'_{x_1}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)+I'_{x_2}x_2'(r_3)r_3'(x_3)+I'_{x_3}\Big]\\
&+r_2'(x_2)x_2'(r_3)r_3'(x_3)r_3(x_3)K+(r_2(x_2)-1)r_3'(x_3)K\\
&\left.\left.-\kappa x_1x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)\right\}+r_3'(x_3)K-\kappa x_2x_2'(r_3)r_3'(x_3)\right\}\\
&+\frac{1}{n_3}\left\{\frac{1}{n_2}\left\{\frac{1}{n_1}\Big[r_1''(x_1)x_1'^2(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)r_2(x_2)r_3(x_3)\right.\right.\\
&+r_1'(x_1)x_1''(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)r_2(x_2)r_3(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2''(x_2)x_2'^2(r_3)r_3'^2(x_3)r_2(x_2)r_3(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2''(r_3)r_3'^2(x_3)r_2(x_2)r_3(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3''(x_3)r_2(x_2)r_3(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)r_2'(x_2)x_2'(r_3)r_3'(x_3)r_3(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)r_2(x_2)r_3'(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)r_3(x_3)\\
&+(r_1(x_1)-1)r_2''(x_2)x_2'^2(r_3)r_3'^2(x_3)r_3(x_3)\\
&+(r_1(x_1)-1)r_2'(x_2)x_2''(r_3)r_3'^2(x_3)r_3(x_3)\\
&+(r_1(x_1)-1)r_2'(x_2)x_2'(r_3)r_3''(x_3)r_3(x_3)\\
&+(r_1(x_1)-1)r_2'(x_2)x_2'(r_3)r_3'^2(x_3)\\
&+r_1'(x_1)x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'^2(x_3)r_2(x_2)
\end{aligned}
$$

$$
\begin{aligned}
&+(r_1(x_1)-1)r_2'(x_2)x_2'(r_3)r_3'^2(x_3)\\
&+\left.(r_1(x_1)-1)r_2(x_2)r_3''(x_3)\right]K\\
&+extraTermCountySOC2\\
&-\kappa x_1'^2(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)\\
&-\kappa x_1 x_1''(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)\\
&-\kappa x_1 x_1'(r_2)r_2''(x_2)x_2'^2(r_3)r_3'^2(x_3)\\
&-\kappa x_1 x_1'(r_2)r_2'(x_2)x_2''(r_3)r_3'^2(x_3)\\
&-\kappa x_1 x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3''(x_3)\\
&+r_2''(x_2)x_2'^2(r_3)r_3'^2(x_3)r_3(x_3)K+r_2'(x_2)x_2''(r_3)r_3'^2(x_3)r_3(x_3)K\\
&+r_2'(x_2)x_2'(r_3)r_3''(x_3)r_3(x_3)K+r_2'(x_2)x_2'(r_3)r_3'^2(x_3)K\\
&+\left.r_2'(x_2)x_2'(r_3)r_3'^2(x_3)K+(r_2(x_2)-1)r_3''(x_3)K\right\}\\
&+\left.r_3''(x_3)K-\kappa x_2'^2(r_3)r_3'^2(x_3)-\kappa x_2 x_2''(r_3)r_3'^2(x_3)-\kappa x_2 x_2'(r_3)r_3''(x_3)\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
&extraTermCountySOC2\\
=&\frac{1}{n_1}\left[I''_{x_1,x_1}x_1'^2(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)+I''_{x_1,x_2}x_1'(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)\right.\\
&+I''_{x_1,x_3}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)+I'_{x_1}x_1''(r_2)r_2'^2(x_2)x_2'^2(r_3)r_3'^2(x_3)\\
&+I'_{x_1}x_1'(r_2)r_2''(x_2)x_2'^2(r_3)r_3'^2(x_3)+I'_{x_1}x_1'(r_2)r_2'(x_2)x_2''(r_3)r_3'^2(x_3)+I'_{x_1}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3''(x_3)\\
&+I''_{x_2,x_1}x_1'(r_2)r_2'(x_2)x_2'^2(r_3)r_3'^2(x_3)+I''_{x_2,x_2}x_2'^2(r_3)r_3'^2(x_3)+I''_{x_2,x_3}x_2'(r_3)r_3'(x_3)\\
&+I'_{x_2}x_2''(r_3)r_3'^2(x_3)+I'_{x_2}x_2'(r_3)r_3''(x_3)\\
&+I''_{x_3,x_1}x_1'(r_2)r_2'(x_2)x_2'(r_3)r_3'(x_3)\\
&+I''_{x_3,x_2}x_2'(r_3)r_3'(x_3)\\
&+\left.I''_{x_3,x_3}\right],
\end{aligned}
$$

and

$$
T_3=\frac{\partial^2 P(x_{hij};x_3)}{\partial x_{hij}^2}|_{x_{hij}=x_3}=\int_{\mathbf{R}}(n_3-1)\left[(n_3-2)H(\zeta_{hij})^{n_3-3}h(\zeta_{hij})+H(\zeta_{hij})^{n_3-2}f`(\zeta_{hij})\right]h(\zeta_{hij})d\zeta_{hij},
$$

and

$$
\begin{aligned}
I'_{x_3}&=\delta\left[(n_1-1)(r_1(x_1)-1)r_2(x_2)r_3'K+n_1(n_2-1)(r_2(x_2)-1)r_3'K+n_1n_2(n_3-1)r_3'(x_3)K\right]\\
I''_{x_1x_3}&=\delta(n_1-1)r_1'(x_1)r_2(x_2)r_3'(x_3)K\\
I''_{x_2x_3}&=\delta\left[(n_1-1)(r_1(x_1)-1)r_2'(x_2)r_3'(x_3)K+n_1(n_2-1)r_2'(x_2)r_3'(x_3)K\right]\\
I''_{x_3x_3}&=\delta\left[(n_1-1)(r_1(x_1)-1)r_2(x_2)r_3''(x_3)K+n_1(n_2-1)(r_2(x_2)-1)r_3''(x_3)K+n_1n_2(n_3-1)r_3''(x_3)K\right].
\end{aligned}
$$

C.3.3 Some Derivatives

The rate of return of private wealth is defined as:

$$r_i = \bar{r}_i - \frac{1-e^{-\lambda x_i}}{1+e^{-\lambda x_i}}(\bar{r}_i - \underline{r}_i). \quad i = 1,2,3 \tag{C-38}$$

The first, second, third, and fourth derivatives of rate of return with respect to effort are:

$$r_i'(x_i) = -\frac{2\lambda e^{-\lambda x_i}}{(1+e^{-\lambda x_i})^2}(\bar{r}_i - \underline{r}_i); \tag{C-39}$$

$$r_i''(x_i) = \frac{2\lambda^2 e^{-\lambda x_i}(1-e^{-2\lambda x_i})}{(1+e^{-\lambda x_i})^4}(\bar{r}_i - \underline{r}_i); \tag{C-40}$$

$$r_i'''(x_i) = \frac{2\lambda^3 e^{-\lambda x_i}(-1+3e^{-\lambda x_i}+3e^{-2\lambda x_i}-e^{-3\lambda x_i})}{(1+e^{-\lambda x_i})^5}(\bar{r}_i - \underline{r}_i); \tag{C-41}$$

$$r_i''''(x_i) = \frac{4\lambda^4 e^{-\lambda x_i}(-2+5e^{-\lambda x_i}-5e^{-3\lambda x_i}+2e^{-4\lambda x_i})}{(1+e^{-\lambda x_i})^6}(\bar{r}_i - \underline{r}_i). \tag{C-42}$$

The first, second, and third order derivatives of effort with respect to lower level rate of return are:

$$x_1'(r_2) = \frac{A}{B}; \tag{C-43}$$

$$x_1''(r_2) = \frac{A'B - AB'}{B^2}; \tag{C-44}$$

$$x_1'''(r_2) = \frac{(A''B - AB'')B^2 - (A'B - AB')2BB'}{B^4}; \tag{C-45}$$

$$x_2'(r_3) = \frac{C}{D}; \tag{C-46}$$

$$x_2''(r_3) = \frac{C'D - D'C}{D^2} \tag{C-47}$$

where

$$A = t_1(r_1(x_1)-1) + \frac{1}{n_1}r_1'(x_1) + t_1\left[\delta(n_1-1)(r_1-1) + \delta n_1(n_2-1)\right] + \frac{1}{n_1}\delta(n_1-1)r_1'(x_1);$$

$$B = \frac{\kappa}{r_3 K} - t_1 r_1'(x_1)r_2 - \frac{1}{n_1}r_1''(x_1)r_2 - t_1\delta(n_1-1)r_1'(x_1)r_2 - \frac{1}{n_1}\delta(n_1-1)r_1''(x_1)r_2;$$

$$A' = t_1 r_1'(x_1)x_1'(r_2) + \frac{1}{n_2}r_1''(x_1)x_1'(r_2) + t_1\delta(n_1-1)r_1'(x_1)x_1'(r_2) + \frac{1}{n_1}\delta(n_1-1)r_1''(x_1)x_1'(r_2);$$

$$\begin{aligned} B' = &-\left[t_1 r_1''(x_1)x_1'(r_2)r_2 + t_1 r_1'(x_1) + \frac{1}{n_1}r_1'''(x_1)x_1'(r_2)r_2 + \frac{1}{n_1}r_1''(x_1)\right] \\ &- t_1\delta(n_1-1)r_1''(x_1)x_1'(r_2)r_2 - t_1\delta(n_1-1)r_1'(x_1) \\ &- \frac{1}{n_1}\delta(n_1-1)r_1'''(x_1)x_1'(r_2)r_2 - \frac{1}{n_1}\delta(n_1-1)r_1''(x_1); \end{aligned}$$

$$A'' = t_1 r_1''(x_1)x_1'^2(r_2) + t_1 r_1'(x_1)x_1''(r_2) + \frac{1}{n_1}r_1'''(x_1)x_1'^2(r_2) + \frac{1}{n_1}r_1''(x_1)x_1''(r_2)$$

$$
\begin{aligned}
&+t_1\delta(n_1-1)r_1^{''}(x_1)x_1^{'2}(r_2)+t_1\delta(n_1-1)r_1^{'}(x_1)x_1^{''}(r_2)\\
&+\frac{1}{n_1}\delta(n_1-1)r_1^{'''}(x_1)x_1^{'2}(r_2)+\frac{1}{n_1}\delta(n_1-1)r_1^{''}(x_1)x_1^{''}(r_2);\\
B''=&-\Big[t_1r_1^{'''}(x_1)x_1^{'2}(r_2)r_2+t_1r_1^{''}(x_1)x_1^{''}(r_2)r_2+2t_1r_1^{''}(x_1)x_1^{'}(r_2)\\
&+\frac{1}{n_1}r_1^{''''}(x_1)x_1^{'2}(r_2)r_2+\frac{1}{n_1}r_1^{'''}(x_1)x_1^{''}(r_2)r_2+2\frac{1}{n_1}r_1^{'''}(x_1)x_1^{'}(r_2)\Big]\\
&-t_1\delta(n_1-1)r_1^{'''}(x_1)x_1^{'2}(r_2)r_2-t_1\delta(n_1-1)r_1^{''}(x_1)x_1^{''}(r_2)r_2-t_1\delta(n_1-1)r_1^{''}(x_1)x_1^{'}(r_2)\\
&-t_1\delta(n_1-1)r_1^{''}(x_1)x_1^{'}(r_2)\\
&-\frac{1}{n_1}\delta(n_1-1)r_1^{''''}(x_1)x_1^{'2}(r_2)r_2-\frac{1}{n_1}\delta(n_1-1)r_1^{'''}(x_1)x_1^{''}(r_2)r_2-\frac{1}{n_1}\delta(n_1-1)r_1^{'''}(x_1)x_1^{'}(r_2)\\
&-\frac{1}{n_1}\delta(n_1-1)r_1^{'''}(x_1)x_1^{'}(r_2).
\end{aligned}
$$

Let

$$
\begin{aligned}
&I(x_1,x_2,x_3)\\
=&\delta\left[(n_1-1)(r_1(x_1)-1)r_2(x_2)r_3(x_3)K+n_1(n_2-1)(r_2(x_2)-1)r_3(x_3)K+n_1n_2(n_3-1)(r_3(x_3)-1)K\right]\\
=&\delta\left[(n_1-1)(r_1(x_1)-1)r_2(x_2)r_3K+n_1(n_2-1)(r_2(x_2)-1)r_3K+n_1n_2(n_3-1)(r_3-1)K\right]\\
\equiv&I(x_1,x_2,r_3).
\end{aligned}
$$

The relevant partial derivatives are:

$$
\begin{aligned}
\frac{I_{x_1}^{'}}{K}&=\delta(n_1-1)r_1^{'}(x_1)r_2(x_2)r_3(x_3);\\
\frac{I_{x_2}^{'}}{K}&=\delta(n_1-1)(r_1(x_1)-1)r_2^{'}(x_2)r_3(x_3)+\delta n_1(n_2-1)r_2^{'}(x_2)r_3;\\
\frac{I_{r_3}^{'}}{K}&=\delta(n_1-1)(r_1(x_1)-1)r_2(x_2)+\delta n_1(n_2-1)(r_2-1)+\delta n_1n_2(n_3-1);\\
\frac{I_{x_1,r_3}^{''}}{K}&=\delta(n_1-1)r_1^{'}(x_1)r_2(x_2);\\
\frac{I_{x_2,r_3}^{''}}{K}&=\delta(n_1-1)(r_1(x_1)-1)r_2^{'}(x_2)+\delta n_1(n_2-1)r_2^{'}(x_2);\\
\frac{I_{x_1,x_2}^{''}}{K}&=\delta(n_1-1)r_1^{'}(x_1)r_2^{'}(x_2)r_3(x_3);\\
\frac{I_{x_2,x_2}^{''}}{K}&=\delta(n_1-1)(r_1(x_1)-1)r_2^{''}(x_2)r_3(x_3)+\delta n_1(n_2-1)r_2^{''}(x_2)r_3;\\
\frac{I_{x_1,x_1}^{''}}{K}&=\delta(n_1-1)r_1^{''}(x_1)r_2(x_2)r_3(x_3);\\
\frac{I_{r_3,r_3}^{''}}{K}&=0;\\
\frac{I_{x_1,x_2,r_3}^{'''}}{K}&=\delta(n_1-1)r_1^{'}(x_1)r_2^{'}(x_2);\\
\frac{I_{x_1,x_1,r_3}^{'''}}{K}&=\delta(n_1-1)r_1^{''}(x_1)r_2(x_2);
\end{aligned}
$$

$$
\begin{aligned}
\frac{I'''_{x_1,x_1,x_2}}{K} &= \delta(n_1-1)r''_1(x_1)r'_2(x_2)r_3(x_3);\\
\frac{I'''_{x_1,x_2,x_2}}{K} &= \delta(n_1-1)r'_1(x_1)r''_2(x_2)r_3(x_3);\\
\frac{I'''_{x_2,x_2,r_3}}{K} &= \delta(n_1-1)(r'_1(x_1)-1)r''_2(x_2)+\delta n_1(n_2-1)r''_2(x_2);\\
\frac{I'''_{x_1,r_3,r_3}}{K} &= 0;\\
\frac{I'''_{x_2,r_3,r_3}}{K} &= 0;\\
\frac{I'''_{x_1,x_1,x_1}}{K} &= \delta(n_1-1)r'''_1(x_1)r_2(x_2)r_3(x_3);\\
\frac{I'''_{x_2,x_2,x_2}}{K} &= \delta(n_1-1)(r_1(x_1)-1)r'''_2(x_2)r_3(x_3)+\delta n_1(n_2-1)r'''_2(x_2)r_3;\\
\frac{I'''_{r_3,r_3,r_3}}{K} &= 0.
\end{aligned}
$$

Lastly, the C, D and their derivatives are given as follows.

$$
\begin{aligned}
C =& -\frac{t_2}{n_1}(r_1(x_1)-1)r_2(x_2)-t_2(r_2(x_2)-1)-\frac{1}{n_1n_2}\Big[r'_1(x_1)x'_1(r_2)r'_2(x_2)r_2(x_2)\\
&+(r_1(x_1)-1)r'_2(x_2)\Big]-\frac{1}{n_2}r'_2(x_2)-\frac{t_2}{n_2}\frac{I'_{r_3}}{K}-\frac{1}{n_1n_2}\left[\frac{I''_{x_1,r_3}}{K}x'_1(r_2)r'_2(x_2)+\frac{I''_{x_2,r_3}}{K}\right];\\
D =& \frac{t_2}{n_1}\Big[r'_1(x_1)x'_1(r_2)r'_2(x_2)r_2(x_2)r_3+(r_1(x_1)-1)r'_2(x_2)r_3\\
&+\frac{I'_{x_1}}{K}x'_1(r_2)r'_2(x_2)+\frac{I'_{x_2}}{K}\Bigg]+t_2r'_2(x_2)r_3-t_2\frac{\kappa}{K}x_1x'_1(r_2)r'_2(x_2)\\
&+\frac{1}{n_1n_2}\Big[r''_1(x_1)x'^2_1(r_2)r'^2_2(x_2)r_2(x_2)+r'_1(x_1)x''_1(r_2)r'^2_2(x_2)r_2(x_2)+r'_1(x_1)x'_1(r_2)r''_2(x_2)r_2(x_2)\\
&+2r'_1(x_1)x'_1(r_2)r'^2_2(x_2)+(r_1(x_1)-1)r''_2(x_2)\Big]r_3\\
&+\frac{1}{n_1n_2}\Bigg[\frac{I''_{x_1,x_1}}{K}x'^2_1(r_2)r'^2_2(x_2)+\frac{I''_{x_1,x_2}}{K}x'_1(r_2)r'_2(x_2)+\frac{I'_{x_1}}{K}x''_1(r_2)r'^2_2(x_2)+\frac{I'_{x_1}}{K}x'_1(r_2)r''_2(x_2)\\
&+\frac{I''_{x_2,x_1}}{K}x'_1(r_2)r'_2(x_2)+\frac{I''_{x_2,x_2}}{K}\Bigg]\\
&+\frac{1}{n_2}r''_2(x_2)r_3-\frac{1}{n_2}\frac{\kappa}{K}x'^2_1(r_2)r'^2_2(x_2)-\frac{1}{n_2}\frac{\kappa}{K}x_1x''_1(r_2)r'^2_2(x_2)-\frac{1}{n_2}\frac{\kappa}{K}x_1x'_1(r_2)r''_2(x_2)\\
&-\frac{\kappa}{K};\\
C' =& -\frac{t_2}{n_1}r'_1(x_1)x'_1(r_2)r'_2(x_2)x'_2(r_3)r_2(x_2)-\frac{t_2}{n_1}(r_1(x_1)-1)r'_2(x_2)x'_2(r_3)\\
&-\frac{t_2}{n_1}r'_2(x_2)x'_2(r_3)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{n_1n_2}\Big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)\\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2)+2r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)\\
&+(r_1(x_1)-1)r_2^{''}(x_2)x_2^{'}(r_3)\Big]-\frac{1}{n_2}r_2^{''}(x_2)x_2^{'}(r_3)\\
&-\frac{t_2}{n_2}\left[\frac{I^{''}_{r_3,x_1}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{r_3,x_2}}{K}x_2^{'}(r_3)+\frac{I^{''}_{r_3,r_3}}{K}\right]\\
&-\frac{1}{n_1n_2}\left[\frac{I^{'''}_{x_1,r_3,x_1}}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,r_3,x_2}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,r_3,r_3}}{K}x_1^{'}(r_2)r_2^{'}(x_2)\right.\\
&+\frac{I^{''}_{x_1,r_3}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,r_3}}{K}x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
&\left.+\frac{I^{'''}_{x_2,r_3,x_1}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_2,r_3,x_2}}{K}x_2^{'}(r_3)+\frac{I^{'''}_{x_2,r_3,r_3}}{K}\right];
\end{aligned}
$$

$$
\begin{aligned}
D'=&\frac{t_2}{n_1}\Big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)r_3+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)r_3\\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2)r_3+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_3+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)r_2(x_2)\\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_3+(r_1(x_1)-1)r_2^{''}(x_2)x_2^{'}(r_3)r_3+(r_1(x_1)-1)r_2^{'}(x_2)\\
&+\frac{I^{''}_{x_1,x_1}}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,x_2}}{K}x_2^{'}(r_3)x_1^{'}(r_2)r_2^{'}(x_2)+\frac{I^{''}_{x_1,r_3}}{K}x_1^{'}(r_2)r_2^{'}(x_2)\\
&+\frac{I^{'}_{x_1}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)+\frac{I^{'}_{x_1}}{K}x_1^{'}(r_2)r_2^{''}(x_2)\\
&+\frac{I^{''}_{x_2,x_1}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_2,x_2}}{K}x_2^{'}(r_3)+\frac{I^{''}_{x_2,r_3}}{K}\Big]\\
&\quad+t_2\Big[r_2^{''}(x_2)x_2^{'}(r_3)r_3+r_2^{'}(x_2)-\frac{\kappa}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)\\
&-\frac{\kappa}{K}x_1x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)-\frac{\kappa}{K}x_1x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)\Big]\\
&+\frac{1}{n_1n_2}\Big[r_1^{'''}(x_1)x_1^{'3}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2)+r_1^{''}(x_1)2x_1^{'}(r_2)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2)\\
&+r_1^{''}(x_1)x_1^{'2}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2)+r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)\\
&+r_1^{''}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{''}(r_2)r_2^{'2}(x_2)r_2(x_2)+r_1^{'}(x_1)x_1^{'''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2)\\
&+r_1^{'}(x_1)x_1^{''}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2)+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)\\
&+r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)r_2(x_2)+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)r_2(x_2)\\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'''}(x_2)x_2^{'}(r_3)r_2(x_2)+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)r_2^{'}(x_2)x_2^{'}(r_3)\\
&+2r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)+2r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)+2r_1^{'}(x_1)x_1^{'}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
&+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)+(r_1(x_1)-1)r_2^{'''}(x_2)x_2^{'}(r_3)\Big]r_3\\
&+\frac{1}{n_1n_2}\Big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)r_2(x_2)+r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)r_2(x_2)+r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)r_2(x_2)\\
&+2r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)+(r_1(x_1)-1)r_2^{''}(x_2)\Big]\\
&+extraTermD'
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{n_2}r_2^{'''}(x_2)x_2^{'}(r_3)r_3+\frac{1}{n_2}r_2^{''}(x_2)\\
&-\frac{1}{n_2}\frac{\kappa}{K}2x_1^{'}(r_2)x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{'2}(x_2)-\frac{1}{n_2}\frac{\kappa}{K}x_1^{'2}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
&-\frac{1}{n_2}\frac{\kappa}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{''}(r_2)r_2^{'2}(x_2)-\frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{'''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{'2}(x_2)\\
&-\frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{''}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)-\frac{1}{n_2}\frac{\kappa}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{'}(r_2)r_2^{''}(x_2)\\
&-\frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)-\frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{'}(r_2)r_2^{'''}(x_2)x_2^{'}(r_3)
\end{aligned}
$$

The extra term for D' is as follows.

$$
\begin{aligned}
&extraTermD'\\
=&\frac{1}{n_1n_2}\left[\frac{I^{'''}_{x_1,x_1,x_1}}{K}x_1^{'3}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,x_1,x_2}}{K}x_1^{'2}r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,x_1,r_3}}{K}x_1^{'2}r_2^{'2}(x_2)\right.\\
+&\frac{I^{''}_{x_1,x_1}}{K}2x_1^{'}(r_2)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,x_1}}{K}x_1^{'2}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
+&\frac{I^{'''}_{x_1,x_2,x_1}}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,x_2,x_2}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_1,x_2,r_3}}{K}x_1^{'}(r_2)r_2^{'}(x_2)\\
+&\frac{I^{''}_{x_1,x_2}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,x_2}}{K}x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
+&\frac{I^{''}_{x_1,x_1}}{K}x_1^{'}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)x_1^{''}(r_2)+\frac{I^{''}_{x_1,x_2}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,r_3}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)\\
+&\frac{I^{'}_{x_1}}{K}x_1^{'''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)+\frac{I^{'}_{x_1}}{K}x_1^{''}(r_2)2r_2^{'}r_2^{''}(x_2)x_2^{'}(r_3)\\
+&\frac{I^{''}_{x_1,x_1}}{K}x_1^{'2}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)+\frac{I^{''}_{x_1,x_2}}{K}x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_1,r_3}}{K}x_1^{'}(r_2)r_2^{''}(x_2)\\
+&\frac{I^{'}_{x_1}}{K}x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)+\frac{I^{'}_{x_1}}{K}x_1^{'}(r_2)r_2^{'''}(x_2)x_2^{'}(r_3)\\
+&\frac{I^{'''}_{x_2,x_1,x_1}}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_2,x_1,x_2}}{K}x_1^{'}r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_2,x_1,r_3}}{K}x_1^{'}r_2^{'}(x_2)\\
+&\frac{I^{''}_{x_2,x_1}}{K}x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)+\frac{I^{''}_{x_2,x_1}}{K}x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
+&\left.\frac{I^{'''}_{x_2,x_2,x_1}}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)+\frac{I^{'''}_{x_2,x_2,x_2}}{K}x_2^{'}(r_3)+\frac{I^{'''}_{x_2,x_2,r_3}}{K}\right].
\end{aligned}
$$

C.3.4 The Optimal Set of Incentives

To find the optimal wage scale and minimal fractions of corruption cases, we maximizes the expected total final output net the cost of incentives:

$$
\max_{W_0,W_1,W_2,W_3}\quad E(n_1n_2n_3\mathcal{A}q_1^{\alpha}q_2^{\beta}q_3^{\gamma})-W_0-(n_1-1)W_1-(n_2-1)n_1W_2-(n_3-1)n_1n_2W_3
$$

$$\Leftrightarrow \quad \max_{W_0,W_1,W_2,W_3} \quad n_1 n_2 n_3 \mathcal{A} x_1^{\alpha} x_2^{\beta} x_3^{\gamma} - W_0 - (n_1-1)W_1 - (n_2-1)n_1 W_2 - (n_3-1)n_1 n_2 W_3 \tag{C-48}$$

subject to the incentive compatibility constraints of the provincial governors, the prefectural mayors, and the county heads.

C.4 Empirical Counterpart of the Model Wage Scale

We assume that the four levels of government officials from bottom to top have annual salaries that are 3:5:8:10. Suppose an eventual top leader spend age 22 to 39 in a county post, age 40 to 44 in a prefecture post, age 45 to 54 in a province post, and age 55 to 70 in a central post. An eventual provincial governor spend age 22 to 39 in a county post, age 40 to 44 in a prefecture post, and age 45 to 65 in a province post. An eventual prefectural mayor spend age 22 to 39 in a county post and age 40 to 60 in a prefectural post. An eventual county head spend age 22 to 60 in a county post. The life-time wage incomes of these four types of officials, with a discount rate of 5%, are given in Table C-2.

The retirement ages of a top leader (70), a provincial governor (65), and a prefectural mayor and below (60) are based on *Interim Provisions on the Term of Office of Leading Cadres of the Party and Government* issued by the Central Office of the CPC (2006) No. 9. The age when the transition from a county leader to a prefectural mayor takes place is 40; that when the transition to a provincial governor takes place is 45; and that when the transition to a top leader takes place is 55. These are the average age observed among officials making the transition according to the Data Blog on NetEase.[3]

The life-time wage income of the four levels of government officials from bottom to top are of the proportion 53.6 : 64.8 : 81.3 : 90.1. Taking the first difference, $\Delta W_3 : \Delta W_2 : \Delta W_1 = 11.2 : 16.5 : 8.8 = 1.3 : 1.9 : 1$. The results from the model simulation are $\Delta W_3 : \Delta W_2 : \Delta W_1 = 1.79 : 4.22 : 2.20 = 0.81 : 1.92 : 1$. We conclude that the model simulation replicates relatively well the empirical counterparts of wage differentials across ranks of Chinese bureaucracy.

[3] The article appears on the NetEase Data Blog on Aug 29, 2014: http://data.163.com/14/0829/08/A4Q8LAG800014MTN.html (accessed June 26, 2019).

Table C-2: Empirical Counterparts of W_i

Age	Discount rate	Top leader: Annual wage	Top leader: Present value	Provincial governor: Annual wage	Provincial governor: Present value	Prefectural mayor: Annual wage	Prefectural mayor: Present value	County head: Annual wage	County head: Present value
22	1.0000	3	3.0000	3	3.0000	3	3.0000	3	3.0000
23	0.9524	3	2.8571	3	2.8571	3	2.8571	3	2.8571
24	0.9070	3	2.7211	3	2.7211	3	2.7211	3	2.7211
25	0.8638	3	2.5915	3	2.5915	3	2.5915	3	2.5915
26	0.8227	3	2.4681	3	2.4681	3	2.4681	3	2.4681
27	0.7835	3	2.3506	3	2.3506	3	2.3506	3	2.3506
28	0.7462	3	2.2386	3	2.2386	3	2.2386	3	2.2386
29	0.7107	3	2.1320	3	2.1320	3	2.1320	3	2.1320
30	0.6768	3	2.0305	3	2.0305	3	2.0305	3	2.0305
31	0.6446	3	1.9338	3	1.9338	3	1.9338	3	1.9338
32	0.6139	3	1.8417	3	1.8417	3	1.8417	3	1.8417
33	0.5847	3	1.7540	3	1.7540	3	1.7540	3	1.7540
34	0.5568	3	1.6705	3	1.6705	3	1.6705	3	1.6705
35	0.5303	3	1.5910	3	1.5910	3	1.5910	3	1.5910
36	0.5051	3	1.5152	3	1.5152	3	1.5152	3	1.5152
37	0.4810	3	1.4431	3	1.4431	3	1.4431	3	1.4431
38	0.4581	3	1.3743	3	1.3743	3	1.3743	3	1.3743
39	0.4363	3	1.3089	3	1.3089	3	1.3089	3	1.3089
40	0.4155	5	2.0776	5	2.0776	5	2.0776	3	1.2466
41	0.3957	5	1.9787	5	1.9787	5	1.9787	3	1.1872
42	0.3769	5	1.8844	5	1.8844	5	1.8844	3	1.1307
43	0.3589	5	1.7947	5	1.7947	5	1.7947	3	1.0768
44	0.3418	5	1.7092	5	1.7092	5	1.7092	3	1.0255
45	0.3256	8	2.6046	8	2.6046	5	1.6279	3	0.9767
46	0.3101	8	2.4805	8	2.4805	5	1.5503	3	0.9302
47	0.2953	8	2.3624	8	2.3624	5	1.4765	3	0.8859
48	0.2812	8	2.2499	8	2.2499	5	1.4062	3	0.8437
49	0.2678	8	2.1428	8	2.1428	5	1.3392	3	0.8035
50	0.2551	8	2.0407	8	2.0407	5	1.2755	3	0.7653
51	0.2429	8	1.9436	8	1.9436	5	1.2147	3	0.7288
52	0.2314	8	1.8510	8	1.8510	5	1.1569	3	0.6941
53	0.2204	8	1.7629	8	1.7629	5	1.1018	3	0.6611
54	0.2099	8	1.6789	8	1.6789	5	1.0493	3	0.6296
55	0.1999	10	1.9987	8	1.5990	5	0.9994	3	0.5996
56	0.1904	10	1.9035	8	1.5228	5	0.9518	3	0.5711
57	0.1813	10	1.8129	8	1.4503	5	0.9065	3	0.5439
58	0.1727	10	1.7266	8	1.3813	5	0.8633	3	0.5180
59	0.1644	10	1.6444	8	1.3155	5	0.8222	3	0.4933
60	0.1566	10	1.5661	8	1.2528	5	0.7830	3	0.4698
61	0.1491	10	1.4915	8	1.1932				
62	0.1420	10	1.4205	8	1.1364				
63	0.1353	10	1.3528	8	1.0823				
64	0.1288	10	1.2884	8	1.0307				
65	0.1227	10	1.2270	8	0.9816				
66	0.1169	10	1.1686						
67	0.1113	10	1.1130						
68	0.1060	10	1.0600						
69	0.1009	10	1.0095						
70	0.0961	10	0.9614						
Life-time sum			90.1291		81.3302		64.7913		53.6037

D All Lower-Level Officials Subject to Disciplinary Inspection

In this section, we change the assumption in the baseline model that only losing lower-level officials are subject to disciplinary inspection. Instead, we assume that all lower-level officials are subject to disciplinary inspection. If an official is found to be corrupt, then he loses all his wealth to the inspector.

First of all, the logic behind Proposition 1 and Corollary 1 continue to hold. In other words, the probability of disciplinary inspection that is implemented is one under which officials are indifferent between being corrupt and not corruption. Let's denote the implemented probability of inspection at the provincial and the prefectural level by π_1^* and π_2^*, under which all lower-level officials would be corrupt.

Consider the corruption decision of a provincial governor. If this provincial governor was inspected when he served as a prefectural leader, then he brings no wealth when arriving at the current position. This means corruption is irrelevant or he is indifferent between corrupt or not corrupt under any probability of inspection. If this provincial governor was not inspected at the prefectural level, he brings r_2K to his current position and face the following trade-off when deciding whether to grow his private wealth further by r_1:

$$\frac{1}{n_1}\left(W_0+(1-\pi_1^*)r_1r_2K\right)+\left(1-\frac{1}{n_1}\right)\left(W_1+(1-\pi_1^*)r_1r_2K\right)=\frac{1}{n_1}\left(W_0+r_2K\right)+\left(1-\frac{1}{n_1}\right)\left(W_1+r_2K\right).$$

The LHS describes his payoff if he is corrupt as a provincial governor. With probability $\frac{1}{n_1}$, he will be promoted to the top leadership, but even then there is π_1^* of chance that he will be inspected and lose all his wealth. With the complementary probability, he will not be promoted and will be subject to the same risk of losing his wealth. The RHS his payoff if not corrupt as a provincial governor. It is easy to see that when $\pi_1^*=1-\frac{1}{r_1}$, he is indifferent between corruption or not.

For a prefectural mayor, his corruption decision depends on his outlook of his future career. Let $V_1^{c,i}$ denote the value of becoming a provincial governor for a prefectural mayor who is corrupt and inspected; let $V_1^{c,ni}$ denote the value of becoming a provincial governor for a prefectural mayor who is corrupt and not inspected; let V_1^{nc} denote the value of becoming a provincial governor for a prefectural mayor who is not corrupt. The condition under which a prefectural mayor is indifferent between corrupt and not corrupt is:

$$\frac{1}{n_2}\left(\pi_2^*V_1^{c,i}+(1-\pi_2^*)V_1^{c,ni}\right)+\left(1-\frac{1}{n_2}\right)\left(W_2+(1-\pi_2^*)r_2K\right)=\frac{1}{n_2}V_1^{nc}+\left(1-\frac{1}{n_2}\right)\left(W_2+K\right) \tag{D-1}$$

With probability $\frac{1}{n_2}$, the prefectural mayor is promoted, in which case he receives the expected continuation value where expectation is taken over probabilistic inspection. With the complementary probability, he is not promoted. At this moment, we postulate that the probability of inspection π_2^* which solves the above condition is precisely $\pi_2^*=1-\frac{1}{r_2}$. Once we establish the value functions, we confirm that this is the case.

We solve the model, assuming that all lower-level officials are corrupt and $\pi_i^*=1-\frac{1}{r_i}$, for $i=1,2$.

Provincial governor's problem. In this setting, we have two types of provincial governors: (i) a governor who was corrupt and inspected at the prefectural level; (ii) a governor who was corrupt and not inspected at the prefectural level. Using the established notation, the value functions are $V_1^{c,i}$ and $V_1^{c,ni}$.

$$V_1^{c,i} = \max_{x_j} P(x_j; x_1)(W_0 + I(\overline{\pi}_1, \overline{\pi}_2)) + (1 - P(x_j; x_1))W_1 - c(x_j).$$

$$\begin{aligned} V_1^{c,ni} &= \max_{x_j} P(x_j; x_1)(W_0 + I(\overline{\pi}_1, \overline{\pi}_2) + (1 - \pi_1^*)r_1 r_2 K) + (1 - P(x_j; x_1))(W_1 + (1 - \pi_1^*)r_1 r_2 K) - c(x_j); \\ &= \max_{x_j} P(x_j; x_1)(W_0 + I(\overline{\pi}_1, \overline{\pi}_2) + r_2 K) + (1 - P(x_j; x_1))(W_1 + r_2 K) - c(x_j), \end{aligned}$$

where the probabilities of promotion for both value functions are given by:

$$P(x_j; x_1) = \int_R G(x_j - x_1 + \eta_j)^{n_1 - 1} g(\eta_j) d\eta_j.$$

The first order conditions for the two types of provincial governors are the same in a symmetry equilibrium:

$$x_1^{c,ni} = x_1^{c,i} = x_1^* = \frac{t_1}{\kappa} \Delta W_1,$$

where $\Delta W_1 \equiv W_0 - W_1 + I(\overline{\pi}_1, \overline{\pi}_2))$ and $t_1 = \frac{\partial P(x_j;x_1)}{x_j}$ evaluated at $x_j = x_1 = x_1^*$ as in the baseline. Substituting x_1^* into the value functions,

$$\begin{aligned} V_1^{c,i} &= \frac{1}{n_1}\Delta W_1 + W_1 - \frac{1}{2}\kappa x_1^{*2}; \\ V_1^{c,ni} &= \frac{1}{n_1}\Delta W_1 + W_1 + r_2 K - \frac{1}{2}\kappa x_1^{*2}. \end{aligned}$$

For future use, we also write down the value function of a provincial governor, who was not corrupt as a prefectural mayor:

$$\begin{aligned} V_1^{nc} &= \max_{x_j} P(x_j; x_1)(W_0 + I(\overline{\pi}_1, \overline{\pi}_2) + (1 - \pi_1^*)r_1 K) + (1 - P(x_j; x_1))(W_1 + (1 - \pi_1^*)r_1 K) - c(x_j); \\ &= \max_{x_j} P(x_j; x_1)(W_0 + I(\overline{\pi}_1, \overline{\pi}_2) + K) + (1 - P(x_j; x_1))(W_1 + K) - c(x_j). \end{aligned}$$

It follows that the optimal effort is still x_1^* in this case and the value function becomes:

$$V_1^{nc} = \frac{1}{n_1}\Delta W_1 + W_1 + K - \frac{1}{2}\kappa x_1^{*2}.$$

At this point, we can substitute in the equilibrium value functions and verify that (D-1) indeed holds for $\pi_2^* = 1 - \frac{1}{r_2}$.

Prefectural mayor's problem. A prefectural mayor who is endowed with K faces the following problem:

$$V_2 = \max_{x_{ij}} P(x_{ij}; x_2)\left((1 - \pi_2^*)V_1^{c,ni} + \pi_2^* V_1^{c,i}\right) + (1 - P(x_{ij}; x_2))\left(W_2 + (1 - \pi_2^*)r_2 K\right) - c(x_{ij})$$

$$= P(x_{ij};x_2)\left(\frac{1}{n_1}\Delta W_1 + W_1 + (1-\pi_2^*)r_2K - \frac{1}{2}\kappa x_1^{*2}\right) + (1-P(x_{ij};x_2))\left(W_2 + (1-\pi_2^*)r_2K\right) - c(x_{ij})$$
$$= P(x_{ij};x_2)\left(\frac{1}{n_1}\Delta W_1 + W_1 + K - \frac{1}{2}\kappa x_1^{*2}\right) + (1-P(x_{ij};x_2))\left(W_2 + K\right) - c(x_{ij}).$$

The first condition for the prefecture governors' problem in a symmetry equilibrium is:

$$x_2^* = \frac{t_2}{\kappa}(\frac{1}{n_1}\Delta W_1 + \Delta W_2 - \frac{1}{2}\kappa x_1^{*2}),$$

where $\Delta W_2 \equiv W_1 - W_2$.

The problem of optimal set of incentives is:

$$\max_{W_0,W_1,W_2,\bar{\pi}_1,\bar{\pi}_2} \quad E(n_1 n_2 \mathcal{A} q_1^\alpha q_2^\beta) - W_0 - (n_1-1)W_1 - (n_2-1)n_1 W_2,$$
$$\Leftrightarrow \max_{x_1,x_2,\bar{\pi}_1,\bar{\pi}_2} \quad n_1 n_2 \mathcal{A} x_1^\alpha x_2^\beta - n_1 \frac{1}{t_2}\kappa x_2 - n_1 \frac{1}{2}\kappa x_1^2 + I(\bar{\pi}_1,\bar{\pi}_2).$$

Like in the baseline model, it is optimal to choose $W_2 = 0$ and $\bar{\pi}_1 = \bar{\pi}_2 = 0$. We therefore have $I(\bar{\pi}_1,\bar{\pi}_2) = I(0,0) = \pi_2^* n_1 n_2 r_2 K + \pi_1^*(1-\pi_2^*)n_1 r_1 r_2 K = n_1 n_2 (r_2-1)K + n_1(r_1-1)K$. This is because the inspector obtains collusion income from all inspected prefectural mayors and a fraction of inspected provincial governors who were not inspected previously as prefectural mayors. The FONCs for x_1 and x_2 are both necessary and sufficient:

$$\begin{cases} n_1 n_2 \mathcal{A}\alpha x_1^{\alpha-1} x_2^\beta = n_1 \kappa x_1 \\ n_1 n_2 \mathcal{A} x_1^\alpha \beta x_2^{\beta-1} = n_1 \frac{\kappa}{t_2} \end{cases}.$$

It's easy to get the optimal efforts.

$$x_1^* = \left(\alpha^{1-\beta}\beta^\beta n_2 \mathcal{A} t_2^\beta \kappa^{-1}\right)^{\frac{1}{2-\alpha-2\beta}}$$
$$x_2^* = \left(\alpha^\alpha \beta^{2-\alpha} n_2^2 \mathcal{A}^2 t_2^{2-\alpha}\kappa^{-2}\right)^{\frac{1}{2-\alpha-2\beta}},$$

which are identical to the solutions of the baseline model. Then

$$\Delta W_1^* = \frac{\kappa}{t_1}x_1^*$$
$$\Delta W_2^* = \frac{\kappa}{t_2}x_2^* - \frac{1}{n_1}\frac{\kappa}{t_1}x_1^* + \frac{1}{2}\kappa x_1^{*2}.$$

And

$$W_2 = 0;$$
$$W_1 = \Delta W_2^*;$$
$$W_0 = \Delta W_1^* + W_1 - I(0,0),$$

Let's revisit the Corollary 4:

$$(W_0 - W_1) - (W_1 - W_2) = \Delta W_1^* - \Delta W_2^* - I(0,0)$$
$$= \Delta W_1^* - \Delta W_2^* - n_1\left(n_2(r_2-1) + r_1 - 1\right)K.$$

Since $n_1\left(n_2(r_2-1)+r_1-1\right) > 1$ for any r_1 and r_2 larger than 1, the incremental increase in incentive in the bureaucratic hierarchy is thus decreasing in the private wealth endowment of the officials K. We conclude that the key insights from the baseline model are robustness to changing the assumption to having all officials subject to disciplinary inspection.

References

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