1. We have

$$\frac{(n-1)\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-1}.$$

Thus, an α-level test would be to accept $H_0 \colon \sigma^2 \leq 1$ if

$$\hat{\sigma}^2 \in \left[0, \chi^2_{n-1,1-\alpha} \cdot \frac{1}{n-1}\right],$$

otherwise accept $H_1 \colon \sigma^2 \geq 1$.

2. We have

$$\begin{aligned} P(\mu) &= \mathbb{P}\left\{N(0,1) + \frac{\sqrt{n}\mu}{\sigma} > z_{1-\alpha}\right\} \\ &= \mathbb{P}\left\{N(0,1) > z_{1-\alpha} - \frac{\sqrt{n}\mu}{\sigma}\right\} \\ &= 1 - \Phi\left(z_{1-\alpha} - \frac{\sqrt{n}\mu}{\sigma}\right) \end{aligned}$$

Thus,

$$\frac{\partial}{\partial \mu} P(\mu) = \frac{\sqrt{n}}{\sigma} \phi\left(z_{1-\alpha} - \frac{\sqrt{n}\mu}{\sigma}\right) > 0.$$

3. We have

$$\begin{aligned} \mathbb{P}(P(X) \leq \alpha) &= \mathbb{P}\left(\Phi\left(-\frac{\sqrt{n}\overline{X}}{\sigma}\right) \leq \alpha\right) \\ &= \mathbb{P}\left(-\frac{\sqrt{n}\overline{X}}{\sigma} \leq z_\alpha\right) \\ &\leq \mathbb{P}\left(-\frac{\sqrt{n}\overline{X} - \mu}{\sigma} \leq z_\alpha\right) && \text{(by } H_0 \colon \mu \leq 0\text{)} \\ &= \mathbb{P}(N(0,1) \leq z_\alpha) \\ &= \alpha. \end{aligned}$$

The inequality would be strict if $\mu \neq 0$. Thus, the p-value distribution would be not uniform.