

The following expression for $\text{Var}\mathcal{M}_{\alpha,\gamma}(QI_n)$ has been obtained by processing the formula given in the file `variance_table.txt` with *Mathematica*'s function `FullSimplify`:

$$\text{Var}\mathcal{M}_{\alpha,\gamma}(QI_n) = \binom{n}{4} \frac{N_1 + N_2 + N_3}{D}$$

where:

$$\begin{aligned} D &= 840(\alpha - 7)(\alpha - 6)(\alpha - 5)(\alpha - 4)(\alpha - 3)^2(\alpha - 2)^2 \\ N_1 &= -35(\alpha - 7)(\alpha - 6)(\alpha - 5)(\alpha - 4)(n - 3)(n - 2)(n - 1)n[(-5\alpha + \gamma + 5)q_1 \\ &\quad + 2(\alpha - \gamma)(-\alpha + \gamma + 1)q_2 - (\alpha - 1)(-2\alpha + \gamma + 2)q_3 + (\alpha - \gamma)(2\alpha - \gamma)q_4]^2, \\ N_2 &= 840(\alpha - 7)(\alpha - 6)(\alpha - 5)(\alpha - 4)(\alpha - 3)(\alpha - 2)[(\alpha - \gamma)(-5\alpha + \gamma + 5)q_1^2 \\ &\quad + 2(\alpha - \gamma)(-\alpha + \gamma + 1)q_2^2 - (\alpha - 1)(-2\alpha + \gamma + 2)q_3^2 \\ &\quad + (\alpha - \gamma)(2\alpha - \gamma)q_4^2], \\ N_3 &= -(\alpha - 3)(\alpha - 2)(n - 4)(F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8) \end{aligned}$$

$$\begin{aligned} F_1 &= -(\alpha - \gamma)(q_1^4 F_{1,1} + 4q_2^4 F_{1,2}), \\ F_2 &= 2(-1 + \alpha)(\alpha - \gamma)q_1^2 q_3^2 F_{2,1}, \\ F_3 &= (\alpha - 1)q_3^4 F_{3,1}, \\ F_4 &= 4(\gamma - \alpha)q_2^2(q_1^2 F_{4,1} + (\alpha - 1)q_3^2 F_{4,2}), \\ F_5 &= 2(\alpha - \gamma)(2\alpha - \gamma)q_1^2 q_4^2 F_{5,1}, \\ F_6 &= 4(\alpha - \gamma)(2\alpha - \gamma)q_2^2 q_4^2 F_{6,1}, \\ F_7 &= 2(1 - \alpha)(\alpha - \gamma)(2\alpha - \gamma)(n - 5)q_3^2 q_4^2 F_{7,1}, \\ F_8 &= -(\alpha - \gamma)(2\alpha - \gamma)q_4^4(\alpha^2 F_{8,1} + 14\alpha^2 F_{8,2} + \alpha^3 F_{8,3} + 16F_{8,4} + \alpha F_{8,5}), \end{aligned}$$

$$\begin{aligned}
F_{1,1} = & 7\alpha^5(n(125(n-2)n+1911)-20802) + \alpha^4(7\gamma(n(243-175(n-2)n) \\
& + 11622) + n(5(3578-3325n)n+248781)+674250) + \alpha^3(5(7\gamma(11\gamma+655) \\
& + 22677)n^3 - 10(\gamma(77\gamma+5353)+47301)n^2 - 3(\gamma(1183\gamma+88691)+220023)n \\
& - 6(\gamma(1687\gamma+27291)+235551)) + \alpha^2(-5(\gamma(7\gamma(\gamma+197)+28299)+33271)n^3 \\
& + 10(\gamma(7\gamma(\gamma+293)+64011)+96919)n^2 + 3\gamma(7\gamma(27\gamma+3655)+95433)n \\
& - 42\gamma(\gamma(5\gamma+1257)-2175)+704103n+1986078) + \alpha(\gamma^3(n-5) \\
& (595n^2+825n-2346) + 14\gamma^2(n(5n(533n-2746)+1587)+9894) \\
& + \gamma(n(5n(40223n-201118)+39969)-99678)+80n^2(535n-7628) \\
& - 816(713n+2548)) + 16(-5(\gamma(\gamma(34\gamma+385)+976)-324)n^3 \\
& + 10(\gamma(\gamma(103\gamma+1015)+2617)+612)n^2 - 3(\gamma(\gamma(237\gamma+1463)+997)-5742)n \\
& - 315(\gamma(\gamma(3\gamma+14)-33)-194)), \\
F_{1,2} = & 7\alpha^5(5n((n-2)n+15)-618) - 7\alpha^4(3\gamma(n(5(n-2)n+43)-482) \\
& + n(n(95n-118)-591)-3702) + \alpha^3(7(5\gamma(3\gamma+55)+639)n^3 \\
& - 2(35\gamma(3\gamma+43)+6897)n^2 - 3(7\gamma(\gamma+877)+3371)n - 6(7\gamma(161\gamma+797)+13355)) \\
& + \alpha^2(-7\gamma^3(n-5)(5n(n+3)-6) + \gamma^2(n((4094-1855n)n+21795)+2406) \\
& + 3\gamma(n((15082-3989n)n+5381)+19826) + 7(n((3970-1181n)n+2685)+23106)) \\
& + \alpha(\gamma^3(n-5)(595n^2+825n-2346) + 2\gamma^2(n(n(5155n-24326)+2601)+4698) \\
& + 3\gamma(n(n(5217n-19306)-3325)-30042) + 8(n(n(505n-2324)-4368)-25500)) \\
& + 8(-3\gamma(474\gamma^2+374\gamma-893)n-1890\gamma(\gamma^2-6) - (\gamma(10\gamma(34\gamma+89)+463)-48)n^3 \\
& + 4(\gamma(\gamma(515\gamma+904)+596)+120)n^2+900(3n+14)), \\
F_{2,1} = & 14\alpha^4(n(25(n-2)n+267)-2634) + \alpha^3(7\gamma(n(183-35(n-2)n)+1422) \\
& + 2n(5(946-665n)n+29961)+141924) + \alpha^2(7\gamma^2(n-5)(5n(n+3)-6) \\
& + 6\gamma(n(15n(49n-130)-7747)+1658) + 6(n(5n(1483n-5206)-14411)-36070)) \\
& + \alpha(-5(\gamma(119\gamma+4929)+15526)n^3 + 50(\gamma(43\gamma+2349)+5678)n^2 + 9\gamma(719\gamma-519)n \\
& - 6\gamma(1955\gamma+7997)-20346n+151692) + 16(5(34\gamma^2+292\gamma+493)n^3 \\
& - 10(\gamma(103\gamma+682)+853)n^2 + 3(\gamma(237\gamma+859)+899)n+315(\gamma-1)(3\gamma+8)),
\end{aligned}$$

$$\begin{aligned}
F_{3,1} = & 5n^3((\alpha - 1)(7(\alpha - 17)\alpha + 544)\gamma^2 - 4(\alpha(7\alpha((\alpha - 19)\alpha + 123) - 1549) + 868)\gamma \\
& + 4(\alpha - 1)(\alpha(\alpha(7(\alpha - 19)\alpha + 879) - 1693) + 1012)) \\
& - 10n^2((\alpha - 16)(\alpha - 1)(7\alpha - 103)\gamma^2 - 4(\alpha(\alpha(7(\alpha - 19)\alpha + 1569) - 2641) + 1036)\gamma \\
& + 4(\alpha - 1)(\alpha(\alpha(\alpha(7\alpha - 109) + 1359) - 2293) + 820)) \\
& + 3n(-3(\alpha - 1)(\alpha(63\alpha - 719) - 1264)\gamma^2 - 4(\alpha(\alpha(7\alpha(5\alpha + 241) - 451) - 5741) \\
& + 6300)\gamma + 4\alpha(\alpha(\alpha(\alpha(91\alpha + 1132) - 988) - 7076) + 15133)) \\
& + 6(5(\alpha - 1)(\alpha(7\alpha - 391) + 504)\gamma^2 \\
& + 4\alpha(\alpha(21\alpha(17\alpha - 35) - 839) + 5567)\gamma - 4\alpha(\alpha(\alpha(\alpha(413\alpha - 1628) + 704) + 9444) \\
& - 22793)) - 144(1260\gamma + 691n + 2310), \\
F_{4,1} = & 7\alpha^5(n(25(n - 2)n + 363) - 3426) - 7\alpha^4(\gamma(n(55(n - 2)n + 333) - 5478) \\
& + n(n(475n - 566) - 4791) - 16494) + \alpha^3(5(49\gamma(\gamma + 29) + 4497)n^3 \\
& - 2(\gamma(245\gamma + 6193) + 39981)n^2 - 3(\gamma(595\gamma + 28479) + 24963)n \\
& - 6(\gamma(1407\gamma + 18551) + 39675)) + \alpha^2(9\gamma(63\gamma^2 + 5475\gamma + 9979)n - 5(\gamma(7\gamma(\gamma + 125) \\
& + 8859) + 7595)n^3 + 2(\gamma(\gamma(35\gamma + 6151) + 94191) + 77143)n^2 - 6\gamma(\gamma(35\gamma + 4199) \\
& - 22081) + 53043n + 248694) + \alpha(\gamma^3(n - 5)(595n^2 + 825n - 2346) \\
& + 2\gamma^2(n(n(11905n - 60218) + 6855) + 36978) + \gamma(n(n(55615n - 254654) + 6381) \\
& - 86886) + 8(n(34n(65n - 307) - 3489) - 15915)) + 16(-5(\gamma(\gamma + 1)(34\gamma + 203) \\
& - 12)n^3 + (\gamma(\gamma(1030\gamma + 5979) + 4823) + 348)n^2 - 9(\gamma(\gamma(79\gamma + 275) + 91) - 95)n \\
& - 315(\gamma - 1)(\gamma(3\gamma + 10) + 5)), \\
F_{4,2} = & 6(966\alpha^4 - 7\alpha^3(197\gamma + 542) + \alpha^2((2942 - 35\gamma)\gamma + 6010) + \alpha(\gamma(1955\gamma - 2763) - 4022) \\
& - 840(\gamma - 1)(3\gamma + 1)) - 5n^3((7(\alpha - 17)\alpha + 544)\gamma^2 + 3(\alpha(-7(\alpha - 18)\alpha - 743) + 768)\gamma \\
& + 2(\alpha - 1)(\alpha(7(\alpha - 18)\alpha + 759) - 928)) + 2n^2(5(\alpha - 16)(7\alpha - 103)\gamma^2 \\
& - 3(\alpha(35\alpha - 547) + 7064)(\alpha - 1)\gamma + 2(7\alpha(5\alpha - 61) + 5864)(\alpha - 1)^2) \\
& - 3n(3((719 - 63\alpha)\alpha + 1264)\gamma^2 + (\alpha(1279 - \alpha(161\alpha + 6334)) + 3536)\gamma \\
& + 2(7\alpha(17\alpha + 207) + 2216)(\alpha - 1)^2), \\
F_{5,1} = & 7\alpha^4(n(25(n - 2)n + 459) - 5466) + 6\alpha^3(7\gamma(n(33 - 5(n - 2)n) + 234) + n(5(82 - 105n)n \\
& + 11849) + 24858) + \alpha^2(7\gamma^2(n - 5)(5n(n + 3) - 6) + \gamma(5n(n(749n - 2074) - 9561) \\
& + 12006) + 3(n(5n(1321n - 7154) - 46473) - 77010)) + \alpha(\gamma^2(-(n - 5))(595n^2 + 825n \\
& - 2346) - \gamma(n(5n(4075n - 21686) + 357) + 54234) + 16(n(-535n^2 + 8180n + 6354) \\
& + 10344)) + 8(5(\gamma(68\gamma + 349) - 207)n^3 - 10(\gamma(206\gamma + 1069) + 321)n^2 \\
& + 3(\gamma(474\gamma + 1259) - 1523)n + 1890(\gamma - 1)(\gamma + 3)),
\end{aligned}$$

$$\begin{aligned}
F_{6,1} &= 7\alpha^4(n(5(n-2)n+87)-906) - 14\alpha^3(\gamma(n(5(n-2)n+27)-582) + 3n(n(15n-14) \\
&\quad - 239) - 1902) + \alpha^2(5(7\gamma(\gamma+35) + 783)n^3 - 2(\gamma(35\gamma+1081) + 8811)n^2 \\
&\quad - 3(\gamma(189\gamma+6775) + 5757)n + 6(\gamma(35\gamma-2599) - 7745)) + \alpha(\gamma^2(-(n-5))(595n^2 \\
&\quad + 825n - 2346) + \gamma(n((36646 - 6875n)n + 8151) + 10326) + 8(n(10(242 - 37n)n \\
&\quad + 1569) + 4533)) + 8(5(\gamma(68\gamma+53) - 9)n^3 - 2(2\gamma(515\gamma+587) + 141)n^2 \\
&\quad + 3(\gamma(474\gamma-17) - 247)n + 630(\gamma-1)(3\gamma+2)), \\
F_{7,1} &= 5n^2(\alpha(14\alpha^2 - 7\alpha(\gamma+36) + 119\gamma + 1542) - 8(68\gamma+163)) + 15n(\alpha(\alpha(14\alpha-7\gamma-316) \\
&\quad + 55\gamma+326) + 192\gamma) + 6\alpha(\alpha(322\alpha+7\gamma-1028) - 391\gamma+1210) + 72(42\gamma-5n-42), \\
F_{8,1} &= 5(7\gamma(\gamma+51) + 1376)n^3 - 10(\gamma(7\gamma+453) + 5072)n^2 - 9\gamma(63\gamma+3533)n \\
&\quad + 6\gamma(35\gamma-887) - 48(1021n+990), \\
F_{8,2} &= n(5(n-2)n+111) - 1434, \\
F_{8,3} &= 21\gamma(n(17-5(n-2)n) + 434) + 2n(5(46-119n)n + 19863) + 62604, \\
F_{8,4} &= \gamma^2(n-5)(10n(17n-18) - 189) - 18\gamma(2n(5n(n+3) + 17) + 35) \\
&\quad + 9(n(5n(n+10) + 269) + 1190), \\
F_{8,5} &= \gamma^2(-(n-5))(595n^2 + 825n - 2346) - 48\gamma(n(5n(37n-240) + 26) + 4) \\
&\quad + 24(n(15n(17n+50) + 283) - 3782),
\end{aligned}$$