Homework 5

1. Consider independent observations $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for $i = 1, ..., n$. Show $\sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$ is independent from $(\hat{\beta}_0, \hat{\beta}_1)$ by calculating $\text{Cov}(\hat{\beta}_0, y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$ and $\text{Cov}(\hat{\beta}_1, y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$.

2. Suppose you collect your regression data from two companies. One has better quality than the other. So you model the data from the first company by $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ independently for $i = 1, ...n_1$, and the data from the second one by $y_i \sim N(\beta_0 + \beta_1 x_i, 2\sigma^2)$ independently for $i = n_1 + 1, ..., n_1 + n_2$. The data from the two companies are also independent from each other.

   (a) Find the MLE for (β_0, β_1).

   (b) Find the mean of the MLE.

   (c) Find the variance of the MLE.

   (d) Find the joint distribution of the MLE.