

Measuring the Natural Rate of Interest of China: A Time Varying Perspective*

Bin Wang[†]
City University of Hong Kong

This Version: January 4, 2019

Abstract

I propose the 5-year forecast of ex ante real interest rate in the time varying parameter vector autoregression model with stochastic volatility as a measure of the natural rate of interest of China. The natural rate of China varied around 1.6% before 2010 when the 4 trillion RMB stimulus ended, and then exhibited an obvious descending trend from around 1.8% to recent 0.4%. This is similar to the descending trend found in the studies of advanced economies.

Keywords: Natural Rate of Interest; TVP-VAR-SV; China.

JEL codes: C11; C32; E43; E52.

*I am grateful to Buben Fu and one anonymous referee for many helpful comments and discussions. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

[†]Corresponding author: Qishanxiangshujun, Dongsheng Road, Liaobu Town, Dongguan, China. Email: workbinwang@gmail.com

1 Introduction

The natural rate of interest is a fundamental concept in interpreting the macroeconomic dynamics and evaluating the stance of the monetary policy. Knut Wicksell introduced the notion of the natural rate of interest in 1898 to be the short-run real interest rate that output attains the potential value and inflation stays constant. Due to the central bank's operations on the short-run nominal interest rate, a fall of the real interest rate below the natural rate of interest stimulates the economy while an increase above the natural rate contracts the economy. This monetary policy rule is called [Taylor \(1993\)](#) rule. Measuring the natural rate of interest is important to the monetary policy conducting of the central bank.

However, the natural rate of interest is not observable and can not be measured directly. The well-known method to measure this rate is [Laubach and Williams \(2003\)](#). They propose a traditional Keynesian approach by specifying a state space system of economic growth, inflation, and real interest rate. Then they estimate the natural rate from this restricted system by maximum likelihood method. [Barsky et al. \(2014\)](#) estimate the canonical DSGE model and measure the natural rate of interest from the micro-founded environment.

A common feature of these approaches is that they impose rich theoretical restrictions on the economic system. [Lubik and Matthes \(2015\)](#) moves to another direction that they impose minimal restrictions by estimating the natural rate in a time varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV). The VAR model featuring time varying parameters can show the underlying regime change, transition, but impose minimal restrictions except identification assumptions.

A strand of studies ([Laubach and Williams \(2016\)](#), [Holston et al. \(2017\)](#), [Lubik and Matthes \(2015\)](#), [Fujiwara et al. \(2016\)](#)) reports that the natural rates of interest in advanced economies were on the descending trend especially after the 2007-2009 financial crisis. However, there is no study about the recent natural rate of interest of China.

This paper contributes to the literature by proposing the 5-year forecast of ex ante real interest rate in the TVP-VAR-SV model as a measure of the natural rate of interest of China along the work of [Lubik and Matthes \(2015\)](#). China is in the process of ongoing reform and transition in various aspects, especially the monetary policy regime ([Chen et al. \(2018\)](#), [Wang and Fu \(2018\)](#)). The VAR approach featuring time varying parameters with minimal restrictions is the best apparatus for the research of China's economy, which can illustrate the underlying nonlinearity of regime change and transition.

Two related studies of the natural rate of China are [Shi et al. \(2006\)](#) and [Cai and Gong](#)

(2016). Shi et al. (2006) follow the state space method of Laubach and Williams (2003) to measure the natural rate of China, and find that the natural rate fluctuated around 2% between 1996 and 2005. Cai and Gong (2016) follow Barsky et al. (2014) and measure the real rate in the flexible-price DSGE model as the natural rate of China. They find that this rate fluctuated around 10%. This high level of natural rate of interest is apparently at odds with the low real interest rate of China. Both studies impose rich restrictions to the economic models and thus may or may not be suitable for China, an economy with ongoing regime change and transition. The VAR method imposing minimal restrictions to measure the natural rate of China in this paper is a complement to the literature.

I first estimate the small-scale TVP-VAR-SV model consisting of GDP growth rate, inflation rate, ex ante real interest rate, and money growth rate as in Wang and Fu (2018) from 1996:Q1 to 2017:Q4. Then the natural rate is calculated as the long-run (5 years) forecast of ex ante real interest rate using the median parameter value of each quarter from 2001:Q3 to 2017:Q4. The natural rate of China exhibited no obvious descending trend before 2010 when the large stimulus ended, but then descended from around 1.8% to recent 0.4%. The latter result is similar to the findings in advanced economies. The real interest rate was 0.8 percentage point higher than the natural rate after 2016, which indicates that the monetary policy was too tight in China.

I conduct two robustness checks. One is to exclude the money growth rate in the VAR and the other is to replace the short-run nominal interest rate with an alternative series, the 7-day China interbank offered rate. The natural rates almost overlap with the benchmark one in these two robustness checks.

The rest of the paper is organized as follows: Section 2 introduces the empirical TVP-VAR-SV model; Section 3 contains the data description and Bayesian estimation; Section 4 discusses the results of the natural rate and robustness checks; Section 5 concludes.

2 The Empirical TVP-VAR-SV Model

The vector autoregression model describes the evolution of a vector of n economic variables y_t at time t as a linear function of its own lags up to order p and a vector u_t of unforecastable disturbances. Both the lag coefficients $B_{j,t}, j = 0, \dots, p$ and variance covariance matrix Ω_t are time varying.

$$y_t = B_{0,t} + \sum_{j=1}^p B_{j,t}y_{t-j} + u_t, u_t \sim N(0, \Omega_t) \quad t = 1, \dots, T. \quad (1)$$

and Ω_t is decomposed as:

$$\Omega_t = A_t^{-1} \Sigma_t \Sigma_t' A_t^{-1'}$$

where A_t is a lower-triangular matrix with ones in the diagonal entries and Σ_t is a diagonal matrix.

$$A_t = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \alpha_{21,t} & 1 & \ddots & 0 \\ \vdots & \ddots & & 0 \\ \alpha_{n1,t} & \cdots & \alpha_{nn-1,t} & 1 \end{bmatrix} \quad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t} & \ddots & 0 \\ \vdots & \ddots & & 0 \\ 0 & \cdots & 0 & \sigma_{n,t} \end{bmatrix}.$$

Let $X_t = (1, y'_{t-1}, \dots, y'_{t-p})$ and $B_t = (B_{0,t}, B_{1,t}, \dots, B_{p,t})'$. Stack all the columns of B_t into a vector $\beta_t = \text{vec}(B_t)$ and then we have the following stacked VAR:

$$y_t = (I_n \otimes X_t) \beta_t + A_t^{-1} \Sigma_t \epsilon_t. \quad (2)$$

Collect all the non-zero and non-one elements into a vector $\alpha_t = (\alpha_t^1, \dots, \alpha_t^{n-1})'$ where α_t^i corresponds to the vector of the non-zero non-one elements of the $i + 1_{th}$ row of A . Collect all the diagonal elements of Σ_t and let $h_t = (\log \sigma_{1,t}, \dots, \log \sigma_{n,t})'$.

It is assumed that the time varying parameters follow random walk processes:

$$\begin{aligned} \beta_t &= \beta_{t-1} + \epsilon_{\beta,t} \\ \alpha_t^i &= \alpha_{t-1}^i + \epsilon_{\alpha,t}^i, \quad i = 1, \dots, n-1 \\ h_t &= h_{t-1} + \epsilon_{h,t}. \end{aligned}$$

I assume that $\epsilon_t, \epsilon_{\beta,t}, \epsilon_{\alpha,t}^i, \epsilon_{h,t}$ are mutually independent.

$$\begin{pmatrix} \epsilon_t \\ \epsilon_{\beta,t} \\ \epsilon_{\alpha,t} \\ \epsilon_{h,t} \end{pmatrix} \sim N(0, V), \quad V = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & \Sigma_{\beta} & 0 & 0 \\ 0 & 0 & \Sigma_{\alpha} & 0 \\ 0 & 0 & 0 & \Sigma_h \end{bmatrix}.$$

Here Σ_{α} is block diagonal with each block corresponding to the non-zero non-one elements

in each row of A_t .

$$\Sigma_\alpha = \begin{bmatrix} \Sigma_{\alpha^1} & 0 & 0 & 0 \\ 0 & \Sigma_{\alpha^2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \Sigma_{\alpha^{n-1}} \end{bmatrix}.$$

The TVP-VAR-SV model is estimated by Bayesian method as in [Primiceri \(2005\)](#).

3 Data and Bayesian Estimation

I incorporate GDP growth rate g_t , inflation rate π_t , ex ante real interest rate r_t ¹, and money growth rate m_t as the benchmark specification in the VAR system. So $n = 4$.

$$y_t = (g_t, \pi_t, r_t, m_t)' \quad (3)$$

I depart from [Lubik and Matthes \(2015\)](#) that I include one more variable the money growth rate in the VAR system because China relied on money growth rate of M2 as the intermediate target to conduct the monetary policy after 2000, different from targeting interest rate in advanced economies. This institution difference is noticed recently in the literature ([Chen et al. \(2018\)](#), [Chen et al. \(2017\)](#), [Wang and Fu \(2018\)](#)). Taking account of China's special institution, I follow them to incorporate money growth rate as the policy rate in the benchmark VAR system.

The real GDP, consumption price index, 7-day Repo rate and M2 data series are from [Chang et al. \(2015\)](#). The data series are quarterly and seasonally adjusted covering the period from 1996:Q1 to 2017:Q4. The starting period of the data is determined by the availability of 7-day Repo rate. CPI excluding food and energy is collected from the CEIC database covering 2006:M1 to 2017:M12. The 7-day China interbank offered rate is from the China Economic Information Database. The latter two series are monthly-measured. I use 3-month average as the quarterly values and do the seasonally-adjustment by X-12-ARIMA method.

Real GDP Growth rate, inflation rate, and money growth rate are calculated as percentage changes on a year-over-year basis. The inflation rate is constructed from CPI excluding food and energy. Prior to 2006, CPI containing all items is used. I use the four-quarter

¹The ex ante real interest rate is defined as the difference between the nominal interest rate and the expected inflation rate.

moving average of past inflation as a proxy for inflation expectation in constructing the ex ante real interest rate².

I follow the conventional practice of the TVP-VAR literature (Primiceri (2005)) to set the priors of the Bayesian estimation. I use the first 5 years as the presample to calibrate the priors. Denote the OLS estimates by marking the subscript “OLS”. The priors are set as:³

$$\begin{aligned}
\beta_0 &\sim N(\hat{\beta}_{OLS}, 4 \cdot \hat{V}_{\beta,OLS}). \\
\alpha_0 &\sim N(\hat{\alpha}_{OLS}, 4 \cdot \hat{V}_{\alpha,OLS}). \\
h_0 &\sim N(\hat{h}_{OLS}, I_4). \\
\Sigma_\beta &\sim IW(k_\beta^2 \cdot 40 \cdot \hat{V}_{\beta,OLS}, 40). \\
\Sigma_{\alpha^1} &\sim IW(k_\alpha^2 \cdot 2 \cdot \hat{V}_{\alpha^1,OLS}, 2). \\
\Sigma_{\alpha^2} &\sim IW(k_\alpha^2 \cdot 3 \cdot \hat{V}_{\alpha^2,OLS}, 3). \\
\Sigma_{\alpha^3} &\sim IW(k_\alpha^2 \cdot 4 \cdot \hat{V}_{\alpha^3,OLS}, 4). \\
\Sigma_h &\sim IW(k_h^2 \cdot 5 \cdot I_4, 5).
\end{aligned}$$

where $k_\beta = 0.01, k_\alpha = 0.1, k_h = 0.01$.

Please refer to Primiceri (2005) and Del Negro and Primiceri (2015) for more detailed descriptions of the Bayesian method of the TVP-VAR-SV model.

4 Results

As in Lubik and Matthes (2015), I define the conditional 5-year forecast of the observed ex ante real interest rate as a measure of natural rate of interest. Forecast y_t in 5 years’ horizon:

$$E_t(y_{t+20}) = (I_n \otimes X_t)\beta_t^{20} \tag{4}$$

For every quarter, the 20-step-ahead forecast of y_t is computed as in Equation (4) using the parameter value β_t of that quarter. The natural rate of interest r_t^* is the third variable in $E_t y_{t+20}$.

²Holston et al. (2017) compares the 4-quarter moving average proxy with the expected inflation built in Laubach and Williams (2003). They find the two proxies behave similarly.

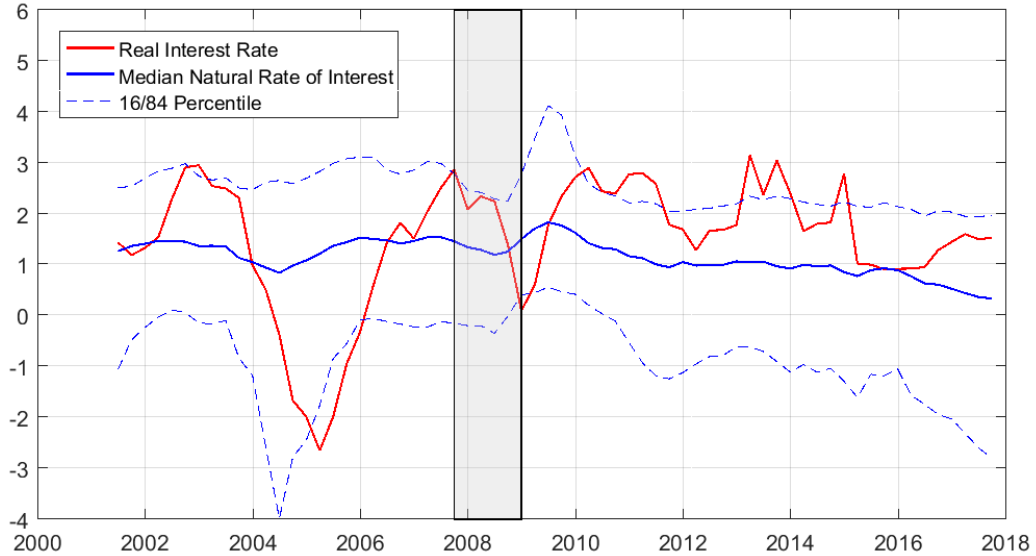
³Primiceri (2005) sets the degree of freedom of the inverse-Wishart distribution of Σ_β as 40, the size of the presample 10 years. However, my presample has only 20 observations less than the number of parameters in β^T 36, which is not permitted in the inverse-Wishart distribution. So I still set the degree of freedom in the prior as 40.

4.1 The Natural Rate of Interest of China

Figure 1 illustrates the natural rate of interest of China from 2001:Q3 to 2017:Q4. One feature stands out that the natural rate of China fluctuated stably around 1.6% before the end of the large stimulus while the real interest rate underwent drastic fluctuations. During the financial crisis (the gray region), the natural rate decreased slightly but bounced back soon after the start of the 4 trillion RMB stimulus in 2008:Q4. After the end of the stimulus in 2010:Q4, the natural rate decreased from 1.8% down to 0.4% recently. This descending trend is also found in the natural rates of other advanced economies (Laubach and Williams (2016), Holston et al. (2017), Lubik and Matthes (2015), Fujiwara et al. (2016)).

Ever since 2016, the real interest rate increased above the natural rate more than 0.8 percentage points. It indicates that the monetary policy was too tight after 2016 and the central bank should reduce the short run nominal interest rate to stabilize the squeezed economy.

Figure 1: The Natural Rate of Interest of China



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (4) by using the parameters of every quarter.

The average natural rate of interest is comparatively low in China where economic growth, nearly 10 % annually, is persistent and high in the last four decades. The neoclassical growth model implies that the steady state natural rate of interest r is positively related to economic

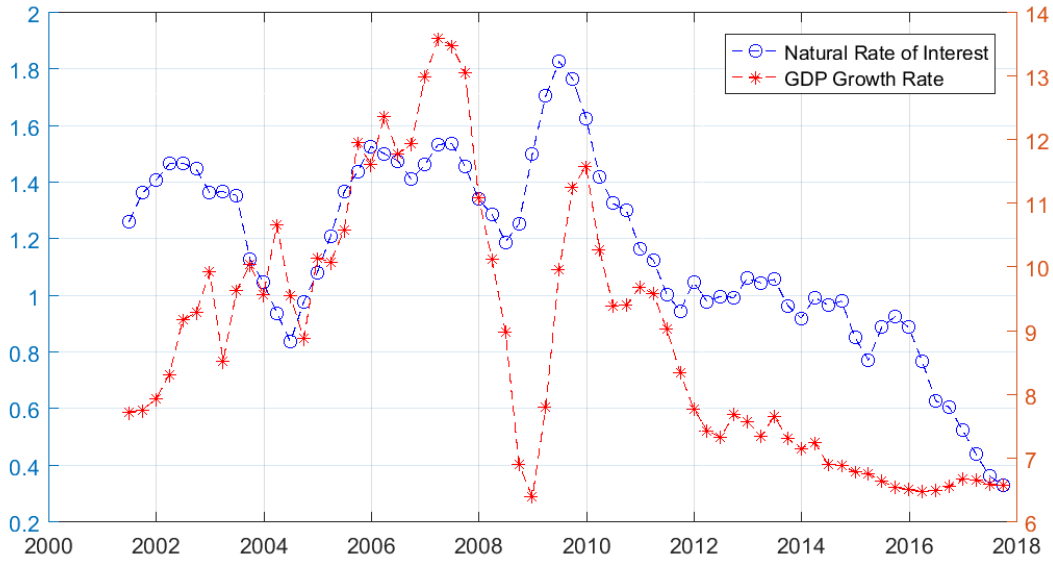
growth rate g in Equation (5).

$$r = \frac{1}{\sigma}g + \theta \quad (5)$$

Theoretically China should have much higher natural rate if the intertemporal elasticity of substitution σ and time preference θ are the same as those of advanced economies. The low level of the natural rate may indicate that either China has a higher intertemporal elasticity of substitution σ or a lower rate of time preference θ .

The descending trend can be explained by the lower GDP growth rate of China after the end of the large stimulus as theoretically implied by Equation (5). Figure 2 embeds the natural rate and GDP growth rate together. The descending trend of the natural rate overlapped with that of the GDP growth rate after 2010.

Figure 2: The Natural Rate of Interest and GDP Growth Rate of China



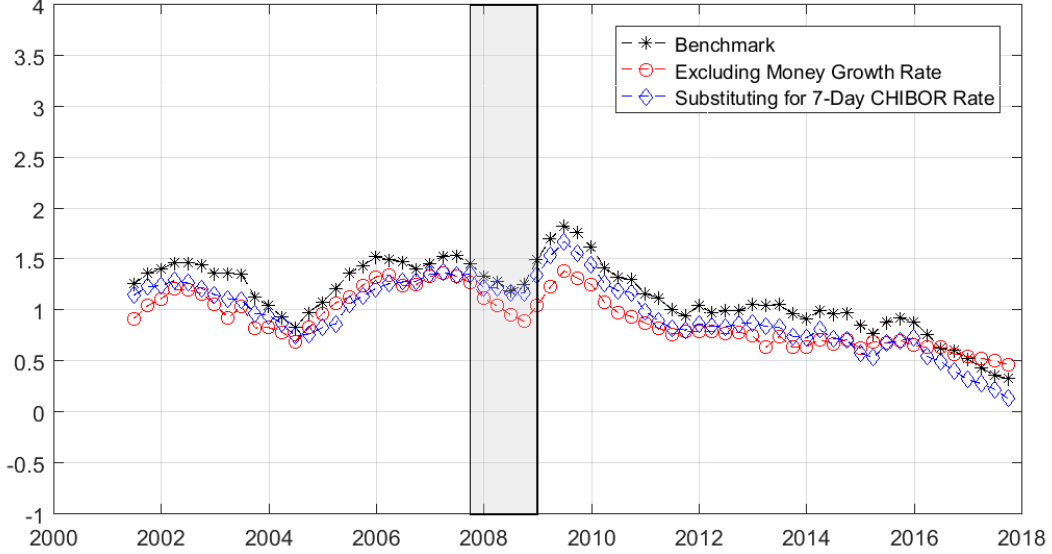
Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (4) by using the median values of the parameters of every quarter. The left vertical axis corresponds to the natural rate and the right vertical axis corresponds to the GDP growth rate.

4.2 Robustness

I conduct two robustness checks in this subsection. The first robustness check is to exclude the money growth rate in the VAR system and set $y_t = (g_t, \pi_t, r_t)'$; the second is to replace

the short-run nominal interest rate with an alternative series, the 7-day China interbank offered rate in the original VAR system. Figure 3 shows the natural rates of interest of China in these two checks. The natural rates in these two checks almost overlap with the benchmark one.

Figure 3: The Natural Rate of Interest of China



Note: I sample 10000 draws from the posterior distribution and discard the first 2000 draws as the burn-in process. The natural interest rate is computed as Equation (4) by using the median values of the parameters of every quarter.

5 Conclusion

I propose a measure of the natural rate of interest of China following the TVP-VAR-SV method of [Lubik and Matthes \(2015\)](#) in this paper. The natural rate of China fluctuated around 1.6% before the end of the 4 trillion RMB stimulus and there was an obvious descending trend after that time in China, similar to the descending trend of the natural rates of advanced economies. The monetary policy was tight after 2016 as the real rate was above the natural rate.

References

- Barsky, R., Justinano, A., and Melosi, L. (2014). The natural rate and its usefulness for monetary policy making. *American Economic Review*, 104(5):37–43.
- Cai, Q. and Gong, M. (2016). Estimating the natural rate in China: Based on dsge model. *Finance and Trade Research (in Chinese)*, 6:29–40.
- Chang, C., Chen, K., Waggoner, D. F., and Zha, T. (2015). Trends and cycles in China’s macroeconomy. *Economic Quarterly*, 30:1–84.
- Chen, K., Higgins, P., Waggoner, D. F., and Zha, T. (2017). Impacts of monetary stimulus on credit allocation and macroeconomy: Evidence from China. *NBER Working Paper*.
- Chen, K., Ren, J., and Zha, T. (2018). The nexus of monetary policy and shadow banking in China. *American Economic Review*, 108(12):3891–3936.
- Del Negro, M. and Primiceri, G. (2015). Time varying structural vector autoregressions and monetary policy: A corrigendum. *Review of Economics Studies*, 82(4):1342–1345.
- Fujiwara, S., Iwasaki, Y., Muto, I., Nishizaki, K., and Sudo, N. (2016). The natural rate and its usefulness for monetary policy making. *Bank of Japan Review*, 2016-E-12.
- Holston, K., Laubach, T., and Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108:59–75.
- Laubach, T. and Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economic Statistics*, 85(4):1063–1070.
- Laubach, T. and Williams, J. C. (2016). Measuring the natural rate of interest redux. *Business Economics*, 51(2):57–67.
- Lubik, T. and Matthes, C. (2015). Calculating the natural rate of interest: A comparison of two alternative approaches. *Federal Reserve Bank of Richmond Economic Brief*, EB15-10.
- Primiceri, G. (2005). Time varying structural vector autoregressions and monetary policy. *Review of Economic Studies*, 72(3):821–852.
- Shi, Z., Deng, C., Liu, J., and Shi, Q. (2006). The relationship of the natural rate of interest with economic growth and inflation. *World Economy (in Chinese)*, 4:12–21.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214.
- Wang, B. and Fu, B. (2018). China’s monetary policy in a time varying parameter vector autoregression model. *Unpublished Manuscript*.