# 12th AIMMS-MOPTA Optimization Modeling Competition

# Vehicle Fleet Sizing, Positioning and Routing

### 1. Problem Description

The goal of the competition this year is to solve an optimal vehicle routing problem where in addition, the location of depots for the vehicles and the fleet size should be optimized.

**Transportation network.** The transportation network is described as an undirected graph (V, E) endowed with edge weight functions  $(c, \delta, \tau)$ . The vertices  $v \in V$  are given as 5-digit zip codes and represent locations. The edges  $e \in E$ , where  $E \subset V \times V$ , represent routes between locations. The function  $\delta : E \mapsto \mathbb{R}$  gives the length of the route as the distance in miles between the two locations. The function  $c : E \mapsto \mathbb{R}$  gives the cost c(e) for a vehicle to move along the route e from one to the other location, in dollars. The function  $\tau : E \mapsto \mathbb{R}$  gives the time  $\tau(e)$  for a vehicle to move along the route e, in hours.

- The file mopta2020\_vertices.csv lists the vertices v ∈ V. There are 4 fields per row: the zipcode (as vertex identifier), location name, longitude (West), latitude (North). The name and geographical coordinates are intended for graphical rendering and interpretation.
- The file mopta2020\_edges.csv lists the undirected edges  $e \in E$  and the distances  $\delta(e)$ . There are 3 fields per row: a zipcode-from and a zipcode-to defining the edge, and the distance in miles.
- To specify c we assume that  $c(e) = 0.70 \cdot \delta(e)$ . The mileage rate (\$0.70/mile) comprises the variable costs such as fuel, tires, repairs, tolls. Note that if the fuel efficiency of the delivery vehicle is 10 miles per gallon, and one gallon of gasoline produces 0.009 metric ton of CO2, then each mile driven generates 0.0009 tons of CO2.
- To specify  $\tau$  we assume that  $\tau(e) = (1/40) \cdot \delta(e)$ , implying that vehicles move at an average speed of 40 miles/hour.

**Depots and vehicles.** The locations of the depots can be described as a subset H of V, to be determined. The cardinality of H will be treated as a problem parameter. An initial number of vehicles should be assigned to each depot. These vehicles remain in place

for the entire duration of the problem, even if they have to remain idle on certain days.

- There is a fixed cost of \$30 per day per vehicle. This cost represents fixed annual costs expressed per day (lease or purchase payments, insurance, licenses, driver wages and benefits).
- The file mopta2020\_depots.csv gives for each zipcode an estimated fixed cost of maintaining a depot at that location. There is one row per zipcode. The first field is the zipcode and the second field is the cost expressed in \$ per day (from the annual costs of depot lease or purchase and personnel wages).

**Demand.** Each day there is a demand at each vertex  $v \in V$  that needs to be served by dispatching a vehicle to v on that day. In general, the demand can be described as a mapping  $D: V \times \mathbb{N} \mapsto \mathbb{R}^3$  with values  $D(v,t) = (q,w_1,w_2)$  indicating that at vertex v on day t there is a demand of amount q to be fulfilled during the time window  $[w_1,w_2]$ , where  $w_1,w_2$  are hours. For the competition we assume that  $w_1 = 8$ am and  $w_2 = 4$ pm when q is nonzero. Let Q(v,t) = q denote the quantity q extracted from D(v,t).

In reality Q(v,t) is only revealed on day t-1, that is, Q(v,t) is a random variable revealed on day t-1. One can view the next-day quantity as a stochastic process  $F_{vt} = Q(v,t+1)$ : on day t at 6pm, one gets the information needed to determine a vehicle routing to serve the demand of day t+1.

• The files mopta2020\_q2018.csv and mopta2020\_q2019.csv provide daily demand data for 2018 and 2019, that is, the realization of Q(v,t) for each  $v \in V$  over  $t=1,\ldots,365$ , with t=1 for January 1st. Each row is relative to a zipcode. The first field is the zipcode v and the subsequent fields are the quantities Q(v,t) for  $t=1,\ldots,365$ .

**Routing.** A routing of one vehicle from its depot  $v_0 \in H$  is defined as a circuit in the graph (V, E) and a delivery amount to each vertex of the circuit. The circuit is described as a sequence of vertices  $(v_0, v_1, \ldots, v_{n-1}, v_n)$  where  $v_n = v_0$ , to which corresponds a sequence of edges  $(e_1, \ldots, e_n)$  where

each  $e_i$  must be in E. The deliveries are described as  $(r_0, r_1, \ldots, r_n)$  where  $r_i$  is the quantity delivered at vertex  $v_i$ . The maximum loading per vehicle is limited to a quantity of 60. However, a vehicle can reload each time it passes by its depot. This means that for each subsequence  $(v_i, v_{i+1}, \ldots, v_j)$  of the circuit such that  $v_i = v_0 = v_j$  and  $v_k \neq v_0$  for  $k = i+1, \ldots, j-1$ , we must have  $\sum_{\ell=i}^{j} r_{\ell} \leq 60$ .

If a vehicle departs from depot  $v_0$  at hour  $h_0$ , the vehicle reaches  $v_i$  at hour  $h_i = h_0 + \sum_{k=1}^i \tau(e_k)$ , delivers  $r_i$  at hour  $h_i$ , and is back to the depot at hour  $h_n$ . The routing cost for this vehicle is  $\sum_{i=1}^n c(e_i)$ . The demand  $D(v_i, t) = (q_i, w_1, w_2)$  is served by the vehicle if  $r_i \geq q_i$  and  $w_1 \leq h_i \leq w_2$ . The overall demand on day t is served if for each  $v \in V$ , there is a vehicle that serves v.

We assume that  $h_0 = 6$ am is the earliest time that a vehicle can leave its depot and  $h_n = 5$ pm is the latest time a vehicle can return. This allows for a maximal total driving time of 11 hours by a single driver.

Modeling and Optimization. Your team is asked:

- 1. To determine, for a given number of depots, the location of the depots and the number of vehicles per depot, so that subsequently, the vehicles can always be routed to serve the 2018 demand. An objective to minimize should be selected. This objective should at least include the fixed costs of the vehicles and the variable costs of the routings.
  - We leave it to you to include or not the fixed cost of the depots, and to explore the impact of the objective on a variety of solution quality metrics as suggested in 4. below. We ask you to assess the realism and practicality of a solution, and to include penalties if necessary. For instance, the depot cost estimates have a real-estate component that may help to discard solutions with the depots installed in prime urban areas.
- 2. To obtain the solutions and objective values for representative values of the number of depots, including the minimal number of depots such that the problem of serving the demand each day remains feasible. We do welcome solution approaches that can decrease computational times and can produce bounds on the optimal total cost. One example is to assign zipcodes to a single depot that serves them for any demand realization, although this may call for having more vehicles per depot.
- 3. To evaluate the robustness of the choices made for the depot locations and number of vehicles,

- by testing them on the 2019 demand data. We also welcome optimization approaches that would produce more robust solutions by exploiting statistical properties estimated from the 2018 data.
- 4. To develop a graphical user interface with visualization and reporting tools (maps, plots, reports) to explore the solutions and the tradeoffs. Examples of metrics that may be worth reporting include: the various components of the cost, the daily costs as a function of the aggregated daily demand, the average duration of a delivery trip (globally and per depot), the fraction of the fleet used averaged over the days (globally and per depot), the total miles driven, the annual CO2 emissions, the routes most impacted by the added traffic, the expected delivery hour at demand sites (averaged globally or per zipcode) as a measure of the quality of service.

#### 2. Deliverables

Your team has to deliver a reasonably complete solution to the problem described above. We expect the submission of the following deliverables:

- An implementation of the model in AIMMS;
- A user interface with graphical and textual output;
- A solution of the model for the given test data sets;
- A report of maximum 15 pages, that describes the mathematical background of the model, the solution techniques used, the results obtained and your team's final recommendations.

You are allowed to use topical literature selected by your team. Please cite all used information sources properly, and distinguish your ideas from ideas found in the literature carefully. The **deadline for submission** is Sunday May 31, 2020, 23:59 Pacific Time.

#### 3. Resources.

The case data are available from competition webpage. The AIMMS software should be downloaded at https://www.aimms.com/english/developers/licensing/free-licenses/. The free academic license should be requested from that page.

### 4. Questions

Questions about the problem or the competition in general should be directed to Prof. Boris Defourny at defourny@lehigh.edu. Please start the subject line of the email with "MOPTA Competition 2020". Questions related to the AIMMS software should be directed to support@aimms.com.