*NOT FOR PUBLICATION

Online Appendix: "A Model of Tournament Incentives with Corruption"

By Bin Wang and Yu Zheng.

Contents

A Patterns of Corruption Charges

The data of corruption charges are collected from the website of the Central Committee for Discipline Inspection of the CCP, which is the Party organ in charge of fighting corruption.[1] This website publishes the official announcements of disciplined cadres. A typical announcement looks as follows:

> Recently, approved by the CCP Central Committee, the CCP Central Commission for Discipline Inspection reviewed the case of WU Tianjun, the former member of the Provincial Standing Committee of the Henan Province and the former secretary of the Political and Legal Committee.
>
> During the course of the investigation, Wu Tianjun violated political discipline, resisted the investigation. He is found to have committed a breach of anti-graft discipline by **taking advantage of his office to seek benefits for his relatives' business** and **requesting his subordinate units to pay for his personal expenses**. He is also found to have committed a breach of work discipline by using fraud when receiving visitors from the masses. He is found to **use his job to seek benefits for others and obtain huge sums of money**. He is currently on suspicion of the criminal offense of **bribery**.
>
> Wu Tianjun, as a senior leading cadre of the Party, lost his faith and beliefs, seriously violated the Party's discipline, and caused serious consequences. Moreover, he did not modify his behaviors after the 18th National Congress of the Party. According to the relevant provisions of the *Communist Party of China Disciplinary Regulations* and other relevant provisions, after the discussion of the Standing Committee of the Central Commission for Discipline Inspection and upon the approval of the Central Committee, it is decided that Wu Tianjun should be expelled from the Party and the public office, with all his unlawful income confiscated, and be transfers to the judiciary with the evidence of the crime. *(Boldfaced letters added by the authors)*

The announcement enumerates the type of breach of discipline that the cadre committed. Our analysis is based on the information in the boldfaced phrases. We collect the announcements of 119 cases where the cadre in question is directly supervised by the Central Organization Committee published in the period from Feb. 8, 2013 to Feb. 23, 2017.[2]

Using a web crawler, we crawled the 119 announcements and summarized the nature of the accusation. When an announcement contains phrases such as or similar to "subordinate units to pay for his personal expenses," "travel fees payed by the public funds," "consumption payed by the public funds," "embezzlement," and "possess public belongings," we count that case towards an instance of embezzlement. When an announcement contains phrases such as or similar to "seek personal interests," "obtain colossal interests," "help relatives' business," "help others to get promoted," and "utilize power to seek interests," we count that case towards an instance of the conflict of interest. Lastly, we also count cases where bribery is present by looking for phrases such as "take colossal bribes," "take cash gift," "take gifts," and "take bribes."

Table A-1 presents our findings. Of the 119 cases, about 21% involve embezzlement while as much as 95% involve some kind of conflict of interest. Bribery is also quite common, which occurs 90.7% of all cases. Note that this is not a breakdown of the cases, it is perfectly possible for a single case to display all three "symptoms." The findings are consistent with a recent article by Guo (2013) that shows embezzlement has

[1] The website of the Central Commission for Discipline Inspection (CCDI) is http://www.ccdi.gov.cn/. The example is taken from the webpage http://www.ccdi.gov.cn/xwtt/201701/t20170123_93321.html (accessed on March 15, 2017).

[2] The announcements are categorized by the organization of the Party: the announcements covering the cadres who are supervised by Central Organization Committee (*zhong guan gan bu*), the announcements covering the cadres who are supervised by Provincial Organization Committee (*sheng guan gan bu*) and the announcements covering the cadres who work in the state-owned enterprises and institutions. We focus on the first group.

given way to the conflict of interest type of corruption as the economy develops, possibly due to better auditing measures in the government.

Table A-1: Patterns of Corruption Charges

Embezzlement	Conflict of Interest	Bribery
21%	95%	90.7%

Notes: The table shows the frequency of phrases describing each type of corruption appearing in the announcements of corruption cases against cadres managed by the Central Organization Committee published by the Central Commission for Discipline Inspection from February 8th, 2013 to February 23rd, 2017.

B The Model: Second-Order Conditions

Here in this section we prove that the agents' problems and principal's problem are concave under some conditions in the benchmark model.

The Provincial Governor's Problem The second order condition for concavity of the provincial governor's problem is:

$$\frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}(W_0 + r_1 r_2 K - W_1 - r_2 K) - \kappa < 0 \tag{1}$$

Evaluated at x_1:

$$T_1(W_0 - W_1 + (r_1 - 1)r_2 K) \equiv T_1 \Delta W_1 < \kappa \tag{2}$$

where

$$T_1 = \frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}|_{x_j = x_1} = \int_{\mathbf{R}} (n_1 - 1)\left[(n_1 - 2)G(\eta_j)^{n_1 - 3} g(\eta_j) + G(\eta_j)^{n_1 - 2} g'(\eta_j)\right] g(\eta_j) d\eta_j. \tag{3}$$

The Prefectural Mayor's Problem The second order condition for concavity of the prefectural Mayor's problem is:

$$\frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}\left[\frac{1}{n_1}(W_0 - W_1 + (r_1 - 1)r_2 K) + W_1 + r_2 K - \frac{1}{2}\kappa x_1^2 - W_2 - K\right] - \kappa < 0 \tag{4}$$

Evaluated at x_2:

$$T_2\left[\frac{1}{n_1}(W_0 - W_1 + (r_1 - 1)r_2 K) + W_1 - W_2 + (r_1 - 1)K - \frac{1}{2}\kappa x_1^2\right] \equiv T_2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2\right) < \kappa \tag{5}$$

where

$$T_2 = \frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}|_{x_{ij} = x_2} = \int_{\mathbf{R}} (n_2 - 1)\left[(n_2 - 2)F(\varepsilon_{ij})^{n_2 - 3} f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2 - 2} f(\varepsilon_{ij})\right] f(\varepsilon_{ij}) d\varepsilon_{ij}. \tag{6}$$

So to guarantee the SOCs of the agents' problems, we need κ sufficiently large:

$$\kappa > \max\left\{T_1 \Delta W_1, T_2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2\right)\right\}.$$

We verify in the simulation that this condition is satisfied at the optimum.

The Principal's Problem The second order condition for concavity of the principal's problem is that the Hessian matrix is negative definite:

$$H(x_1, x_2) = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \tag{7}$$

where $L_{ij} = \frac{\partial^2 L}{\partial x_i \partial x_j}$ and L is the Lagrangian function of the principal's problem.

We have:

$$L_{11} = n_1 n_2 \mathcal{A}\alpha(\alpha-1)x_1^{\alpha-2}x_2^{\beta} - n_1\kappa \tag{8}$$
$$L_{12} = L_{21} = n_1 n_2 \mathcal{A}\alpha\beta x_1^{\alpha-1}x_2^{\beta-1} \tag{9}$$
$$L_{22} = n_1 n_2 \mathcal{A}\beta(\beta-1)x_1^{\alpha}x_2^{\beta-2} \tag{10}$$

It is obvious that $L_{11} < 0$ because $\alpha < 1$.

And the determinant of the Hessian matrix is:

$$\begin{aligned}
|H| =& L_{11}L_{22} - L_{21}L_{12} \\
=& \left(n_1 n_2 \mathcal{A}\alpha(\alpha-1)x_1^{\alpha-2}x_2^{\beta} - n_1\kappa\right) n_1 n_2 \mathcal{A}\beta(\beta-1)x_1^{\alpha}x_2^{\beta-2} - \left(n_1 n_2 \mathcal{A}\alpha\beta x_1^{\alpha-1}x_2^{\beta-1}\right)^2 \\
=& (n_1 n_2 \mathcal{A})^2\alpha\beta(\alpha-1)(\beta-1)x_1^{2\alpha-2}x_2^{2\beta-2} - n_1\kappa(n_1 n_2 \mathcal{A})\beta(\beta-1)x_1^{\alpha}x_2^{\beta-2} - (n_1 n_2 \mathcal{A})^2\alpha^2\beta^2 x_1^{2\alpha-2}x_2^{2\beta-2} \\
=& (n_1 n_2 \mathcal{A})^2\alpha\beta(1-\alpha-\beta)x_1^{2\alpha-2}x_2^{2\beta-2} + n_1\kappa(n_1 n_2 \mathcal{A})\beta(1-\beta)x_1^{\alpha}x_2^{\beta-2} \\
>& 0
\end{aligned}$$

So the Hessian matrix is negative definite and hence the principal's problem is concave globally.

C The Simulation: The Extended Model with a Four-Level Government

C.1 The Data

The county-level output data are from the Financial and Economic Statistics at the City and County Level Sub-database of the EPS China Fiscal and Taxation Database. This is a proprietary database, which consolidates data from the Ministry of Finance of the People's Republic of China and the State Administration of Taxation. Obtain the output data for 2,976 counties in China from 1997 to 2007. We also obtain provincial consumer price index (CPI) from the CEIC China Database, also proprietary, to deflate the output variables.

The sample selection proceeds in several steps. Firstly, after converting the nominal output into real output (normalizing the price level in Anhui in 2007 to 100) and computing the real GDP growth rates, we first trim the top and bottom 5% of the annual growth rates to exclude outliers. The 5th percentile of growth rates corresponds to a gross annual growth rate of 0.8 or a drop of output by 20%, while the 95th percentile of growth rates corresponds to a gross annual growth rate of 1.68 or an increase of output by 68%. Secondly, we drop four municipalities that are under direct control of the central government: Beijing, Tianjin, Shanghai, and Chongqing. Thirdly, we compute the average growth rates for each county over the sample period and drop prefectures under which we have fewer than five county observations. Fourthly, we drop provinces under which we have fewer than five prefectures. The last two steps ensure that each prefecture and each province has sufficiently large numbers of subordinate administrative regions so that it makes sense to compute the variances. We document the change in the number of counties remaining in the sample in Table C-1.

The resulting analysis sample consists of 2,613 counties in 274 prefectures of 25 provinces.

Table C-1: Sample Selection: County-Level Output Data

Operation	Number of counties
(Initial sample)	2,976
Trim top and bottom 5% of county growth rates	2,882
Drop four municipalities under central control	2,840
Drop prefectures with fewer than 5 counties	2,646
Drop provinces with fewer than 5 prefectures	2,613

Notes: The table shows the sample size (i.e. the number of counties) at each operation in the sample selection process.

C.2 The Extended Model with Exogenous Rates of Return

Here we extend the two-stage tournament model in the Model section of the paper to a three-stage tournament with a provincial, a prefectural and a county stage from top to down. In the extended model, we assume that the convicted probability $(\bar{\pi}_1, \bar{\pi}_2, \bar{\pi}_3)$ is the same as the inspection probability $(\pi_1^*, \pi_2^*, \pi_3^*)$. So there is no collusion in the extended model.

C.2.1 Enforcement of Disciplinary Inspection

For a losing county level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_3 + (1-\pi_3) r_3 K = W_3 + K \\ \Rightarrow \quad & \pi_3 = 1 - \frac{1}{r_3} \end{aligned}$$

For a losing prefecture level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_2 + (1-\pi_2) r_2 r_3 K = W_2 + r_3 K \\ \Rightarrow \quad & \pi_2 = 1 - \frac{1}{r_2} \end{aligned}$$

For a province level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_1 + (1-\pi_1) r_1 r_2 r_3 K = W_2 + r_2 r_3 K \\ \Rightarrow \quad & \pi_1 = 1 - \frac{1}{r_1} \end{aligned}$$

C.2.2 The Agents' Problem

Provincial Governor's Problem. For the provincial governor of province j:

$$\begin{aligned} V_1 &= \max_{x_j} P(x_j; x_1)\left(W_0 + r_1 r_2 r_3 K\right) + \left(1 - P(x_j; x_1)\right)\left(W_1 + (1-\pi_1) r_1 r_2 r_3 K\right) - c(x_j) \\ &= \max_{x_j} P(x_j; x_1)\left(W_0 + r_1 r_2 r_3 K\right) + \left(1 - P(x_j; x_1)\right)\left(W_1 + r_2 r_3 K\right) - c(x_j), \end{aligned}$$

where the probability of winning is:

$$P(x_j; x_1) = \int_{\mathbf{R}} G(x_j - x_1 + \eta_j)^{n_1 - 1} g(\eta_j) d\eta_j, \tag{11}$$

and $G(\cdot)$ ($g(\cdot)$) is the cdf (pdf) of η_j. Then,

$$\frac{\partial P(x_j; x_1)}{\partial x_j} = \int_{\mathbf{R}} (n_1 - 1) G(x_j - x_1 + \eta_j)^{n_1 - 2} g(x_j - x_1 + \eta_j) g(\eta_j) d\eta_j.$$

The FOC is

$$\frac{\partial P(x_j; x_1)}{\partial x_j}\left(W_0 + r_1 r_2 r_3 K - W_1 - r_2 r_3 K\right) - c'(x_j) = 0.$$

Evaluated at x_1, it becomes:

$$\int_{\mathbf{R}} (n_1 - 1) G(\eta_j)^{n_1 - 2} g(\eta_j)^2 d\eta_j \left(W_0 + r_1 r_2 r_3 K - W_1 - r_2 r_3 K\right) = c'(x_1) = \kappa x_1.$$

Let $\Delta W_1 = W_0 - W_1 + (r_1 - 1) r_2 r_3 K$, we have:

$$\frac{t_1 \Delta W_1}{\kappa} = x_1. \tag{12}$$

or

$$\Delta W_1 = \frac{\kappa}{t_1} x_1$$

The value function is then

$$V_1 = \frac{1}{n_1}\Delta W_1 + (W_1 + r_2 r_3 K) - \frac{1}{2}\kappa x_1^2. \tag{13}$$

The SOC is:

$$\frac{\partial^2 P(x_j;x_1)}{\partial x_j^2}\left(W_0 + r_1 r_2 r_3 K - W_1 - r_2 r_3 K\right) - c''(x_j) < 0.$$

Evaluated at x_1, it becomes:

$$T_1 \left(W_0 + r_1 r_2 r_3 K - W_1 - r_2 r_3 K\right) < \kappa.$$

where

$$T_1 = \frac{\partial^2 P(x_j;x_1)}{\partial x_j^2}|_{x_j=x_1} = \int_{\mathbf{R}} (n_1-1)\left[(n_1-2)G(\eta_j)^{n_1-3}g(\eta_j) + G(\eta_j)^{n_1-2}g'(\eta_j)\right] g(\eta_j) d\eta_j.$$

The Prefectural Mayor's Problem. For the prefectural mayor i in province j:

$$\begin{aligned} V_2 &= \max_{x_{ij}} P(x_{ij};x_2)V_1 + (1-P(x_{ij};x_2))\left(W_2 + (1-\pi_2)r_2 r_3 K\right) - c(x_{ij}) \\ &= \max_{x_{ij}} P(x_{ij};x_2)V_1 + (1-P(x_{ij};x_2))\left(W_2 + r_3 K\right) - c(x_{ij}), \end{aligned}$$

where $P(x_{ij};x_2)$ is the probability that he will beat all other mayors in province j:

$$P(x_{ij};x_2) = \int_{\mathbf{R}} F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2-1} f(\varepsilon_{ij}) d\varepsilon_{ij}. \tag{14}$$

Here $F(\cdot)$ ($f(\cdot)$) is the cdf (pdf) of ε_{ij}. Then,

$$\frac{\partial P(x_{ij};x_2)}{\partial x_{ij}} = \int_{\mathbf{R}} (n_2-1)F(x_{ij}-x_2+\varepsilon_{ij})^{n_2-2} f(x_{ij}-x_2+\varepsilon_{ij}) f(\varepsilon_{ij}) d\varepsilon_{ij}.$$

The FOC is

$$\frac{\partial P(x_{ij};x_2)}{\partial x_{ij}}\left[V_1 - W_2 - r_3 K\right] - c'(x_{ij}) = 0.$$

Evaluated at x_2, it becomes:

$$\int_{\mathbf{R}} (n_2-1)F(\varepsilon_{ij})^{n_2-2} f(\varepsilon_{ij})^2 d\varepsilon_{ij}\left(V_1 - W_2 - r_3 K\right) = c'(x_2),$$

Let $\Delta W_2 \equiv W_1 - W_2 + (r_2-1)r_3 K$ and we have

$$\frac{t_2}{\kappa}\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2 \Delta W_1^2\right) = x_2, \tag{15}$$

or

$$\Delta W_2 = \frac{\kappa}{t_2} x_2 - \frac{1}{n_1}\frac{\kappa}{t_1} x_1 + \frac{1}{2}\kappa x_1^2.$$

The value function is then:

$$V_2 = \frac{1}{n_2}\left(\frac{1}{n_1}\Delta W_1 + \Delta W_2 - \frac{1}{2}\kappa x_1^2\right) + W_2 + r_3 K - \frac{1}{2}\kappa x_2^2. \tag{16}$$

The SOC is:

$$\frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}\left[V_1 - W_2 - r_3 K\right] < \kappa.$$

Evaluated at x_2, it becomes:

$$T_2\left[V_1 - W_2 - r_3 K\right] - c''(x_{ij}) < 0.$$

where

$$T_2 = \frac{\partial^2 P(x_{ij}; x_2)}{\partial x_{ij}^2}|_{x_{ij}=x_2} = \int_{\mathbf{R}} (n_2 - 1)\left[(n_2-2)F(\varepsilon_{ij})^{n_2-3} f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2-2} f(\varepsilon_{ij})\right] f(\varepsilon_{ij}) d\varepsilon_{ij}.$$

The County Governor's Problem. For the county governor h in prefeture of i province j:

$$\begin{aligned} V_3 &= \max_{x_{ij}} P(x_{hij}; x_3)V_2 + (1 - P(x_{hij}; x_3))\left(W_3 + (1-\pi_3)r_3 K\right) - c(x_{hij}) \\ &= \max_{x_{hij}} P(x_{hij}; x_2)V_2 + (1 - P(x_{hij}; x_3))\left(W_3 + K\right) - c(x_{hij}), \end{aligned}$$

where $P(x_{hij}; x_3)$ is the probability that he will beat all other county governors in prefecture i of province j:

$$P(x_{hij}; x_3) = \int_{\mathbf{R}} H(x_{hij} - x_3 + \varepsilon_{hij})^{n_3-1} h(\zeta_{hij}) d\zeta_{ij}. \tag{17}$$

Here $H(\cdot)$ ($h(\cdot)$) is the cdf (pdf) of ζ_{hij}. Then,

$$\frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}} = \int_{\mathbf{R}} (n_3 - 1) H(x_{hij} - x_3 + \zeta_{hij})^{n_3-2} h(x_{hij} - x_2 + \zeta_{hij}) h(\zeta_{ij}) d\zeta_{hij}.$$

The FOC is

$$\frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}}\left[V_2 - W_3 - K\right] - c'(x_{hij}) = 0.$$

Evaluated at x_3, it becomes:

$$\int_{\mathbf{R}} (n_3 - 1) F(\zeta_{hij})^{n_3-2} f(\zeta_{hij})^2 d\zeta_{hij}\left(V_2 - W_3 - K\right) = c'(x_2),$$

Let $\Delta W_3 \equiv W_2 - W_3 + (r_3 - 1)K$ and we have

$$\frac{t_3}{\kappa}\left[\frac{1}{n_2}\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2 \Delta W_1^2\right) + \Delta W_3 - \frac{1}{2\kappa}t_2^2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right)^2\right] = x_3, \tag{18}$$

or

$$\Delta W_3 = \frac{\kappa}{t_3}x_3 - \frac{1}{n_2}\frac{\kappa}{t_2}x_2 + \frac{1}{2}\kappa x_2^2.$$

The SOC is:

$$\frac{\partial^2 P(x_{hij};x_3)}{\partial x_{hij}^2}\left[V_2 - W_3 - K\right] - c''(x_{hij}) < 0.$$

Evaluated at x_3, it becomes:

$$T_3\left(V_2 - W_3 - K\right) < \kappa,$$

where

$$T_3 = \frac{\partial^2 P(x_{hij};x_3)}{\partial x_{hij}^2}|_{x_{hij}=x_3} = \int_{\mathbf{R}}(n_3-1)\left[(n_3-2)H(\zeta_{hij})^{n_3-3}h(\zeta_{hij}) + H(\zeta_{hij})^{n_3-2}f(\zeta_{hij})\right]h(\zeta_{hij})d\zeta_{hij}.$$

C.2.3 The Principal's Problem.

The principal maximizes the expected total final output net the cost of incentives:

$$\begin{aligned}
&\max_{W_0,W_1,W_2,W_3} \quad E(n_1n_2n_3\mathcal{A}q_1^\alpha q_2^\beta q_3^\gamma) - W_0 - (n_1-1)W_1 - (n_2-1)n_1W_2 + (n_3-1)n_2W_3 + \pi_1(n_1-1)r_1r_2r_3K \\
&\qquad + \pi_2 n_1(n_2-1)r_2r_3K + \pi_3 n_2(n_3-1)r_3K \\
\Leftrightarrow\ &\max_{W_0,W_1,W_2} \quad n_1n_2n_3\mathcal{A}x_1^\alpha x_2^\beta x_3^\gamma - W_0 - (n_1-1)W_1 - (n_2-1)n_1W_2 + (n_3-1)n_2W_3 + (n_1-1)(r_1-1)r_2r_3K \\
&\qquad + n_1(n_2-1)(r_2-1)r_3K + n_2(n_3-1)(r_3-1)K \\
\Leftrightarrow\ &\max_{\Delta W_1,\Delta W_2,\Delta W_3,W_2\geq 0} \quad n_1n_2n_3\mathcal{A}\frac{t_1^\alpha t_2^\beta t_3^\gamma}{\kappa^{\alpha+\beta+\gamma}}\Delta W_1^\alpha\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right)^\beta \qquad (19) \\
&\qquad * \left[\frac{1}{n_2}\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right) + \Delta W_3 - \frac{1}{2\kappa}t_2^2\left(\Delta W_2 + \frac{1}{n_1}\Delta W_1 - \frac{1}{2\kappa}t_1^2\Delta W_1^2\right)^2\right]^\gamma \\
&\qquad - \Delta W_1 - n_1\Delta W_2 - n_1n_2\Delta W_3 - n_1n_2n_3W_3 + n_1\left((r_1-1)r_2r_3 + n_2(r_2-1)r_3 + n_2n_3(r_3-1)\right)K \qquad (20)
\end{aligned}$$

subject to the incentive compatibility constraints of the provincial governors, prefectural mayors and county governors, (12), (15) and (18). This is a standard problem. The principal optimally sets W_3 to 0. Then the problem equals to the one which optimizes on x_1, x_2, x_3 after substituting $\Delta W_1, \Delta W_2, \Delta W_3$ for x_1, x_2, x_3.

$$\begin{aligned}
&\max_{x_1,x_2,x_3\geq 0} \quad n_1n_2n_3\mathcal{A}x_1^\alpha x_2^\beta x_3^\gamma - \frac{1}{2}n_1\kappa x_1^2 - \frac{1}{2}n_1n_2\kappa x_2^2 - n_1n_2\frac{\kappa}{t_3}x_3 \qquad (21) \\
&\qquad + n_1\left((r_1-1)r_2r_3 + n_2(r_2-1)r_3 + n_2n_3(r_3-1)\right)K.
\end{aligned}$$

The FOC with respect to x_1, x_2, x_3 are:

$$x_1 : n_1n_2n_3\mathcal{A}\alpha x_1^{\alpha-1}x_2^\beta x_3^\gamma - n_1\kappa x_1 = 0. \qquad (22)$$

$$x_2 : n_1n_2n_3\mathcal{A}\beta x_1^\alpha x_2^{\beta-1}x_3^\gamma - n_1n_2\kappa x_2 = 0. \qquad (23)$$

$$x_1 : n_1n_2n_3\mathcal{A}\gamma x_1^\alpha x_2^\beta x_3^{\gamma-1} - n_1n_2\frac{\kappa}{t_3} = 0. \qquad (24)$$

The optimal efforts in three levels are:

$$x_1 = \left(\mathcal{A}\alpha^{1-\frac{\beta}{2}-\gamma}\beta^{\frac{\beta}{2}}\gamma^{\gamma}n_2^{1-\frac{\beta}{2}-\gamma}n_3t_3^{\gamma}\kappa^{-1}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{25}$$

$$x_2 = \left(\mathcal{A}\alpha^{\frac{\alpha}{2}}\beta^{1-\frac{\alpha}{2}-\gamma}\gamma^{\gamma}n_2^{\frac{\alpha}{2}}n_3t_3^{\gamma}\kappa^{-1}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{26}$$

$$x_3 = \left(\mathcal{A}^2\alpha^{\alpha}\beta^{\beta}\gamma^{2-\alpha-\beta}n_2^{\alpha}n_3^2t_3^{2-\alpha-\beta}\kappa^{-2}\right)^{\frac{1}{2-\alpha-\beta-2\gamma}}. \tag{27}$$

C.3 The Extended Model with Endogenous Rates of Return

Here in this section, we characterize the solution of the principal-agent problem with endogenous rate of return on private wealth. Basically, we first nail the disciplinary probability of each level down and then obtain the optimal effort of agents of each level by solving the Bellman equations. Then the principal maximizes his profit given the optimal effort of the agents.

C.3.1 Enforcement of Disciplinary Inspection

For a losing county level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_3 + (1-\pi_3^*(\cdot))r_3(x_{hij})K = W_3 + K \\ \Rightarrow \quad & \pi_3^*(x_{hij}) = 1 - \frac{1}{r_3(x_{hij})} \end{aligned}$$

For a losing prefecture level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_2 + (1-\pi_2^*(\cdot))r_2(x_ij)r_3(x_{hij})K = W_2 + r_3(x_{hij})K \\ \Rightarrow \quad & \pi_2^*(x_{ij}) = 1 - \frac{1}{r_2(x_{ij})} \end{aligned}$$

For a losing province level official, he is indifferent between corrupt and not corrupt if:

$$\begin{aligned} & W_1 + (1-\pi_1^*(\cdot))r_1(x_j)r_2(x_{ij})r_3(x_{hij})K = W_2 + r_2(x_{ij})r_3(x_{hij})K \\ \Rightarrow \quad & \pi_1^*(x_j) = 1 - \frac{1}{r_1(x_j)} \end{aligned}$$

As before, we simplify the problem by setting the contracted convicted probability $\bar{\pi}$ as the inspection probability π^* so that there is no collusion between the officials and the the inspector.

C.3.2 The Agents' Problem

We solve the agents' problem by backward induction of the Bellman equations. Specifically, at each level, the agents consider the efforts and rates of return of private wealth at lower levels (obtained in the past) as given. Then agents' objective function of each level are maximized over efforts.

Provincial Governor's Problem The governor of province j maximizes the expected payoff by exerting effort to compete.

$$
\begin{aligned}
V_1 = &\max_{x_j} P(x_j;x_1)\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K\right) \\
&+ (1-P(x_j;x_1))\left(W_1 + (1-\pi_1(x_j))r_1(x_j)r_2(x_2)r_3(x_3)K\right) - c(x_j) \\
&= \max_{x_j} P(x_j;x_1)\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K\right) + (1-P(x_j;x_1))\left(W_1 + r_2(x_2)r_3(x_3)K\right) - c(x_j),
\end{aligned} \tag{28}
$$

where the probability of winning is:

$$P(x_j;x_1) = \int_{\mathbf{R}} G(x_j - x_1 + \eta_j)^{n_1-1} g(\eta_j) d\eta_j,$$

and $G(\cdot)$ ($g(\cdot)$) is the cdf (pdf) of η_j. Then,

$$\frac{\partial P(x_j;x_1)}{\partial x_j} = \int_{\mathbf{R}} (n_1-1)G(x_j - x_1 + \eta_j)^{n_1-2} g(x_j - x_1 + \eta_j) g(\eta_j) d\eta_j.$$

The FOC is:

$$
\begin{aligned}
&\frac{\partial P(x_j;x_1)}{\partial x_j}\left(W_0 + r_1(x_j)r_2(x_2)r_3(x_3)K - W_1 - r_2(x_2)r_3(x_3)K\right) \\
&+ P(x_j;x_1)r_1'(x_j)r_2(x_2)r_3(x_3)K - c'(x_j) = 0.
\end{aligned}
$$

Evaluated at x_1:

$$t_1\left(W_0 - W_1 + \big(r_1(x_1)-1\big)r_2(x_2)r_3(x_3)K\right) + \frac{1}{n_1}r_1'(x_1)r_2(x_2)r_3(x_3)K = \kappa x_1 \tag{29}$$

where

$$t_1 = \int_{\mathbf{R}} (n_1-1)G(\eta_j)^{n_1-2} g(\eta_j)^2 d\eta_j \tag{30}$$

The value function is

$$V_1 = \frac{1}{n_1}\left(W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K\right) + W_1 + r_2(x_2)r_3(x_3)K - \frac{1}{2}\kappa x_1^2. \tag{31}$$

The SOC is:

$$\frac{\partial^2 P(x_j;x_1)}{\partial x_j^2}(W_0 - W_1 + (r_1(x_j)-1)r_2(x_2)r_3(x_3)K) \tag{32}$$

$$+2\frac{\partial P(x_j;x_1)}{\partial x_j}r_1'(x_j)r_2(x_2)r_3(x_3)K + P(x_j;x_1)r_1''(x_j)r_2(x_2)r_3(x_3)K - \kappa \tag{33}$$

Evaluated at x_1:

$$T_1(W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K) \tag{34}$$

$$+2t_1 r_1'(x_1)r_2(x_2)r_3(x_3)K + \frac{1}{n_1}r_1''(x_1)r_2(x_2)r_3(x_3)K - \kappa < 0 \tag{35}$$

where

$$T_1 = \frac{\partial^2 P(x_j; x_1)}{\partial x_j^2}|_{x_j=x_1} = \int_{\mathbf{R}} (n_1 - 1)\left[(n_1-2)G(\eta_j)^{n_1-3}g(\eta_j) + G(\eta_j)^{n_1-2}g'(\eta_j)\right] g(\eta_j)d\eta_j. \tag{36}$$

Prefectoral Mayor's Problem. The mayor of prefecture i in province j maximizes the expected payoff of the competition.

$$\begin{aligned} V_2 =& \max_{x_{ij}} P(x_{ij}; x_2)V_1 + (1 - P(x_{ij}; x_2))\left(W_2 + (1-\pi_2(x_{ij}))r_2(x_{ij})r_3(x_3)K\right) - c(x_{ij}) \qquad (37) \\ =& \max_{x_{ij}} P(x_{ij}; x_2)\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1(x_{ij})) - 1)r_2(x_{ij})r_3(x_3)K\right] + W_1 \\ & + r_2(x_{ij})r_3(x_3)K - c(x_1(x_{ij}))\big\} + (1 - P(x_{ij}; x_2))\left(W_2 + r_3(x_3)K\right) - c(x_{ij}), \end{aligned}$$

where $P(x_{ij}; x_2)$ is the probability that ij will come out first in his group:

$$P(x_{ij}; x_2) = \int_{\mathbf{R}} F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2-1} f(\varepsilon_{ij})d\varepsilon_{ij}.$$

Here $F(\cdot)$ ($f(\cdot)$) is the cdf (pdf) of ε_{ij}. Then,

$$\frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} = \int_{\mathbf{R}} (n_2 - 1)F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2-2} f(x_{ij} - x_2 + \varepsilon_{ij})f(\varepsilon_{ij})d\varepsilon_{ij}.$$

The FOC is

$$\begin{aligned} & \frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}}\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1(x_{ij})) - 1)r_2(x_{ij})r_3(x_3)K\right] + W_1 + r_2(x_{ij})r_3(x_3)K - c(x_1(x_{ij})) \\ & - W_2 - r_3(x_3)K\big\} + P(x_{ij}; x_2)\big\{\frac{1}{n_1}[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_{ij})r_2(x_{ij}) + (r_1(x_1) - 1)r_2^{'}(x_{ij})]r_3(x_3)K \\ & + r_2^{'}(x_{ij})r_3(x_3)K - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_{ij})\big\} = \kappa x_{ij} \end{aligned}$$

Evaluated at x_2:

$$\begin{aligned} & t_2\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1) - 1)r_2(x_2)r_3(x_3)K\right] + W_1 - W_2 + (r_2(x_2) - 1)r_3(x_3)K - \frac{1}{2}\kappa x_1^2\big\} \\ & + \frac{1}{n_2}\big\{\frac{1}{n_1}[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)r_2(x_2) + (r_1(x_1) - 1)r_2^{'}(x_2)]r_3(x_3)K + r_2^{'}(x_2)r_3(x_3)K \\ & - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)\big\} = \kappa x_2 \end{aligned} \tag{38}$$

where

$$t_2 = \int_{\mathbf{R}} (n_2 - 1)F(\varepsilon_{ij})^{n_2-2} f(\varepsilon_{ij})^2 d\varepsilon_{ij}, \tag{39}$$

The value function is:

$$\begin{aligned} V_2 =& \frac{1}{n_2}\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1) - 1)r_2(x_2)r_3(x_3)K\right] + W_1 - W_2 + (r_2(x_2) - 1)r_3(x_3)K - \frac{1}{2}\kappa x_1^2\big\} \\ & + W_2 + r_3(x_3)K - \frac{1}{2}\kappa x_2^2 \end{aligned} \tag{40}$$

The SOC of the second level officials' problem is:

$$
\begin{aligned}
&\frac{\partial^2 P(x_{ij};x_2)}{\partial x_{ij}^2}\Big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1(x_{ij})) - 1)r_2(x_{ij})r_3(x_3)K\right] + W_1 + r_2(x_{ij})r_3(x_3)K - \frac{1}{2}x_1^2 \\
&\qquad - W_2 - r_3(x_3)K\Big\} + \\
&2\frac{\partial P(x_{ij};x_2)}{\partial x_{ij}}\Big\{\frac{1}{n_1}[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_{ij})r_2(x_{ij}) + (r_1(x_1)-1)r_2^{'}(x_{ij})]r_3(x_3)K \\
&\qquad + r_2^{'}(x_{ij})r_3(x_3)K - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_{ij})\Big\} + \\
&P(x_{ij};x_2)\Big\{\frac{1}{n_1}[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_{ij})r_2(x_{ij}) + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_{ij})r_2(x_{ij}) \\
&\qquad + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_{ij})r_2(x_{ij}) + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_{ij}) + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_{ij}) \\
&\qquad + (r_1(x_1)-1)r_2^{''}(x_{ij})]r_3(x_3)K + r_2^{''}(x_{ij})r_3(x_3)K - \kappa x_1^{'2}(r_2)r_2^{'2}(x_{ij}) \\
&\qquad - \kappa x_1 x_1^{''}(r_2)r_2^{'2}(x_{ij}) - \kappa x_1 x_1^{'}(r_2)r_2^{''}(x_{ij})\Big\} - \kappa \\
&\qquad < 0
\end{aligned}
\tag{41}
$$

Evalutaed at x_2:

$$
\begin{aligned}
\kappa >& T_2\Big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1) - 1)r_2(x_2)r_3(x_3)K\right] + W_1 + r_2(x_2)r_3(x_3)K - \frac{1}{2}x_1^2 \\
&- W_2 - r_3(x_3)K\Big\} \\
&+ 2t_2\Big\{\frac{1}{n_1}[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)r_2(x_2) + (r_1(x_1)-1)r_2^{'}(x_2)]r_3(x_3)K \\
&+ r_2^{'}(x_2)r_3(x_3)K - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)\Big\} \\
&+ \frac{1}{n_2}\Big\{\frac{1}{n_1}[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)r_2(x_2) + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)r_2(x_2) \\
&+ r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)r_2(x_2) + 2r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2) \\
&+ (r_1(x_1)-1)r_2^{''}(x_2)]r_3(x_3)K + r_2^{''}(x_2)r_3(x_3)K - \kappa x_1^{'2}(r_2)r_2^{'2}(x_2) \\
&- \kappa x_1 x_1^{''}(r_2)r_2^{'2}(x_2) - \kappa x_1 x_1^{'}(r_2)r_2^{''}(x_2)\Big\}
\end{aligned}
\tag{42}
$$

where

$$
T_2 = \frac{\partial^2 P(x_{ij};x_2)}{\partial x_{ij}^2}|_{x_{ij}=x_2} = \int_{\mathbf{R}} (n_2 - 1)\left[(n_2-2)F(\varepsilon_{ij})^{n_2-3}f(\varepsilon_{ij}) + F(\varepsilon_{ij})^{n_2-2}f(\varepsilon_{ij})\right] f(\varepsilon_{ij})d\varepsilon_{ij}.
\tag{43}
$$

County Leader's Problem. The leader of county h in prefecture i of province j maximizes his expected payoff by exerting effort in the competition.

$$
\begin{aligned}
V_3 = & \max_{x_{hij}} P(x_{hij}; x_3)V_2 + (1 - P(x_{hij}; x_3)) \left(W_3 + (1 - \pi_3(x_{hij}))r_3(x_{hij})K\right) - c(x_{ij}) & (44) \\
= & \max_{x_{iij}} P(x_{iij}; x_2)\Big\{\frac{1}{n_2}\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1) - 1)r_2(x_2)r_3(x_{hij})K\right] + W_1 - W_2 & (45) \\
& + (r_2(x_2) - 1)r_3(x_{hij})K - \frac{1}{2}\kappa x_1^2\} + W_2 + r_3(x_{hij})K - \frac{1}{2}\kappa x_2^2\Big\} \\
& + (1 - P(x_{hij}; x_3))W_3 - \frac{1}{2}\kappa x_{hij}^2,
\end{aligned}
$$

where $P(x_{hij}; x_3)$ is the probability that hij will come out first in his group:

$$P(x_{hij}; x_3) = \int_{\mathbf{R}} H(x_{hij} - x_3 + \zeta_{hij})^{n_3 - 1} h(\zeta_{hij}) d\zeta_{hij}.$$

Here $H(\cdot)$ ($h(\cdot)$) is the cdf (pdf) of ζ_{hij}. Then,

$$\frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}} = \int_{\mathbf{R}} (n_3 - 1)H(x_{hij} - x_3 + \zeta_{ij})^{n_3 - 2} h(x_{hij} - x_3 + \zeta_{hij}) h(\zeta_{hij}) d\zeta_{hij}.$$

The FOC is:

$$
\begin{aligned}
& \frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}}\Big\{\frac{1}{n_2}\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1) - 1)r_2(x_2)r_3(x_{hij})K\right] + W_1 - W_2 + (r_2(x_2) - 1)r_3(x_{hij})K & (46) \\
& \quad - \frac{1}{2}\kappa x_1^2\} + W_2 - W_3 + (r_3(x_{hij}) - 1)K - \frac{1}{2}\kappa x_2^2\Big\} + \\
& P(x_{hij}; x_3)\Big\{\frac{1}{n_2}\{\frac{1}{n_1}\big[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
& \quad + (r_1(x_1) - 1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij}) + (r_1(x_1) - 1)r_2(x_2)r_3^{'}(x_{hij})\big]K \\
& \quad + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})r_3(x_{hij})K + (r_2(x_2) - 1)r_3^{'}(x_{hij})K \\
& \quad - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_{hij})\} + r_3^{'}(x_{hij})K - \kappa x_2 x_2^{'}(r_3)r_3^{'}(x_{hij})\Big\} \\
& \quad - \kappa x_{hij} = 0
\end{aligned}
$$

Evaluated at x_3:

$$
\begin{aligned}
t_3\Big\{&\frac{1}{n_2}\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K\right] + W_1 - W_2 + (r_2(x_2)-1)r_3(x_3)K \\
&-\frac{1}{2}\kappa x_1^2\big\} + W_2 - W_3 + (r_3(x_3)-1)K - \frac{1}{2}\kappa x_2^2\Big\} + \\
\frac{1}{n_3}\Big\{&\frac{1}{n_2}\big\{\frac{1}{n_1}\big[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_2(x_2)r_3(x_3) \\
&+(r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3) + (r_1(x_1)-1)r_2(x_2)r_3^{'}(x_3)\big]K \\
&+r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3)K + (r_2(x_2)-1)r_3^{'}(x_3)K \\
&-\kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)\big\} + r_3^{'}(x_3)K - \kappa x_2 x_2^{'}(r_3)r_3^{'}(x_3)\Big\} \\
&-\kappa x_3 = 0
\end{aligned}
\tag{47}
$$

where

$$
t_3 = \frac{\partial P(x_{hij}; x_3)}{\partial x_{hij}}|_{x_{hij}=x_3} = \int_{\mathbf{R}} (n_3-1)H(\zeta_{hij})^{n_3-2}h(\zeta_{hij})^2 d\zeta_{hij}, \tag{48}
$$

The SOC is:

$$
\begin{aligned}
&\frac{\partial^2 P(x_{hij};x_3)}{\partial x_{hij}^2}\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_{hij})K\right] + W_1 - W_2 + (r_2(x_2)-1)r_3(x_{hij})K \\
&\quad - \frac{1}{2}\kappa x_1^2\big\} + W_2 - W_3 + (r_3(x_{hij})-1)K - \frac{1}{2}\kappa x_2^2\Big\} + \\
&2\frac{\partial P(x_{hij};x_3)}{\partial x_{hij}}\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\big[r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_3(x_{hij}) + (r_1(x_1)-1)r_2(x_2)r_3^{\prime}(x_{hij})\big]K \\
&\quad + r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_3(x_{hij})K + (r_2(x_2)-1)r_3^{\prime}(x_{hij})K \\
&\quad - \kappa x_1 x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})\big\} + r_3^{\prime}(x_{hij})K - \kappa x_2 x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})\Big\} + \\
&P(x_{hij};x_3)\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\big[r_1^{\prime\prime}(x_1)x_1^{\prime 2}(r_2)r_2^{\prime 2}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime\prime}(r_2)r_2^{\prime 2}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime\prime}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime\prime}(r_3)r_3^{\prime 2}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime\prime}(x_{hij})r_2(x_2)r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_3(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime}(x_{hij})r_2(x_2)r_3^{\prime}(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime 2}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_3(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2^{\prime\prime}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_3(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2^{\prime}(x_2)x_2^{\prime\prime}(r_3)r_3^{\prime 2}(x_{hij})r_3(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime\prime}(x_{hij})r_3(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad + r_1^{\prime}(x_1)x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime 2}(x_{hij})r_2(x_2) \\
&\quad + (r_1(x_1)-1)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad + (r_1(x_1)-1)r_2(x_2)r_3^{\prime\prime}(x_{hij})\big]K \\
&\quad - \kappa x_1^{\prime 2}(r_2)r_2^{\prime 2}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad - \kappa x_1 x_1^{\prime\prime}(r_2)r_2^{\prime 2}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad - \kappa x_1 x_1^{\prime}(r_2)r_2^{\prime\prime}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad - \kappa x_1 x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime\prime}(r_3)r_3^{\prime 2}(x_{hij}) \\
&\quad - \kappa x_1 x_1^{\prime}(r_2)r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime\prime}(x_{hij}) \\
&\quad + r_2^{\prime\prime}(x_2)x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij})r_3(x_{hij})K \\
&\quad + r_2^{\prime}(x_2)x_2^{\prime\prime}(r_3)r_3^{\prime 2}(x_{hij})r_3(x_{hij})K \\
&\quad + r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime\prime}(x_{hij})r_3(x_{hij})K \\
&\quad + r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime 2}(x_{hij})K \\
&\quad + r_2^{\prime}(x_2)x_2^{\prime}(r_3)r_3^{\prime 2}(x_{hij})K \\
&\quad + (r_2(x_2)-1)r_3^{\prime\prime}(x_{hij})K\big\} \\
&\quad + r_3^{\prime\prime}(x_{hij})K - \kappa x_2^{\prime 2}(r_3)r_3^{\prime 2}(x_{hij}) - \kappa x_2 x_2^{\prime\prime}(r_3)r_3^{\prime 2}(x_{hij}) - \kappa x_2 x_2^{\prime}(r_3)r_3^{\prime\prime}(x_{hij})\Big\} \\
&\quad - \kappa < 0
\end{aligned} \tag{49}
$$

Evaluated at x_3:

$$
\begin{aligned}
\kappa > & T_2\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\left[W_0 - W_1 + (r_1(x_1)-1)r_2(x_2)r_3(x_3)K\right] + W_1 - W_2 + (r_2(x_2)-1)r_3(x_3)K \\
& - \frac{1}{2}\kappa x_1^2\big\} + W_2 - W_3 + (r_3(x_3)-1)K - \frac{1}{2}\kappa x_2^2\Big\} + \\
& 2t_3\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\big[r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_2(x_2)r_3(x_3) \\
& + (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3) + (r_1(x_1)-1)r_2(x_2)r_3^{'}(x_3)\big]K \\
& + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3)K + (r_2(x_2)-1)r_3^{'}(x_3)K \\
& - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)\big\} + r_3^{'}(x_3)K - \kappa x_2 x_2^{'}(r_3)r_3^{'}(x_3)\Big\} \\
& \frac{1}{n_3}\Big\{\frac{1}{n_2}\big\{\frac{1}{n_1}\big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_2(x_2)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_2(x_2)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_2(x_2)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_3)r_2(x_2)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_3)r_2(x_2)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_3(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'}(x_3)r_2(x_2)r_3^{'}(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_3(x_3) \\
& + (r_1(x_1)-1)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_3(x_3) \\
& + (r_1(x_1)-1)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_3)r_3(x_3) \\
& + (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_3)r_3(x_3) \\
& + (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_3) \\
& + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_3)r_2(x_2) \\
& + (r_1(x_1)-1)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_3) \\
& + (r_1(x_1)-1)r_2(x_2)r_3^{''}(x_3)\big]K \\
& - \kappa x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3) \\
& - \kappa x_1 x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3) \\
& - \kappa x_1 x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3) \\
& - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_3) \\
& - \kappa x_1 x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_3) \\
& + r_2^{''}(x_2)x_2^{'2}(r_3)r_3^{'2}(x_3)r_3(x_3)K \\
& + r_2^{'}(x_2)x_2^{''}(r_3)r_3^{'2}(x_3)r_3(x_3)K \\
& + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{''}(x_3)r_3(x_3)K \\
& + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_3)K \\
& + r_2^{'}(x_2)x_2^{'}(r_3)r_3^{'2}(x_3)K \\
& + (r_2(x_2)-1)r_3^{''}(x_3)K\big\} \\
& + r_3^{''}(x_3)K - \kappa x_2^{'2}(r_3)r_3^{'2}(x_3) - \kappa x_2 x_2^{''}(r_3)r_3^{'2}(x_3) - \kappa x_2 x_2^{'}(r_3)r_3^{''}(x_3)\Big\}
\end{aligned} \tag{50}
$$

where

$$T_3 = \frac{\partial^2 P(x_{hij}; x_3)}{\partial x_{hij}^2}|_{x_{hij}=x_3} = \int_{\mathbf{R}} (n_3 - 1) \left[(n_3 - 2) H(\zeta_{hij})^{n_3-3} h(\zeta_{hij}) + H(\zeta_{hij})^{n_3-2} f(\zeta_{hij}) \right] h(\zeta_{hij}) d\zeta_{hij}. \tag{51}$$

C.3.3 Some Derivatives

The rate of return of private wealth is defined as:

$$r_q = \bar{r}_q - \frac{1 - e^{-\lambda x_q}}{1 + e^{-\lambda x_q}} (\bar{r}_q - \underline{r}_q) \quad q = 1, 2, 3 \tag{52}$$

The first, second, third and fourth derivatives of rate of return with respect to effort are:

$$r_i'(x_i) = -\frac{2\lambda e^{-\lambda x_i}}{(1 + e^{-\lambda x_i})^2} (\bar{r}_i - \underline{r}_i) \tag{53}$$

$$r_i''(x_i) = \frac{2\lambda^2 e^{-\lambda x_i} (1 - e^{-2\lambda x_i})}{(1 + e^{-\lambda x_i})^4} (\bar{r}_i - \underline{r}_i) \tag{54}$$

$$r_i'''(x_i) = \frac{2\lambda^3 e^{-\lambda x_i} (-1 + 3e^{-\lambda x_i} + 3e^{-2\lambda x_i} - e^{-3\lambda x_i})}{(1 + e^{-\lambda x_i})^5} (\bar{r}_i - \underline{r}_i) \tag{55}$$

$$r_i''''(x_i) = \frac{4\lambda^4 e^{-\lambda x_i} (-2 + 5e^{-\lambda x_i} - 5e^{-3\lambda x_i} + 2e^{-4\lambda x_i})}{(1 + e^{-\lambda x_i})^6} (\bar{r}_i - \underline{r}_i) \tag{56}$$

The first ,second and third third order derivatives of effort with respect to lower level rate of return are:

$$x_1'(r_2) = \frac{A}{B} \tag{57}$$

$$x_1''(r_2) = \frac{A' * B + A * B'}{B^2} \tag{58}$$

$$x_1''(r_2) = \frac{(A'' * B + 2A'B' + A * B'')B^2 - (A'B + AB')2BB'}{B^4} \tag{59}$$

$$x_2'(r_3) = \frac{C}{D} \tag{60}$$

$$x_2''(r_3) = \frac{C' * D - D' * C}{D^2} \tag{61}$$

where

$$A = t_1(r_1(x_1) - 1) + \frac{1}{n_1} r_1^{'}(x_1) \tag{62}$$

$$B = \frac{\kappa}{r_3 K} - t_1 r_1^{'}(x_1) r_2 - \frac{1}{n_1} r_1^{''}(x_1) r_2 \tag{63}$$

$$A' = t_1 r_1^{'}(x_1) x_1^{'}(r_2) + \frac{1}{n_2} r_1^{''}(x_1) x_1^{'}(r_2) \tag{64}$$

$$B' = t_1 r_1^{''}(x_1) x_1^{'}(r_2) r_2 + t_1 r_1^{'}(x_1) + \frac{1}{n_1} r_1^{'''}(x_1) x_1^{'}(r_2) r_2 + \frac{1}{n_1} r_1^{''}(x_1) \tag{65}$$

$$A'' = t_1 r_1^{''}(x_1) x_1^{'2}(r_2) + t_1 r_1^{'}(x_1) x_1^{''}(r_2) + \frac{1}{n_1} r_1^{'''}(x_1) x_1^{'2}(r_2) + \frac{1}{n_1} r_1^{''}(x_1) x_1^{''}(r_2) \tag{66}$$

$$\begin{aligned} B'' = {} & t_1 r_1^{'''}(x_1) x_1^{'2}(r_2) r_2 + t_1 r_1^{''}(x_1) x_1^{''}(r_2) r_2 + 2 t_1 r_1^{''}(x_1) x_1^{'}(r_2) \\ & + \frac{1}{n_1} r_1^{''''}(x_1) x_1^{'2}(r_2) r_2 + \frac{1}{n_1} r_1^{'''}(x_1) x_1^{''}(r_2) r_2 + 2 \frac{1}{n_1} r_1^{'''}(x_1) x_1^{'}(r_2) \end{aligned} \tag{67}$$

and

$$C = -\frac{t_2}{n_1}(r_1(x_1) - 1) r_2(x_2) - t_2(r_2(x_2) - 1) - \frac{1}{n_1 n_2}[r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'}(x_2) r_2(x_2) \tag{68}$$

$$+ (r_1(x_1) - 1) r_2^{'}(x_2)] - \frac{1}{n_2} r_2^{'}(x_2) \tag{69}$$

$$\begin{aligned} D = {} & \frac{t_2}{n_1}[r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'}(x_2) r_2(x_2) r_3 + (r_1(x_1) - 1) r_2^{'}(x_2) r_3] + t_2 r_2^{'}(x_2) r_3 - t_2 \frac{\kappa}{K} x_1 x_1^{'}(r_2) r_2^{'}(x_2) \\ & + \frac{1}{n_1 n_2}\Big[r_1^{''}(x_1) x_1^{'2}(r_2) r_2^{'2}(x_2) r_2(x_2) + r_1^{'}(x_1) x_1^{''}(r_2) r_2^{'2}(x_2) r_2(x_2) + r_1^{'}(x_1) x_1^{'}(r_2) r_2^{''}(x_2) r_2(x_2) \\ & + 2 r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'2}(x_2) + (r_1(x_1) - 1) r_2^{''}(x_2)\Big] r_3 \\ & + \frac{1}{n_2} r_2^{''}(x_2) r_3 - \frac{1}{n_2} \frac{\kappa}{K} x_1^{'2}(r_2) r_2^{'2}(x_2) - \frac{1}{n_2} \frac{\kappa}{K} x_1 x_1^{''}(r_2) r_2^{'2}(x_2) - \frac{1}{n_2} \frac{\kappa}{K} x_1 x_1^{'}(r_2) r_2^{''}(x_2) \\ & - \frac{\kappa}{K} \end{aligned} \tag{70}$$

$$\begin{aligned} C' = {} & -\frac{t_2}{n_1} r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'}(x_2) x_2^{'}(r_3) r_2(x_2) - \frac{t_2}{n_1}(r_1(x_1) - 1) r_2^{'}(x_2) x_2^{'}(r_3) \\ & - \frac{t_2}{n_1} r_2^{'}(x_2) x_2^{'}(r_3) \\ & - \frac{1}{n_1 n_2}\Big[r_1^{''}(x_1) x_1^{'2}(r_2) r_2^{'2}(x_2) x_2^{'}(r_3) r_2(x_2) + r_1^{'}(x_1) x_1^{''}(r_2) r_2^{'2}(x_2) x_2^{'}(r_3) r_2(x_2) \\ & + r_1^{'}(x_1) x_1^{'}(r_2) r_2^{''}(x_2) x_2^{'}(r_3) r_2(x_2) + 2 r_1^{'}(x_1) x_1^{'}(r_2) r_2^{'2}(x_2) x_2^{'}(r_3) \\ & + (r_1(x_1) - 1) r_2^{''}(x_2) x_2^{'}(r_3)\Big] - \frac{1}{n_2} r_2^{''}(x_2) x_2^{'}(r_3) \end{aligned} \tag{71}$$

$$
\begin{aligned}
D' =& \frac{t_2}{n_1}\big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)r_3 + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_2(x_2)r_3 \qquad (72)\\
&+ r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2)r_3 + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_3 + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)r_2(x_2)\\
&+ r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)r_3 + (r_1(x_1)-1)r_2^{''}(x_2)x_2^{'}(r_3)r_3 + (r_1(x_1)-1)r_2^{'}(x_2)\big]\\
&+ t_2\big[r_2^{''}(x_2)x_2^{'}(r_3)r_3 + r_2^{'}(x_2) - \frac{\kappa}{K}x_1^{'2}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3)\\
&- \frac{\kappa}{K}x_1x_1^{''}(r_2)r_2^{'2}(x_2)x_2^{'}(r_3) - \frac{\kappa}{K}x_1x_1^{'}(r_2)r_2^{''}(x_2)x_2^{'}(r_3)\big]\\
&+ \frac{1}{n_1n_2}\big[r_1^{'''}(x_1)x_1^{'3}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2) + r_1^{''}(x_1)2x_1^{'}(r_2)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2)\\
&+ r_1^{''}(x_1)x_1^{'2}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2) + r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)\\
&+ r_1^{''}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{''}(r_2)r_2^{'2}(x_2)r_2(x_2) + r_1^{'}(x_1)x_1^{'''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)r_2(x_2)\\
&+ r_1^{'}(x_1)x_1^{''}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)r_2(x_2) + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3)\\
&+ r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)r_2(x_2) + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2)r_2(x_2)\\
&+ r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'''}(x_2)x_2^{'}(r_3)r_2(x_2) + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)r_2^{'}(x_2)x_2^{'}(r_3)\\
&+ 2r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3) + 2r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'3}(x_2)x_2^{'}(r_3) + 2r_1^{'}(x_1)x_1^{'}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
&+ r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2) + (r_1(x_1)-1)r_2^{'''}(x_2)x_2^{'}(r_3)\big]r_3\\
&+ \frac{1}{n_1n_2}\big[r_1^{''}(x_1)x_1^{'2}(r_2)r_2^{'2}(x_2)r_2(x_2) + r_1^{'}(x_1)x_1^{''}(r_2)r_2^{'2}(x_2)r_2(x_2) + r_1^{'}(x_1)x_1^{'}(r_2)r_2^{''}(x_2)r_2(x_2)\\
&+ 2r_1^{'}(x_1)x_1^{'}(r_2)r_2^{'2}(x_2) + (r_1(x_1)-1)r_2^{''}(x_2)\big]\\
&+ \frac{1}{n_2}r_2^{'''}(x_2)x_2^{'}(r_3)r_3 + \frac{1}{n_2}r_2^{''}(x_2)\\
&- \frac{1}{n_2}\frac{\kappa}{K}2x_1^{'}(r_2)x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{'2}(x_2) - \frac{1}{n_2}\frac{\kappa}{K}x_1^{'2}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3)\\
&- \frac{1}{n_2}\frac{\kappa}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{''}(r_2)r_2^{'2}(x_2) - \frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{'''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{'2}(x_2)\\
&- \frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{''}(r_2)2r_2^{'}(x_2)r_2^{''}(x_2)x_2^{'}(r_3) - \frac{1}{n_2}\frac{\kappa}{K}x_1^{'}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)x_1^{'}(r_2)r_2^{''}(x_2)\\
&- \frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{''}(r_2)r_2^{'}(x_2)x_2^{'}(r_3)r_2^{''}(x_2) - \frac{1}{n_2}\frac{\kappa}{K}x_1x_1^{'}(r_2)r_2^{'''}(x_2)x_2^{'}(r_3)
\end{aligned}
$$

C.3.4 The Principal's Problem.

The principal maximizes the expected total final output net the cost of incentives:

$$
\begin{aligned}
&\max_{W_0,W_1,W_2,W_3} \quad E(n_1n_2n_3\mathcal{A}q_1^{\alpha}q_2^{\beta}q_3^{\gamma}) - W_0 - (n_1-1)W_1 - (n_2-1)n_1W_2 - (n_3-1)n_1n_2W_3\\
&\qquad + \pi_1(n_1-1)r_1(x_1)r_2(x_2)r_3(x_3)K + \pi_2n_1(n_2-1)r_2(x_2)r_3(x_3)K + \pi_3n_1n_2(n_3-1)r_3(x_3)K\\
\Leftrightarrow\ &\max_{W_0,W_1,W_2,W_3} \quad n_1n_2n_3\mathcal{A}x_1^{\alpha}x_2^{\beta}x_3^{\gamma} - W_0 - (n_1-1)W_1 - (n_2-1)n_1W_2 - (n_3-1)n_1n_2W_3\\
&\qquad + (n_1-1)(r_1(x_1)-1)r_2(x_2)r_3(x_3)K + n_1(n_2-1)(r_2(x_2)-1)r_3(x_3)K \qquad (73)\\
&\qquad + n_1n_2(n_3-1)(r_3(x_3)-1)K
\end{aligned}
$$

subject to the incentive compatibility constraints of the provincial governors, the prefectural mayors, and the county leaders.

References

Guo, Y. (2013). Six tendencies of corruption and anti-corruption in contemporary China. *Chinese Public Administration*, 331:60–63.