Measuring the Natural Rate of Interest of OECD and BRICS Economies: A Time Varying Perspective*

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Abstract

This paper measures the natural rates of interest of eleven economies including six OECD economies and five emerging economies in a coherent time varying parameter vector autoregression framework. I find that the natural rates of interest in OECD economies have been descending especially since the 2007-2009 financial crisis. There is no such descending trend in BRICS economies except China. The descending trend of the natural rate of interest is a regional phenomenon instead of a global one.

Keywords: Natural Rate of Interest; TVP-VAR-SV; OECD; BRICS; Emerging Markets.

JEL codes: C11; C32; E43; E52.

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1 Introduction

One decade after the Great Recession has witnessed an unusual episode in the world that advanced economies expirenced a slow but long recovery, full employment but low inflation, and espcially a low interest rate environment. The US kept the norminal short-term interest rate right above the zero lower bound for a long time until recently the Fed lifted it to above 2%. The US is not alone in the low interest rate world that the UK, EU,and Japan still set the norminal short-term interest rate a little bit above zero.

One way to explain the low norminal interest rate links to the low natural rate of interest. Knut Wisksell introduced the notion that the natural rate of interest is the real short-term interest rate that the inflation stays the same and the output attains the potential. If the present natural rate of interest in the world is so low as a bit above zero, central bankers have no reason to raise the norminal one to supress the economy. The real rate gap which is the difference between the real interest rate and the natural rate of interest is an important index in evaluating the status of the macroeconomy and can help central banks to adjust the stance of monetary policy to stabilize the economy. Measuring the natural rate of interest is the key to clear the controversy whether central banks should raise the nominal short-term interest rate or not.

Recently some studies like Holston et al. (2017), Lubik and Matthes (2015), Negro et al. (2017) point out that the natural rates of several advanced economies have been decreasing especially since the Great Recession. However, they exlude the exploration of developing economies except Wang (2019) who finds the natural rate of China has been decreasing since 2009. Is the natural rate of interest really on a descending trend in the world including developed economies and especially developing economies? I explore this question in this paper.

The difficulty to compare the natural rates is on the measurement because the natural rate of interest can not be directly observed. One famous method to estimate the natural rate is Laubach and Williams (2003) who propose a few restrictions on the trend growth rate, natural rate of interest, and inflation rate of the economic system in a backward old-Keynsian style. Another method is to establish the natural rate in the fully-blown DSGE model such as Giammarioli and Valla (2004) and Barsky et al. (2014). One common feasure of these methods is that plenty restrictions are imposed on the economic system and are accomodated by the advanced economies, which makes the highly-structured models may or may not be suitable for developing economies.

Lubik and Matthes (2015) moves to another direction that they impose minimal restrictions on the vector autoregression system in which the parameters and variance are time varying to manifest the regime-changing in the underlying economic system. Wang (2019) applies their method to estimate the natural rate of China and finds that China's natural rate resembles the descending trend of advanced economies.

I contribute to the literature by comparing the natural rates of different kinds of economies including advanced and developing economies in a coherent econometric framework. I estimate the natural rates of interest of eleven economies including OECD and BRICS economies in the the time varying parameter vector autoregression model with stochastic volatility, following Lubik and Matthes (2015) in this paper. The natural rate is proposed as the long run (5 years) forecast of the ex ante real interest rate in the TVP-VAR-SV model. I find that the natural rates of interest in OECD economies have been descending but there is no such descending trend in BRICS economies except China.

For the eleven economies Canada, Euro Area, Japan, South Korea, the U.K., the U.S., Brazil, China, India, Russia, and South Africa, I estimate the small-scale TVP-VAR-SV model consisting of GDP growth rate, inflation rate, and ex ante real interest rate, as in Lubik and Matthes (2015). Then the natural rate is calculated as the long run (5 years) forecast of ex ante real interest rate using the median parameter value of each quarter. The results show that the natural rates in OECD economies have been descreasing while those in BRICS economies do not share the same pattern except the natural rate of China has been decreasing since 2009.

Overally the natural rates of all economies comove with the GDP growth rate as theoretically implied by the neoclassical growth model. The natural rate series exibit unit root and all are $I(1)$. The correlation matrix of the Hodrick-Prescott trend of the natural rates show that the OECD economies share the same descending trend but it is not the case in the BRICS economies. In the BRICS economies, the natural rate of China has been on a descending trend since 2009, which replicates the result in Wang (2019). The natural rate trend of India is ascending and opposite to the trends of OECD economies. The natural rates of Brasil and Russia bounced back in the aftermath of the Great Recession while the natural rate of South Africa does not show any obvious trend. This correlation evidence indicates that descending trend of the natural rate is a regional phenomenon instead of a gloal one.

I reestimate the natural rates of all the economies by the Holston et al. (2017) method for the robustness check. The estimated natural rates of Canada, Euro Area, the UK, and

the US are nearly the same as the rates estimated in the TVP-VAR-SV method in the paper. The natural rates are on a descending trend since 2000s. But the estimated natural rates of other economies including Japan, South Korea, and all BRICS economies are different from those estimated in the TVP-VAR-SV method. This result is not so suprising because Japan, South Korea, and all BRICS economies have been undergoing fierce institutional change[1] during the estimated sample. The restrictions applied in Holston et al. (2017) may not be suitable to the economies in which the institutions undergo dramatic changes.

The rest of the paper is organized as follows: Section 2 introduces the econometric framework to estimate the natural rate of interest; Section 3 contains the estimation results; Section 4 describes the robustness check; Section 5 concludes.

2 The Econometric Framework for Estimating the Natural Rate of Interest

My empirical method to measure the natural rate of interest is based on the time varying parameter vector autoregression with stochastic volatility model. The VAR framework is flexible to describe the dynamics of the macroeconomy and requires only minimal identification restrictions. The time varying parameter VAR model with stochastic volatility can manifest the underlying regime switch and thus nonlinearity of the economy, which is suitable for comparison study of multi-economies with different institutions.

The VAR model describes the evolution of a vector of n economic variables y_t as a linear function of its own lags up to order p and a vector of innovations u_t. Here in this study, I include GDP growth rate g_t, inflation rate π_t, and ex ante real interest rate r_t in the small scale VAR system so that $y_t = (g_t, \pi_t, r_t)'$ and I select 2 lags following the TVP-VAR convention.

$$y_t = B_{0,t} + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + u_t, u_t \sim N(0, \Omega_t) \qquad t = 1, \cdots, T. \tag{1}$$

Both the lag coefficients $B_{0,t}, B_{1,t}, B_{2,t}$ and variance covariance matrix Ω_t are time varying. The variance covariance matrix Ω_t is decomposed as:

$$\Omega_t = A_t^{-1}\Sigma_t\Sigma_t'A_t^{-1'}.$$

[1] Japan and South Korea grew from developing countries to developed countries during the estimated samle period. The BRICS countries are still in the process.

where

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix}; \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix}.$$

Let $x_t' = (1, y_{t-1}', y_{t-2}')$ and $B_t = (B_{0,t}, B_{1,t}, B_{2,t})'$. Stack all the columns of B_t into a vector $\beta_t = vec(B_t)$ and then we have the following stacked VAR:

$$y_t = X_t'\beta_t + A_t^{-1}\Sigma_t\epsilon_t. \tag{2}$$

where $X_t' = I_n \otimes x_t'$

Collect all the non-zero and non-one elements into a vector $\alpha_t = (\alpha_t^1, \alpha_t^2)'$ where α_t^1, α_t^2 corresponds to the vector of the non-zero non-one elements of the second and third row of A_t respectively. Collect all the diagonal elements of Σ_t and let $h_t = (\log \sigma_{1,t}, \log \sigma_{2,t}, \log \sigma_{3,t})'$.

It is assumed that the time varying parameters follow random walk processes:

$$\begin{aligned} \beta_t &= \beta_{t-1} + \epsilon_{\beta,t} \\ \alpha_t^i &= \alpha_{t-1}^i + \epsilon_{\alpha,t}^i, \quad i = 1, 2 \\ h_t &= h_{t-1} + \epsilon_{h,t}. \end{aligned}$$

I assume that $\epsilon_t, \epsilon_{\beta,t}, \epsilon_{\alpha^i,t}, \epsilon_{h,t}$ are mutually independent.

$$\begin{pmatrix} \epsilon_t \\ \epsilon_{\beta,t} \\ \epsilon_{\alpha,t} \\ \epsilon_{h,t} \end{pmatrix} \sim N(0, V), \quad V = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & \Sigma_\beta & 0 & 0 \\ 0 & 0 & \Sigma_\alpha & 0 \\ 0 & 0 & 0 & \Sigma_h \end{bmatrix}.$$

Here Σ_α is block diagonal with each block corresponding to the non-zero non-one elements in each row of A_t.

$$\Sigma_\alpha = \begin{bmatrix} \Sigma_{\alpha^1} & 0 \\ 0 & \Sigma_{\alpha^2} \end{bmatrix}.$$

I estimate the parameters by Bayesian methodology following Primiceri (2005). The order of Gibbs sampling is corrected as Del Negro and Primiceri (2015). The first 10 years is used as the presample to calibrate the priors except that 5 years is instead used for economies

with less than 100 observations. The prior distributions are elaborated below.

$$
\begin{aligned}
\beta_0 &\sim N(\hat{\beta}_{OLS}, 4 \cdot \hat{V}_{\beta,OLS}). \\
\alpha_0 &\sim N(\hat{\alpha}_{OLS}, 4 \cdot \hat{V}_{\alpha,OLS}). \\
h_0 &\sim N(\hat{h}_{OLS}, I_3). \\
\Sigma_\beta &\sim IW(k_\beta^2 \cdot 40 \cdot \hat{V}_{\beta,OLS}, 40). \\
\Sigma_{\alpha^1} &\sim IW(k_\alpha^2 \cdot 2 \cdot \hat{V}_{\alpha^1,OLS}, 2). \\
\Sigma_{\alpha^2} &\sim IW(k_\alpha^2 \cdot 3 \cdot \hat{V}_{\alpha^2,OLS}, 3). \\
\Sigma_h &\sim IW(k_h^2 \cdot 4 \cdot I_3, 4).
\end{aligned}
$$

where $k_\beta = 0.01, k_\alpha = 0.1, k_h = 0.01$.

As in Lubik and Matthes (2015), I define the conditional 5-year forecast of the observed ex ante real interest rate as a measure of natural rate of interest in the small scale VAR system. The natural rate of interest is:

$$r_t^* = (0, 0, 1) * E_t(y_{t+20}) = (0, 0, 1) * X_t' \beta_t^{20} \tag{3}$$

For every quarter, the 20-step-ahead forecast of y_t is computed using the very parameter β_t of that quarter as in Equation (3). The natural rate r_t^* is the third varaible in the 20-step-ahead forecast of y_t.

3 Estimation Results

I select 11 advanced and emerging economies to shed light on the natural rate of interest in the world. The advanced economies are the US, the UK, the Euro area, Canada, Japan, and South Korea from the OECD. The emerging economies are the five BRICS countries Brazil, Russia, India, China, and South Africa. The data sources are elaborated in Appendix A.

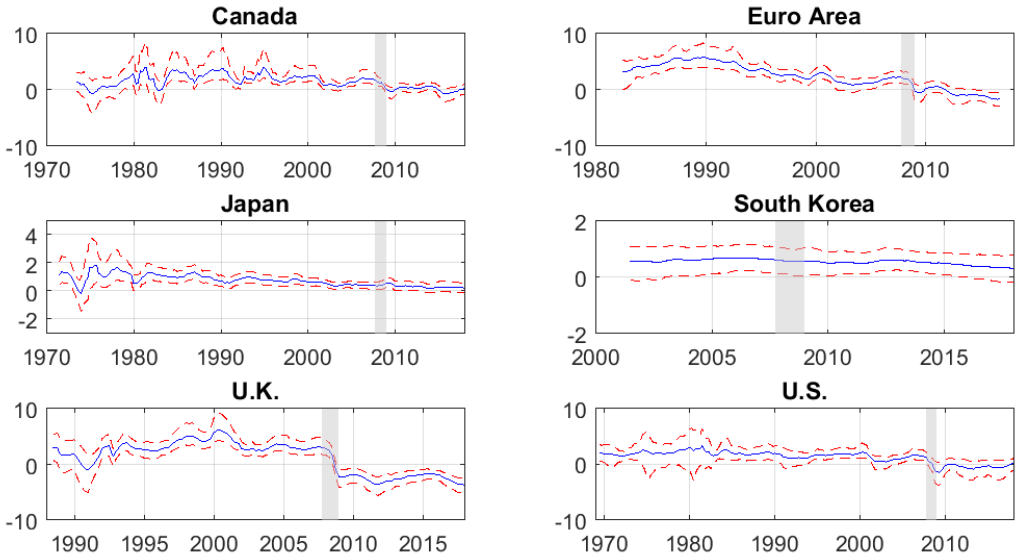
3.1 Estimates of the Natural Rate of Interest

Figure 1 illustrates the natural rates of interest of OECD economies . The natural rates of interest in these six OECD economies are roughly decreasing especially after 2000 and goes negative after the 2007-2009 financial crisis. This descending trend is also found in other studies (Laubach and Williams (2016), Holston et al. (2017), Lubik and Matthes (2015), Fujiwara et al. (2016)) applying various methods to measure this rate of different economies. The natural rates of Canada, Euro Area, U.K., and U.S. measured in this paper are smoother than those measured by Holston et al. (2017). The descending trends of the natural rates

replicate their result in this paper.

I explore two more economies Japan and South Korea for the OECD countries group. The natural rate of Japan fluctuated around 1% before the beginning of lost decades in 1990 and then descended along a slow trend. After 2000, this rate was around nearly zero. Fujiwara et al. (2016) which applies Lauback-Williams model finds similar result. For South Korea, the natural rate decreased after the financial crisis. In all the six OECD economies, the natural rates all plummeted sharply during the Great Recession except Japan.

Figure 1: The Natural Rate of Interest of BRICS Economies



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (3) by using median and 16th/84th percentiles of the parameter of every quarter.

Table 1 displays the yearly average of the natural rate estimates of the OECD economies in three specific years 1995, 2008, and 2016. A clear descending trend can be seen from this simple illustration of natual rate yearly averages of three years. After the 2007-2009 financial crisis, the natural rate of US decreased to be negative. In the aftermath of the crisis, the natural rates of interest of all OECD economies continued to descend. The natural rates for Canada, Euro Area, and UK further declined to the negative region along with the US in 2016.

Figure 2 illustrates the natural rates of interest of BRICS economies. Distinct from the OECD economies, there is no common pattern for the natural rates of interest within BRICS

Table 1: Natural Rate Estimates of OECD Economies

Economy	1995	2008	2016	1995-2008	2008-2016
Canada	3.21	1.18	-0.50	-2.03	-1.68
Euro Area	3.60	1.76	-1.60	-1.84	-3.36
Japan	0.83	0.44	0.31	-0.39	-0.13
South Korea	n/a	0.58	0.40	n/a	-0.18
U.K.	2.53	1.50	-2.21	-1.04	-3.71
U.S.	1.69	-0.03	-0.57	-1.72	-0.55

Note: All numbers are yearly averages of the median estimates of the natural rates of interest from the TVP-VAR-SV model. The last two columns are the changes of the natural rates of the two specific years.

countires. Like the OECD economies, the natural rate of China started to decrease after the end of the 4 trillion stimulus in 2010 as found in Wang (2019). Brazil and Russia have much higher level of natural rate. The natural rate descended from 7% to 5% for Brazil while it decreased from 5% to 1% for Russia during the estimated sample period but the natural rates bounced back during the aftermath of the financial crisis. For India, the natural rate decreased until the end of the Great Recession and then bounced back. During most of the estimated period, the natural rate of India is negative. For South Africa, the rate increased from 1980s to 2000s and then decreased after 2005.

During the Great Recession in the shaded area, the natural rates of interest plummeted for Brasil, India, Russia, and South Korea but China experienced sudden drop at first but bounced back promptly after the large stimulus.

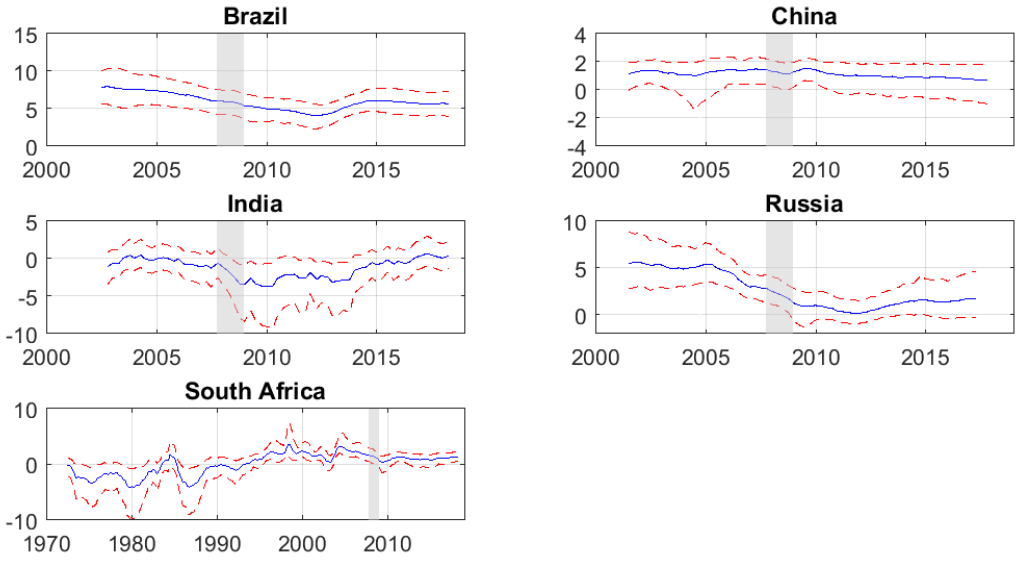
There is no universal common trend for the BRICS emerging economies. Table 2 furthter illustrates this point by showing the yearly averages of the BRICS economies in 2003, 2010, and 2016. The yearly averages are not universally descending like the OECD economies.

Table 2: Natural Rate Estimates of BRICS Economies

Economy	2003	2010	2016	2003-2010	2010-2016
Brazil	7.67	4.92	5.82	-2.75	0.90
China	1.19	1.25	0.84	0.07	-0.42
India	-0.20	-3.06	-0.30	-2.86	2.75
Russia	5.08	0.85	1.52	-4.23	0.67
South Africa	0.82	0.90	1.15	0.08	0.25

Note: All numbers are yearly averages of the median estimates of the natural rates of interest from the TVP-VAR-SV model. The last two columns are the changes of the natural rates of the two specific years.

Figure 2: The Natural Rate of Interest of BRICS Economies



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (3) by using median and 16th/84th percentiles of the parameter of every quarter.

3.2 Hodrick-Prescott Trends of the Natural Rates

The above visual displays show the natural rates of OECD economoies are descending after 2000s but there is no common trend in the BRICS economies. In this section, I more formally show the interdependence of the natural rates among the eleven economies through the correlations of the trends of the natural rates.

The first two columns of Table 3 show the Augmented Dickey-Fuller test statistics for the eleven economies and the natural rates of all economies exhibit unit root. After first differencing, the natural rates are all stationary shown by the statistics in the last two column statistics. So all the natural rate series are $I(1)$. The natural rates of all economies have a trend but it doesn't mean that they have the same asending or descending trend.

I use the Hodrick-Prescott filter with a smoothing parameter 1600 to obtain the trends of all the natural rates. Because the natural rates are trend-stationary, the correlation parameter of the trend series will be highly close to 1 if two trend series share the same linear trend. Otherwise, the correlation parameter will be a small number or even be negative if two trend series go to opposite directions. Table 4 shows the correlation of the Hodrick-Prescott trends of the natural rates of all the 11 economies in the period from 2008:Q1 to 2016:Q4.

Table 3: Augmented Dickey-Fuller Test Statistics

Economy	Level: t	Level: p	1st Difference: t	1st Difference: p
Canada	-1.64	0.11	-3.34	0.00
Euro Area	-1.34	0.19	-3.53	0.00
Japan	-0.64	0.52	-4.53	0.00
South Korea	-0.99	0.33	-4.37	0.00
U.K.	-1.07	0.29	-3.60	0.00
U.S.	-1.72	0.09	-3.02	0.00
Brazil	-0.92	0.36	-2.16	0.04
China	-0.87	0.39	-4.22	0.00
India	-0.93	0.36	-5.02	0.00
Russia	-1.89	0.06	-2.79	0.01
South Africa	-1.37	0.18	-4.38	0.00

Note: The T statistics show that the natural rates of interest of all economies are all unstationary under 95% confidence level. After first differencing, the series are stationary.

We can see that the correlation between the natural rate trends of OECD economies are close to 1 but the correlations between those of OECD economies and BRICS economies are not the case. For example, the correlation between the natural rate trend of the US and other five OECD economies are all above 0.7. But the numbers between US and BRICS economies are 0.22, 0.86, -0.32, 0.71, and 0.98 respectively. The US and Brazil are weakly related while the US and india have opposite trends of the natural rates.

This is the direct statistical evidence that the common desending trend of the natural rate exists in advanced economies. But it is a regional phenomenon. There is no universal descending trend if we expand to emerging economies.

3.3 Discussion: Factors that Influce the Natural Rate of Interest

The neoclassical growth model concludes that the steady state natural rate of interest r is positively related to economic growth rate g given intertemporal elasticity of substitution σ, and the rate of time preference θ as in Equation (4).

$$r = \frac{1}{\sigma} g + \theta \tag{4}$$

The faster the economy grows, the higher the natural rate of interest should be keeping other two factors the same. Bullard (2018) summerizes three factors affecting the long term natural rate as: labor productivity growth rate, labor force growth rate, and investors' desire

for safe assets.

This implication can be illustrated by the display of economic growth rates and natural rates of interest. Figure 3 shows the median natural rate of interest and economic growth rate for each estimated economy. The trends of the natural rates interwinde with those of the GDP growth rates in the OECD and BRICS countries overally. Especially the descending trend of OECD countries overlaps with the decrease of economic growth rates. In BRICS countries, the natural rates also go along the growth rates but feature different patterns. During the Great Recession, the natural rates of interest plummeted with the sudden fall of economic growth in all economies.

4 Robustness

I reestimate the natural rates of interest of the eleven economies by the Holston et al. (2017) (later as HLW) estimation [2] for the robustness check. Figure 4 and 5 show the natural rates estimated by our benchmark VAR method and HLW method.

The natural rates of interest estimated by the two methods are similar in the economies of Canada, Euro Area, the UK, and the US, coincidently the four economies explored by Holston et al. (2017). And the natural rates in these four economies have been decreasing since 2000s.

For other economies including two OECD economies Japan and South Korea and all BRICS economies, the natural rates of interest estimated by the two methods are different in terms of magnitude and dynamics. The natural rate of Japan estimated by the HLW method is below zero ever since 1980s. For South Korea, the natural rate has been decreasing from above 5% in early 2000s to nearly 0 recently. For China and India, the maganitude of the natural rate estimated by the HLW method are much higher than those estimated from the TVP-VAR-SV method. For Russia, the natural rate estimated by HLW is absurdly volatile.

One explanation for this distinction lies in the institution change of different economies. The HLW method imposes heavy restrictions on the economic system, which may or may not conform to the real economy. It was firstly applied in the US in Laubach and Williams (2003). Other three economies Canada, the Euro Area, and the UK are all advanced economies before the start of the sample. It is not surprising that the same restrictions imposed by the HLW method also applies in these three economies. However, Japan, Sourth Korea, and other five BRICS economies all underwent fierce change in the economc institution. The former two

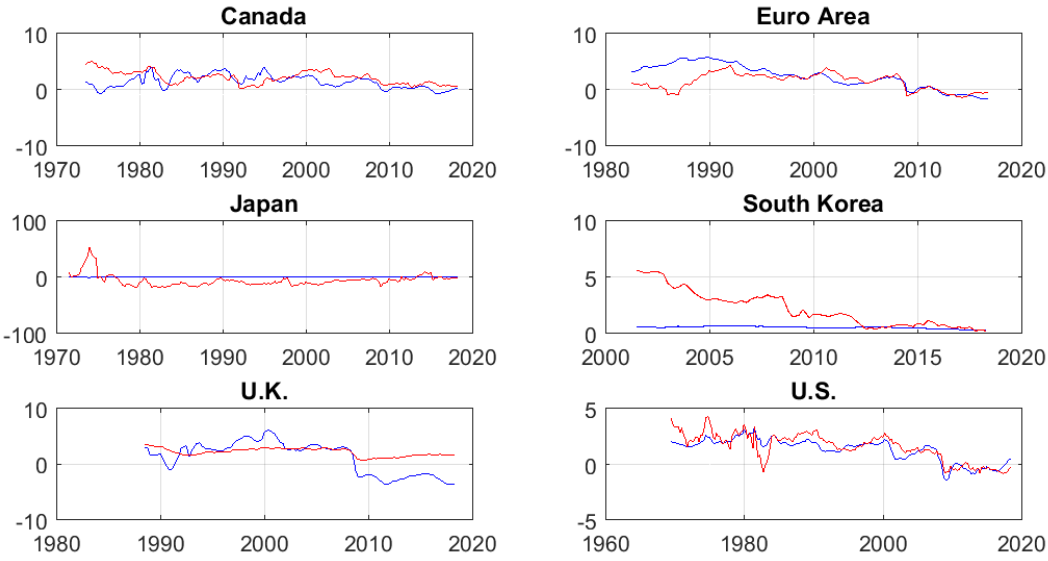
[2]The Holston et al. (2017) program is downloaded from the New York Fed website: https://www.newyorkfed.org/medialibrary/media/research/economists/williams/data/HLW_Code.zip

Figure 3: The Natural Rate of Interest and GDP Growth Rate of OECD and BRICS Economies

Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (3) by using the median values of the parameter of every quarter. The left vertical axis corresponds to the natural rate (blue line) and the right vertical axis corresponds to the GDP growth rate (red line).

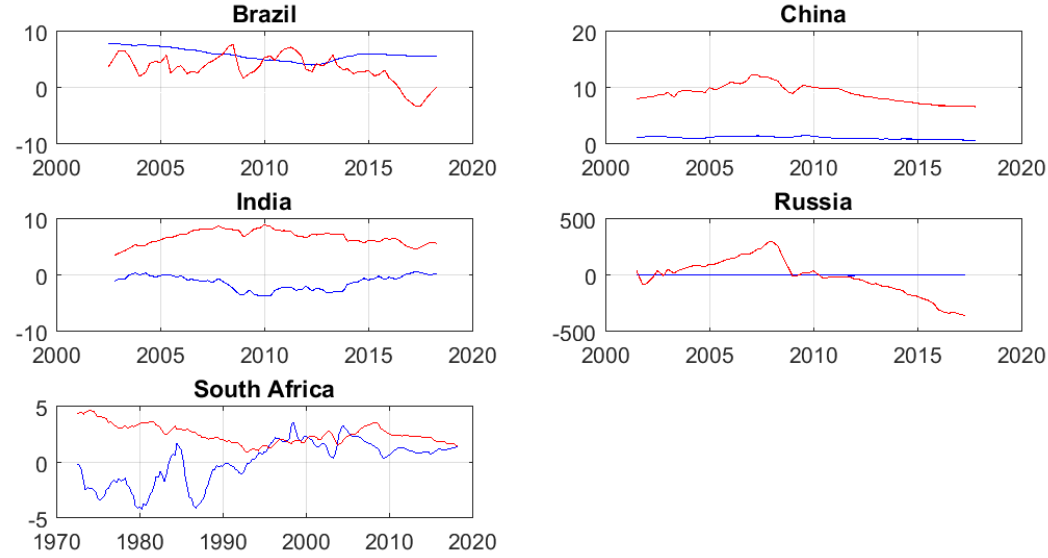
economies grew from developing economies to developed economies during the sample period. The BRICS economies are still in the process of developing. The HLW restrictions may not resemble the institutions of these economies for some time or all time. The TVP-VAR-SV

Figure 4: The Natural Rates of OECD Economies Estimated by TVP-VAR-SV v.s. HLW



Note: The blue line corresponds to the median natural rate estimated by TVP-VAR-SV method and the red line corresponds to the natural rate estimated by the HLW method.

Figure 5: The Natural Rates of BRICS Economies Estimated by TVP-VAR-SV v.s. HLW



Note: The blue line corresponds to the median natural rate estimated by TVP-VAR-SV method and the red line corresponds to the natural rate estimated by the HLW method.

method is more appropriate to deal with the situation of institutional change especially the international comparison of economies with different institutions.

5 Conclusion

I propose the 5-year forecast of the ex ante real interest rate in a time varying parameter vector autoregression model with stochastic volatility as the natural rate of interest following Lubik and Matthes (2015) and measure the rate for 11 economies including OECD countries and BRICS countries. I find that the natural rates of interest in OECD economies have been descending especially since the 2007-2009 financial crisis. There is no such descending trend in BRICS economies except China. Overall movements of the natural rates in all countries overlap with those of economic growth rates of corresponding countries.

Table 4: Correlation Matrix of the Trends of Natural Rates: 2008:Q1-2016:Q4

	Canada	Euro Area	Japan	South Korea	U.K.	U.S.	Brazil	China	India	Russia	South Africa
Canada	1.00	0.98	0.95	0.97	0.74	0.86	-0.29	0.97	-0.74	0.27	0.75
Euro Area	0.98	1.00	0.99	0.93	0.77	0.88	-0.26	1.00	-0.72	0.29	0.77
Japan	0.95	0.99	1.00	0.89	0.76	0.87	-0.25	1.00	-0.71	0.29	0.76
South Korea	0.97	0.93	0.89	1.00	0.54	0.70	-0.52	0.92	-0.88	0.02	0.56
U.K.	0.74	0.77	0.76	0.54	1.00	0.98	0.42	0.75	-0.12	0.84	0.99
U.S.	0.86	0.88	0.87	0.70	0.98	1.00	0.22	0.86	-0.32	0.71	0.98
Brazil	-0.29	-0.26	-0.25	-0.52	0.42	0.22	1.00	-0.28	0.85	0.84	0.39
China	0.97	1.00	1.00	0.92	0.75	0.86	-0.28	1.00	-0.73	0.27	0.75
India	-0.74	-0.72	-0.71	-0.88	-0.12	-0.32	0.85	-0.73	1.00	0.44	-0.13
Russia	0.27	0.29	0.29	0.02	0.84	0.71	0.84	0.27	0.44	1.00	0.83
South Africa	0.75	0.77	0.76	0.56	0.99	0.98	0.39	0.75	-0.13	0.83	1.00

Note: The trend of the natural rate is computed from the Hodrick-Prescott filter with a smoothing parameter 1600.

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Appendix A Data

I collect three series: real GDP, central bank target interest rate or short-term interest rate, and CPI excluding food and energy or CPI containing all items for each economy in this study. Real GDP Growth rate and inflation rate are computed as percentage changes on a year-over-year basis. I use the CPI excluding food and energy to construct the inflation rate. If the core CPI series is shorter, I use CPI containing all items. I use a four-quarter moving average of past inflation as a proxy for inflation expectations in constructing the ex ante real interest rate.[3] The quarterly interest rate series is calculated as the average of monthly series within each quarter.

All the series are collected from the FRED data base except the data of Euro area and China. The data of Euro area are from the Area-wide Model (AWM) database (Fagan et al. (2005))). The data of China are from Chang et al. (2015). The starting period is determined by the availability of the dataset. The specific short-term nominal interest rates series for each economy are listed in Table 5.

Table 5: Data Source

Economy	Short-Term Nominal Interest Rate
Canada	Immediate Rates of less than 24 Hours, Central Bank Rates for Canada
Euro Area	Nominal Short-Term Interest Rate, Euribor 3-month, the AWM database
Japan	Immediate Rates of less than 24 Hours, Central Bank Rates for Japan
South Korea	Interest Rates, Discount Rate for Republic of Korea
U.K.	Immediate Rate of less than 24 Hours:, Call Money/Interbank Rate for the UK
U.S.	Effective Federal Funds Rate
Brazil	Immediate Rates of less than 24 Hours: Federal Funds Rate for Brazil
China	7-Day Repo Rate
India	Immediate Rates: Less than 24 Hours: Federal Funds Rate for India
Russia	Immediate Rates: Less than 24 Hours: Federal Funds Rate for the Russia
South Africa	Immediate Rates: Less than 24 Hours: Federal Funds Rate for South Africa

Notes: Interest rates data are collected from FRED database except Euro area and China. Euro area data is from the AWM database. China data is from Chang et al. (2015).

[3] Holston et al. (2017) compares the 4-quarter moving average proxy with the expected inflation built in Laubach and Williams (2003). They find the two proxies behave similarly.