Generation Investment Equilibria With Strategic Producers—Part I: Formulation

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***Abstract*—The first of this two-paper series proposes a methodology to characterize generation investment equilibria in a pool-based network-constrained electricity market, where the producers behave strategically. To this end, the investment problem of each strategic producer is represented using a bilevel model, whose upper-level problem determines the optimal investment and the supply offering curves to maximize its profit, and whose several lower-level problems represent different market clearing scenarios. This model is transformed into a mathematical program with equilibrium constraint (MPEC) through replacing the lower-level problems by their optimality conditions. The joint consideration of all producer MPECs, one per producer, constitutes an equilibrium problem with equilibrium constraints (EPEC). To identify the solutions of this EPEC, each MPEC problem is replaced by its Karush-Kuhn-Tucker (KKT) conditions, which are in turn linearized. The resulting mixed-integer linear system of equalities and inequalities allows determining the EPEC equilibria through an auxiliary MILP problem.**

***Index Terms*—Bilevel model, equilibrium problem with equilibrium constraints (EPEC), generation investment equilibria, mathematical program with equilibrium constraints (MPEC), strategic producer.**

NOTATION

For quick reference, the main notation used in the paper is stated in this section. Other symbols are defined as needed throughout the text.

A. Indices:

t	Index for demand blocks.
y	Index for producers.
i	Index for candidate generating units.
k	Index for existing generating units.
d	Index for demands.
n/m	Indices for buses.

B. Sets:

Ψ^{S}	Set of candidate units.
Ψ^{E}	Set of existing units.
Ψ^{D}	Set of demands.
Ω_y	Set of units owned by producer y.
Φ_n	Set of buses adjacent to bus n.

Sets Ψ^{S}, Ψ^{E} and Ψ^{D} include subscript n if referring to the set of units/demands located at bus n.

C. Parameters:

σ_t	Weighting factor of demand block t [hour].
K_i	Annualized capital cost of candidate unit $i \in \Psi^{\mathrm{S}}$ [€/MW].
$K^{\max}$	Available investment budget [€].
$X_i^{\max}$	Maximum power of candidate unit $i \in \Psi^{\mathrm{S}}$ [MW].
$P_k^{\mathrm{E}^{\max}}$	Capacity of existing unit $k \in \Psi^{\mathrm{E}}$ [MW].
$P_{td}^{\mathrm{D}^{\max}}$	Maximum load of demand $d \in \Psi^{\mathrm{D}}$ in block t [MW].
C_i^{S}	Production cost of candidate unit $i \in \Psi^{\mathrm{S}}$ [€/MWh].
C_k^{E}	Production cost of existing unit $k \in \Psi^{\mathrm{E}}$ [€/MWh].
U_{td}^{D}	Price bid of demand $d \in \Psi^{\mathrm{D}}$ in block t [€/MWh].
B_{nm}	Susceptance of transmission line (n,m) [p.u.].
$F_{nm}^{\max}$	Capacity of transmission line (n,m) [MW].

D. Variables:

X_i	Capacity of candidate unit $i \in \Psi^{\mathrm{S}}$ [MW].
O_{ti}^{S}	Price offer by candidate unit $i \in \Psi^{\mathrm{S}}$ in demand block t [€/MWh].
O_{tk}^{E}	Price offer by existing unit $k \in \Psi^{\mathrm{E}}$ in demand block t [€/MWh].
P_{ti}^{S}	Power produced by candidate unit $i \in \Psi^{\mathrm{S}}$ in demand block t [MW].
P_{tk}^{E}	Power produced by existing unit $k \in \Psi^{\mathrm{E}}$ in demand block t [MW].
P_{td}^{D}	Power consumed by demand $d \in \Psi^{\mathrm{D}}$ in block t [MW].
θ_{tn}	Voltage angle of bus n in demand block t [rad].
λ_{tn}	Locational marginal price at bus n in demand block t [€/MWh].

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I. Introduction

A. Motivation and Aim

THE aim of a producer intending to expand its generation portfolio is to maximize its profit through its investment and operation strategies. Since the strategies of any producer are interrelated with those of other producers by the market clearing algorithm, decisions made by one producer may influence the strategies of other producers. Therefore, a number of investment equilibria may exist, where each producer cannot increase its profit by changing unilaterally its strategies [1].

An equilibrium analysis is useful for a market regulator to gain insight into the investment behavior of the producers and the generation investment evolution. Such insight may allow the market regulator to better design market rules, which in turn may contribute to increase the competitiveness of the market and to stimulate optimal investment in generation capacity.

The producers considered in this paper are "strategic", i.e., they can alter the formation of the market clearing prices through their strategies. "Strategic offering" and "strategic investment" refer to the offering and investment decisions of a strategic producer, respectively.

A pool-based electricity market is considered where a market operator clears the pool once a day, one day ahead, on an hourly basis, and using a dc representation of the network. The market operator seeks to maximize the social welfare (SW) considering the stepwise supply function offers and the demand bids submitted by the strategic producers and the consumers, respectively. The market clearing results are hourly productions/consumptions and locational marginal prices (LMPs).

As it is customary in the large-scale generation investment studies [2]–[4] and pursuing an appropriate tradeoff between accuracy and computational tractability, a static approach is used in this paper. In the static approach, only the last year of a planning horizon is considered as the target year. Then, based on the existing generation portfolio of the initial year, the capacity and mix of the generation investment required to attain the optimal solution of the target year is computed. Note that in this approach, the building path from an initial year to the target year is not explicitly represented.

Contrary to the static approach, the building schedule from the initial year to the last year of the planning horizon is derived in a multi-stage dynamic approach. This approach provides higher accuracy but at the cost of potential intractability.

As an input of the static approach, Fig. 1 depicts the load-duration curve of the system for the last year of the planning horizon, approximated through a number of stepwise demand blocks. Note that the weighting factor corresponding to demand block t (σ_t) refers to a portion of the hours in the year, for which the load of the system is approximated through the demand of that block. Clearly, the summation of the weighting factors of all demand blocks equals the number of hours in a year, i.e., $\sum_t \sigma_t = 8760$.

For the sake of clarity, the following terms are further explained:

1) Demand blocks: these blocks are obtained from a stepwise approximation of the load-duration curve as illustrated in Fig. 1. Each demand block may include several demands located at different buses of the network.

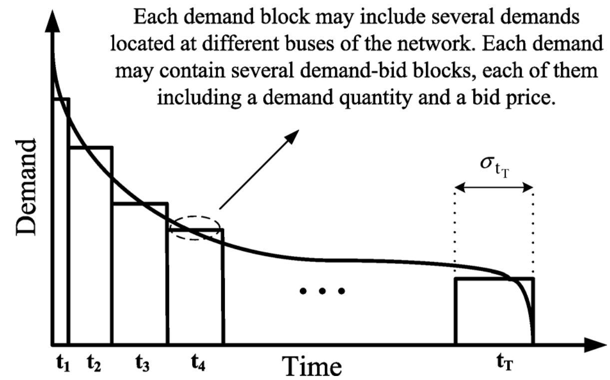

Fig. 1. Load-duration curve for the last year of the planning horizon approximated through a number of stepwise demand blocks.

2) Demand-bid blocks: each demand may contain several demand-bid blocks, each of them including a demand quantity and its associated bid price.
3) Production-offer blocks: each generator's offer may consist of several production-offer blocks derived from a stepwise linearization of its quadratic cost curve. Thus, each production-offer block includes a production quantity and its corresponding production cost. Note that since the generators are considered strategic, they can offer their blocks at prices different than their actual production costs.

The investment decision making and the strategic offering of each strategic producer is represented through a bilevel model, whose upper-level (UL) problem decides the optimal investment and the supply offering curves for maximizing its profit, and whose several lower-level (LL) problems represent different market clearing scenarios, one per demand block. Replacing the LL problems by their optimality conditions renders a mathematical program with equilibrium constraint (MPEC). The joint consideration of all producer MPECs, one per producer, constitutes an equilibrium problem with equilibrium constraints (EPEC).

The problem considered in this paper can be classified as a generalized Nash equilibrium (GNE) problem [5] and [6] since the feasibility regions of the producers' problems are interrelated by their strategies. Moreover, the proposed model is a GNE with shared constraints in which the market clearing conditions are common to all producers.

We characterize the GNE problem by its associated optimality conditions which are obtained by concatenating all the Karush-Kuhn-Tucker (KKT) conditions from all MPECs, one per strategic producer. However, due to the non-convex nature of the MPECs, standard constraint qualifications are generally not met and therefore the solution of the KKT system may include multiple equilibrium points as well as saddle points. For this reason, in order to verify that each solution attained is actually a Nash equilibrium, we perform an ex-post analysis based on a diagonalization algorithm, i.e., an iterative method in which each producer determines sequentially or in parallel its investment decisions considering other producers' strategies fixed. Diagonalization algorithms are clearly inefficient with respect to the proposed approach since they are iterative and heuristic, and they provide, if convergence is achieved, at most one single equilibrium point.

The specific details of the modeling and solution approaches are described in the next subsection.

B. Approach

The approach used in this paper is similar to those used in [7] and [8]. Such references analyze the equilibria reached by strategic producers in a network-constrained pool in which the behavior of each producer is represented by an MPEC. Note that [7] and [8] address an operation, not an investment problem. Similarly to the model developed in [8], the EPEC is characterized in this paper by solving the optimality conditions of all MPECs, which are formulated as a mixed-integer linear programming (MILP), and diverse linear objective functions are used to obtain different equilibria. The methodology presented in this paper extends the model in [8] by incorporating generation investment decisions and analyzing the impact that these decisions have on the competitiveness of the market. The detailed steps of this approach are listed below:

1) To formulate a bilevel model for each strategic producer, whose UL problem determines the optimal investment (capacity and location) and the stepwise supply offering curves to maximize its profit, and whose several LL problems represent the market clearing conditions, one per demand block.
2) To transform each bilevel model into a single-level problem by replacing the LL problems by their primal-dual optimality conditions. The resulting model is an MPEC.
3) To simultaneously consider all producer MPECs, one per producer, and thus to formulate an EPEC.
4) To derive the optimality conditions of the EPEC by replacing each MPEC with its KKT conditions. This results in a collection of nonlinear systems of equalities and inequalities.
5) To linearize the optimality conditions of the EPEC obtained in the previous step without approximation through a) a linearization of the complementarity conditions, and b) a parameterization approach. Thus, the resulting conditions become mixed-integer and linear.
6) To detect meaningful equilibria by formulating and solving an MILP problem whose constraints are the system of equalities and inequalities characterizing the EPEC and whose linear objective function is selected targeting a particular equilibrium.

C. Literature Review and Contributions

Reference [2] proposes a bilevel generation capacity expansion game in a perfectly competitive market and in open-loop and closed-loop Cournot duopolies. In the open-loop duopoly, production and investment decisions are made simultaneously. However, in the closed-loop duopoly, decisions for installing new capacities are made in the first stage and in the second stage the units compete in a spot market. All three models are static, and uncertainties as well as the network constraints are not modeled.

Reference [3] considers a static but stochastic capacity expansion equilibria in an electricity market. The objective of this reference is to investigate the impact of four key parameters on the generation investment equilibria and on the supply security. Such parameters are: 1) profit risk, 2) investment incentives, 3) market organization (energy-only or energy and capacity) and price caps, and 4) carbon trading issues. In the capacity expansion equilibria reported in [3], fuel prices and climate change policies are considered as uncertain parameters, while the demand is assumed known. The producers are considered to be price-takers, but subject to regulatory imperfections. The network is disregarded, and a small case study is analyzed.

The equilibrium of generation investment considering both forward and spot markets is studied in [4], where a Cournot model is used. This reference shows that forward contracts may not mitigate market power in the spot market, in case that the production capacities of the producers are endogenous and constrain the production level.

A multi-stage generation investment equilibrium considering uncertain demand is addressed in [9]. This reference proposes a Markov chain to model the strategic interaction between a short-run capacity-constrained Cournot game and a long-run generation investment game. It is concluded that a socially optimal level of capacity is not built, and that the distance between the capacity built and the optimal capacity is largely dependent on investment profitability.

In [10], an EPEC is proposed to identify the generation capacity investment equilibria. To this end, a bilevel optimization problem is formulated so that each producer selects capacities in an UL problem maximizing its profit and anticipating the equilibrium outcomes of the LL problems, in which production quantities and prices are determined by a conjectured-price response approach.

In [11], the capacity expansion decisions are made by a leader (market regulator) on behalf of the market agents. The aim of this model is to maximize the total welfare of all market participants, which renders a bilevel model, whose UL problem determines the investment decisions, and whose LL problems represent the operation decisions of each market participant. Such bilevel model is transformed into a mathematical program with complementarity constraints (MPCC).

Investment incentives are studied in [12] using a simple strategic dynamic model with random demand growth. This model is based on a non-collusive Markovian equilibrium in which the investment decisions of each producer depend on its existing capacity. In the same vein, reference [13] formulates a dynamic capacity investment equilibrium considering a hydrothermal duopoly under uncertain demand. Both Markov perfect and open-loop equilibria are modeled in this reference and then the incentives needed to promote investment are studied.

The work reported in this two-part paper differs from [2]–[4] and [9]–[13]. Such references make use of comparatively simpler approaches (e.g., Cournot, Bertrand or conjectural variations) to model oligopolistic markets than the stepwise supply function model proposed in this two-part paper. For the sake of clarity, the relevant features of the model proposed in this paper and other works reported in the literature are summarized in Table I.

In addition, there are several papers in the literature pertaining to electricity market equilibria from the operational point of view, e.g., [7], [8], [14], and [15].

Moreover, several papers are found in the literature referring to the generation investment decision-making problem of a particular producer (e.g., [16] and [17]). Reference [16] proposes

TABLE I
RELEVANT FEATURES OF THE PAPERS REPORTED IN THE LITERATURE REVIEW AND THIS TWO-PART PAPER

Ref.	Model	Bilevel	Strategic offering	Transmission constraints	Static/dynamic	Uncertainty	Different investment technologies
[2]	Cournot (MPEC)	Yes	No	No	Static	No	Yes
[3]	Complementarity	No	No	No	Static	Yes	Yes
[4]	Cournot (EPEC)	No	No	No	Static	Yes	Yes
[9]	Cournot-Marcov chain	No	No	No	Dynamic	Yes	No
[10]	Conjectural variations (EPEC)	Yes	No	No	Dynamic	No	Yes
[11]	Cournot (MPCC)	Yes	No	Yes	Static	No	Yes
[12]	Non-collusive Markov	No	No	No	Dynamic	Yes	No
[13]	Open-loop and Markov perfect	No	No	No	Dynamic	Yes	Yes
This paper	Stepwise supply function (EPEC)	Yes	Yes	Yes	Static	No	Yes

a bilevel model to assist a producer in making multi-stage generation investment decisions considering the investment uncertainty of rival producers. In the UL problem, the producer maximizes its expected profit, while the LL problem represents the market clearing conditions characterized by a conjectural variations model and including no network constraint. In [17], a static but stochastic bilevel investment model for a strategic producer participating in a pool with stepwise supply function offers is proposed. The models proposed in [16] and [17] render MPECs.

Considering the context above, the main contributions of this two-part paper are twofold:

1) To develop a methodology for representing the interactions among a number of strategic investors in a network-constrained electricity market as a game-theoretic model.
2) To mathematically identify generation investment equilibria. This is done through the formulation of an EPEC and its solution using MILP techniques.

Additionally, an ex-post analysis is required to identify which of these equilibria are meaningful and may actually occur in practice. However, such analysis is outside the scope of this work.

D. Paper Organization

The rest of this paper is organized as follows. Section II characterizes a single-producer problem by its bilevel model and the corresponding MPEC. Section III presents the multiple producers problem characterized by the EPEC resulting from the joint consideration of the MPECs of all producers. In addition, the EPEC is linearized in Section III. Two different linear objective functions for equilibria detection are selected in Section IV. A single-iteration diagonalization algorithm is presented in Section V to check if each solution obtained by the proposed approach is a Nash equilibrium. Finally, some relevant conclusions are drawn in Section VI.

II. SINGLE-PRODUCER PROBLEM

A. Bilevel Model

The problem solved by each strategic producer y to determine its best investment and offering decisions is formulated as a bilevel model. The structure of the proposed bilevel model is illustrated in Fig. 2 and described below. The UL problem represents the minus-profit minimization for the producer subject to 1) the UL constraints and 2) a set of LL problems. The UL constraints include bounds on investment options, investment budget limit, minimum available capacity imposed by the market regulator and the non-negativity of strategic offers. On the other hand, an LL problem per demand block represents the clearing of the pool minimizing the minus-SW subject to the market operation conditions, i.e., balance constraints at every bus, production/consumption power limits, transmission line capacity limits, voltage angle limits and the reference bus identification.

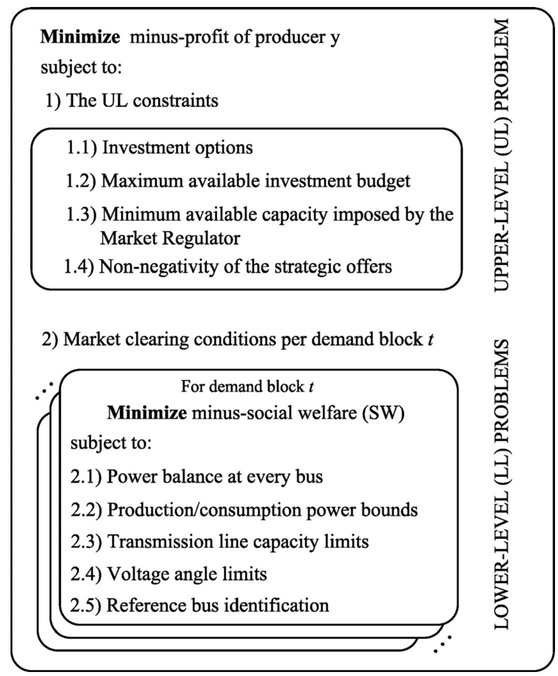

Fig. 2. Structure of the proposed bilevel model solved by each strategic producer.

Note that the UL and the LL problems are interrelated as illustrated in Fig. 3. On one hand, the LL problems determine the LMPs and the production quantities, that directly influence the producer's profit in the UL problem. On the other hand, the strategic offering and investment decisions made by the strategic producer at the UL problem affect the market clearing outcomes in the LL problems. The formulation of the bilevel model corresponding to strategic producer y is given below by (1).

The UL problem includes (1a)–(1f), while (1g)–(1n) pertain to the LL problems, one per demand block t. Objective function (1a) and constraints (1b) correspond to the strategic producer y, while other UL constraints (1c)–(1f) and the LL problems

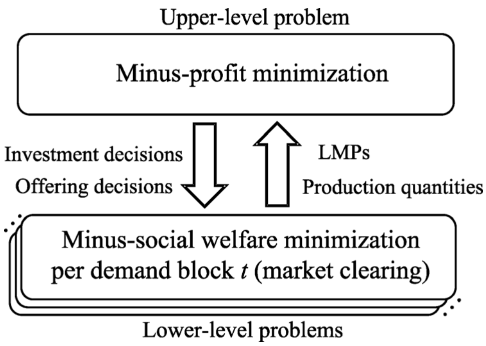

Fig. 3. Interrelation between the upper-level and lower-level problems.

(1g)–(1n) are common to all strategic producers. Dual variables of each LL problem are indicated at their corresponding constraints following a colon:

$$\underset{\Xi^{\mathrm{UL}}}{\text{Minimize}}$$

$$\sum_{i\in(\Psi^{\mathrm{S}}\cap\Omega_y)} K_i X_i - \sum_t \sigma_t \left[\sum_{i\in(\Psi^{\mathrm{S}}\cap\Omega_y)} P^{\mathrm{S}}_{ti}\left(\lambda_{t(n:i\in\Psi^{\mathrm{S}}_n)} - C^{\mathrm{S}}_i\right) + \sum_{k\in(\Psi^{\mathrm{E}}\cap\Omega_y)} P^{\mathrm{E}}_{tk}\left(\lambda_{t(n:k\in\Psi^{\mathrm{E}}_n)} - C^{\mathrm{E}}_k\right)\right] \quad (1a)$$

subject to :

$$0 \le X_i \le X^{\max}_i \quad \forall i \in (\Psi^{\mathrm{S}}\cap\Omega_y) \quad (1b)$$

$$\sum_{i\in\Psi^{\mathrm{S}}} K_i X_i \le K^{\max} \quad (1c)$$

$$\left(\sum_{i\in\Psi^{\mathrm{S}}} X_i + \sum_{k\in\Psi^{\mathrm{E}}} P^{\mathrm{E^{\max}}}_k\right) \ge \Upsilon \times \sum_{d\in\Psi^{\mathrm{D}}} P^{\mathrm{D^{\max}}}_{td} \quad t = t_1 \quad (1d)$$

$$O^{\mathrm{S}}_{ti} \ge 0 \quad \forall t,\ \forall i\in\Psi^{\mathrm{S}} \quad (1e)$$

$$O^{\mathrm{E}}_{tk} \ge 0 \quad \forall t, \forall k\in\Psi^{\mathrm{E}} \quad (1f)$$

$$\lambda_{tn}, P^{\mathrm{S}}_{ti}, P^{\mathrm{E}}_{tk} \in \arg \min_{\Xi^{\mathrm{P}}_{\mathrm{t}},\forall t} \Bigg\{$$

$$\sum_{i\in\Psi^{\mathrm{S}}} O^{\mathrm{S}}_{ti} P^{\mathrm{S}}_{ti} + \sum_{k\in\Psi^{\mathrm{E}}} O^{\mathrm{E}}_{tk} P^{\mathrm{E}}_{tk} - \sum_{d\in\Psi^{\mathrm{D}}} U^{\mathrm{D}}_{td} P^{\mathrm{D}}_{td} \quad (1g)$$

subject to :

$$\sum_{d\in\Psi^{\mathrm{D}}_n} P^{\mathrm{D}}_{td} + \sum_{m\in\Phi_n} B_{nm}(\theta_{tn}-\theta_{tm}) - \sum_{i\in\Psi^{\mathrm{S}}_n} P^{\mathrm{S}}_{ti} - \sum_{k\in\Psi^{\mathrm{E}}_n} P^{\mathrm{E}}_{tk} = 0 \ : \lambda_{tn},\ \forall n \quad (1h)$$

$$0 \le P^{\mathrm{S}}_{ti} \le X_i \ : \mu^{\mathrm{S^{\min}}}_{ti}, \mu^{\mathrm{S^{\max}}}_{ti}, \quad \forall i\in\Psi^{\mathrm{S}} \quad (1i)$$

$$0 \le P^{\mathrm{E}}_{tk} \le P^{\mathrm{E^{\max}}}_k \ : \mu^{\mathrm{E^{\min}}}_{tk}, \mu^{\mathrm{E^{\max}}}_{tk}, \quad \forall k\in\Psi^{\mathrm{E}} \quad (1j)$$

$$0 \le P^{\mathrm{D}}_{td} \le P^{\mathrm{D^{\max}}}_{td} \ : \mu^{\mathrm{D^{\min}}}_{td}, \mu^{\mathrm{D^{\max}}}_{td}, \quad \forall d\in\Psi^{\mathrm{D}} \quad (1k)$$

$$-F^{\max}_{nm} \le B_{nm}(\theta_{tn}-\theta_{tm}) \le F^{\max}_{nm} \ : \nu^{\min}_{tnm}, \nu^{\max}_{tnm}, \quad \forall n, \forall m\in\Phi_n \quad (1l)$$

$$-\pi \le \theta_{tn} \le \pi \ : \xi^{\min}_{tn}, \xi^{\max}_{tn}, \quad \forall n \quad (1m)$$

$$\left.\theta_{tn} = 0 \ : \xi^{1}_{t}, \quad n = 1\right\} \forall t. \quad (1n)$$

The primal optimization variables of each LL problem are included in the set $\Xi^{\mathrm{P}}_{\mathrm{t}}=\{P^{\mathrm{S}}_{ti}, P^{\mathrm{E}}_{tk}, P^{\mathrm{D}}_{td}, \theta_{tn}\}$, while the set of its dual variables is $\Xi^{\mathrm{D}}_{\mathrm{t}} = \{\lambda_{tn}, \mu^{\mathrm{S^{\min}}}_{ti}, \mu^{\mathrm{S^{\max}}}_{ti}, \mu^{\mathrm{E^{\min}}}_{tk}, \mu^{\mathrm{E^{\max}}}_{tk}, \mu^{\mathrm{D^{\min}}}_{td}, \mu^{\mathrm{D^{\max}}}_{td}, \tau^{\min}_{tnm}, \tau^{\max}_{tnm}, \xi^{\min}_{tn}, \xi^{\max}_{tn}, \xi^{1}_{t}\}$.

Producer y behaves strategically through its following strategic decisions made at the UL problem (1a)–(1f):

a) Strategic investment decisions, i.e., $X_i\ \forall i \in (\Psi^{\mathrm{S}}\cap\Omega_y)$.
b) Strategic offering decisions, i.e., $O^{\mathrm{S}}_{ti}\ \forall t, \forall i \in (\Psi^{\mathrm{S}}\cap\Omega_y)$ and $O^{\mathrm{E}}_{tk}\ \forall t, \forall k \in (\Psi^{\mathrm{E}}\cap\Omega_y)$.

Note that strategic producer y anticipates the market outcomes, e.g., LMPs and production quantities, versus its offering and investment strategies. To this end, constraining the UL problem, the pool is cleared in each LL problem for given investment and offering decisions. This allows each strategic producer to obtain feedback regarding how its offering and investment actions affect the market. Thus, X_i, O^{S}_{ti} and O^{E}_{tk} are variables in the UL problem while they are parameters in the LL problems. Note that this makes the LL problems (1g)–(1n) linear and thus convex. In addition, since the LL problems constrain the UL problem, the LL variable sets $\Xi^{\mathrm{P}}_{\mathrm{t}}$ and $\Xi^{\mathrm{D}}_{\mathrm{t}}$ are included in the variable set of the UL problem as well. Thus, the primal variables of the UL problem are those in the set $\Xi^{\mathrm{UL}} = \{\Xi^{\mathrm{P}}_{\mathrm{t}}, \Xi^{\mathrm{D}}_{\mathrm{t}}, X_i, O^{\mathrm{S}}_{ti}, O^{\mathrm{E}}_{tk}\}$.

The objective function (1a) of the UL problem seeks to minimize the minus-profit, i.e., the investment cost of strategic producer y ($\sum_{i\in(\Psi^{\mathrm{S}}\cap\Omega_y)} K_i X_i$) minus its operations revenue. Note that λ_{tn} representing the LMPs are endogenously generated within the LL problems as dual variables. Constraints (1b) bound the capacity of the candidate units. Constraints (1c) and (1d) enforce the upper and lower bounds on the total capacity to be built by all producers. On one hand, the available investment budget of all producers considered in (1c) imposes a cap on total investment by all producers, which reflects the limited financial resources available to the market as a whole. On the other hand, constraint (1d) enforces a minimum available capacity (including existing and newly built units) to ensure supply security. Such condition is adjusted through a non-negative factor Υ that multiplies the peak demand level (demand of the first block, $t = t_1$). In addition, constraints (1e) and (1f) enforce the non-negativity of the offer curves of all strategic producers.

The objective function of each LL problem (1g) is the minus-SW. Constraints (1h) represent the energy balance at each node, being the associated dual variables LMPs. Constraints (1i)–(1k) enforce the lower and upper production/consumption bounds of candidate and existing units/demands. Limits on transmission line capacities and voltage angles of nodes are enforced through (1l) and (1m), respectively. Finally, constraints (1n) identify $n = 1$ as the reference bus.

Note that for notational clarity, additional subscripts have not been included in (1) to represent production-offer blocks of each unit and demand-bid blocks of each demand. However, the former are considered in the case studies analyzed in the companion paper [18].

In this work, demands are considered to be elastic to prices. Thus, they are not necessarily supplied at their maximum levels. Additionally, no constraint is included in the model to force the supply of a minimum demand level. However, note that such constraint can be easily added to the formulation. Note also that in this work, the demands do not behave strategically.

B. MPEC

In this subsection, the bilevel problem (1) of each strategic producer y is transformed into a single-level problem. Since the LL problems (1g)–(1n) are linear and thus convex, they can be replaced by their first-order optimality conditions rendering an MPEC.

The optimality conditions of a linear optimization problem can be formulated through two alternative approaches:

a) KKT conditions;
b) Primal-dual transformation, i.e., enforcing primal constraints, dual constraints and the strong duality equality.

In the first approach (KKT conditions), some equalities are obtained from differentiating the corresponding Lagrangian with respect to the decision variables, and such equalities are equivalent to the set of primal and dual constraints of the second approach (primal-dual transformation). In addition, the set of complementarity conditions obtained by the first approach (KKT conditions) is equivalent to the corresponding strong duality equality of the primal-dual transformation [19].

In this section, the second approach (primal-dual transformation) is used to avoid the use of non-convex and difficult to handle complementarity conditions, but at the cost of the nonlinearities introduced by the strong duality equalities. Fig. 4 depicts the transformation of bilevel problem (1) into its corresponding MPEC. The MPEC derived from bilevel problem (1), corresponding to strategic producer y is given below by (2). Dual variables of problem (2) are indicated at their corresponding constraints following a colon:

$$\underset{\Xi^{\mathrm{UL}}}{\text{Minimize}}$$

$$\sum_{i\in(\Psi^{\mathrm{S}}\cap\Omega_y)} K_i X_i - \sum_t \sigma_t \left[\sum_{i\in(\Psi^{\mathrm{S}}\cap\Omega_y)} P_{ti}^{\mathrm{S}} \left(\lambda_{t(n:i\in\Psi_n^{\mathrm{S}})} - C_i^{\mathrm{S}}\right) + \sum_{k\in(\Psi^{\mathrm{E}}\cap\Omega_y)} P_{tk}^{\mathrm{E}} \left(\lambda_{t(n:k\in\Psi_n^{\mathrm{E}})} - C_k^{\mathrm{E}}\right)\right] \quad (2a)$$

subject to:

$$0 \le X_i \le X_i^{\max} \ : \Lambda_{yi}^{\min}, \Lambda_{yi}^{\max}, \quad \forall i \in (\Psi^{\mathrm{S}}\cap\Omega_y) \quad (2b)$$

$$\sum_{i\in\Psi^{\mathrm{S}}} K_i X_i \le K^{\max} \ : \Delta_y \quad (2c)$$

$$\left(\sum_{i\in\Psi^{\mathrm{S}}} X_i + \sum_{k\in\Psi^{\mathrm{E}}} P_k^{\mathrm{E}^{\max}}\right) \ge \Upsilon \times \sum_{d\in\Psi^{\mathrm{D}}} P_{td}^{\mathrm{D}^{\max}} \ : \Gamma_y, \ t = t_1 \quad (2d)$$

$$O_{ti}^{\mathrm{S}} \ge 0 \ : \eta_{yti}^{\alpha^{\mathrm{S}}}, \quad \forall t, \ \forall i \in \Psi^{\mathrm{S}} \quad (2e)$$

$$O_{tk}^{\mathrm{E}} \ge 0 \ : \eta_{ytk}^{\alpha^{\mathrm{E}}}, \quad \forall t, \ \forall k \in \Psi^{\mathrm{E}} \quad (2f)$$

$$\sum_{d\in\Psi_n^{\mathrm{D}}} P_{td}^{\mathrm{D}} + \sum_{m\in\Phi_n} B_{nm}(\theta_{tn} - \theta_{tm}) - \sum_{i\in\Psi_n^{\mathrm{S}}} P_{ti}^{\mathrm{S}} - \sum_{k\in\Psi_n^{\mathrm{E}}} P_{tk}^{\mathrm{E}} = 0 \ : \beta_{ytn}, \quad \forall t, \ \forall n \quad (2g)$$

$$0 \le P_{ti}^{\mathrm{S}} \le X_i \ : \gamma_{yti}^{\mathrm{S}^{\min}}, \gamma_{yti}^{\mathrm{S}^{\max}}, \quad \forall t, \forall i \in \Psi^{\mathrm{S}} \quad (2h)$$

$$0 \le P_{tk}^{\mathrm{E}} \le P_k^{\mathrm{E}^{\max}} \ : \gamma_{ytk}^{\mathrm{E}^{\min}}, \gamma_{ytk}^{\mathrm{E}^{\max}}, \quad \forall t, \forall k \in \Psi^{\mathrm{E}} \quad (2i)$$

$$0 \le P_{td}^{\mathrm{D}} \le P_{td}^{\mathrm{D}^{\max}} \ : \gamma_{ytd}^{\mathrm{D}^{\min}}, \gamma_{ytd}^{\mathrm{D}^{\max}}, \quad \forall t, \forall d \in \Psi^{\mathrm{D}} \quad (2j)$$

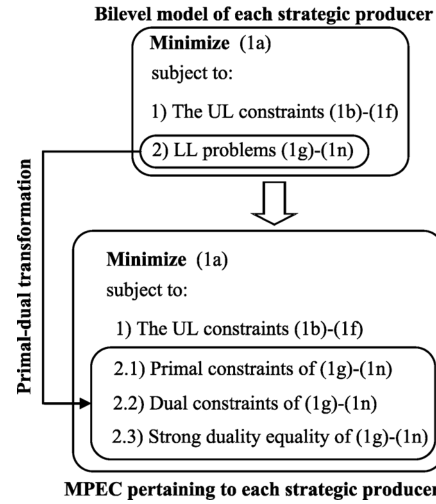

Fig. 4. Transformation of the bilevel model of a strategic producer into its corresponding MPEC (primal-dual transformation).

$$-F_{nm}^{\max} \le B_{nm}(\theta_{tn} - \theta_{tm}) \le F_{nm}^{\max} \ : \tau_{ytnm}^{\min}, \tau_{ytnm}^{\max}, \quad \forall t, \ \forall n, \ \forall m \in \Phi_n \quad (2k)$$

$$-\pi \le \theta_{tn} \le \pi \ : \delta_{ytn}^{\min}, \delta_{ytn}^{\max}, \quad \forall t, \forall n \quad (2l)$$

$$\theta_{tn} = 0 \ : \delta_{yt}^{1}, \quad \forall t, n = 1 \quad (2m)$$

$$-U_{td}^{\mathrm{D}} + \lambda_{t(n:d\in\Psi_n^{\mathrm{D}})} + \mu_{td}^{\mathrm{D}^{\max}} - \mu_{td}^{\mathrm{D}^{\min}} = 0 \ : \rho_{ytd}^{\mathrm{D}}, \quad \forall t, \ \forall d \in \Psi^{\mathrm{D}} \quad (2n)$$

$$O_{ti}^{\mathrm{S}} - \lambda_{t(n:i\in\Psi_n^{\mathrm{S}})} + \mu_{ti}^{\mathrm{S}^{\max}} - \mu_{ti}^{\mathrm{S}^{\min}} = 0 \ : \rho_{yti}^{\mathrm{S}}, \quad \forall t, \ \forall i \in \Psi^{\mathrm{S}} \quad (2o)$$

$$O_{tk}^{\mathrm{E}} - \lambda_{t(n:k\in\Psi_n^{\mathrm{E}})} + \mu_{tk}^{\mathrm{E}^{\max}} - \mu_{tk}^{\mathrm{E}^{\min}} = 0 \ : \rho_{ytk}^{\mathrm{E}}, \quad \forall t, \forall k \in \Psi^{\mathrm{E}} \quad (2p)$$

$$\sum_{m\in\Phi_n} B_{nm}(\lambda_{tn} - \lambda_{tm}) + \sum_{m\in\Phi_n} B_{nm}\left(\nu_{tnm}^{\max} - \nu_{tmn}^{\max}\right) - \sum_{m\in\Phi_n} B_{nm}\left(\nu_{tnm}^{\min} - \nu_{tmn}^{\min}\right) + \xi_{tn}^{\max} - \xi_{tn}^{\min} + \left(\xi_t^1\right)_{n=1} = 0 \ : \rho_{ytn}^{\theta}, \quad \forall t, \forall n \quad (2q)$$

$$\mu_{ti}^{\mathrm{S}^{\min}} \ge 0; \mu_{ti}^{\mathrm{S}^{\max}} \ge 0 \ : \eta_{yti}^{\mathrm{S}^{\min}}, \eta_{yti}^{\mathrm{S}^{\max}}, \quad \forall t, \ \forall i \in \Psi^{\mathrm{S}} \quad (2r)$$

$$\mu_{tk}^{\mathrm{E}^{\min}} \ge 0; \mu_{tk}^{\mathrm{E}^{\max}} \ge 0 \ : \eta_{ytk}^{\mathrm{E}^{\min}}, \eta_{ytk}^{\mathrm{E}^{\max}}, \quad \forall t, \forall k \in \Psi^{\mathrm{E}} \quad (2s)$$

$$\mu_{td}^{\mathrm{D}^{\min}} \ge 0; \mu_{td}^{\mathrm{D}^{\max}} \ge 0 \ : \eta_{ytd}^{\mathrm{D}^{\min}}, \eta_{ytd}^{\mathrm{D}^{\max}}, \quad \forall t, \forall d \in \Psi^{\mathrm{D}} \quad (2t)$$

$$\nu_{tnm}^{\min} \ge 0; \ \nu_{tnm}^{\max} \ge 0 \ : \eta_{ytnm}^{\nu^{\min}}, \eta_{ytnm}^{\nu^{\max}}, \quad \forall t, \forall n, \forall m \in \Phi_n \quad (2u)$$

$$\xi_{tn}^{\min} \ge 0; \xi_{tn}^{\max} \ge 0 \ : \eta_{ytn}^{\xi^{\min}}, \eta_{ytn}^{\xi^{\max}}, \quad \forall t, \ \forall n \quad 2v)$$

$$\sum_{i\in\Psi^{\mathrm{S}}} O_{ti}^{\mathrm{S}} P_{ti}^{\mathrm{S}} + \sum_{k\in\Psi^{\mathrm{E}}} O_{tk}^{\mathrm{E}} P_{tk}^{\mathrm{E}} - \sum_{d\in\Psi^{\mathrm{D}}} U_{td}^{\mathrm{D}} P_{td}^{\mathrm{D}} + \sum_{i\in\Psi^{\mathrm{S}}} \mu_{ti}^{\mathrm{S}^{\max}} X_i + \sum_{k\in\Psi^{\mathrm{E}}} \mu_{tk}^{\mathrm{E}^{\max}} P_k^{\mathrm{E}^{\max}} + \sum_{d\in\Psi^{\mathrm{D}}} \mu_{td}^{\mathrm{D}^{\max}} P_{td}^{\mathrm{D}^{\max}} + \sum_{n(m\in\Phi_n)} \left(\nu_{tnm}^{\min} + \nu_{tnm}^{\max}\right) F_{nm}^{\max} + \sum_n \left(\xi_{tn}^{\min} + \xi_{tn}^{\max}\right)\pi = 0 \ : \phi_{yt}^{\mathrm{SD}}, \ \forall t. \quad (2w)$$

The primal optimization variable set of MPEC (2) is identical to that of bilevel problem (1), i.e., the set Ξ^{UL}, while the set of its dual variables is $\Xi^{\mathrm{Dual}} = \{\Lambda_{yi}^{\min}, \Lambda_{yi}^{\max}, \Delta_y, \Gamma_y, \eta_{yti}^{\alpha^{\mathrm{S}}}, \eta_{ytk}^{\alpha^{\mathrm{E}}},$

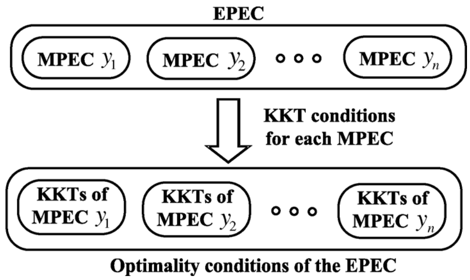

Fig. 5. EPEC and its optimality conditions.

$\beta_{ytn}, \gamma_{yti}^{\mathrm{S}^{\min}}, \gamma_{yti}^{\mathrm{S}^{\max}}, \gamma_{ytk}^{\mathrm{E}^{\min}}, \gamma_{ytk}^{\mathrm{E}^{\max}}, \gamma_{ytd}^{\mathrm{D}^{\min}}, \gamma_{ytd}^{\mathrm{D}^{\max}}, \tau_{ytnm}^{\min}, \tau_{ytnm}^{\max}, \delta_{ytn}^{\min}, \delta_{ytn}^{\max}, \delta_{yt}^{1}, \rho_{ytd}^{\mathrm{D}}, \rho_{yti}^{\mathrm{S}}, \rho_{ytk}^{\mathrm{E}}, \rho_{ytn}^{\theta}, \eta_{yti}^{\mathrm{S}^{\min}}, \eta_{yti}^{\mathrm{S}^{\max}}, \eta_{ytk}^{\mathrm{E}^{\min}}, \eta_{ytk}^{\mathrm{E}^{\max}}, \eta_{ytd}^{\mathrm{D}^{\min}}, \eta_{ytd}^{\mathrm{D}^{\max}}, \eta_{ytnm}^{\nu^{\min}}, \eta_{ytnm}^{\nu^{\max}}, \eta_{ytn}^{\xi^{\min}}, \eta_{ytn}^{\xi^{\max}}, \phi_{yt}^{\mathrm{SD}}\}$.

The structure of MPEC (2) is explained below. The objective function (2a) is identical to the objective function of problem (1), i.e., (1a). The constraints (2b)–(2f) are the UL constraints of problem (1), i.e., constraints (1b)–(1f). In addition, each LL problem (1g)–(1n), one per demand block, is replaced by its primal constraints (2g)–(2m) [equivalent to (1h)–(1n)], its dual constraints (2n)–(2v) and associated strong duality equality (2w), that enforces the equality of the primal and dual objective function values at the optimal solution.

III. Multiple Producers Problem

A. EPEC

The joint consideration of all producer MPECs (2), one per producer, constitutes an EPEC. This is depicted by the upper plot of Fig. 5.

The EPEC solution identifies the market equilibria. To attain such solution, the optimality conditions associated to the EPEC, i.e., the optimality conditions of all producer MPECs, need to be derived.

To formulate the optimality conditions of all producer MPECs, it is important to note that MPECs (2) are nonlinear and thus the application of the primal-dual transformation (second approach explained in Section II-B) is not straightforward. Therefore, all MPECs (2) are replaced by their corresponding KKT conditions (first approach explained in Section II-B) rendering the optimality conditions of the EPEC [20]. This transformation is illustrated in Fig. 5.

The optimality conditions associated with the EPEC include the conditions below.

1) Primal equality constraints of MPECs (2).
2) Equality constraints obtained from differentiating the corresponding Lagrangian associated to the MPECs (2) with respect to the variables in Ξ^{UL}.
3) Complementarity conditions related to the inequality constraints of MPECs (2).

The primal equality constraints of all MPECs (2) are given by (3):

$$(2\mathrm{g}), (2\mathrm{m}) - (2\mathrm{q}), (2\mathrm{w}), \quad \forall y. \tag{3}$$

The equality constraints (4) are obtained from differentiating the Lagrangian with respect to the variables in Ξ^{UL}. Note that $\mathcal{L}_y$ is the Lagrangian function of the MPEC (2) pertaining to the strategic producer y:

$$\frac{\partial \mathcal{L}_y}{\partial P_{ti}^{\mathrm{S}}} = -\sigma_t\left(\lambda_{t(n:i\in\Psi_n^{\mathrm{S}})} - C_i^{\mathrm{S}}\right) - \beta_{yt(n:i\in\Psi_n^{\mathrm{S}})} + \gamma_{yti}^{\mathrm{S}^{\max}} - \gamma_{yti}^{\mathrm{S}^{\min}} + \phi_{yt}^{\mathrm{SD}} O_{ti}^{\mathrm{S}} = 0 \ \forall y,\ \forall t,\ \forall i \in (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{4a}$$

$$\frac{\partial \mathcal{L}_y}{\partial P_{ti}^{\mathrm{S}}} = -\beta_{yt(n:i\in\Psi_n^{\mathrm{S}})} + \gamma_{yti}^{\mathrm{S}^{\max}} - \gamma_{yti}^{\mathrm{S}^{\min}} + \phi_{yt}^{\mathrm{SD}} O_{ti}^{\mathrm{S}} = 0 \ \forall y,\ \forall t,\ \forall i \notin (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{4b}$$

$$\frac{\partial \mathcal{L}_y}{\partial P_{tk}^{\mathrm{E}}} = -\sigma_t\left(\lambda_{t(n:k\in\Psi_n^{\mathrm{E}})} - C_k^{\mathrm{E}}\right) - \beta_{yt(n:k\in\Psi_n^{\mathrm{E}})} + \gamma_{ytk}^{\mathrm{E}^{\max}} - \gamma_{ytk}^{\mathrm{E}^{\min}} + \phi_{yt}^{\mathrm{SD}} O_{tk}^{\mathrm{E}} = 0 \ \forall y,\ \forall t,\ \forall k \in (\Psi^{\mathrm{E}} \cap \Omega_y) \tag{4c}$$

$$\frac{\partial \mathcal{L}_y}{\partial P_{tk}^{\mathrm{E}}} = -\beta_{yt(n:k\in\Psi_n^{\mathrm{E}})} + \gamma_{ytk}^{\mathrm{E}^{\max}} - \gamma_{ytk}^{\mathrm{E}^{\min}} + \phi_{yt}^{\mathrm{SD}} O_{tk}^{\mathrm{E}} = 0 \quad \forall y,\ \forall t,\ \forall k \notin (\Psi^{\mathrm{E}} \cap \Omega_y) \tag{4d}$$

$$\frac{\partial \mathcal{L}_y}{\partial P_{td}^{\mathrm{D}}} = \beta_{yt(n:d\in\Psi_n^{\mathrm{D}})} + \gamma_{ytd}^{\mathrm{D}^{\max}} - \gamma_{ytd}^{\mathrm{D}^{\min}} - \phi_{yt}^{\mathrm{SD}} U_{td}^{\mathrm{D}} = 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{4e}$$

$$\frac{\partial \mathcal{L}_y}{\partial X_i} = K_i + \Lambda_{yi}^{\max} - \Lambda_{yi}^{\min} + \Delta_y - \Gamma_y - \sum_t \gamma_{yti}^{\mathrm{S}^{\max}} + \sum_t \phi_{yt}^{\mathrm{SD}} \mu_{ti}^{\mathrm{S}^{\max}} = 0 \quad \forall y,\ \forall i \in (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{4f}$$

$$\frac{\partial \mathcal{L}_y}{\partial X_i} = \Delta_y - \Gamma_y - \sum_t \gamma_{yti}^{\mathrm{S}^{\max}} + \sum_t \phi_{yt}^{\mathrm{SD}} \mu_{ti}^{\mathrm{S}^{\max}} = 0 \quad \forall y, \forall i \notin (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{4g}$$

$$\frac{\partial \mathcal{L}_y}{\partial O_{ti}^{\mathrm{S}}} = -\eta_{yti}^{\alpha^{\mathrm{S}}} + \rho_{yti}^{\mathrm{S}} + \phi_{yt}^{\mathrm{SD}} P_{ti}^{\mathrm{S}} = 0 \ \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{4h}$$

$$\frac{\partial \mathcal{L}_y}{\partial O_{tk}^{\mathrm{E}}} = -\eta_{ytk}^{\alpha^{\mathrm{E}}} + \rho_{ytk}^{\mathrm{E}} + \phi_{yt}^{\mathrm{SD}} P_{tk}^{\mathrm{E}} = 0 \ \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{4i}$$

$$\frac{\partial \mathcal{L}_y}{\partial \theta_{tn}} = \sum_{m\in\Phi_n} B_{nm}(\beta_{ytn} - \beta_{ytm}) + \sum_{m\in\Phi_n} B_{nm}(\tau_{ytnm}^{\max} - \tau_{ytmn}^{\max}) - \sum_{m\in\Phi_n} B_{nm}(\tau_{ytnm}^{\min} - \tau_{ytmn}^{\min}) + \delta_{ytn}^{\max} - \delta_{ytn}^{\min} + (\delta_{yt}^{1})_{n=1} = 0 \quad \forall y,\ \forall t,\ \forall n \tag{4j}$$

$$\frac{\partial \mathcal{L}_y}{\partial \lambda_{tn}} = -\sigma_t\left(\sum_{i\in(\Psi_n^{\mathrm{S}}\cap\Omega_y)} P_{ti}^{\mathrm{S}} + \sum_{k\in(\Psi_n^{\mathrm{E}}\cap\Omega_y)} P_{tk}^{\mathrm{E}}\right) + \sum_{d\in\Psi_n^{\mathrm{D}}} \rho_{ytd}^{\mathrm{D}} - \sum_{i\in\Psi_n^{\mathrm{S}}} \rho_{yti}^{\mathrm{S}} - \sum_{k\in\Psi_n^{\mathrm{E}}} \rho_{ytk}^{\mathrm{E}} + \sum_{m\in\Phi_n} B_{nm}(\rho_{ytn}^{\theta} - \rho_{ytm}^{\theta}) = 0 \ \forall y,\ \forall t,\ \forall n \tag{4k}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{ti}^{\mathrm{S}^{\min}}} = -\rho_{yti}^{\mathrm{S}} - \eta_{yti}^{\mathrm{S}^{\min}} = 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{4l}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{ti}^{\mathrm{S}^{\max}}} = \rho_{yti}^{\mathrm{S}} - \eta_{yti}^{\mathrm{S}^{\max}} + \phi_{yt}^{\mathrm{SD}} X_i = 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{4m}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{tk}^{\mathrm{E}^{\min}}} = -\rho_{ytk}^{\mathrm{E}} - \eta_{ytk}^{\mathrm{E}^{\min}} = 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{4n}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{tk}^{\mathrm{E}^{\max}}} = \rho_{ytk}^{\mathrm{E}} - \eta_{ytk}^{\mathrm{E}^{\max}} + \phi_{yt}^{\mathrm{SD}} P_k^{\mathrm{E}^{\max}} = 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{4o}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{td}^{\mathrm{D}^{\min}}} = -\rho_{ytd}^{\mathrm{D}} - \eta_{ytd}^{\mathrm{D}^{\min}} = 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{4p}$$

$$\frac{\partial \mathcal{L}_y}{\partial \mu_{td}^{\mathrm{D}^{\max}}} = \rho_{ytd}^{\mathrm{D}} - \eta_{ytd}^{\mathrm{D}^{\max}} + \phi_{yt}^{\mathrm{SD}} P_{td}^{\mathrm{D}^{\max}} = 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{4q}$$

$$\frac{\partial \mathcal{L}_y}{\partial \nu_{tnm}^{\min}} = -B_{nm}(\rho_{ytn}^{\theta} - \rho_{ytm}^{\theta}) - \eta_{ytnm}^{\nu^{\min}} + \phi_{yt}^{\mathrm{SD}} F_{nm}^{\max} = 0 \quad \forall y,\ \forall t,\ \forall n,\ \forall m \in \Phi_n \tag{4r}$$

$$\frac{\partial \mathcal{L}_y}{\partial \nu_{tnm}^{\max}} = B_{nm}(\rho_{ytn}^{\theta} - \rho_{ytm}^{\theta}) - \eta_{ytnm}^{\nu^{\max}} + \phi_{yt}^{\mathrm{SD}} F_{nm}^{\max} = 0 \quad \forall y,\ \forall t,\ \forall n,\ \forall m \in \Phi_n \tag{4s}$$

$$\frac{\partial \mathcal{L}_y}{\partial \xi_{tn}^{\min}} = -\rho_{ytn}^{\theta} - \eta_{ytn}^{\xi^{\min}} + \phi_{yt}^{\mathrm{SD}} \pi = 0 \quad \forall y,\ \forall t,\ \forall n \tag{4t}$$

$$\frac{\partial \mathcal{L}_y}{\partial \xi_{tn}^{\max}} = \rho_{ytn}^{\theta} - \eta_{ytn}^{\xi^{\max}} + \phi_{yt}^{\mathrm{SD}} \pi = 0 \quad \forall y,\ \forall t,\ \forall n \tag{4u}$$

$$\frac{\partial \mathcal{L}_y}{\partial \xi_t^{1}} = \rho_{yt(n=1)}^{\theta} = 0 \quad \forall y,\ \forall t. \tag{4v}$$

The complementarity conditions included in the KKT conditions associated to the EPEC are given by (5). Note that each complementarity condition of the form $0 \le a \perp b \ge 0$ is equivalent to $a \ge 0, b \ge 0$ and $ab = 0$:

$$0 \le X_i \perp \Lambda_{yi}^{\min} \ge 0 \quad \forall y, \forall i \in (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{5a}$$

$$0 \le (X_i^{\max} - X_i) \perp \Lambda_{yi}^{\max} \ge 0 \quad \forall y, \forall i \in (\Psi^{\mathrm{S}} \cap \Omega_y) \tag{5b}$$

$$0 \le \left(K^{\max} - \sum_{i \in \Psi^{\mathrm{S}}} K_i X_i\right) \perp \Delta_y \ge 0 \quad \forall y \tag{5c}$$

$$0 \le \left[\sum_{i \in \Psi^{\mathrm{S}}} X_i + \sum_{k \in \Psi^{\mathrm{E}}} P_k^{\mathrm{E}^{\max}} - \left(\Upsilon \times \sum_{d \in \Psi^{\mathrm{D}}} P_{td}^{\mathrm{D}^{\max}}\right)\right] \perp \Gamma_y \ge 0 \quad \forall y, t = t_1 \tag{5d}$$

$$0 \le O_{ti}^{\mathrm{S}} \perp \eta_{yti}^{\alpha^{\mathrm{S}}} \ge 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{5e}$$

$$0 \le O_{tk}^{\mathrm{E}} \perp \eta_{ytk}^{\alpha^{\mathrm{E}}} \ge 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{5f}$$

$$0 \le P_{ti}^{\mathrm{S}} \perp \gamma_{yti}^{\mathrm{S}^{\min}} \ge 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{5g}$$

$$0 \le \left(X_i - P_{ti}^{\mathrm{S}}\right) \perp \gamma_{yti}^{\mathrm{S}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{5h}$$

$$0 \le P_{tk}^{\mathrm{E}} \perp \gamma_{ytk}^{\mathrm{E}^{\min}} \ge 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{5i}$$

$$0 \le \left(P_k^{\mathrm{E}^{\max}} - P_{tk}^{\mathrm{E}}\right) \perp \gamma_{ytk}^{\mathrm{E}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{5j}$$

$$0 \le P_{td}^{\mathrm{D}} \perp \gamma_{ytd}^{\mathrm{D}^{\min}} \ge 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{5k}$$

$$0 \le \left(P_{td}^{\mathrm{D}^{\max}} - P_{td}^{\mathrm{D}}\right) \perp \gamma_{ytd}^{\mathrm{D}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{5l}$$

$$0 \le [F_{nm}^{\max} + B_{nm}(\theta_{tn} - \theta_{tm})] \perp \tau_{ytnm}^{\min} \ge 0 \quad \forall y,\ \forall t,\ \forall n,\ \forall m \in \Phi_n \tag{5m}$$

$$0 \le [F_{nm}^{\max} - B_{nm}(\theta_{tn} - \theta_{tm})] \perp \tau_{ytnm}^{\max} \ge 0 \quad \forall y,\ \forall t,\ \forall n,\ \forall m \in \Phi_n \tag{5n}$$

$$0 \le (\pi + \theta_{tn}) \perp \delta_{ytn}^{\min} \ge 0 \quad \forall y,\ \forall t,\ \forall n \tag{5o}$$

$$0 \le (\pi - \theta_{tn}) \perp \delta_{ytn}^{\max} \ge 0 \quad \forall y,\ \forall t,\ \forall n \tag{5p}$$

$$0 \le \mu_{ti}^{\mathrm{S}^{\min}} \perp \eta_{yti}^{\mathrm{S}^{\min}} \ge 0; \quad 0 \le \mu_{ti}^{\mathrm{S}^{\max}} \perp \eta_{yti}^{\mathrm{S}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{5q}$$

$$0 \le \mu_{tk}^{\mathrm{E}^{\min}} \perp \eta_{ytk}^{\mathrm{E}^{\min}} \ge 0; \quad 0 \le \mu_{tk}^{\mathrm{E}^{\max}} \perp \eta_{ytk}^{\mathrm{E}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{5r}$$

$$0 \le \mu_{td}^{\mathrm{D}^{\min}} \perp \eta_{ytd}^{\mathrm{D}^{\min}} \ge 0; \quad 0 \le \mu_{td}^{\mathrm{D}^{\max}} \perp \eta_{ytd}^{\mathrm{D}^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{5s}$$

$$0 \le \nu_{tnm}^{\min} \perp \eta_{ytnm}^{\nu^{\min}} \ge 0; \quad 0 \le \nu_{tnm}^{\max} \perp \eta_{ytnm}^{\nu^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall n,\ \forall m \in \Phi_n \tag{5t}$$

$$0 \le \xi_{tn}^{\min} \perp \eta_{ytn}^{\xi^{\min}} \ge 0; \quad 0 \le \xi_{tn}^{\max} \perp \eta_{ytn}^{\xi^{\max}} \ge 0 \quad \forall y,\ \forall t,\ \forall n. \tag{5u}$$

Note finally that the optimality conditions (3)–(5) are nonlinear and highly nonconvex due to both products of variables and complementarity conditions.

B. EPEC Linearization

The optimality conditions (3)–(5) include the following nonlinearities:

a) The complementarity conditions (5).
b) The products of variables involved in the strong duality equalities (2w) included in (3).
c) The products of variables in (4a)–(4d), (4f)–(4i), and (4m). Observe that the common variables of such non-linear terms are the dual variables ϕ_{yt}^{SD}.

1) Linearizing the Complementarity Conditions (5): Each complementarity condition of the form $0 \le a \perp b \ge 0$ is replaced by $a \ge 0, b \ge 0, a \le \psi M$ and $b \le (1-\psi)M$, where ψ and M are a binary variable and a large enough positive constant, respectively [21].

2) Linearizing the Strong Duality Equalities (2w): Unlike complementarity conditions (5) that can be easily linearized by the approach explained in Section III-B1 through auxiliary binary variables, the strong duality equalities (2w) cannot be easily linearized due the nature of the nonlinearities, i.e., the product of continuous variables.

As explained in Section II-B, the strong duality equality resulting from the primal-dual transformation is equivalent to the set of complementarity conditions obtained by the KKT conditions. Hence, pursuing linearity, the strong duality equalities (2w) are replaced by their equivalent complementarity conditions (6) below related to the inequality constraints (1i)–(1m). Note that these complementarity conditions can be linearized as explained in Section III-B1:

$$0 \le P_{ti}^{\mathrm{S}} \perp \mu_{ti}^{\mathrm{S}^{\min}} \ge 0 \quad \forall t, \forall i \in \Psi^{\mathrm{S}} \tag{6a}$$

$$0 \le \left(X_i - P_{ti}^{\mathrm{S}}\right) \perp \mu_{ti}^{\mathrm{S}^{\max}} \ge 0 \quad \forall t,\ \forall i \in \Psi^{\mathrm{S}} \tag{6b}$$

$$0 \le P_{tk}^{\mathrm{E}} \perp \mu_{tk}^{\mathrm{E}^{\min}} \ge 0 \quad \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{6c}$$

$$0 \le \left(P_k^{\mathrm{E}^{\max}} - P_{tk}^{\mathrm{E}}\right) \perp \mu_{tk}^{\mathrm{E}^{\max}} \ge 0 \quad \forall t,\ \forall k \in \Psi^{\mathrm{E}} \tag{6d}$$

$$0 \le P_{td}^{\mathrm{D}} \perp \mu_{td}^{\mathrm{D}^{\min}} \ge 0 \quad \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{6e}$$

$$0 \le \left(P_{td}^{\mathrm{D}^{\max}} - P_{td}^{\mathrm{D}}\right) \perp \mu_{td}^{\mathrm{D}^{\max}} \ge 0 \quad \forall t,\ \forall d \in \Psi^{\mathrm{D}} \tag{6f}$$

$$0 \le [F_{nm}^{\max} + B_{nm}(\theta_{tn} - \theta_{tm})] \perp \nu_{tnm}^{\min} \ge 0 \quad \forall t, \forall n, \forall m \in \Phi_n \tag{6g}$$

$$0 \le [F_{nm}^{\max} - B_{nm}(\theta_{tn} - \theta_{tm})] \perp \nu_{tnm}^{\max} \ge 0 \quad \forall t, \forall n, \forall m \in \Phi_n \tag{6h}$$

$$0 \leq (\pi + \theta_{tn}) \perp \xi_{tn}^{\min} \geq 0 \quad \forall t, \forall n \tag{6i}$$
$$0 \leq (\pi - \theta_{tn}) \perp \xi_{tn}^{\max} \geq 0 \quad \forall t, \forall n. \tag{6j}$$

3) Linearizing the Nonlinear Terms Involving ϕ_{yt}^{SD}: From a mathematically point of view, we can parameterize the model in the variables ϕ_{yt}^{SD} because these are dual variables associated to MPECs (2), which are non-regular, i.e., the Mangasarian-Fromovitz constraint qualification (MFCQ) is not satisfied at any feasible point [22]. In other words, the dual variable set associated to MPECs (2) at any solution is not unique and forms a ray [20]. This redundancy allows the parameterization of variables ϕ_{yt}^{SD}.

Hence, nonlinear terms of (4a)–(4d), (4f)–(4i), and (4m) become linear if problem (3)-(5) is parameterized in dual variables ϕ_{yt}^{SD}.

Regarding the value selection for dual variables ϕ_{yt}^{SD}, note that the combination of constraints (4l), (4m), and (5q) requires that dual variables ϕ_{yt}^{SD} to be non-negative.

IV. Searching for Equilibria

Note that the optimality conditions (3)–(5) developed and linearized in Section III constitute a system of linear equalities and inequalities that involves continuous and binary variables and that generally has multiple solutions. To explore for such solutions, it is possible to formulate an optimization problem considering the mixed-integer linear form of conditions (3)–(5) as constraints. In addition, several objective functions can be considered to identify different equilibria [8], for example:

1) Total profit (TP) of all producers.
2) Annual true social welfare (ATSW) considering the production costs of the generation units.
3) Annual SW considering the strategic offer prices of the generation units.
4) Minus payment of the demands.
5) Profit of a given producer.
6) Minus payment of a given demand.

In this two-part paper, the first two objectives are selected because i) they can be formulated linearly; and ii) they refer to general market measures. Thus, the optimization problem to find equilibria is formulated as follows:

$$\text{Maximize TP or ATSW} \tag{7a}$$
$$\text{subject to:}$$
$$\text{Mixed-integer linear form of the conditions } (3)\text{–}(5). \tag{7b}$$

The primal optimization variable set of problem (7) includes 1) variable set Ξ^{UL}, 2) variable set Ξ^{Dual} except dual variables ϕ_{yt}^{SD} that have been parameterized in Section III-B3, and 3) the set of binary variables introduced in Section III-B1 to linearize the complementarity conditions (5) and (6).

The two linear objective functions (TP and ATSW) to be included in (7a) are described in the two following subsections.

A. Objective Function (7a): TP

The summation of (1a) for all producers provides the minus-TP of all producers, but such expression is nonlinear due to the product of continuous variables in the term $\sum_{i\in\Psi^{\mathrm{S}}} P_{ti}^{\mathrm{S}} \lambda_{t(n:i\in\Psi_n^{\mathrm{S}})} + \sum_{k\in\Psi^{\mathrm{E}}} P_{tk}^{\mathrm{E}} \lambda_{t(n:k\in\Psi_n^{\mathrm{E}})}$ denoted here as Z_t.

To linearize Z_t, the exact linearization approach proposed in [23] is used. Its application to the generation investment problem is available in [17]. Using this approach, the following linear equivalent for Z_t (denoted as Z_t^{Lin}) is obtained:

$$\begin{aligned} Z_t^{\mathrm{Lin}} = &\sum_{d\in\Psi^{\mathrm{D}}} U_{td}^{\mathrm{D}} P_{td}^{\mathrm{D}} - \sum_{d\in\Psi^{\mathrm{D}}} \mu_{td}^{\mathrm{D}^{\max}} P_{td}^{\mathrm{D}^{\max}} \\ &- \sum_{n(m\in\Phi_n)} \left(\nu_{tnm}^{\min} + \nu_{tnm}^{\max}\right) F_{nm}^{\max} - \sum_{n} \left(\xi_{tn}^{\min} + \xi_{tn}^{\max}\right) \pi \\ &\forall t. \end{aligned} \tag{8a}$$

Therefore, the linear form of the TP to be included in (7a) is

$$\begin{aligned} \mathrm{TP} = \sum_t \sigma_t \left(Z_t^{\mathrm{Lin}} - \sum_{i\in\Psi^{\mathrm{S}}} P_{ti}^{\mathrm{S}} C_i^{\mathrm{S}} - \sum_{k\in\Psi^{\mathrm{E}}} P_{tk}^{\mathrm{E}} C_k^{\mathrm{E}} \right) \\ - \sum_{i\in\Psi^{\mathrm{S}}} K_i X_i. \end{aligned} \tag{8b}$$

B. Objective Function (7a): ATSW

The linear term of the ATSW to be included in (7a) is

$$\begin{aligned} \mathrm{ATSW} = \sum_t \sigma_t \Bigg(&\sum_{d\in\Psi^{\mathrm{D}}} U_{td}^{\mathrm{D}} P_{td}^{\mathrm{D}} - \sum_{i\in\Psi^{\mathrm{S}}} C_i^{\mathrm{S}} P_{ti}^{\mathrm{S}} \\ &- \sum_{k\in\Psi^{\mathrm{E}}} C_{tk}^{\mathrm{E}} P_{tk}^{\mathrm{E}} \Bigg). \end{aligned} \tag{9}$$

Note that instead of the strategic offers of the generation units (O_{ti}^{S} and O_{tk}^{E}), their true production costs (C_i^{S} and C_k^{E}) are considered in formulating the ATSW.

V. Algorithm for the Diagonalization Checking

In this work, we carry out a single-iteration of the diagonalization algorithm, because this allows us to check whether or not each solution obtained by the proposed approach is, in fact, a Nash equilibrium. Note that if under the diagonalization framework no producer desires to deviate from its actual strategy, then the set of strategies of all producers satisfies the definition of Nash equilibrium.

For example, assume a triopoly market with three strategic producers (Producers 1, 2, and 3) with investment decisions X_1^*, X_2^* and X_3^*, respectively, obtained by the proposed approach. In order to verify that this solution constitutes a Nash equilibrium, the following steps are carried out.

a) Consider the mixed-integer linear form (as in [17]) of MPEC (2) pertaining to Producer 1 (e.g., MPEC1).
b) Set the investment decisions of other producers (Producers 2 and 3) to those obtained by the proposed approach (i.e., X_2^* and X_3^*), and then solve MPEC1. Note that its solution provides the investment decisions of Producer 1, which we denote as $\hat{X}_1$.
c) Repeat the two steps above for every producer. For the example considered, this step results in deriving the optimal investment decisions $\hat{X}_2$ and $\hat{X}_3$ pertaining to Producers 2 and 3, respectively.
d) Compare the results obtained from the previous steps of the diagonalization algorithm ($\hat{X}_1, \hat{X}_2$, and $\hat{X}_3$) with those achieved from the proposed approach (X_1^*, X_2^*, and

X_3^*). If the investment results of each producer obtained from the single-iteration diagonalization algorithm are identical to those attained by the proposed approach (i.e., $\hat{X}_1 = X_1^*, \hat{X}_2 = X_2^*$ and $\hat{X}_3 = X_3^*$), then the proposed solution (X_1^*, X_2^* and X_3^*) is a Nash equilibrium because each producer cannot increase its profit by changing its strategy unilaterally.

VI. Conclusion

This paper provides a methodology to characterize the interactions among strategic producers and to find generation investment equilibria in a network-constrained electricity pool. The conclusions below can be drawn from the structure of the developed model:

1) An MPEC is developed to model the strategic behavior of each producer in its offering and investment actions. This MPEC explicitly considers stepwise supply function offers, which constitutes a more accurate description of the functioning of real-world electricity markets if compared with other competition models such as Cournot, Bertrand, or conjectural variations.
2) The joint consideration of all producer MPECs, one per producer, allows building an EPEC to represent the equilibrium problem.
3) The EPEC equilibria are identified by considering the linearized form of the optimality conditions of all MPECs.

To numerically validate the proposed methodology, the companion paper [18] provides results from a small-scale illustrative example and a large-scale case study.

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