Column generation methods for Multi-package Delivery

Bin Yao

Problem Statement

- Multi-package delivery
- Identical time window for both vehicles and customers
    - E.g. Vehicle: 6am-5pm. Customer: 8am-4pm.
- CVRP with identical vehicle capacity
- Multiple depots with unlimited inventory
- Multi-trip (reload allowed)

Problem Statement

- There is a fixed daily cost for maintaining a vehicle even it is kept idle.

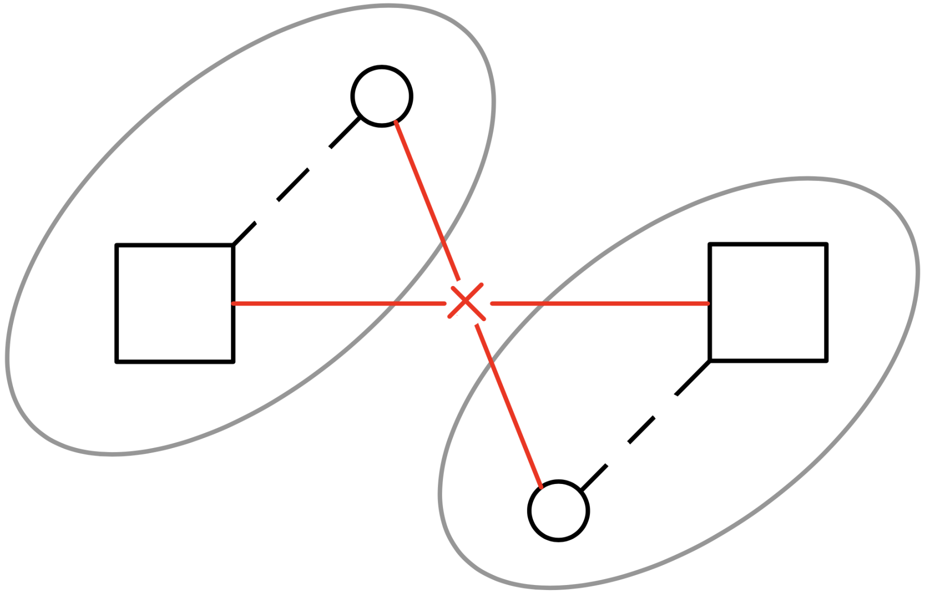
- Routing cost is proportional to routing distance.

- Task 1: Provide packing and routing solution every day with objective to minimize the total cost (routing cost + vehicle cost).
- Task 2: Make decision on fleet size for each depot.

Clustering

- For simplicity, I decompose the multi-depot problem into single-depot problem.

- Assign each customer to the nearest depot.



Model I: pack-first, route-second

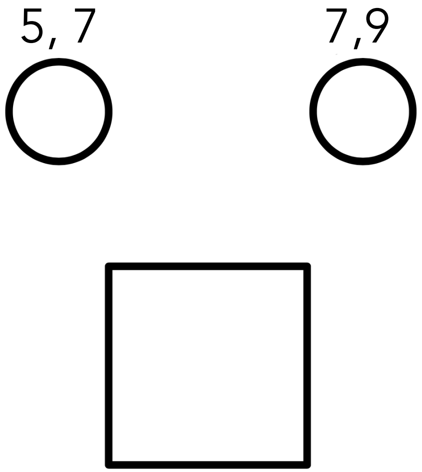
- Given the order and address ahead, solve the following bin-packing problem first.

$$\text{minimize } K = \sum_{j=1}^{n} y_j$$

$$\begin{aligned}
\text{subject to } & K \geq 1, & \\
& \sum_{i \in I}^{n} s(i) x_{ij} \leq B y_j, \ \forall j \in \{1, \ldots, n\} & \\
& \sum_{j=1}^{n} x_{ij} = 1, & \forall i \in I \\
& y_j \in \{0, 1\}, & \forall j \in \{1, \ldots, n\} \\
& x_{ij} \in \{0, 1\}, & \forall i \in I \, \forall j \in \{1, \ldots, n\}
\end{aligned}$$

Model I: pack-first, route-second

- Alternative: formulate it as cutting-stock problem and solve it using column-generation method.



Size	5	7	9
Demand	1	2	1

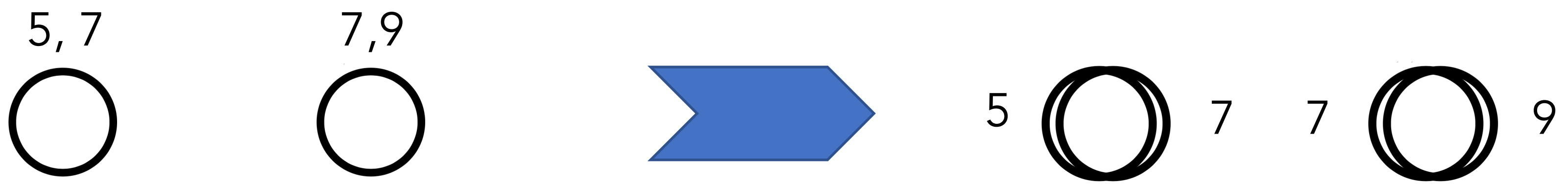
Model I: pack-first, route-second

- Given the packing solution, VRP becomes solving several TSP.

- Since multi-trip is allowed, a vehicle may serve more than one trip.
    - Heuristics: order the trips with increasing time and combine the trip in order to see whether time window are satisfied.
    - Integer programming model

Model II: pack and route simultaneously

- Each customer with multiple package will be split to several duplicates with the same address but only single order.



- Use column generation to solve the whole problem.

- Each column corresponds to a feasible multi-trip route.
  - Capacity
  - Time window

Notation

- $G(\mathrm{D}, \mathrm{C}, \mathrm{E})$: A graph with D as set of depots, C as set of customers and E as the set of all edges.
- C_d: the set of customers associate with depot d.
- E_d: the set of edges among $C_d \cup d$.
- c_{veh}: vehicle cost for single vehicle a day.
- Ω: set of all generated routes.
- τ: index of generated route.
- Cap: capacity of vehicle.
- $[t_v, T_v], [t_c, T_c]$: time window of vehicles and customers.

Model II: pack and route simultaneously

- For each depot d, solve the following master problem

$$\begin{aligned} \min \quad & \sum_{\tau \in \Omega} (c_\tau + c_{veh}) x_\tau \\ \text{subject to} \quad & \sum_{\tau \in \Omega} a_{i\tau} x_\tau \geq 1 \quad \forall i \in C_d \\ & x_\tau \geq 0 \quad \forall \tau \in \Omega. \end{aligned}$$

- For each column, $a_{i\tau}$ is binary indicating whether order i is served by route τ.
- Check whether there exists a multi-trip route τ that satisfies the following:

$$c_\tau + c_{veh} - p^T a_\tau < 0$$

- Subproblem

$$\begin{aligned} \min \quad & c_\tau + c_{veh} - p^T a \\ \text{subject to} \quad & c_\tau = \alpha \sum_{(i,j)\in E_d} dist(i,j) x_{ij} \end{aligned}$$

Degree

$$\sum_{j\in C_d} x_{sj} = \sum_{j\in C_d} x_{jt} = 1$$

$$\sum_{j\in C_d} x_{oj} = \sum_{j\in C_d} x_{jo}$$

$$\sum_{j\in C_d\cup[d]} x_{ij} = \sum_{j\in C_d\cup[d]} x_{ji} = a_i \qquad \forall i \in C_d$$

Capacity

$$l_s = l_o = Cap$$

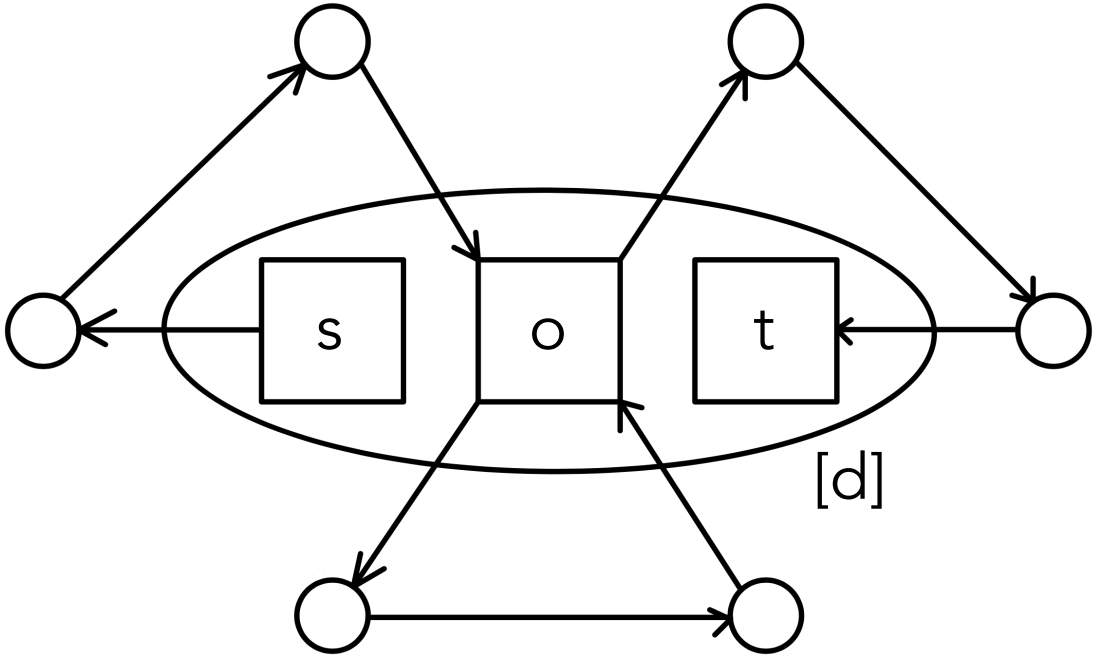
$$l_j + d_i - l_i \leq (1 - x_{ij})M \qquad i,j \in C_d \cup \{d\}$$

Subtour-Elimination

$$\sum_{i,j\in S} x_{ij} \leq |S| - 1 \qquad \forall S \in C_d, |S| \geq 2.$$

$$x_{ij}, a_i \in \{0,1\} \qquad \forall (i,j) \in E_d, i \in C_d$$

$$l_i \geq 0 \qquad \forall i \in C_d$$



Time

$$\sum_{(i,j)\in E_d} t_{ij}x_{ij} \leq T_v - t_v$$

$$\sum_{(i,j)\in E_d} t_{ij}x_{ij} - \sum_{(s,j)\in E_d} t_{sj}x_{sj} \leq T_v - t_c$$

$$\sum_{(i,j)\in E_d} t_{ij}x_{ij} - \sum_{(j,t)\in E_d} t_{jt}x_{jt} \leq T_c - t_v$$

$$\sum_{(i,j)\in E_d} t_{ij}x_{ij} - \sum_{(s,j)\in E_d} t_{sj}x_{sj} - \sum_{(j,t)\in E_d} t_{jt}x_{jt} \leq T_c - t_c$$

Future steps

- Experiment results show that the running time for the subproblem increases dramatically.
- Figure out a way to speed up the algorithm to find the column. (e.g. formulate as shortest path problem and solve it with DP)

- Interested in other methods like branch-and-price.

- May consider how to add interaction between different depots into the model.