

COSC 341 – Tutorial 3

1. Show that the set of even natural numbers is countable.
2. Show that the set of even integers is countable.
3. Show that the set $\{f|f : \mathbb{N} \rightarrow \mathbb{N}\}$ of all functions from \mathbb{N} to \mathbb{N} is uncountable.
4. Show that the power set $\mathcal{P}(\mathbb{N})$ of \mathbb{N} is uncountable.
5. Show that, for any set A , $|A| < |\mathcal{P}(A)|$.

Homework

1. Show that the set of total functions from \mathbb{N} to $\{0, 1\}$ is uncountable.
2. We can define the set \mathbb{N} of natural numbers as:

$$0 \in \mathbb{N}$$

$$\text{If } n \in \mathbb{N}, \text{ then } n + 1 \in \mathbb{N}$$

We call this a *recursive* definition.

Give recursive definitions of:

- (a) The set of even natural numbers $EN = \{2n | n \in \mathbb{N}\}$
- (b) The set $P = \{1, 2, 4, 8, 16, \dots\}$ of powers of 2 within \mathbb{N}