Measuring the Natural Rate of Interest of OECD and BRICS Economies: A Time Varying Perspective*

Bin Wang†
City University of Hong Kong

This Version: June 23, 2019

Abstract

This paper measures the natural rates of interest of eleven economies including six OECD economies and five emerging economies of BRICS in a coherent time varying parameter vector autoregression framework. I find that the natural rates of interest in OECD economies have been descending especially since the 2007-2009 financial crisis. The trends of natural rates of BRICS economies do not share one common pattern. The descending trend of the natural rate of interest is a regional phenomenon instead of a global one. The demographic structure is the dominant factor to affect the variations of natural rate compared with other two factors economic growth and desire to safte assets in all the eleven economies.

Keywords: Natural Rate of Interest; TVP-VAR-SV; OECD; BRICS; Emerging Markets.

JEL codes: C11; C32; E43; E52.

---

*This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

†Corresponding author: Qishanxiangshujun, Dongsheng Road, Liaobu Town, Dongguan, China. Email: workbinwang@gmail.com, Website: https://sites.google.com/view/workbinwang

1 Introduction

One decade after the Great Recession has witnessed an unusual episode in the world that advanced economies expirenced a slow but long recovery, full employment but low inflation, and espcially a low interest rate environment. The US kept the norminal short-term interest rate right above the zero lower bound for a long time until recently the Fed lifted it to above 2%. The U.S. is not alone in the low interest rate world that the U.K., the European Union,and Japan still set the norminal short-term interest rate a little bit above zero.

One way to explain the low nominal interest rate links to the low natural rate of interest. Knut Wisksell introduced the notion that the natural rate of interest is the real short-term interest rate that the inflation stays the same and the output attains the potential. If the present natural rate of interest in the world is so low as a bit above zero, central bankers have no reason to raise the norminal one to supress the economy. The real rate gap which is the difference between the real interest rate and the natural rate of interest is an important index in evaluating the status of the macroeconomy and can help central banks to adjust the stance of monetary policy to stabilize the economy. Measuring the natural rate of interest is the key to clear the controversy whether central banks should raise the nominal short-term interest rate or not.

Recently some studies like Lubik and Matthes (2015), Holston et al. (2017), and Negro et al. (2017) point out that the natural rates of several advanced economies have been decreasing especially since the Great Recession. However, they exlude the exploration of developing economies except Wang (2019) who finds the natural rate of China has been decreasing since 2009. Is the natural rate of interest really on a descending trend in the world including developed economies and especially developing economies? I explore this question in this paper.

The difficulty to compare the natural rates is on the measurement because the natural rate of interest can not be directly observed. One famous method to estimate the natural rate is Laubach and Williams (2003) who propose a few restrictions on the trend growth rate, natural rate of interest, and inflation rate of the economic system in a backward old-Keynsian style. Another method is to establish the natural rate in the fully-blown DSGE model such as Giammarioli and Valla (2004) and Barsky et al. (2014). One common feasure of these methods is that plenty restrictions are imposed on the economic system and are accomodated by the advanced economies, which makes the highly-structured models may or may not be suitable for developing economies.

Lubik and Matthes (2015) moves to another direction that they impose minimal restrictions on the vector autoregression system in which the parameters and variance are time varying to manifest the regime-changing in the underlying economic system. Wang (2019) applies their method to estimate the natural rate of China and finds that China's natural rate resembles the descending trend of advanced economies.

I contribute to the literature by estimating the natural rates of different kinds of economies including advanced and developing economies in a coherent econometric framework-the time varying parameter vector autoregression model with stochastic volatility, following Lubik and Matthes (2015) in this paper. For the eleven economies Canada, Euro Area, Japan, South Korea, the U.K., the U.S., Brazil, China, India, Russia, and South Africa, I estimate the small-scale TVP-VAR-SV model consisting of GDP growth rate, inflation rate, and ex ante real interest rate, as in Lubik and Matthes (2015). Then the natural rate is calculated as the long run (5 years) forecast of ex ante real interest rate using the median parameter value of each quarter. The results show that the natural rates in OECD economies have been descreasing while those in BRICS economies do not share the same pattern.

The natural rate series of all the eleven economies exibit unit root and all are $I(1)$. We establish the conclusion that the natural rates of OECD economies are decreasing but there is no common pattern among BRICS economies by a series of statistical results. First, we compare the yearly averages of natural rates of three specific years for the eleven economies. Second, we regress the natural rates on the time periods to get the determinisic trends and show the sign of time trend parameter along with the standard errors. Third, we compute the Hodrick-Prescott trends of the natural rate and show the correlation matrix among the eleven natural rate trends. The three pieces of evidence indicate that descending trend of the natural rate is a regional phenomenon instead of a gloal one. There is no common patterns for the natural rate trends of BRICS economies. For China, the natural rate was indeed on a descending trend after the financial crisis as found in Wang (2019). But the natural rate of India significantly exibited an ascending deterministic trend after 2007:Q4.

In the literature, the desire for safe assets, economic growth, and demographic structure are three factors to affect the variations of the natural rate of interest. We apply a bayesian VAR model consisted of the three factors and natural rates to analyze which factor is the most important for each economy. Different from Holston et al. (2017) who only consider economic growth, our results show that demographic structure is the dominant factor to affect natural rates.

I reestimate the natural rates of all the economies by the Holston et al. (2017) method for the robustness check. The estimated natural rates of Canada, Euro Area, the U.K., and the U.S. are nearly the same as the rates estimated in the TVP-VAR-SV method in the paper. The natural rates are on a descending trend since 2000s. But the estimated natural rates of other economies including Japan, South Korea, and all BRICS economies are different from those estimated in the TVP-VAR-SV method. This result is not so suprising because Holston et al. (2017) impose the restriction that the natural rate of interest is the sum of trend growth rate of potential output and a random shock. When the GDP growth rate is high, the estimated natural rate of interest is also high in magnitude even though the short-run real interest rate is low in magnitude. Japan, South Korea, and all BRICS economies have been undergoing high economic growth rate epsiode[1] during the estimated sample. Our TVP-VAR-SV method doesn't impose this restriction and be more assumption-free in the estimation of natural rate of interest.

The rest of the paper is organized as follows: Section 2 introduces the econometric framework to estimate the natural rate of interest; Section 3 contains the estimation results; Section 4 discusses the factors to influence the natural rate of interes; Section 5 describes the robustness check; Section 6 concludes.

2 The Econometric Framework for Estimating the Natural Rate of Interest

My empirical method to measure the natural rate of interest is based on the time varying parameter vector autoregression with stochastic volatility model. The VAR framework is flexible to describe the dynamics of the macroeconomy and requires only minimal identification restrictions. The time varying parameter VAR model with stochastic volatility can manifest the underlying regime switch and thus nonlinearity of the economy, which is suitable for comparison study of multi-economies with different institutions.

The VAR model describes the evolution of a vector of n economic variables y_t as a linear function of its own lags up to order p and a vector of innovations u_t. Here in this study, I include GDP growth rate g_t, inflation rate π_t, and ex ante real interest rate r_t in the small scale VAR system so that $y_t = (g_t, \pi_t, r_t)'$ and I select 2 lags following the TVP-VAR convention.

$$y_t = B_{0,t} + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + u_t, u_t \sim N(0, \Omega_t) \qquad t = 1, \cdots, T. \tag{1}$$

[1]Japan and South Korea grew from developing countries to developed countries during the estimated samle period. The BRICS countries are still in the process.

Both the lag coefficients $B_{0,t}, B_{1,t}, B_{2,t}$ and variance covariance matrix Ω_t are time varying. The variance covariance matrix Ω_t is decomposed as:

$$\Omega_t = A_t^{-1}\Sigma_t\Sigma_t' A_t^{-1'}.$$

where

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix}; \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix}.$$

Let $x_t' = (1, y_{t-1}', y_{t-2}')$ and $B_t = (B_{0,t}, B_{1,t}, B_{2,t})'$. Stack all the columns of B_t into a vector $\beta_t = vec(B_t)$ and then we have the following stacked VAR:

$$y_t = X_t'\beta_t + A_t^{-1}\Sigma_t\epsilon_t. \tag{2}$$

where $X_t' = I_n \otimes x_t'$

Collect all the non-zero and non-one elements into a vector $\alpha_t = (\alpha_t^1, \alpha_t^2)'$ where α_t^1, α_t^2 corresponds to the vector of the non-zero and non-one elements of the second and third row of A_t respectively. Collect all the diagonal elements of Σ_t and let $h_t = (\log\sigma_{1,t}, \log\sigma_{2,t}, \log\sigma_{3,t})'$.

It is assumed that the time varying parameters follow random walk processes:

$$\begin{aligned} \beta_t &= \beta_{t-1} + \epsilon_{\beta,t} \\ \alpha_t^i &= \alpha_{t-1}^i + \epsilon_{\alpha,t}^i, \quad i = 1,2 \\ h_t &= h_{t-1} + \epsilon_{h,t}. \end{aligned}$$

I assume that $\epsilon_t, \epsilon_{\beta,t}, \epsilon_{\alpha^i,t}, \epsilon_{h,t}$ are mutually independent.

$$\begin{pmatrix} \epsilon_t \\ \epsilon_{\beta,t} \\ \epsilon_{\alpha,t} \\ \epsilon_{h,t} \end{pmatrix} \sim N(0,V), \quad V = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & \Sigma_\beta & 0 & 0 \\ 0 & 0 & \Sigma_\alpha & 0 \\ 0 & 0 & 0 & \Sigma_h \end{bmatrix}.$$

Here Σ_α is block diagonal with each block corresponding to the non-zero and non-one elements in each row of A_t.

$$\Sigma_\alpha = \begin{bmatrix} \Sigma_{\alpha^1} & 0 \\ 0 & \Sigma_{\alpha^2} \end{bmatrix}.$$

I estimate the parameters by Bayesian methodology following Primiceri (2005). The order of Gibbs sampling is corrected as Del Negro and Primiceri (2015). The first 10 years is used as the presample to calibrate the priors except that 5 years is instead used for economies with less than 100 observations. The prior distributions are elaborated below.

$$\begin{aligned}
\beta_0 &\sim N(\hat{\beta}_{OLS}, 4 \cdot \hat{V}_{\beta,OLS}).\\
\alpha_0 &\sim N(\hat{\alpha}_{OLS}, 4 \cdot \hat{V}_{\alpha,OLS}).\\
h_0 &\sim N(\hat{h}_{OLS}, I_3).\\
\Sigma_\beta &\sim IW(k_\beta^2 \cdot 40 \cdot \hat{V}_{\beta,OLS}, 40).\\
\Sigma_{\alpha^1} &\sim IW(k_\alpha^2 \cdot 2 \cdot \hat{V}_{\alpha^1,OLS}, 2).\\
\Sigma_{\alpha^2} &\sim IW(k_\alpha^2 \cdot 3 \cdot \hat{V}_{\alpha^2,OLS}, 3).\\
\Sigma_h &\sim IW(k_h^2 \cdot 4 \cdot I_3, 4).
\end{aligned}$$

where $k_\beta = 0.01, k_\alpha = 0.1, k_h = 0.01$.

As in Lubik and Matthes (2015), I define the conditional 5-year forecast of the observed ex ante real interest rate as a measure of natural rate of interest in the small scale VAR system. The natural rate of interest is:

$$r_t^* = (0,0,1) * E_t(y_{t+20}) = (0,0,1) * X_t' \beta_t^{20} \tag{3}$$

For every quarter, the 20-step-ahead forecast of y_t is computed using the very parameter β_t of that quarter as in Equation (3). The natural rate r_t^* is the third varaible in the 20-step-ahead forecast of y_t.

3 Estimation Results

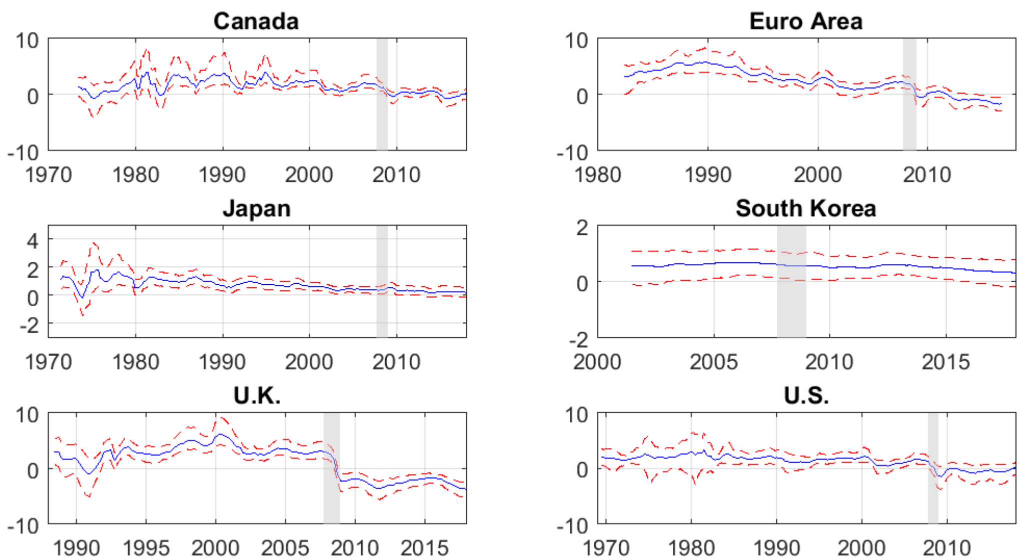
I select 11 advanced and emerging economies to shed light on the natural rate of interest in the world. The advanced economies are Canada, Euro Area, Japan, South Korea, the U.K., and the U.S.. The emerging economies are the five BRICS countries Brazil, China, India, Russia, and South Africa. The data sources are elaborated in Appendix A.

3.1 Estimates of the Natural Rate of Interest

Figure 1 illustrates the natural rates of interest of OECD economies . The natural rates of interest in these six OECD economies are roughly decreasing especially after 2000 and goes negative after the 2007-2009 financial crisis. This descending trend is also found in other studies (Laubach and Williams (2016), Holston et al. (2017), Lubik and Matthes (2015), Fujiwara et al. (2016)) applying various methods to measure this rate of different economies. The natural rates of Canada, Euro Area, the U.K., and the U.S. measured in this paper are smoother than those measured by Holston et al. (2017). The descending trends of the natural rates replicate their result in this paper.

I explore two more economies Japan and South Korea for the OECD countries group. The natural rate of Japan fluctuated around 1% before the beginning of lost decades in 1990 and then descended along a slow trend. After 2000, this rate was around nearly zero. Fujiwara et al. (2016) who applies Lauback-Williams model finds similar result. For South Korea, the natural rate decreased after the financial crisis. In all the six OECD economies, the natural rates all plummeted sharply during the Great Recession except Japan.

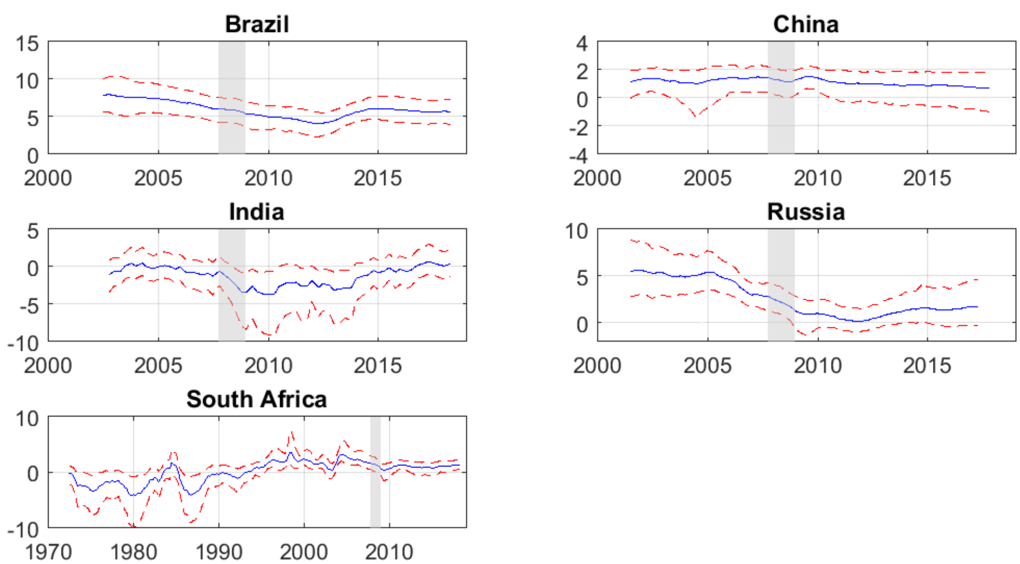
Figure 1: The Natural Rate of Interest of OECD Economies



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (3) by using median and 16th/84th percentiles of the parameter of every quarter.

Figure 2 illustrates the natural rates of interest of BRICS economies. Distinct from the OECD economies, there is no common pattern for the natural rates of interest within BRICS countires. Like the OECD economies, the natural rate of China started to decrease after the end of the 4 trillion stimulus in 2010 as found in Wang (2019). Brazil and Russia have much higher level of natural rate. The natural rate descended from 7% to 5% for Brazil. For Russian, the natural rate decreased from 5% to 1% . The natural rates of these two economies bounced back after the end of the financial crisis. During most of the estimated period, the natural rate of India is negative. The natural rate decreased until the end of the Great Recession and then bounced back to the level before the financial crisis. For South Africa, the rate increased from 1980s to 2000s and then decreased after 2005. During the Great Recession in the shaded area, the natural rates of interest plummeted for all BRICS economies.

Figure 2: The Natural Rate of Interest of BRICS Economies



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (3) by using median and 16th/84th percentiles of the parameter of every quarter.

3.2 Time Trends of the Natural Rates

The above visual displays show the natural rates of OECD economoies are descending after 2000s but there is no common pattern in the BRICS economies. In this section, I more formally show the time trends of the natural rates of the eleven economies by comparing the natural rate averages of three specific year, exporing the deterministic trends and Hodrick-Prescott trends.

3.2.1 Natural Rate Averages of Three Specific Years

Table 1 displays the yearly average of the natural rate estimates of the OECD economies in three specific years 1995, 2008, and 2016. A clear descending trend can be seen from this simple illustration of natual rate yearly averages of three years. After the 2007-2009 financial crisis, the natural rate of the U.S. decreased to be negative. In the aftermath of the crisis, the natural rates of interest of all OECD economies continued to descend. The natural rates for Canada, Euro Area, and the U.K. further declined to the negative region along with the U.S. in 2016.

Table 1: Natural Rate Estimates of OECD Economies

Economy	1995	2008	2016	1995-2008	2008-2016
Canada	3.21	1.18	-0.50	-2.03	-1.68
Euro Area	3.60	1.76	-1.60	-1.84	-3.36
Japan	0.83	0.44	0.31	-0.39	-0.13
South Korea	n/a	0.58	0.40	n/a	-0.18
U.K.	2.53	1.50	-2.21	-1.04	-3.71
U.S.	1.69	-0.03	-0.57	-1.72	-0.55

Note: All numbers are yearly averages of the median estimates of the natural rates of interest from the TVP-VAR-SV model. The last two columns are the changes of the natural rates of the two specific years.

There is no universal common pattern for the BRICS emerging economies. Table 2 furthter illustrates this point by showing the yearly averages of the BRICS economies in 2003, 2010, and 2016. The yearly averages are not universally descending like the OECD economies.

Table 2: Natural Rate Estimates of BRICS Economies

Economy	2003	2010	2016	2003-2010	2010-2016
Brazil	7.67	4.92	5.82	-2.75	0.90
China	1.19	1.25	0.84	0.07	-0.42
India	-0.20	-3.06	-0.30	-2.86	2.75
Russia	5.08	0.85	1.52	-4.23	0.67
South Africa	0.82	0.90	1.15	0.08	0.25

Note: All numbers are yearly averages of the median estimates of the natural rates of interest from the TVP-VAR-SV model. The last two columns are the changes of the natural rates of the two specific years.

3.2.2 Deterministic Time Trend

Here we show the evidence that the natural rates of all eleven economies are all first-difference-stationary and then explore the trend pattern by finding the deterministic time trends.

The first two columns of Table B-1 in Appendix B show the Augmented Dickey-Fuller test statistics for the eleven economies and the natural rates of all economies exhibit unit root. After first differencing, the natural rates are all stationary shown by the statistics in the last two column statistics. So all the natural rate series are $I(1)$. The natural rates of all economies have a trend but it doesn't mean that they have the same asending or descending trend.

I regress the natural rate against time periods to obtain the deterministic trends and show the coefficients along with the standard errors for all the eleven economies in Table 3 and 4. The top section of these two tables corresponds to the statistics of the sample between 1995:Q1 to the end[2]. The lower section displays the statistics of the sample after the start of the financial crisis. We can see that there are slow descending trends for the natural rates of all the six OECD economies. But there is no common pattern for the BRICS group. China is similar to the OECD economies with a slow descending trend of the natural rate over the whole period and the period after 2008.. For Brazil, India, Russia, and South Africa, the natural rates are descending over the the period after 1995. But the descending trend of India is not significant. After the financial crisis, the natural rates of all these four economies were asending but the asending trends for Russia and South Africa are not significant.

Table 3: Time Trend of OECD Economies

	Canada	Euro Area	Japan	South Korea	U.K.	U.S.
Period: After 1995:Q1						
Coefficient	-0.0304*	-0.0533*	-0.0061*	-0.0037*	-0.0989*	-0.0297*
St..Dev.	0.0022	0.0026	0.0003	0.0004	0.006	0.0023
Period: After 2007:Q4						
Coefficient	-0.0249*	-0.0865*	-0.0058*	-0.0057*	-0.0720*	0.0021
St..Dev..	0.0056	0.0082	0.0008	0.0007	0.0167	0.0063

Note: The coefficients with corresponding standard errors are obtained from regressing the natural rates on time periods. "" stands for statistical significance under 95% confidence level. The top section displays the results of the sample after 1995:Q1 while the bottome section displays thoses of the sample period after 2007:Q4.*

3.2.3 Hodrick-Prescott Trend

It is difficult to distingusih whether the trend series has deterministic trend or stochastic trend. The macroeconomic series are usually modeled as variables with stochastic trends. So

[2] If the starting period is after 1995:Q1, the we select the whole sample for this economy.

Table 4: Time Trend of BRICS Economies

	Brazil	China	India	Russia	South Africa
Period: After 1995:Q1					
Coefficient	-0.0376*	-0.0089*	-0.0020	-0.0850*	-0.0138*
St..Dev.	0.0056	0.0010	0.0090	0.0071	0.0024
Period: After 2007:Q4					
Coefficient	0.0145	-0.0166*	0.0801*	0.0004	0.0024
St..Dev..	0.0074	0.0013	0.0111	0.0092	0.0036

Note: The coefficient with corresponding standard error is obtained from regressing the natural rate on time periods. "" stands for statistical significance under 95% confidence level. The top section displays the results of the sample period after 1995:Q1 while the bottome section displays thoses of the period after 2007:Q4.*

I use the Hodrick-Prescott filter with a smoothing parameter 1600 to obtain the trends of all the natural rates for an alternative illustration of the deterministic trend results. Because the natural rates are trend-stationary, the correlation parameter of the trend series will be highly close to 1 if two trend series share the same direction. Otherwise, the correlation parameter will be a small number or even be negative if two trend series go to opposite directions. Table 5 shows the correlation matrix of the Hodrick-Prescott trends of the natural rates of all the 11 economies in the period from 2008:Q1 to 2016:Q4. We can see that the correlation between the natural rate trends of OECD economies are close to 1 but the correlations between those of OECD economies and BRICS economies are not the case. For example, the correlation between the natural rate trend of the US and other five OECD economies are all above 0.7. But the numbers between US and BRICS economies are 0.22, 0.86, -0.32, 0.71, and 0.98 respectively. The US and Brazil are weakly related while the US and india have opposite trends of the natural rates.

This is the direct statistical evidence that the common desending trend of the natural rate exists in advanced economies. But it is a regional phenomenon. There is no universal descending trend if we expand to emerging economies.

4 Discussion: Factors that Affect the Natural Rate of Interest

The neoclassical growth model concludes that the steady state natural rate of interest r^* is positively related to economic growth rate g given intertemporal elasticity of substitution σ, and the rate of time preference θ as in Equation (4).

$$r^* = \frac{1}{\sigma} g + \theta \tag{4}$$

The less risk averse is, or the faster the economy grows or the less patient consumers are, the higher the natural rate of interest should be. The three parameters correspond to three probable factors that affect the natural rate of interest in the literature: desire for safe assets, economic growth and demographic structure[3]. Bullard (2018) summerizes three factors affecting the long term natural rate as: labor productivity growth rate, labor force growth rate, and investors' desire for safe assets. The former two combines to be the economic growth rate g, and the third one is related to the risk aversion σ. Holston et al. (2017) attributes the descending trend of four developed economies to the lower economic growth rate. Negro et al. (2017) conclude that the descending trend in the advanced economies was driven by an increase in the convenience yield for safety and liquidity and by lower global economic growth.

I run an VAR consisting of convenince yield, economic growth rate, the ratio of working over young population, and natural rate of interest to show the contribution of each factor on the variation of natural rate. Negro et al. (2017) define the convenience yield as the spread between long-term Baa corporate bond yield and the treasury bill of the U.S., which is a proxy for the desire for safe assets. I use this conveniece yield as a proxy for risk averse parameter σ of each economy considering that foreign reserves of majority economies are still US treasury bills or bonds. I use the ratio of working population (those between 15 and 64) to young population (those between 0 and 14) as a proxy for the demographic structure [4]. The economic growth rate and natural rate of interest are obtained from the TVP-VAR-SV part.

The VAR is identified by the recursive tradition with the assumption that natural rate responds to other three long-term factors contemporaneously but the three factors respond to natural rate with lags. So the natural rate of interest is placed last. The shocks related to the three factors are not interesting in this paper so the order within the three variables is not essential. It does no harm to place the four variables as:

$$d_t = (cy_t, g_t, wyr_t, r_t^*)' \tag{5}$$

where cy_t is the conveniency yield, g_t is the GDP growth rate, wyr_t is the ratio of working over young population, and r_t^* is the natural rate of interest.

[3] If people more intently desire for safe assets, people are more risk averse and σ is higher. If old people account more in a population, people save more to deal with the aging demographic structure and then the rate of time preference θ is smaller.

[4] Favero et al. (2016) find that one variable effectively summarizes the information between demographic composition and real interest rates—the ratio of middle-aged (those between 40 and 49) over young individuals (those between 20 and 29)

I estimate the VAR by bayesian method and Minnisota prior is assumed. The priors and estimation details are elaborated in Appendix C. We sample 12000 draws from the posterior distribution and the first 2000 draws are discarded. We use the median of all the remaining draws to compute the forecast error variance decomposition of the natural rate of interest.

The left sections of Tables 6 and 7 display the forecast error decompostioin of the natural rate of the whole sample period while the right sections show that of the period after the financial crisis. We can see that the variations of the natural rates are overwhelmingly dominated by the ratio of working over young population which is an indicater of the demographic structure.

For the OECD group, the demographic structure acounts for over 70% of the variance of natural rates for Canada, Euro Area, the UK, and the US at long run horizon. This result is different from Holston et al. (2017) who only exploit the factor of slowing economic growth. My result attributes tiny role to economic growth. For Japan and South Korea, the demographic structure is still the dominant role to the variations of natural rates among the three factors but the magnitudes are far less than the other four economies, 38.28% for Japan and 13.03% for South Korea. When the samples are shortened to the period after the financial crisis, the demographic structure is still the dominant factor though the propotions decrease, especially for Japan in which the proportion decreased from 38.28% to 1.86%. For Japan and South Korea, larger proportion of the variations of natural rates are not explained by the three factors.

For the BRICS group, the demographic structure is nearly the unique factor that determines the natural rates at long term horizon, with nearly 100% contributions to the variance of natural rates of all the six economies.

5 Robustness

I reestimate the natural rates of interest of the eleven economies by the Holston et al. (2017) (later as HLW) estimation [5] for the robustness check. Figure D-1 and D-2 in Appendix D embed the natural rates estimated by our benchmark VAR method and HLW method for OECD group and BRICS group.

The natural rates of interest estimated by the two methods are similar in the economies of Canada, Euro Area, the U.K., and the U.S., coincidently the four economies explored by Holston et al. (2017). And the natural rates in these four economies have been decreasing

[5]The Holston et al. (2017) program is downloaded from the New York Fed website: `https://www.newyorkfed.org/medialibrary/media/research/economists/williams/data/HLW_Code.zip`

since 2000s.

For other economies including two OECD economies Japan and South Korea and all BRICS economies, the natural rates of interest estimated by the two methods are different in terms of magnitude and dynamics. The natural rate of Japan esitmated by HLW method began at a high value in the beggining of 1970s and then decreased to below zero ever since 1980s. For South Korea, the natural rate by HLW method has been decreasing from above 5% in early 2000s to nearly 0 recently. For China and India, the maganitudes of the natural rates estimated by the HLW method are much higher than those estimated from the TVP-VAR-SV method. For Russia, the natural rate estimated by HLW is highly volatile.

These seemingly abnormality can be seen from the restrictions imposed by Holston et al. (2017). Equation (6) and (9) in their paper restrict that the natural rate is the sum of the trend growth rate of the potential output and other determinates of r_t^*.

$$\begin{aligned} r_t^* &= g_t + z_t &\quad (6) \\ z_t &= z_{t-1} + \epsilon_{z,t} &\quad (7) \end{aligned}$$

So the estimated natural rate of interest comoves with the growth rate of potential output closely. We know that the growth rate of Japan in the 1970s was quite high and then the esimated natural rate by HLW comoved with the high growth rate instead of near to the "normal" level of real interest rate. The same reason can explain the abnormalities of India, China, and Russia. The GDP growth rate of india and China are quite high across the whole sample period. But the real interest rates are comparatively lower. Then the estimated natural rates of interest estimated by HLW are of high magnitudes. For Russia, the GDP growth rate is really volatile so the estimated natural rate by HLW behaved the same.

The natural rates of interest in TVP-VAR-SV are long run forecast of the real interest rate. Then our estimated natural rate of interest behaves more close to the real interest rate. This is different from Holston et al. (2017) who restrict the natural rate of interest behaves more close to the growth rate. If the magnitude of growth rate is not far away from the real interest rate, the TVP-VAR-SV estimate is similar to HLW, such as Canada, Euro Area, the U.K., and the U.S.. Otherwise, the estimated natural rates by the two methods are different such as China and India.

It is not suprising that the estimated natural rate of interest by HLW method exhibits a decending trend when the GDP growth rate is on a descending trend. Our method of

TVP-VAR-SV does not impose the restriction that the natural rate shoud be around the trend of potential output as Equation (6). We believe our result is based on a more general setup.

6 Conclusion

I propose the 5-year forecast of the ex ante real interest rate in a time varying parameter vector autoregression model with stochastic volatility as the natural rate of interest following Lubik and Matthes (2015) and measure the rate for 11 economies including OECD countries and BRICS countries. I find that the natural rates of interest in OECD economies have been descending especially since the 2007-2009 financial crisis. There is no common pattern in BRICS economies. Among three probable factors that affect the natural rate, the demographic factor is the dominant one compared with the other two: desire for safe assets and economic growth.

Table 5: Correlation Matrix of the Trends of Natural Rates: 2008:Q1-2016:Q4

	Canada	Euro Area	Japan	South Korea	U.K.	U.S.	Brazil	China	India	Russia	South Africa
Canada	1.00	0.98	0.95	0.97	0.74	0.86	-0.29	0.97	-0.74	0.27	0.75
Euro Area	0.98	1.00	0.99	0.93	0.77	0.88	-0.26	1.00	-0.72	0.29	0.77
Japan	0.95	0.99	1.00	0.89	0.76	0.87	-0.25	1.00	-0.71	0.29	0.76
South Korea	0.97	0.93	0.89	1.00	0.54	0.70	-0.52	0.92	-0.88	0.02	0.56
U.K.	0.74	0.77	0.76	0.54	1.00	0.98	0.42	0.75	-0.12	0.84	0.99
U.S.	0.86	0.88	0.87	0.70	0.98	1.00	0.22	0.86	-0.32	0.71	0.98
Brazil	-0.29	-0.26	-0.25	-0.52	0.42	0.22	1.00	-0.28	0.85	0.84	0.39
China	0.97	1.00	1.00	0.92	0.75	0.86	-0.28	1.00	-0.73	0.27	0.75
India	-0.74	-0.72	-0.71	-0.88	-0.12	-0.32	0.85	-0.73	1.00	0.44	-0.13
Russia	0.27	0.29	0.29	0.02	0.84	0.71	0.84	0.27	0.44	1.00	0.83
South Africa	0.75	0.77	0.76	0.56	0.99	0.98	0.39	0.75	-0.13	0.83	1.00

Note: The trend of the natural rate is computed from the Hodrick-Prescott filter with a smoothing parameter 1600.

Table 6: Forecast Error Variance Decompostion of Natural Rates of Interest of OECD Economies

	Horizon	cy	g	wyr	r^*	cy	g	wyr	r^*
		The Whole Sample				After 2007:Q4			
Canada	1	0.12	0.10	2.55	97.15	0.05	0.02	1.39	98.47
	4	0.89	0.28	23.50	74.97	0.21	0.05	6.35	93.21
	12	1.01	0.16	72.93	25.46	0.31	0.10	17.70	81.65
	∞	0.69	0.10	82.24	16.69	0.32	0.10	20.13	79.10
Euro Area	1	1.45	0.59	5.34	92.15	3.05	0.59	5.54	90.09
	4	5.54	0.60	21.39	70.37	3.39	0.50	27.45	67.49
	12	3.26	0.58	61.69	32.36	1.63	0.61	53.84	43.18
	∞	1.60	0.31	80.39	16.26	0.77	0.36	74.15	24.31
Japan	1	0.00	0.00	0.35	99.64	0.10	0.00	0.08	99.79
	4	0.01	0.05	3.31	96.61	0.36	0.00	0.52	99.00
	12	0.06	0.10	17.38	82.40	0.34	0.01	1.39	98.06
	∞	0.08	0.08	38.28	61.53	0.33	0.01	1.86	97.59
South Korea	1	0.00	0.00	0.68	99.32	0.00	0.00	0.95	99.04
	4	0.00	0.00	1.65	98.34	0.01	0.00	1.68	98.30
	12	0.01	0.00	3.73	96.24	0.01	0.00	3.12	96.83
	∞	0.04	0.00	13.03	86.89	0.02	0.01	7.92	92.03
U.K.	1	0.37	0.20	41.21	57.88	0.42	0.08	80.64	18.79
	4	0.64	0.20	85.34	13.40	0.32	0.09	91.51	7.98
	12	0.24	0.08	97.20	2.33	0.46	0.16	94.31	4.92
	∞	0.20	0.05	98.55	1.05	0.41	0.13	95.28	4.00
U.S.	1	0.13	0.01	9.83	89.99	0.04	0.03	42.60	57.19
	4	0.19	0.02	19.84	79.86	0.31	0.10	60.97	38.37
	12	0.20	0.04	39.58	60.05	0.40	0.13	76.55	22.63
	∞	0.10	0.02	76.56	23.26	0.38	0.13	82.48	16.78

Note: The left section displays the forecast error variance decomposition of the natural rate of interest for the whole sample period while the right section corresponds to the same statistics of the period after the financial crisis.

Table 7: Forecast Error Variance Decompostion of Natural Rates of Interest of BRICS Economies

	Horizon	cy	g	wyr	r^*	cy	g	wyr	r^*
		The Whole Sample				After 2007:Q4			
Brazil	1	0.00	0.00	97.65	2.35	0.00	0.00	98.46	1.54
	4	0.00	0.00	99.69	0.30	0.00	0.00	99.72	0.29
	12	0.00	0.00	99.90	0.10	0.00	0.00	99.88	0.12
	∞	0.00	0.00	99.94	0.06	0.00	0.00	99.89	0.11
China	1	0.00	0.01	87.84	12.14	0.00	0.00	90.03	9.96
	4	0.00	0.01	97.95	2.04	0.00	0.00	99.40	0.60
	12	0.00	0.00	98.99	1.00	0.00	0.00	99.78	0.22
	∞	0.00	0.00	99.15	0.84	0.00	0.00	99.84	0.16
India	1	0.00	0.00	99.96	0.04	0.00	0.00	99.99	0.01
	4	0.00	0.00	100.00	0.00	0.00	0.00	100.00	0.00
	12	0.00	0.00	100.00	0.00	0.00	0.00	100.00	0.00
	∞	0.00	0.00	100.00	0.00	0.00	0.00	100.00	0.00
Russia	1	0.03	0.00	79.56	20.40	0.05	0.00	80.25	19.67
	4	0.01	0.00	95.49	4.50	0.03	0.00	97.42	2.53
	12	0.01	0.00	98.27	1.72	0.01	0.00	99.18	0.80
	∞	0.00	0.00	99.77	0.23	0.01	0.00	99.25	0.73
South Africa	1	0.00	0.00	99.67	0.32	0.00	0.00	99.88	0.12
	4	0.00	0.00	99.98	0.02	0.00	0.00	99.96	0.04
	12	0.00	0.00	100.00	0.00	0.00	0.00	99.99	0.01
	∞	0.00	0.00	100.00	0.00	0.00	0.00	99.99	0.01

Note: The left section displays the forecast error variance decomposition of the natural rate of interest for the whole sample period while the right section corresponds to the same statistics of the period after the financial crisis.

References

Barsky, R., Justinano, A., and Melosi, L. (2014). The natural rate and its usefulness for monetary policy making. *American Economic Review*, 104(5):37–43.

Bullard, J. B. (2018). R-star wars: the phantom menace. *Business Economics*, 53(2):60–65.

Chang, C., Chen, K., Waggoner, D. F., and Zha, T. (2015). Trends and cycles in china's macroeconomy. *Economic Quarterly*, 30:1–84.

Del Negro, M. and Primiceri, G. (2015). Time varying structural vector autoregressions and monetary policy: A corrigendum. *Review of Economics Studies*, 82(4):1342–1345.

Fagan, G., Henry, J., and Mestre, R. (2005). An area-wide model for the euro area. *Economic Modelling*, 22(1):39–59.

Favero, C. A., Gozluklu, A. E., and Yang, H. (2016). Demographics and the Behavior of Interest Rates. *IMF Economic Review*, 64(4):732–776.

Fujiwara, S., Iwasaki, Y., Muto, I., Nishizaki, K., and Sudo, N. (2016). The natural rate and its usefulness for monetary policy making. *Bank of Japan Review*, 2016-E-12.

Giammarioli, N. and Valla, N. (2004). The natural real interest rate and monetary policy: a review. *Journal of Policy Modeling*, 26(5):641–660.

Holston, K., Laubach, T., and Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108:59–75.

Laubach, T. and Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economic Statistics*, 85(4):1063–1070.

Laubach, T. and Williams, J. C. (2016). Measuring the natural rate of interest redux. *Business Economics*, 51(2):57–67.

Lubik, T. and Matthes, C. (2015). Calculating the natural rate of interest: A comparison of two alternative approaches. *Federal Reserve Bank of Richmond Economic Brief*, EB15-10.

Negro, M. D., Giannone, D., Giannoni, M. P., and Tambalotti, A. (2017). Safety, Liquidity, and the Natural Rate of Interest. *Brookings Papers on Economic Activity*, 48(1 (Spring):235–316.

Primiceri, G. (2005). Time varying structural vector autoregressions and monetary policy. *Review of Economic Studies*, 72(3):821–852.

Wang, B. (2019). Measuring the natural rate of interest of china: A time varying perspective. *Economics Letters*, 176:117–120.

Appendix A Data

I collect three series: real GDP, central bank target interest rate or short-term interest rate, and CPI excluding food and energy or CPI containing all items for each economy in this study. Real GDP Growth rate and inflation rate are computed as percentage changes on a year-over-year basis. I use the CPI excluding food and energy to construct the inflation rate. If the core CPI series is shorter, I use CPI containing all items. I use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate.[6] The quarterly interest rate series is calculated as the average of monthly series within each quarter.

All the series are collected from the FRED data base except the data of Euro area and China. The data of Euro area are from the Area-wide Model (AWM) database (Fagan et al. (2005))). The data of China are from Chang et al. (2015). The starting period is determined by the availability of the dataset. The specific short-term nominal interest rates series for each economy are listed in Table 8.

Table 8: Data Source

Economy	Short-Term Nominal Interest Rate
Canada	Immediate Rates of less than 24 Hours, Central Bank Rates for Canada
Euro Area	Nominal Short-Term Interest Rate, Euribor 3-month, the AWM database
Japan	Immediate Rates of less than 24 Hours, Central Bank Rates for Japan
South Korea	Interest Rates, Discount Rate for Republic of Korea
U.K.	Immediate Rate of less than 24 Hours:, Call Money/Interbank Rate for the UK
U.S.	Effective Federal Funds Rate
Brazil	Immediate Rates of less than 24 Hours: Federal Funds Rate for Brazil
China	7-Day Repo Rate
India	Immediate Rates: Less than 24 Hours: Federal Funds Rate for India
Russia	Immediate Rates: Less than 24 Hours: Federal Funds Rate for the Russia
South Africa	Immediate Rates: Less than 24 Hours: Federal Funds Rate for South Africa

Notes: Interest rates data are collected from FRED database except Euro area and China. Euro area data is from the AWM database. China data is from Chang et al. (2015).

[6] Holston et al. (2017) compares the 4-quarter moving average proxy with the expected inflation built in Laubach and Williams (2003). They find the two proxies behave similarly.

Appendix B Dickey-Fuller Statistics

Table B-1: Augmented Dickey-Fuller Test Statistics

Economy	Level: t	Level: p	1st Difference: t	1st Difference: p
Canada	-1.64	0.11	-3.34	0.00
Euro Area	-1.34	0.19	-3.53	0.00
Japan	-0.64	0.52	-4.53	0.00
South Korea	-0.99	0.33	-4.37	0.00
U.K.	-1.07	0.29	-3.60	0.00
U.S.	-1.72	0.09	-3.02	0.00
Brazil	-0.92	0.36	-2.16	0.04
China	-0.87	0.39	-4.22	0.00
India	-0.93	0.36	-5.02	0.00
Russia	-1.89	0.06	-2.79	0.01
South Africa	-1.37	0.18	-4.38	0.00

Note: The T statistics show that the natural rates of interest of all economies are all unstationary under 95% confidence level. After first differencing, the series are stationary.

Appendix C Bayesian VAR with Minnesota Prior

All the data of this section are downloaded from the FRED database of St.Louis federal researve bank. For the convenience yield data, the Baa bond yield and treasury bill yield are monthly. I use the average within each quarter as the quarterly observation. For the demographic data, we can only obtain the yearly data. We use the linear interpolation to transform the yearly observations into quarterly observations.

The vector autoregression model describes an $n \times 1$ vector of macroeconomic endogenous variables d_t as a function of its own lags up to p and an $n \times 1$ vector of innovations u_t.

$$d_t = c + \sum_{l=1}^{p} A_l d_{t-l} + u_t, \quad t = 1, \cdots, T. \tag{8}$$

where $u_t \sim N(0, \Omega)$.

The VAR reduced-form variance Ω is estimated by the OLS. The VAR intercept c and lag coefficients $A_l, l = 1, \cdots, p$ are estimated by the Bayesian method with Minnesota priors. The priors are set as:

$$c \sim N(0, \lambda_1 \cdot I_n) \tag{9}$$

$$A_l \sim \begin{cases} N(I_n, V_l) & l = 1 \\ N(0_n, V_l) & l \neq 1 \end{cases} \tag{10}$$

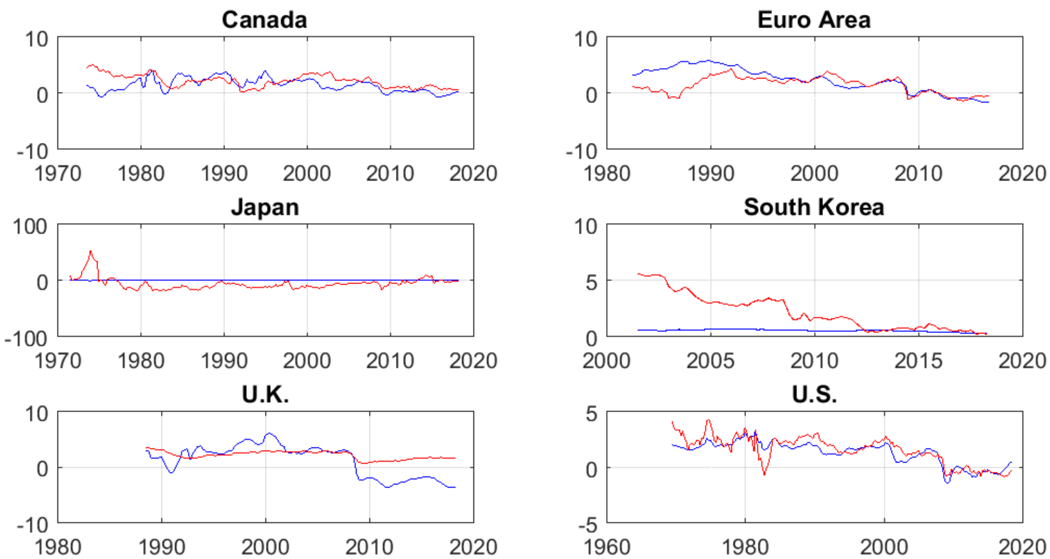
where each element $v_{ij,l}$ in V_l is

$$v_{ij,l} = \begin{cases} \frac{\lambda_2}{l^2} & i = j \\ \frac{\lambda_3 \Omega_{ii}}{l^2 \Omega_{jj}} & i \neq j \end{cases} \tag{11}$$

We set the maximum lag as $p = 2$ and the hyper parameters as $\lambda_1 = 100, \lambda_2 = 0.5, \lambda_3 = 0.5$. We sample 12000 draws from the posterior distribution and discard the first 2000 draws as the burn-in sample. Convergence diagnosis shows that the chain is convergent.

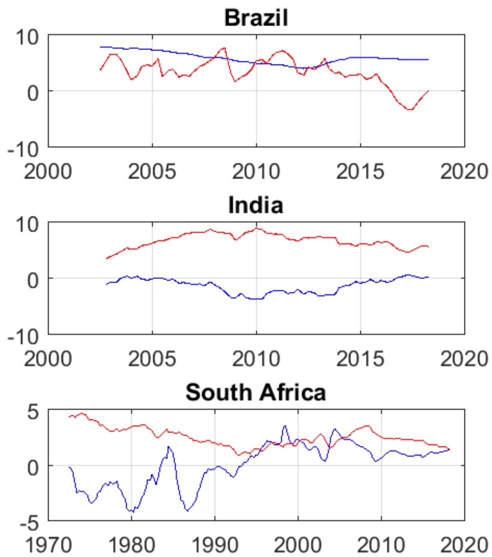
Appendix D   Robustness Figures

Figure D-1: The Natural Rates of OECD Economies Estimated by TVP-VAR-SV v.s. HLW



Note: The blue line corresponds to the median natural rate estimated by TVP-VAR-SV method and the red line corresponds to the natural rate estimated by the HLW method.

Figure D-2: The Natural Rates of BRICS Economies Estimated by TVP-VAR-SV v.s. HLW



Note: The blue line corresponds to the median natural rate estimated by TVP-VAR-SV method and the red line corresponds to the natural rate estimated by the HLW method.