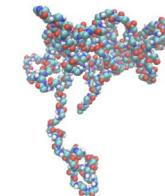
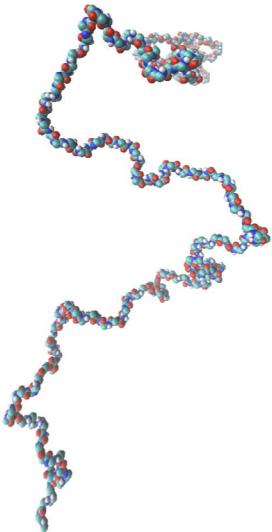


# Advanced topics 4: Flexible systems and intrinsically disordered proteins

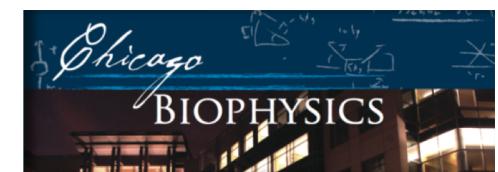


Tobin R. Sosnick

Dept. of Biochemistry & Mol. Biology, Chair  
Institute for Biophysical Dynamics  
University of Chicago



BioSAXS 5  
Nov. 6, 2019 Argonne Nat. Lab



Graduate Program in  
Biophysical Sciences

# Advanced topics 4: Flexible systems and intrinsically disordered proteins

## References:

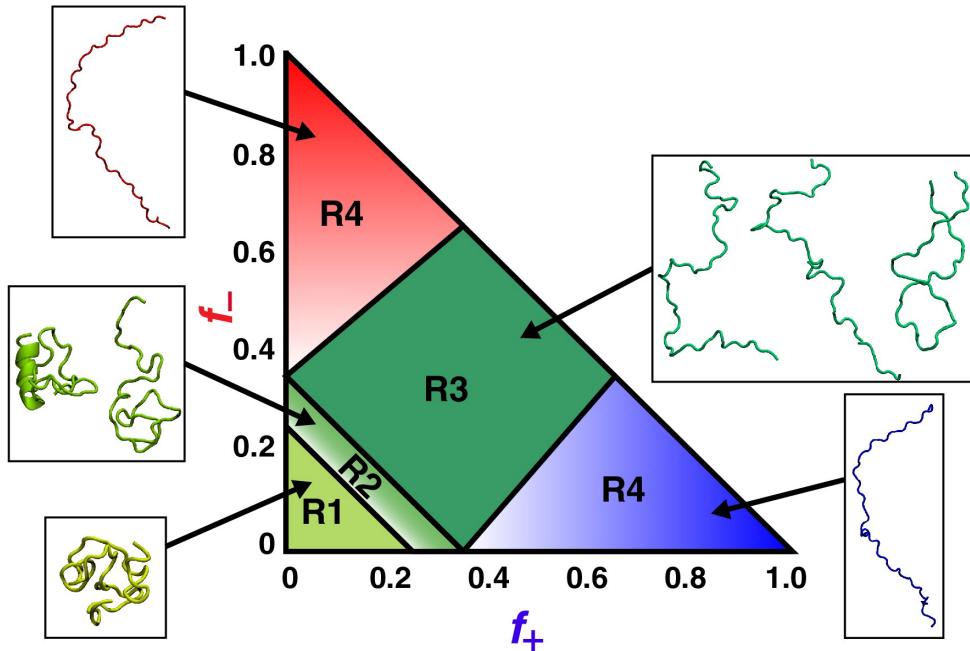
### Small-angle Scattering

1. Riback, J.A., et al., *Innovative scattering analysis shows that hydrophobic disordered proteins are expanded in water*. Science, 2017. **358**(6360): p. 238-241.
2. Rambo, R.P. and J.A. Tainer, *Characterizing flexible and intrinsically unstructured biological macromolecules by SAS using the Porod-Debye law*. Biopolymers, 2011. **95**(8): p. 559-71.
3. Bernado, P. and D.I. Svergun, *Analysis of intrinsically disordered proteins by small-angle X-ray scattering*. Methods Mol Biol, 2012. **896**: p. 107-22.
4. Receveur-Brechot, V. and D. Durand, *How random are intrinsically disordered proteins? A small angle scattering perspective*. Curr Protein Pept Sci, 2012. **13**(1): p. 55-75.

### Disordered proteins

1. Das, R.K., K.M. Ruff, and R.V. Pappu, *Relating sequence encoded information to form and function of intrinsically disordered proteins*. Curr Opin Struct Biol, 2015. **32**: p. 102-12.
2. van der Lee, R., et al., *Classification of intrinsically disordered regions and proteins*. Chem Rev, 2014. **114**(13): p. 6589-631.

# Unfolded and disordered proteins



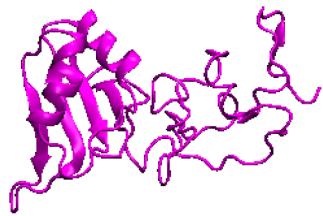
Das, Ruff, & Pappu  
(2015) Curr Op Str Biol

## Relevance:

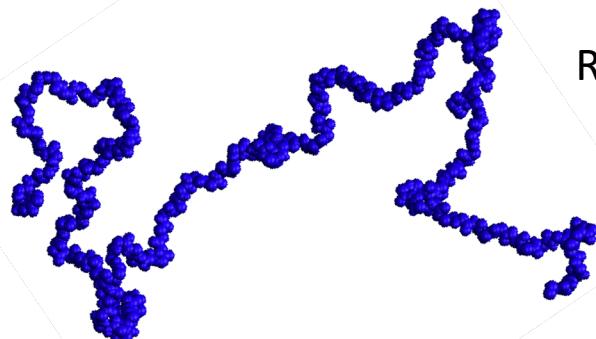
- Does protein folding typically initiate with a rapid hydrophobic collapse?
- ~1/3 of the proteome is intrinsically disordered: **IDP**(roteins), **IDR**(egions)
- Folding upon binding
- phase separation & disease
- Is water a good or poor solvent for proteins?

# Scattering: compact versus unfolded polypeptides

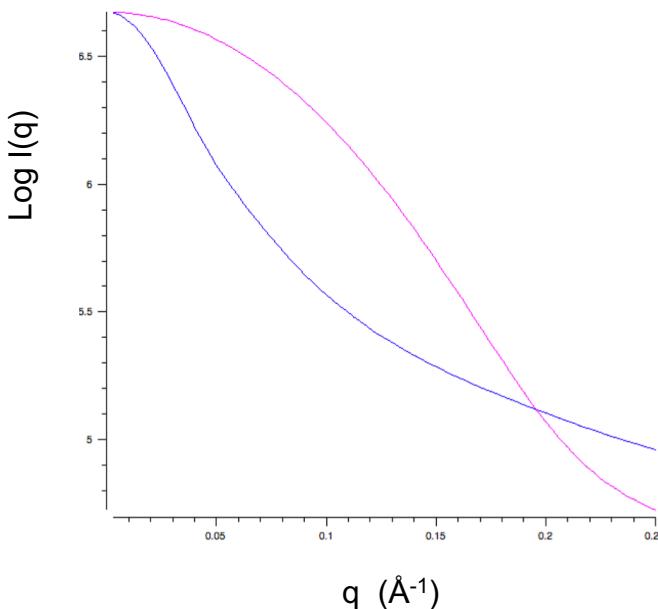
$R_g=19 \text{ \AA}$



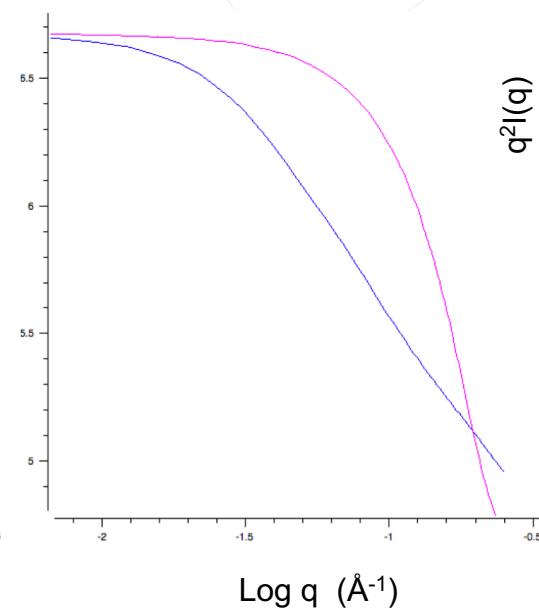
$R_g^{\text{ensemble}}=49 \text{ \AA}$



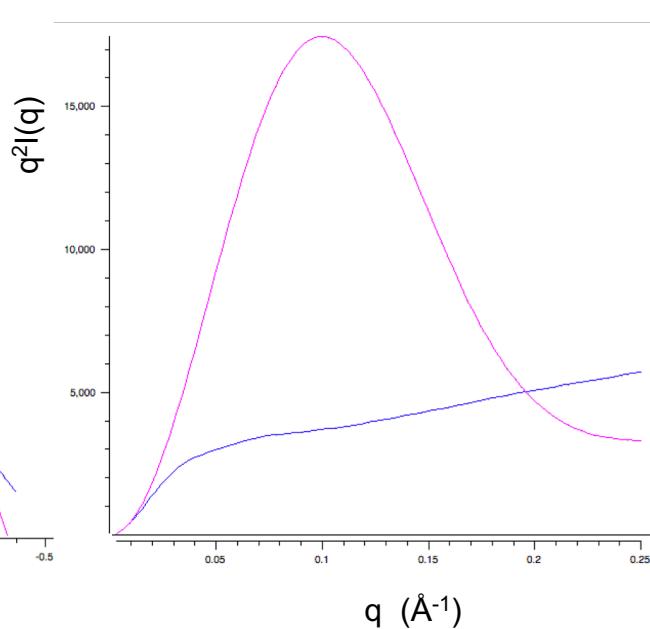
log-linear



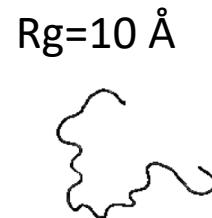
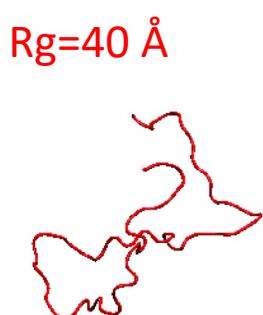
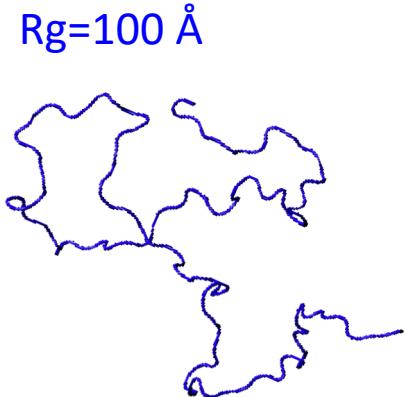
log-log plot



Kratky plot



# Rescaling to remove size but keep shape information: Dimensionless Kratky Plot

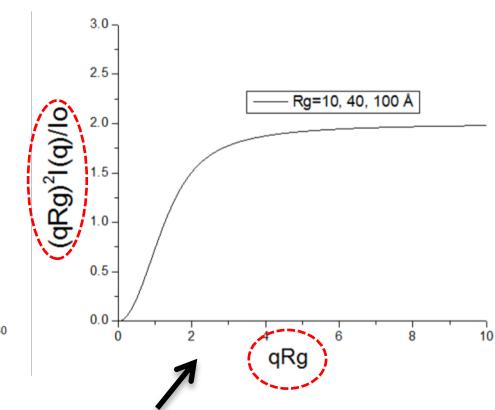
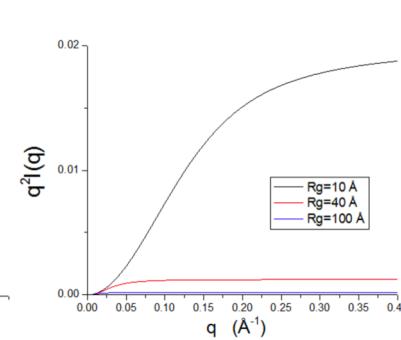
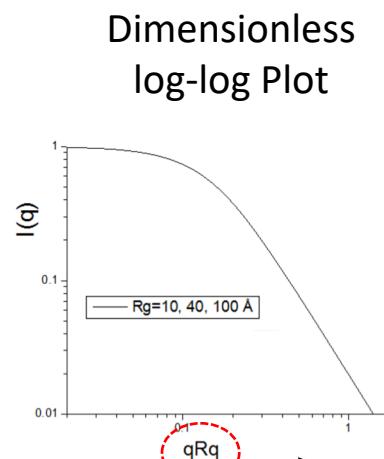
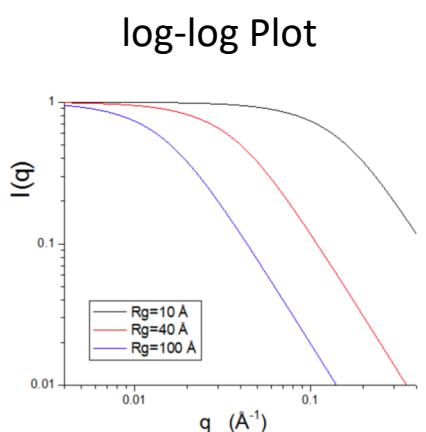


Debye formula for random walk:

$$I(q) = \frac{2I_o \left( e^{-(qRg)^2} - 1 + (qRg)^2 \right)}{(qRg)^4}$$

$$x = qRg$$

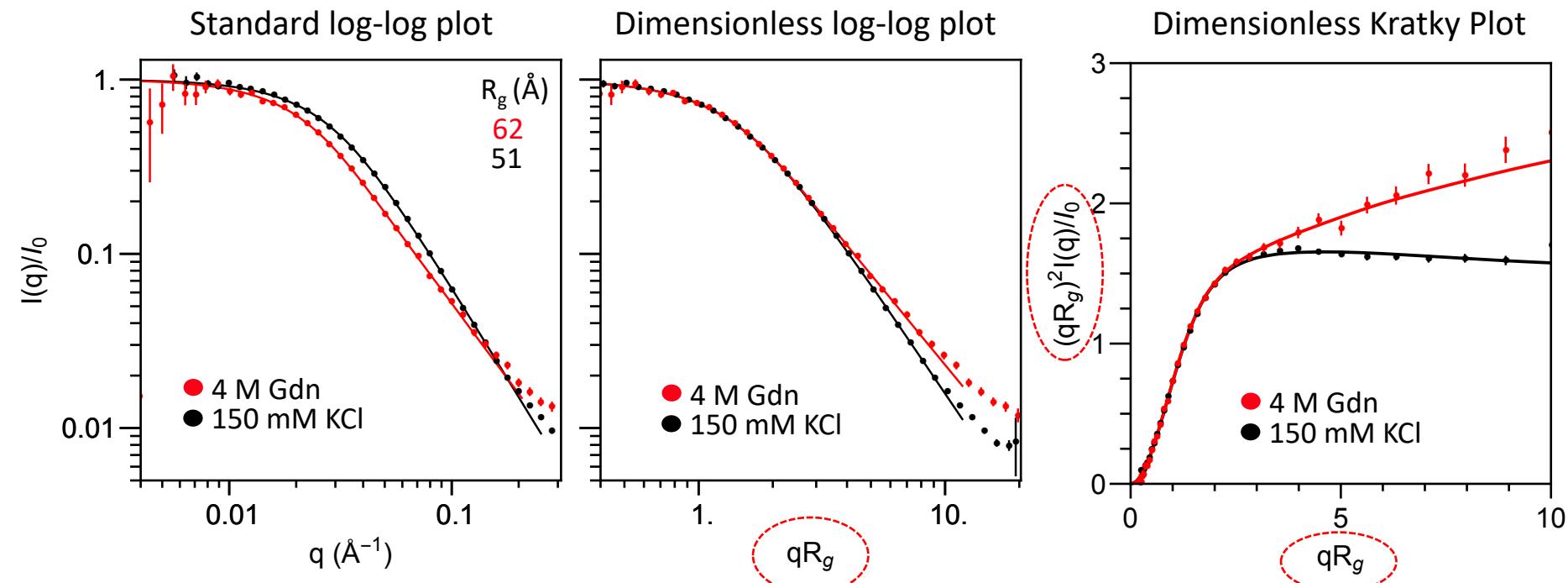
$$I(q) = \frac{2I_o \left( e^{-x^2} - 1 + x^2 \right)}{x^4}$$



Report only on shape

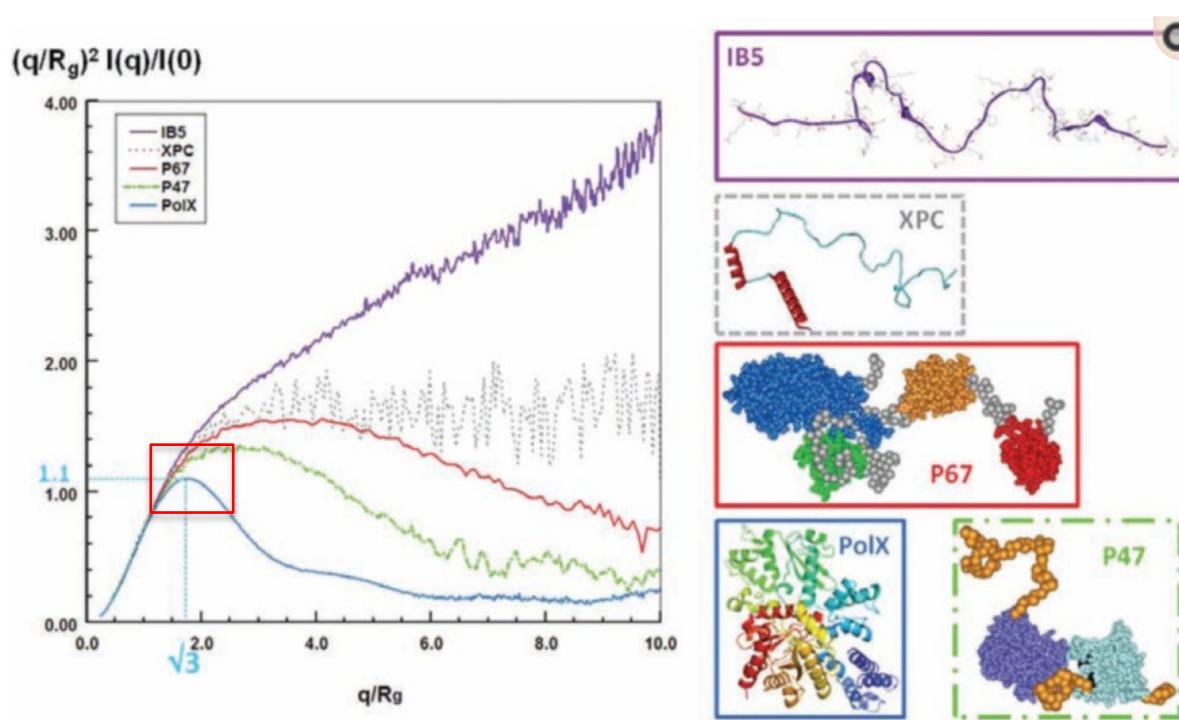
# More on scaling and shape

IDP in water and high denaturant concentration



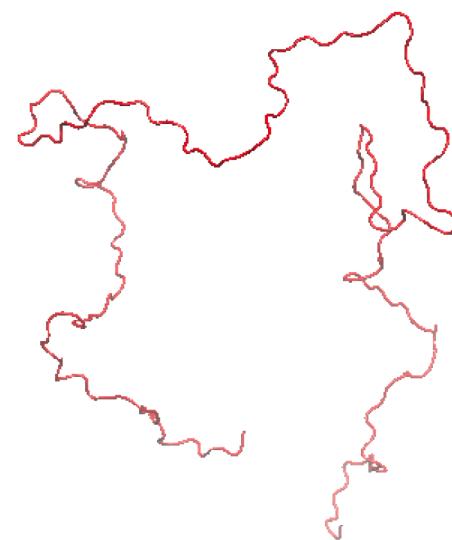
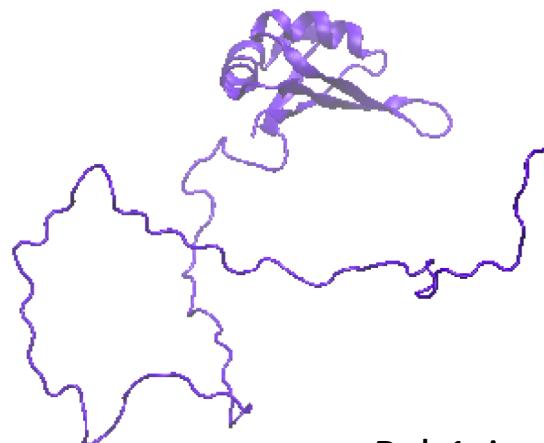
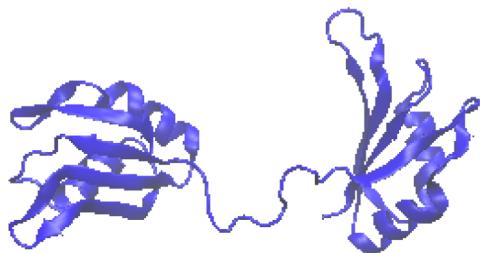
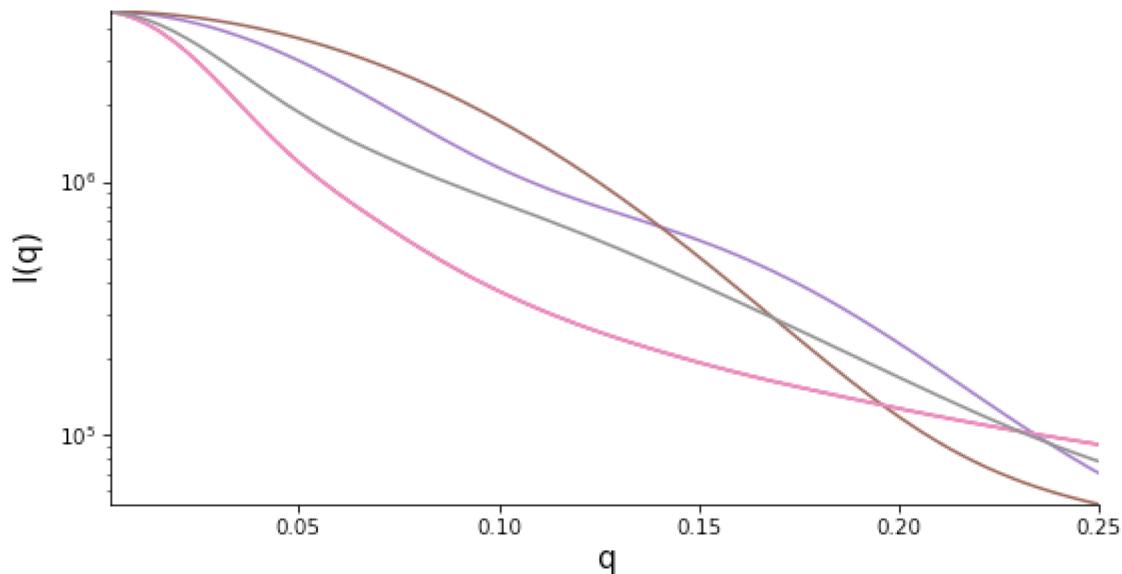
Shapes are different – but how?

# Useful examples from globular to unfolded



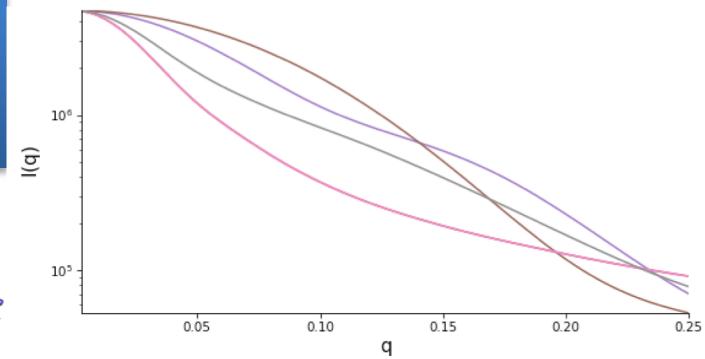
Normalized Kratky plots. The scattering pattern of globular proteins in a normalized Kratky plot exhibits a bell-shaped profile with a clear maximum value of 1.104 for  $qR_g = \sqrt{3}$ , regardless of the size of the protein, and are all nearly superimposable in the  $q$  range  $0 < qR_g < 3$ . Conversely, for a random chain, the curve rises with increasing angle, to nearly reach a plateau between 1.5 and 2 and may further increase at  $q > 0.2-0.3 \text{ \AA}^{-1}$ , depending on the persistence length and the internal structure of the protein. Bell-shaped profile of a globular protein (PolX, blue line); curve of a protein consisting of several domains tethered by linkers with rather compact conformations ( $p47^{\text{phox}}$ , dotted green line) or extended conformations ( $p67^{\text{phox}}$ , continue red line); curve of a fully disordered protein with very short elements of secondary structure (XPC dotted grey line); and curve of a fully disordered and extended protein with short segments of proline repeats (salivary protein IB5, continue purple line).

# Examples of mixed systems

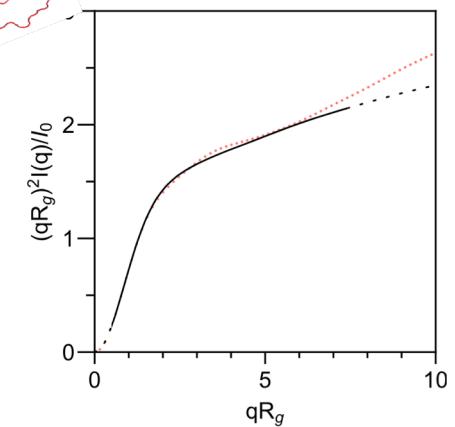
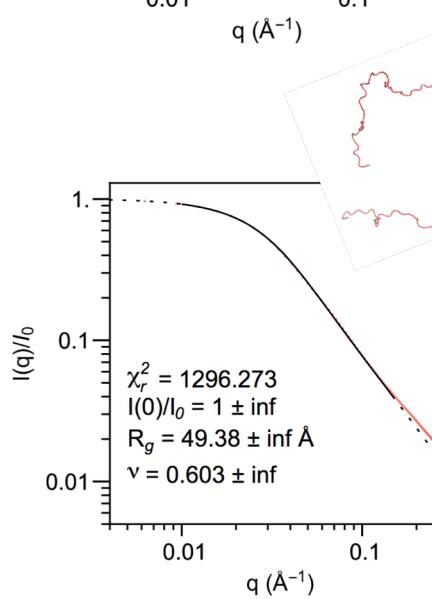
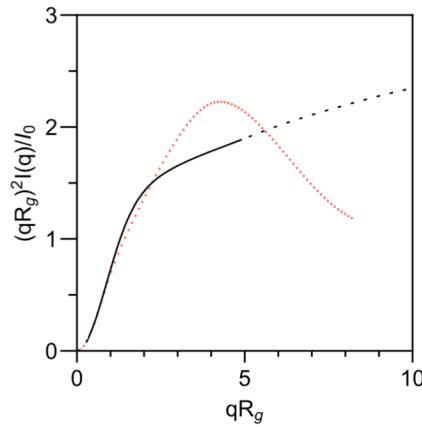
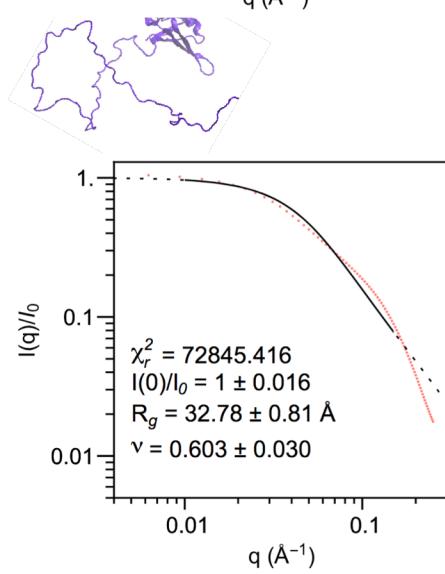
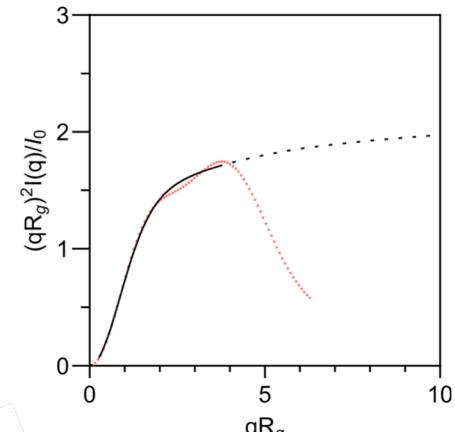
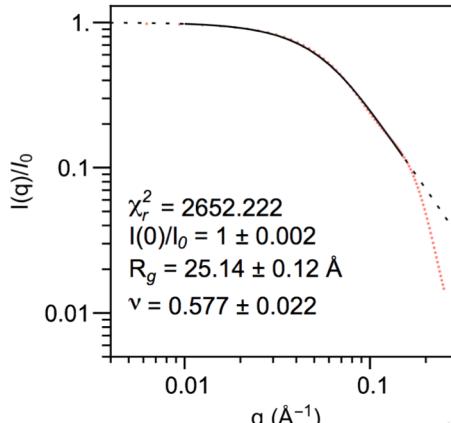
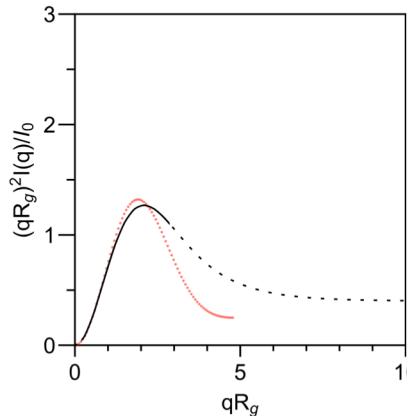
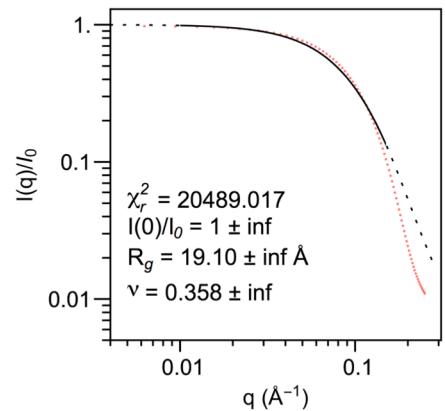


Pab1sims.vmd

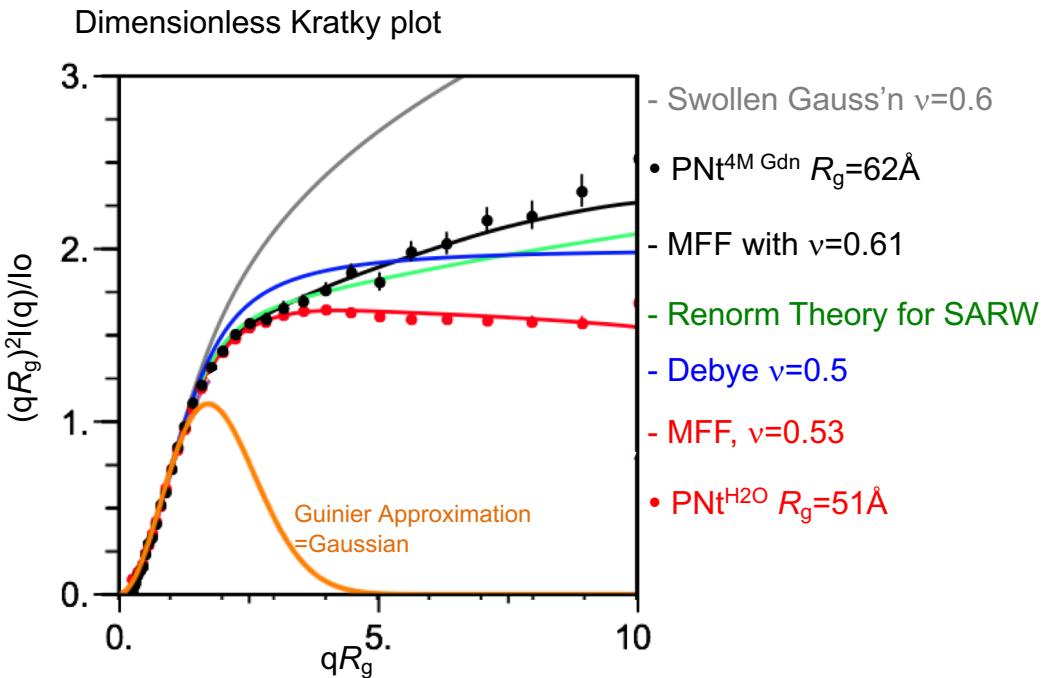
# Examples of mixed systems



(shapes, Fit)



# Scattering from Random Walks (allowed to cross) and self-avoiding RW (not allowed to cross)



Debye formula for random walk:

$$I_{Debye}(q) = \frac{2I_o \left( e^{-(qRg)^2} - 1 + (qRg)^2 \right)}{(qRg)^4}$$

Swollen Gaussian coil model

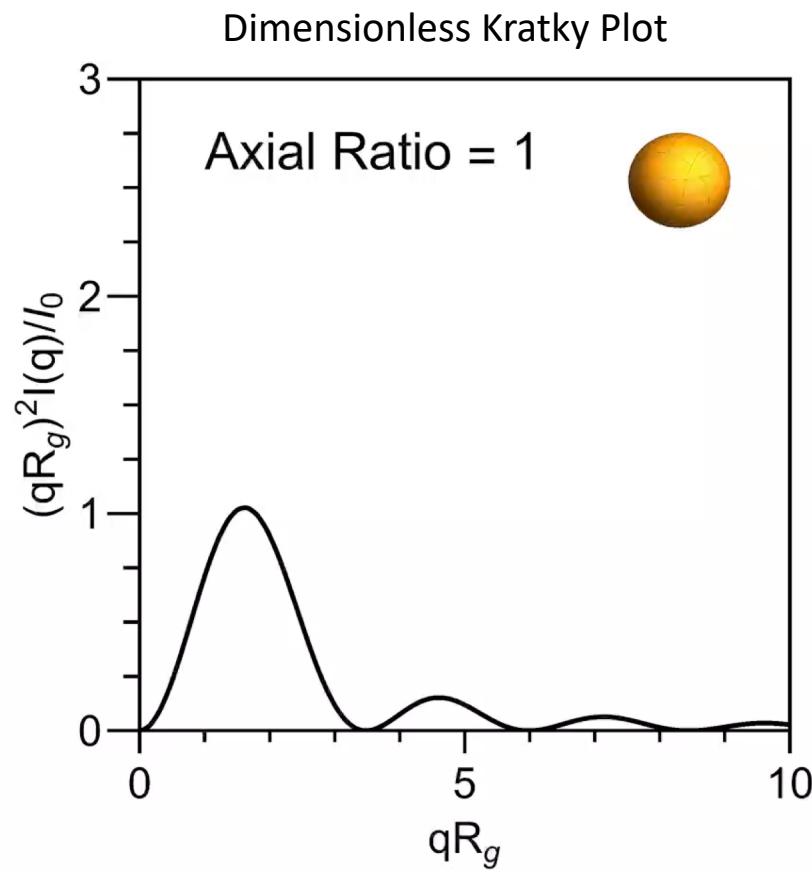
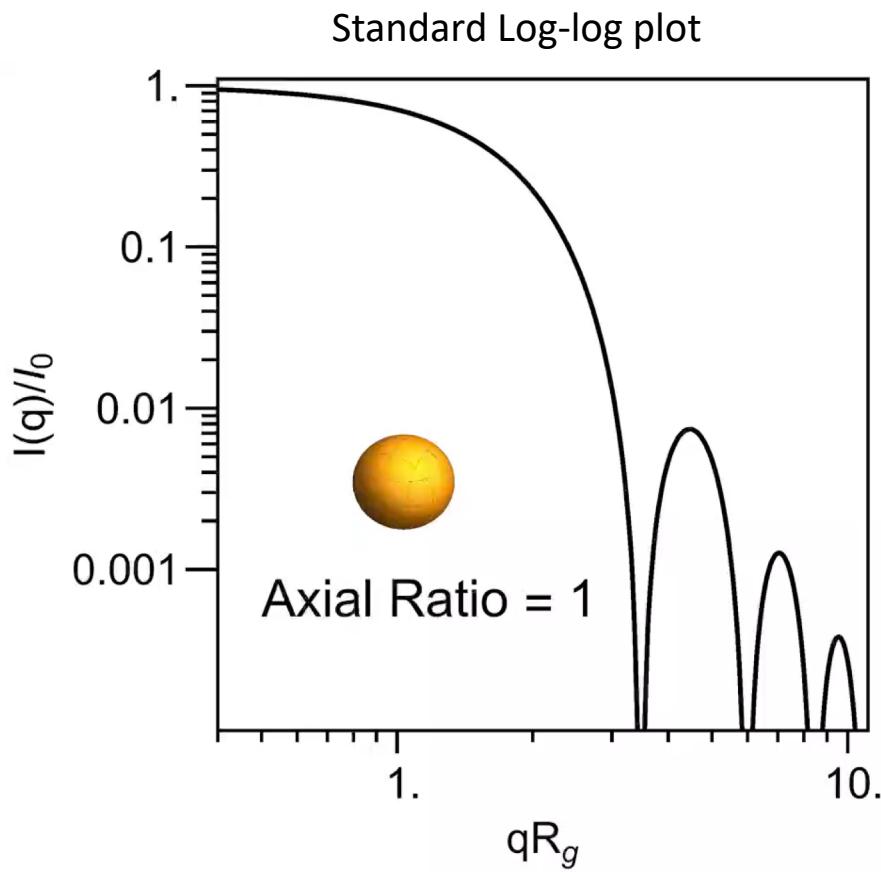
$$P_{SwollenGausCoil}(R_{ij}) \propto e^{-\left(\frac{3R_{ij}^2}{3\langle R_{ij}^2 \rangle}\right)}$$

$$\langle R_{ij}^2 \rangle = a^2 |i - j|^{2v}$$

$a$ =segment length

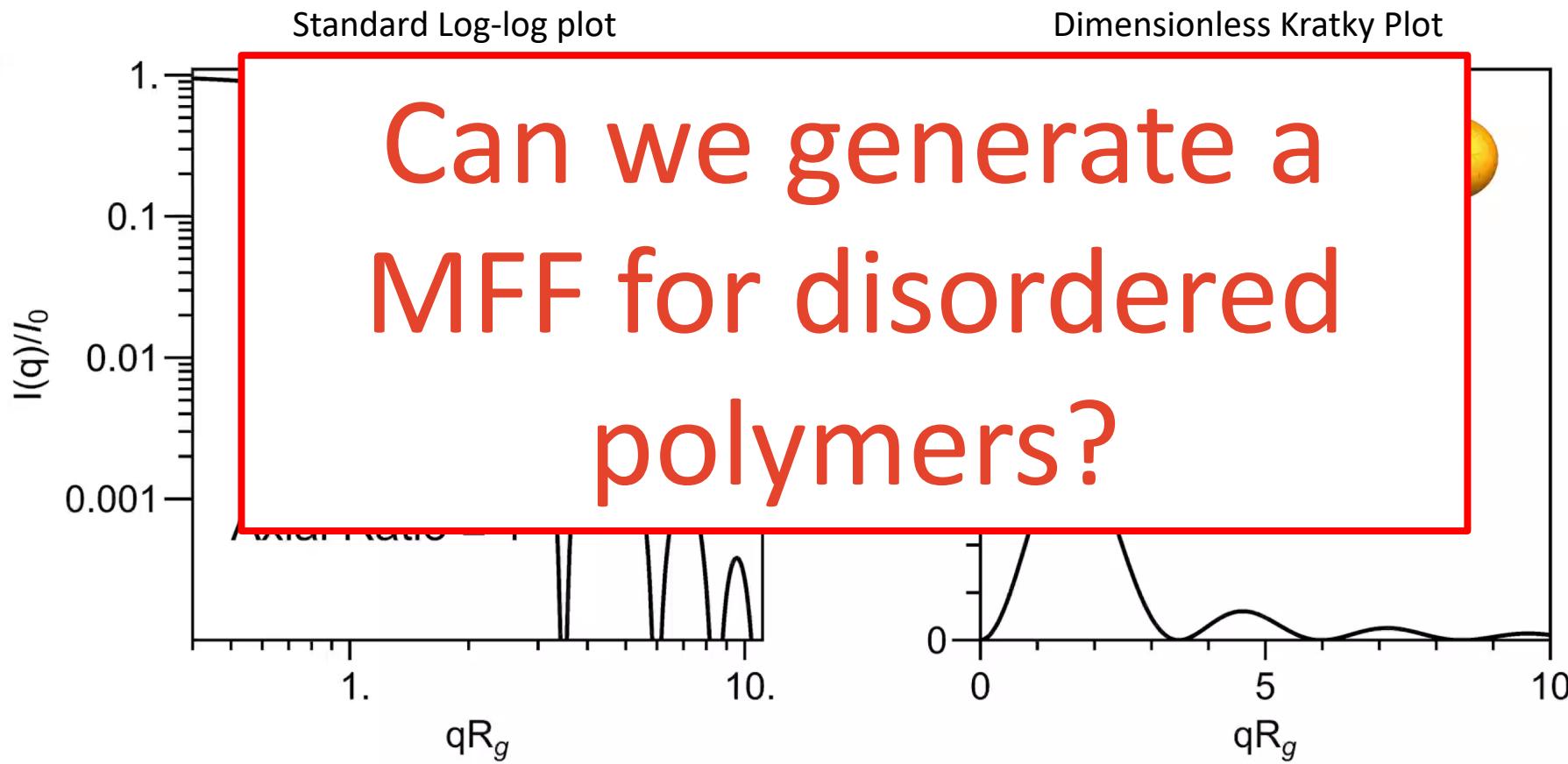
# Molecular Form Factors (MFF) describe shape of objects

E.g., ellipsoid, axial ratio



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E.g., ellipsoid, axial ratio

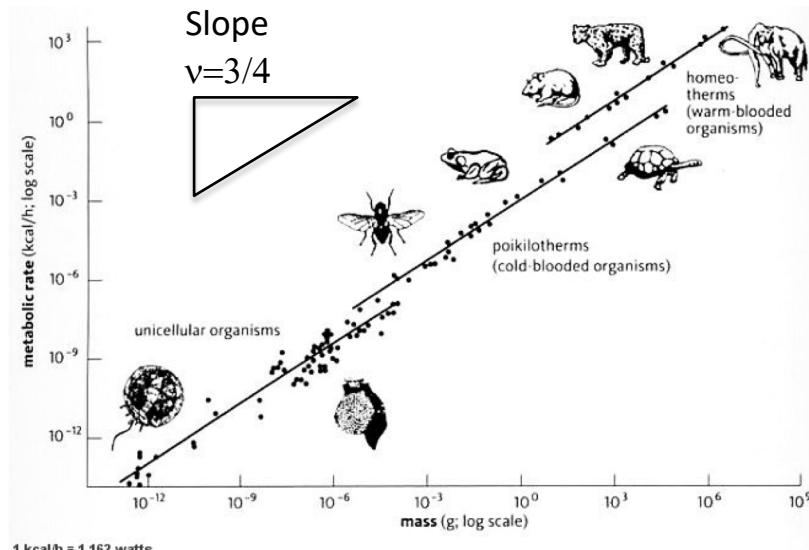


# Allometry and scaling laws in Biology

General form for a biological variable for animals

$$\text{Variable} \propto (\text{Size})^v$$

Kleiber's Law: Metabolic Rate  $\propto (\text{Mass})^{3/4}$



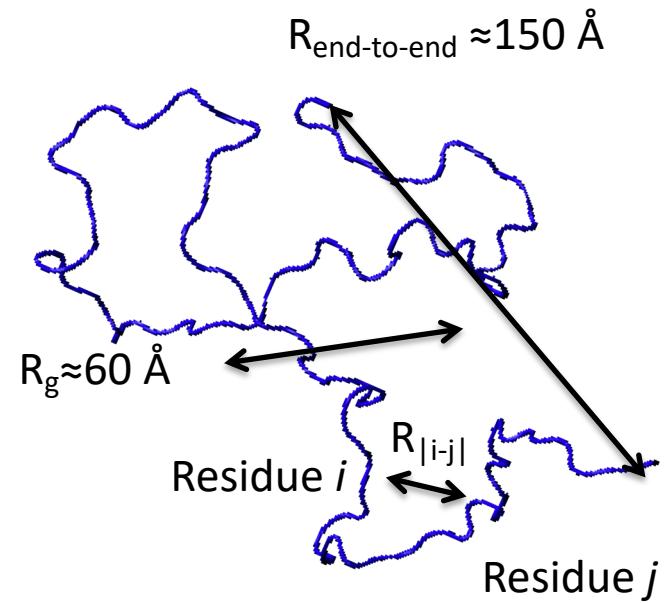
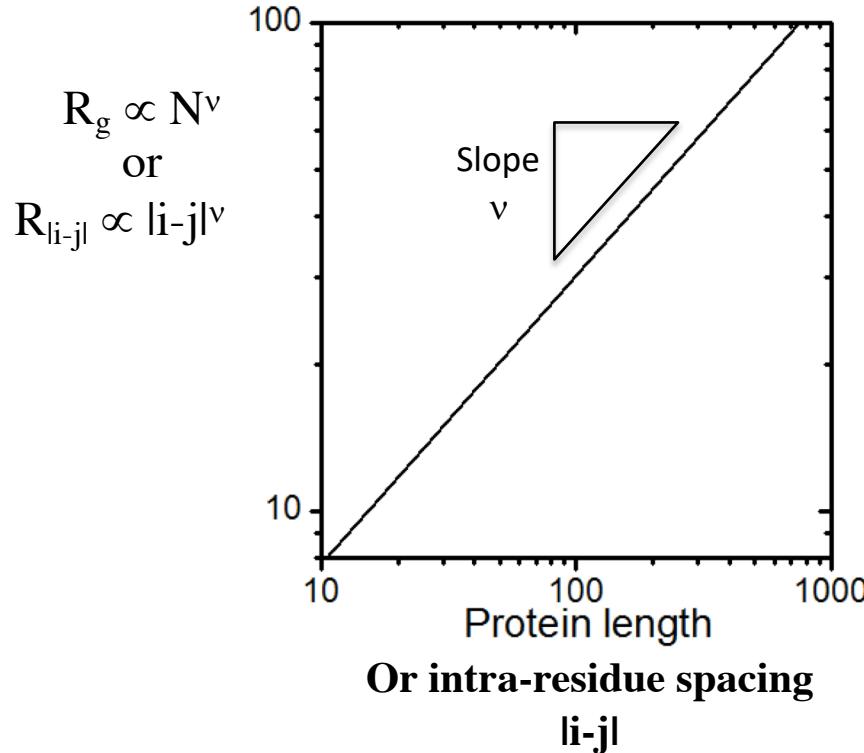
<http://universe-review.ca/I10-83-metabolic.jpg>

**Spheres**  
 $\text{Mass} \propto R^3$   
or  
 $R \propto M^{1/3}$   
 $v=1/3$

# Polymer Physics: scaling laws and Flory exponent $\nu$

## Good and poor solvents

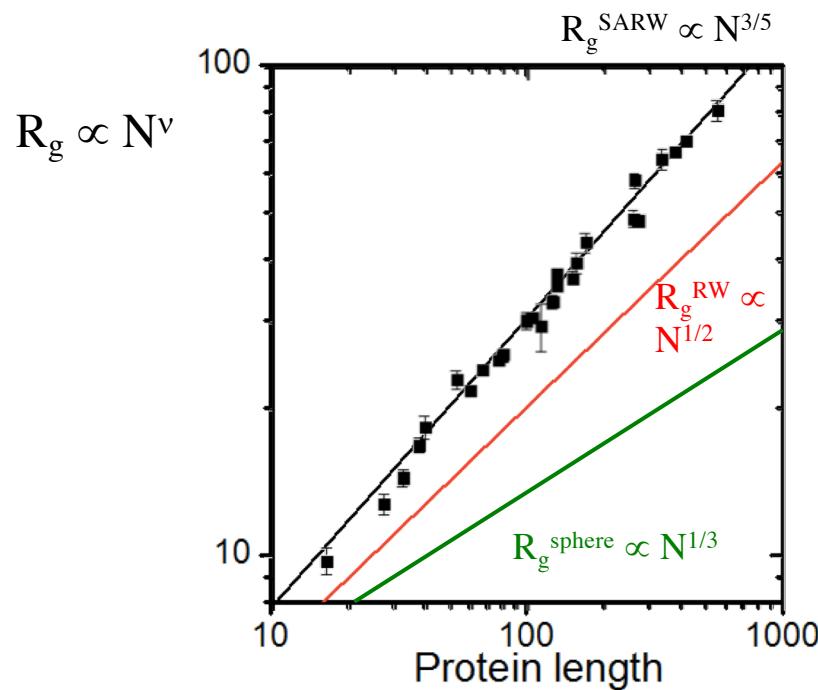
Solvent quality: Flory exponent  $\nu$



# Polymer Physics: scaling laws and Flory exponent $\nu$

## Good and poor solvents

Solvent quality:  $\nu$



$\nu$

Good solvent  
(contracted)  
 $0.5 < \nu \leq 0.6$

1/2

Poor  
(collapsed)  
 $1/3 < \nu < 1/2$

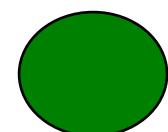
1/3

Self Avoiding  
random walk  
(SARW)  
 $E_{\text{prot-water}} >$   
 $E_{\text{prot-prot}}$

Random Walk  
behavior:  
θ solvent  
 $E_{\text{prot-prot}} \sim$   
 $E_{\text{prot-water}}$

Self-avoiding  
Random Walk

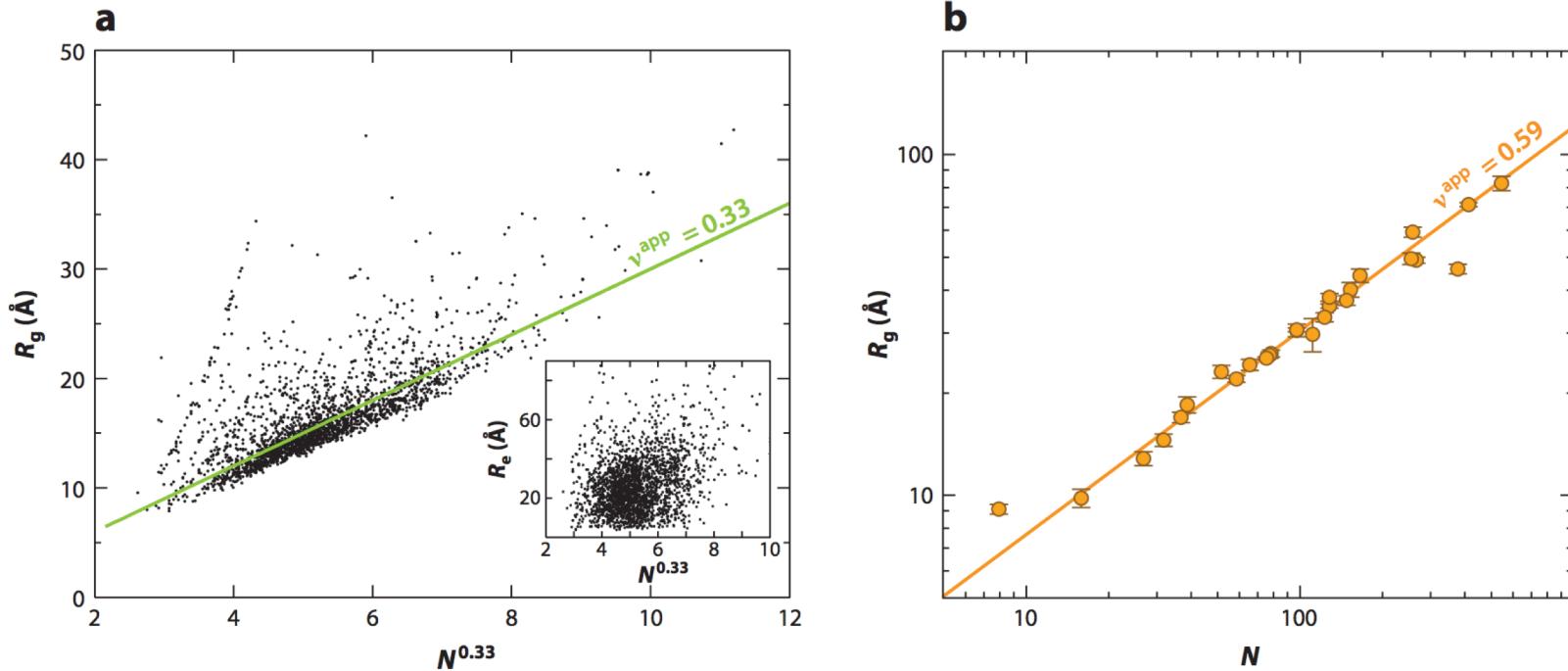
Random Walk  
(non-physical)



$$V = \frac{4}{3}\pi R^3$$

**Chemically denatured Proteins:**  
 $R_g$  scales as a self-avoiding random walk

# General scaling behavior of proteins



**Figure 3**

(a) Scaling of radius of gyration ( $R_g$ ) with chain length  $N$  for folded proteins based on  $\sim 2,400$  nonredundant structures taken from PDBSELECT25 (50). While the radius of gyration shows reasonable agreement with  $\nu_{app} \approx 0.33$ , the end-to-end distance shows a poor correlation (inset). (b) Scaling behavior for chemically denatured proteins based on data from Reference 74. The unfolded state under strongly denaturing conditions is well described by a self-avoiding random chain ( $\nu_{app} \approx 0.59$ ).

# Obtaining a MFF for disordered polymers

## Polymer Physics: scaling laws and Flory exponent $\nu$

### Good and poor solvents

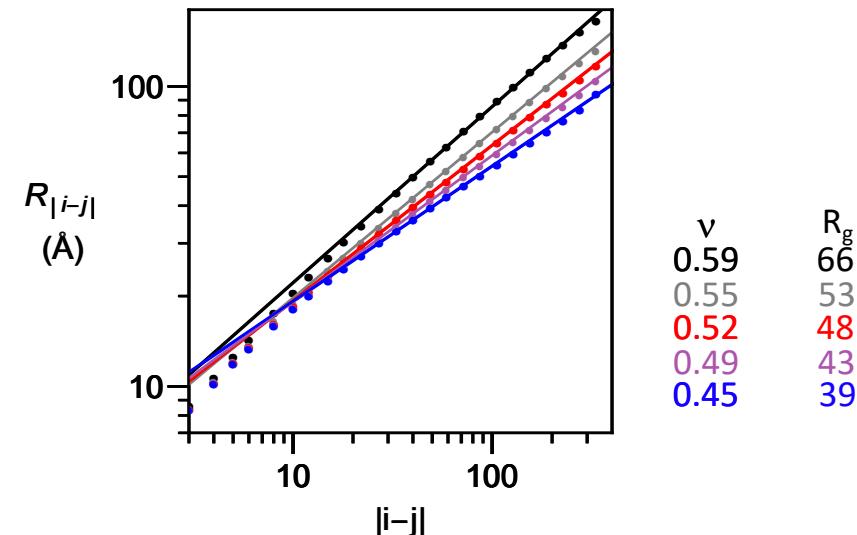
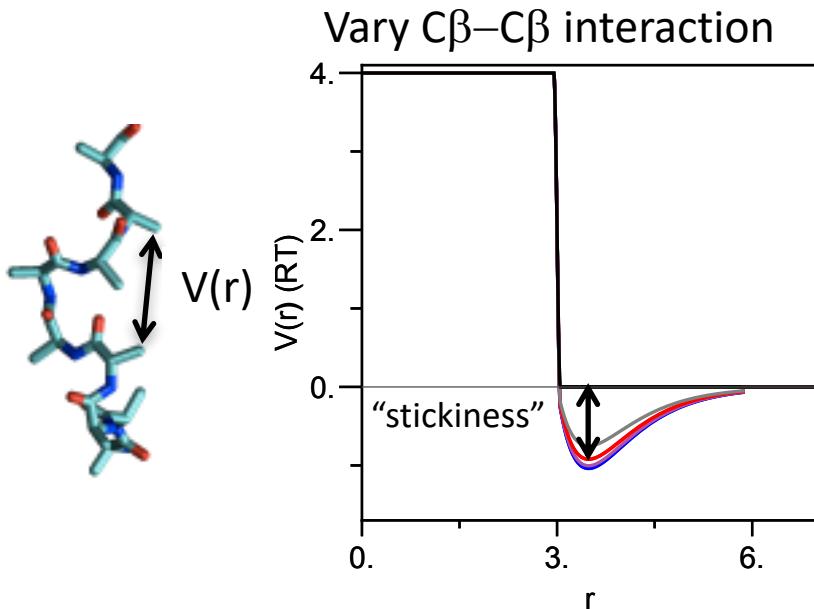
Debye formula for random walk

$$I_{Debye}(q) = \frac{2I_o \left( e^{-(qRg)^2} - 1 + (qRg)^2 \right)}{(qRg)^4}$$

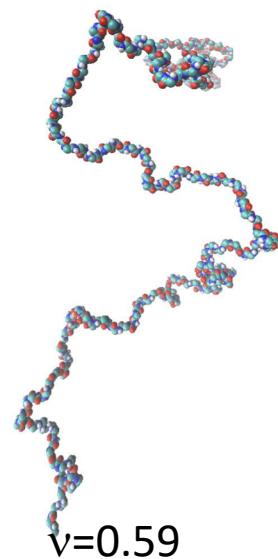
non-physical inter-penetration of chain

Can we generate a general  
MFF for realistic polymers?

# Simulations generate ensembles used to make an MFF



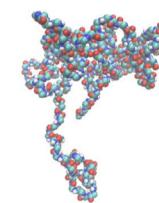
Hard sphere



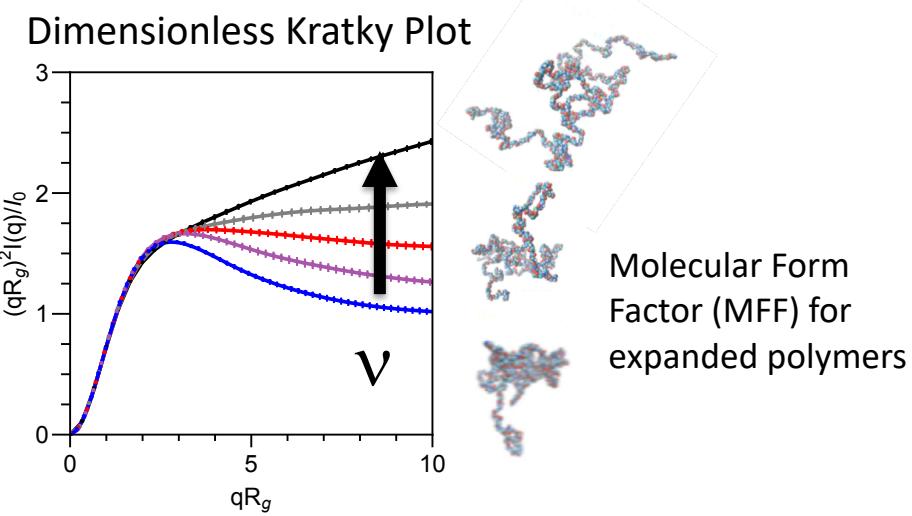
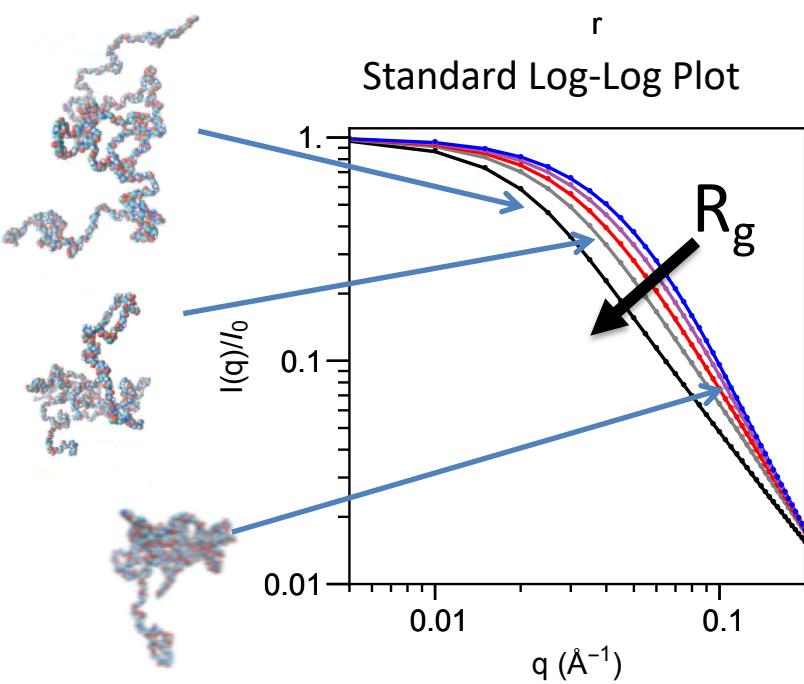
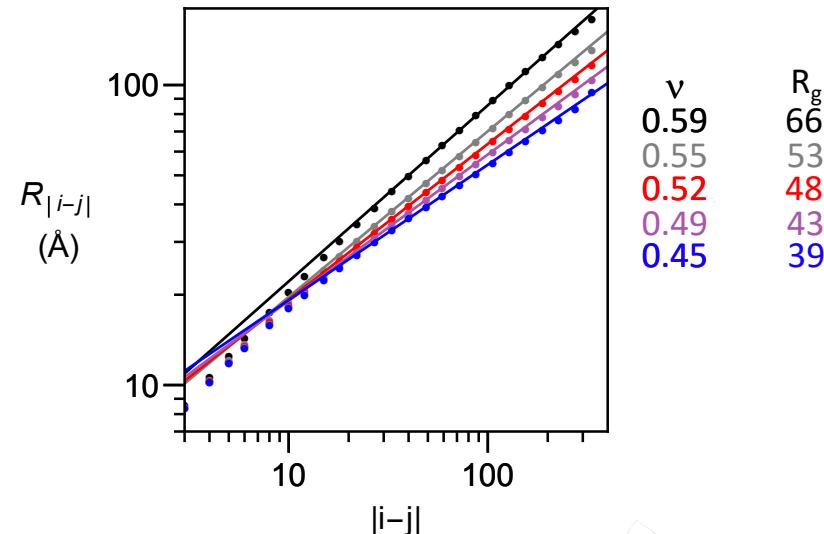
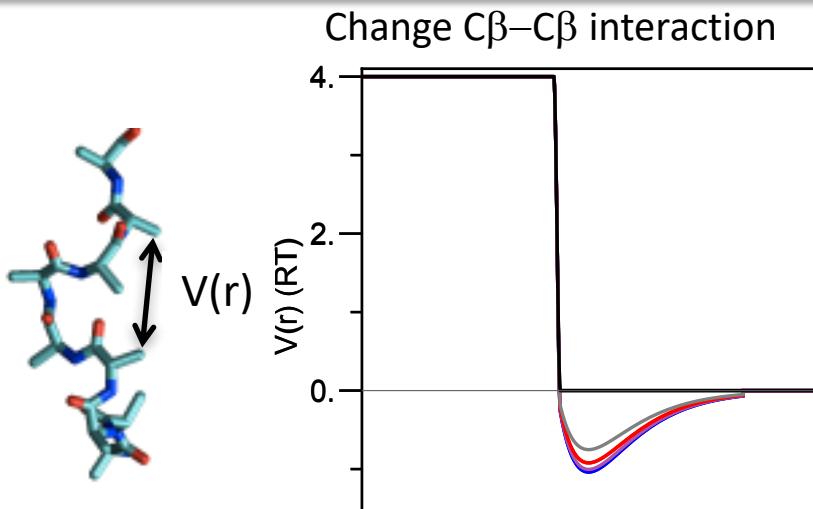
A little sticky



A little more sticky

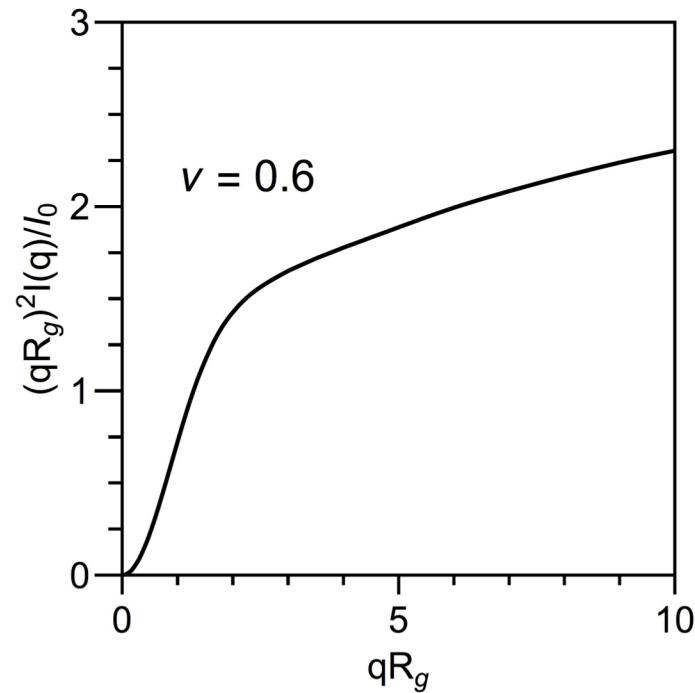
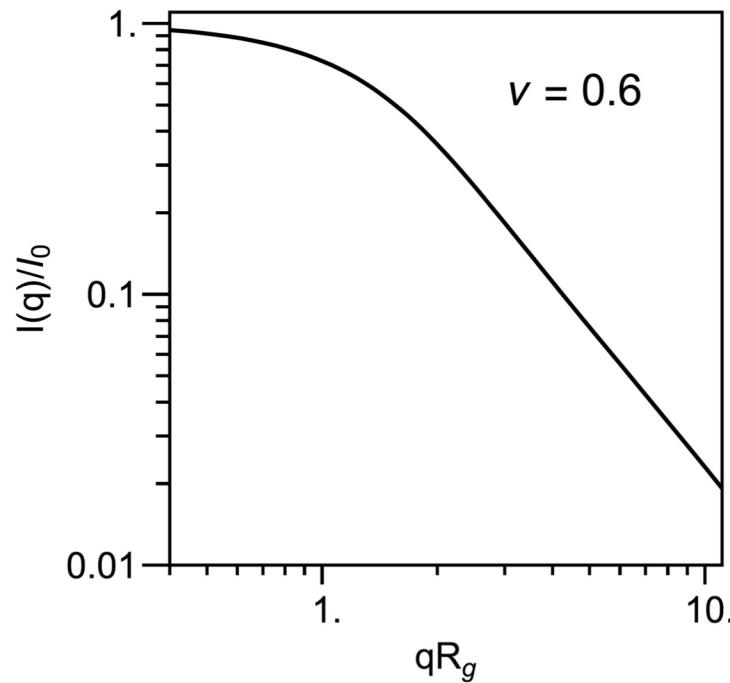


# Simulations generate ensembles used to make an MFF



Molecular Form Factor (MFF) for expanded polymers

# MFF for disorder systems: Obtain $R_g$ and $\nu$ from a single SAS measurement

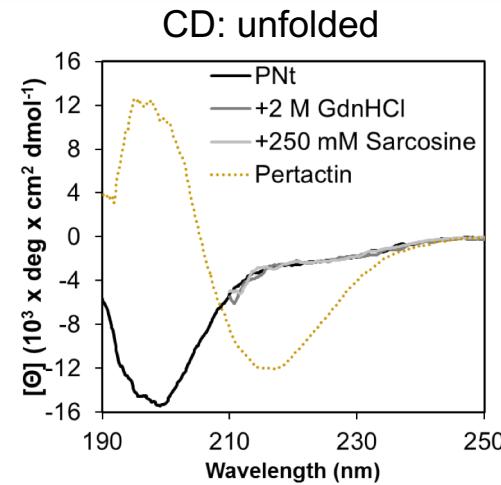
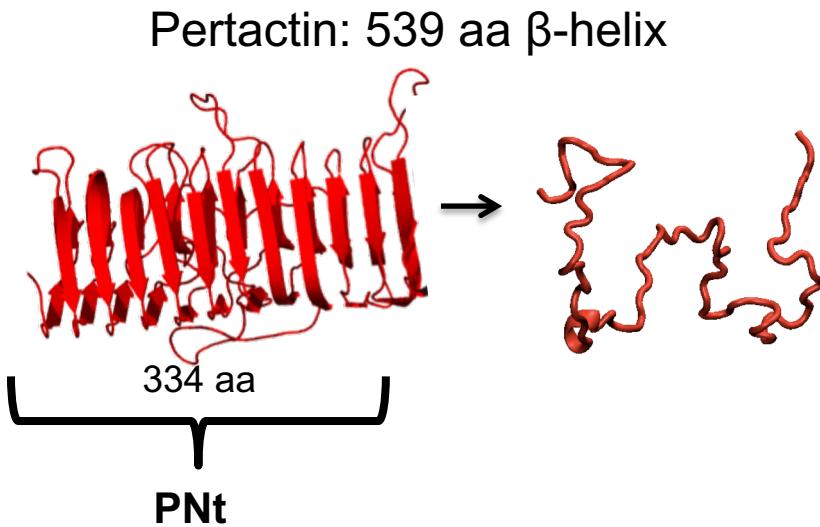


How well does it work?

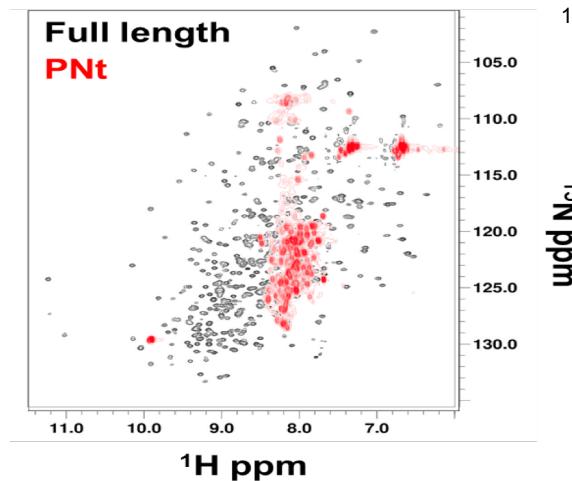
# How well does our MFF work?

PNt, 334 residue, low charge hydrophobic IDP

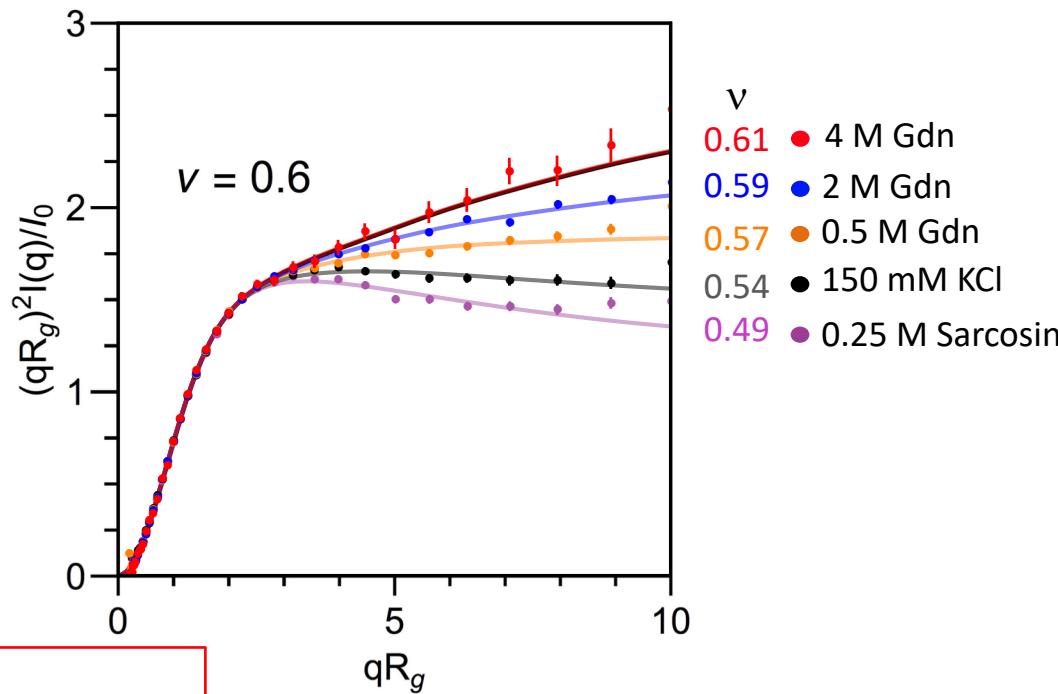
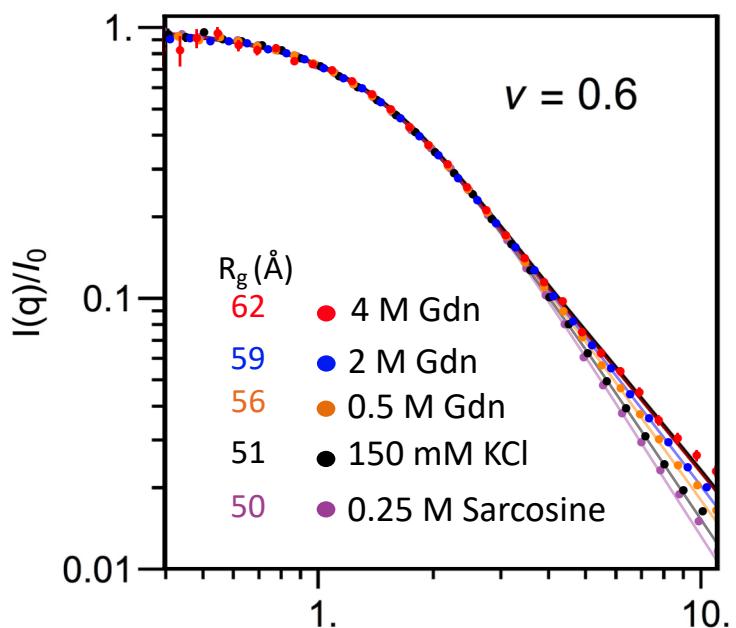
P. Clark (U Notre Dame)



Chemical shifts typical of IDP



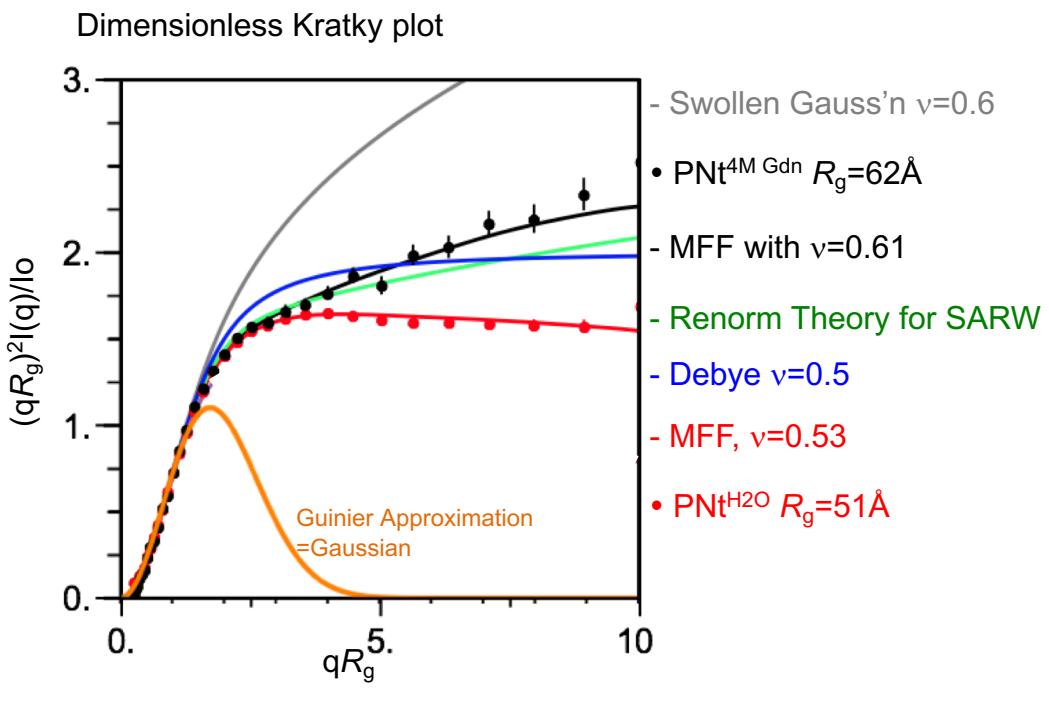
# Measuring $R_g$ and $\nu$ for an IDP using the MFF



**IT WORKS!**  
**LET'S USE IT!**

Fit out to  $qR_g \sim 5$  is generally sufficient  
Hydration may be an issue at  $q > 0.15 \text{ \AA}^{-1}$

# Scattering from Random Walks (allowed to cross) and self-avoiding RW (not allowed)



Debye formula for random walk:

$$I_{Debye}(q) = \frac{2I_o \left( e^{-(qRg)^2} - 1 + (qRg)^2 \right)}{(qRg)^4}$$

Swollen Gaussian coil model

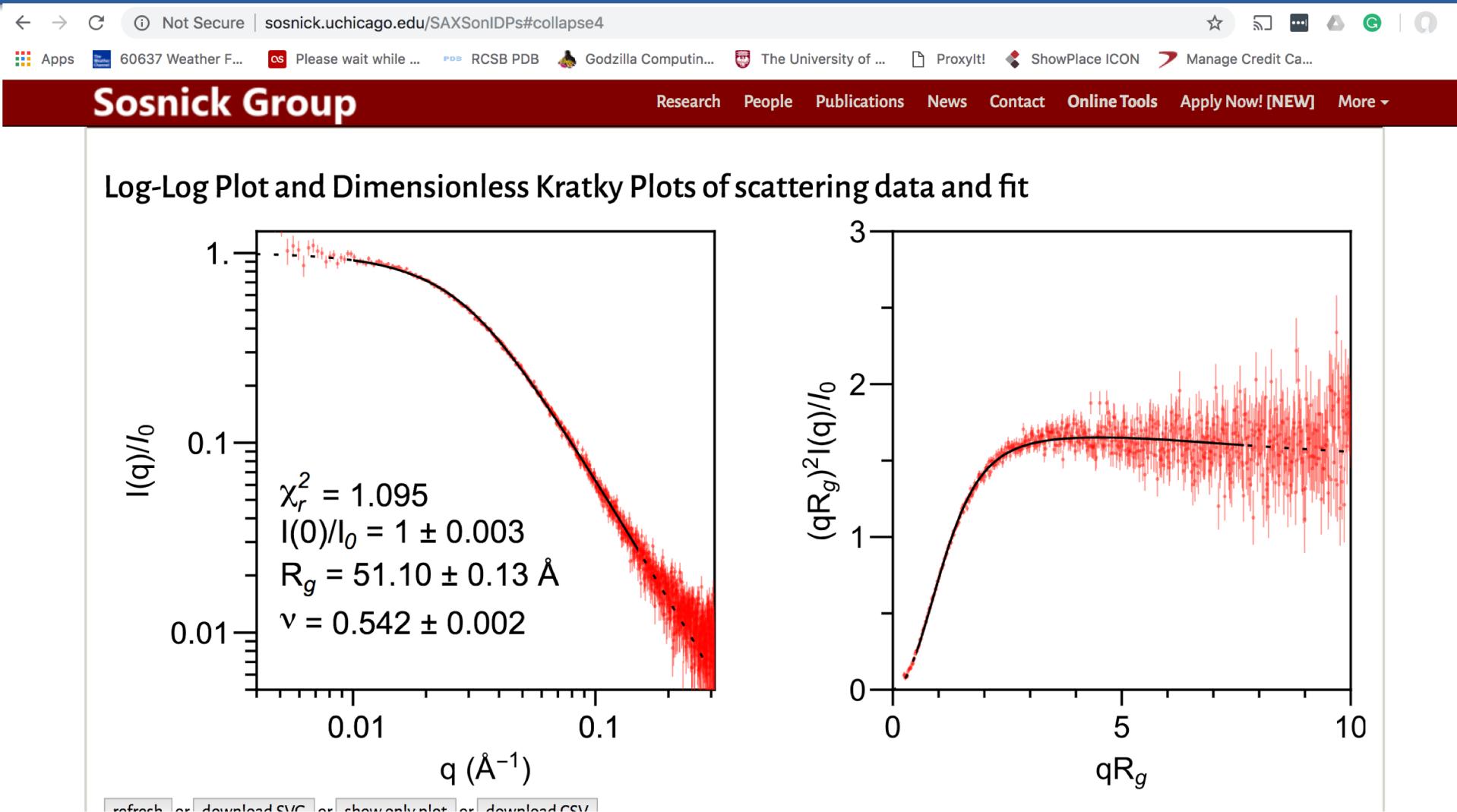
$$P_{SwollenGausCoil}(R_{ij}) \propto e^{-\left(\frac{3R_{ij}^2}{3\langle R_{ij}^2 \rangle}\right)}$$

$$\langle R_{ij}^2 \rangle = a^2 |i-j|^{2\nu}$$

$a$ =segment length

# Website for fitting disordered polymers

<http://sosnick.uchicago.edu/SAXSonIDPs>



## Acknowledgements

### Collaborators:

Notre Dame: Prof. Patricia Clark,  
Dr. Micayla Bowman

UChicago: Prof. Karl Freed,



**Drs. Joshua Riback, John Jumper  
Adam Zmyslowski**



Srinivas Chakravarthy BioCAT  
Beamline  
APS: DOE, NIH