Knapsack Problem

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Presentation of the problem

- Input:

  For any n object set, we have size set S, weight set W and value set V for those objects.

  S = $\{s_i | \, where \ i \ from \ 1 \ to \ n\}$ = $\{s_1, s_2, s_3, \ldots, s_n\}$

  W = $\{w_i | \, where \ i \ from \ 1 \ to \ n\}$ = $\{w_1, w_2, w_3, \ldots, w_n\}$

  V = $\{v_i | \, where \ i \ from \ 1 \ to \ n\}$ = $\{v_1, v_2, v_3, \ldots, v_n\}$

  You have a capacity of C which is a size limit.

- Output:

  A subset of S

  T = $\{s_i | \, where \ i \in \mathrm{T}\}$

- Metric

  Max$\sum_{i \in \mathrm{T}} v_i$

Important of this problem

- Any arrangement and schedule problem with a threshold
  1. If you have an exam tomorrow, how could you select the most value part in limited time.
  2. How could you schedule your travel plan with limited money?
  3. Data download management
  4. Cache select

ILP formulation

$$Maxmize \sum_i x_i * v_i \, where \; 1 \le i \le n$$

$$subject\ to \; \sum_i x_i * s_i \le C_S \;\; where \; 1 \le i \le n$$

$$\sum_i x_i * w_i \le C_w \;\; where \; 1 \le i \le n$$

$$x_i \in \{0,1\}$$

x_i=1 means object was picked, otherwise object was not picked

Summary of the complexity of the ILP

- Coefficients

  $w_i, s_i\ and\ v_i$
- Variables

  x_i
- Constraints

  Two constraints $\sum_i x_i * s_i \leq C_s$, $\sum_i x_i * w_i \leq C_w$
- Integer variables

  x_i is integer variable

Why the ILP works

The constrain $\sum_i x_i * s_i \leq C_s$accounts for the size threshold. The constrain $\sum_i x_i * w_i \leq C_w$accounts for the weight threshold. Under those two thresholds, we have the max value.

For example if we have a solution of {0,1,0,0,0}, we know only second item was picked and the $x_2 * s_2 \leq C_s$, $x_2 * w_2 \leq C_w$ and the $x_2 * v_2$ is max value.