Mallard Duck Wing Kinematics

Team 3

In [1]:

```
#pip install pynamics
```

In [2]:

```
%matplotlib inline
import scipy.optimize
import pynamics
from pynamics.frame import Frame
from pynamics.variable_types import Differentiable,Constant
from pynamics.system import System
from pynamics.body import Body
from pynamics.dyadic import Dyadic
from pynamics.output import Output,PointsOutput
from pynamics.particle import Particle
import pynamics.integration
import numpy
import sympy
import matplotlib.pyplot as plt
from matplotlib.patches import Arc
plt.ion()
from math import pi, sin, cos, tan, asin, acos, atan, degrees, radians, pi
```

In [3]:

```
def prettyplot(x,y): # Function plots mechanism and generates labels
    # Create Figure
    Fig1 = plt.figure(figsize=[15,10])
    ax = Fig1.add_axes([0.0, 0.0, 1.0, 1.0], aspect='equal')
    #ax.patch.set_alpha(0.0)

    # plot links and joints
    plt.plot(x[0:3],y[0:3],color='tab:blue')
    plt.plot(x[2:5],y[2:5],color='tab:green')
    plt.plot(x[4:],y[4:],color='k')
    plt.scatter(x,y,color='k')
    plt.axis('equal')
    plt.xlim([-0.1,0.5])

    # label joints
    for i in range(0, len(x)-1):
        plt.text(x[i]-0.004, y[i]-0.004, "p"+str(i), verticalalignment='top', horizontal
alignment='right', fontsize=14)

    # plot COMs
    comx = [(x[0]+x[1])/2, (x[1]+x[2])/2, (x[4]+x[3])/2, (x[3]+x[2])/2, (x[4]+x[0])/2]
    comy = [(y[0]+y[1])/2, (y[1]+y[2])/2, (y[4]+y[3])/2, (y[3]+y[2])/2, (y[4]+y[0])/2]
    plt.scatter(comx,comy,color='k')

    # label COMs
    linknames = ['A', 'B', 'C', 'D', 'E']
    for i in range(0, len(comx)):
        plt.text(comx[i]+0.004, comy[i], linknames[i], verticalalignment='bottom', horiz
ontalalignment='left', weight='bold', fontsize=16)

    # label properties
    propertynames = ['La', 'Lb', 'Lc', 'Ld', 'Le']
    for i in range(0, len(comx)):
        plt.text(comx[i]+0.016, comy[i], propertynames[i], verticalalignment='bottom', h
orizontalalignment='left', fontsize=14)
```

```
    # plot frame axes
    alen = 0.04
    plt.arrow(0, 0, alen, 0,head_width=0.008,width=0.0004,color='r')
    plt.arrow(0, 0, 0, alen,head_width=0.008,width=0.0004,color='r')
    vx = []
    vy = []
    for i in range(1, len(x)-2):
        if i <= 2:
            vecx = numpy.array([x[i]-x[i-1], y[i]-y[i-1]])
            vecx = vecx/(vecx.dot(vecx)**0.5)
            vecy = numpy.cross([0,0,1], vecx)[0:2]
            plt.arrow(x[i], y[i], alen*vecx[0], alen*vecx[1],head_width=0.008,width=0.00
04,color='b')
            plt.arrow(x[i], y[i], alen*vecy[0], alen*vecy[1],head_width=0.008,width=0.00
04,color='b')
            vx.append(vecx)
            vy.append(vecy)
        if i >= 2:
            vecx = numpy.array([x[i]-x[i+1], y[i]-y[i+1]])
            vecx = vecx/(vecx.dot(vecx)**0.5)
            vecy = numpy.cross([0,0,1], vecx)[0:2]
            plt.arrow(x[i], y[i], alen*vecx[0], alen*vecx[1],head_width=0.008,width=0.00
04,color='g')
            plt.arrow(x[i], y[i], alen*vecy[0], alen*vecy[1],head_width=0.008,width=0.00
04,color='g')
            vx.append(vecx)
            vy.append(vecy)

    # label frames
    lablen = 0.06

    plt.text(x[0]+0.06, y[0], 'Nx', verticalalignment='center', horizontalalignment='cen
ter', color='r', fontsize=14)
    plt.text(x[0], y[0]+0.06, 'Ny', verticalalignment='center', horizontalalignment='cen
ter', color='r', fontsize=14)

    plt.text(x[1]+lablen*vx[0][0], y[1]+lablen*vx[0][1], 'Ax', verticalalignment='center
', horizontalalignment='center', color='b', fontsize=14)
    plt.text(x[1]+lablen*vy[0][0], y[1]+lablen*vy[0][1], 'Ay', verticalalignment='center
', horizontalalignment='center', color='b', fontsize=14)

    plt.text(x[2]+lablen*vx[1][0], y[2]+lablen*vx[1][1], 'Bx', verticalalignment='center
', horizontalalignment='center', color='b', fontsize=14)
    plt.text(x[2]+lablen*vy[1][0], y[2]+lablen*vy[1][1], 'By', verticalalignment='center
', horizontalalignment='center', color='b', fontsize=14)

    plt.text(x[3]+lablen*vx[3][0], y[3]+lablen*vx[3][1], 'Cx', verticalalignment='center
', horizontalalignment='center', color='g', fontsize=14)
    plt.text(x[3]+lablen*vy[3][0], y[3]+lablen*vy[3][1], 'Cy', verticalalignment='center
', horizontalalignment='center', color='g', fontsize=14)

    plt.text(x[2]+lablen*vx[2][0], y[2]+lablen*vx[2][1], 'Dx', verticalalignment='center
', horizontalalignment='center', color='g', fontsize=14)
    plt.text(x[2]+lablen*vy[2][0], y[2]+lablen*vy[2][1], 'Dy', verticalalignment='center
', horizontalalignment='center', color='g', fontsize=14)

    # plot angles
    Aang1 = 0
    Aang2 = degrees(atan(vx[0][1]/vx[0][0]))
    ax.add_patch(Arc((0, 0), 0.04, 0.04, theta1=min(Aang1, Aang2), theta2=max(Aang1, Aan
g2), edgecolor='k', linewidth=0.0006, label='q1'))

    Bang1 = degrees(atan(vx[0][1]/vx[0][0]))
    Bang2 = degrees(atan(vx[1][1]/vx[1][0]))
    ax.add_patch(Arc((x[1], y[1]), 0.04, 0.04, theta1=min(Bang1, Bang2), theta2=max(Bang
1, Bang2), edgecolor='k', linewidth=0.0006))

    plt.plot([0,0.04],[0.04,0.04],color='k',linewidth=1)
    Cang1 = 0
    Cang2 = degrees(atan(vx[3][1]/vx[3][0]))
    ax.add_patch(Arc((x[4], y[4]), 0.04, 0.04, theta1=min(Cang1, Cang2), theta2=max(Cang
```

```
1, Cang2), edgecolor='k', linewidth=0.0006))

    Dang1 = degrees(atan(vx[3][1]/vx[3][0]))
    Dang2 = degrees(atan(vx[2][1]/vx[2][0]))
    ax.add_patch(Arc((x[3], y[3]), 0.04, 0.04, theta1=min(Dang1, Dang2), theta2=max(Dang
1, Dang2), edgecolor='k', linewidth=0.0006))

    # label angles
    plt.text(x[0]+0.032*cos(radians((Aang1+Aang2)/2)), y[0]+0.032*sin(radians((Aang1+Aan
g2)/2)), 'qA', verticalalignment='center', horizontalalignment='center', fontsize=14)
    plt.text(x[1]+0.032*cos(radians((Bang1+Bang2)/2)), y[1]+0.032*sin(radians((Bang1+Ban
g2)/2)), 'qB', verticalalignment='center', horizontalalignment='center', fontsize=14)
    plt.text(x[4]+0.032*cos(radians((Cang1+Cang2)/2)), y[4]+0.032*sin(radians((Cang1+Can
g2)/2)), 'qC', verticalalignment='center', horizontalalignment='center', fontsize=14)
    plt.text(x[3]+0.032*cos(radians((Dang1+Dang2)/2)), y[3]+0.032*sin(radians((Dang1+Dan
g2)/2)), 'qD', verticalalignment='center', horizontalalignment='center', fontsize=14)

    # Label inputs and outputs
    plt.text(x[0]-0.004, y[0]-0.004, "Motor A", verticalalignment='bottom', horizontal
ignment='right', fontsize=14, weight='bold')
    plt.text(x[4]-0.004, y[4]-0.004, "Motor C", verticalalignment='bottom', horizontal
ignment='right', fontsize=14, weight='bold')
    plt.text(x[2]+0.004, y[2]+0.004, "End Effector (x,y)", verticalalignment='bottom', h
orizontalalignment='left', fontsize=14, weight='bold')
```

1. Create a figure

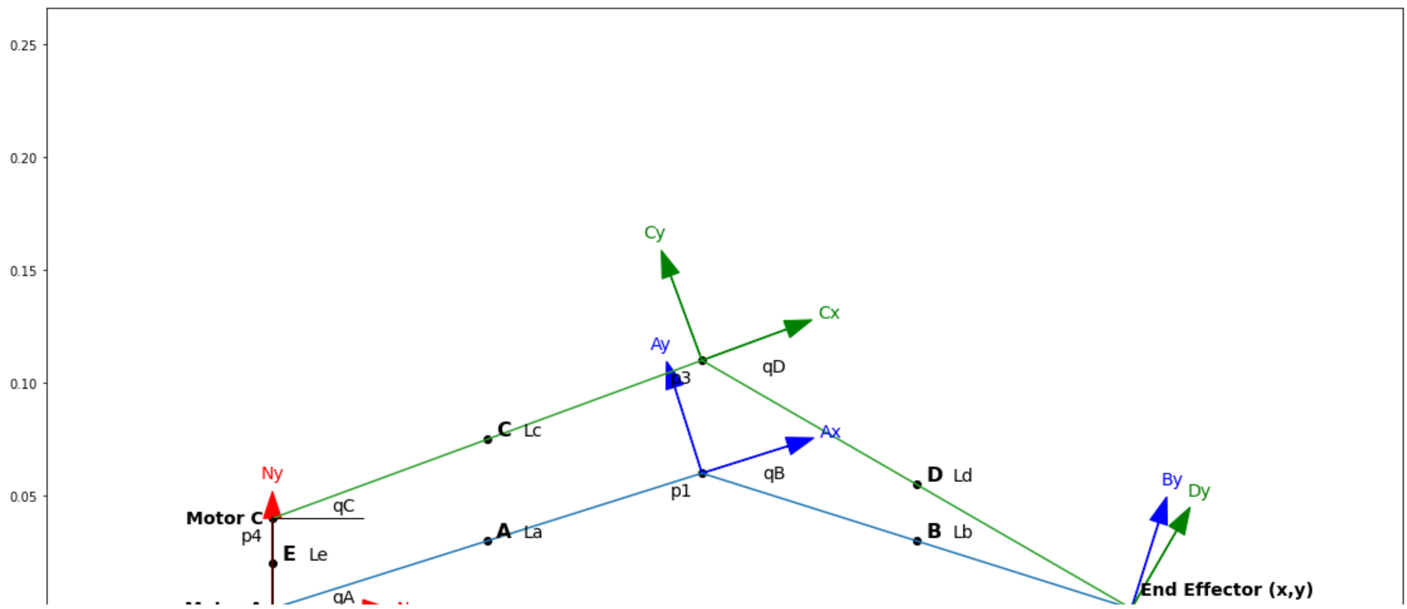
Create a figure (either in python, in a vector-based drawing program like inkscape or illustrator, or as a solidworks rendering) of your system kinematics. Annotate the image to include:

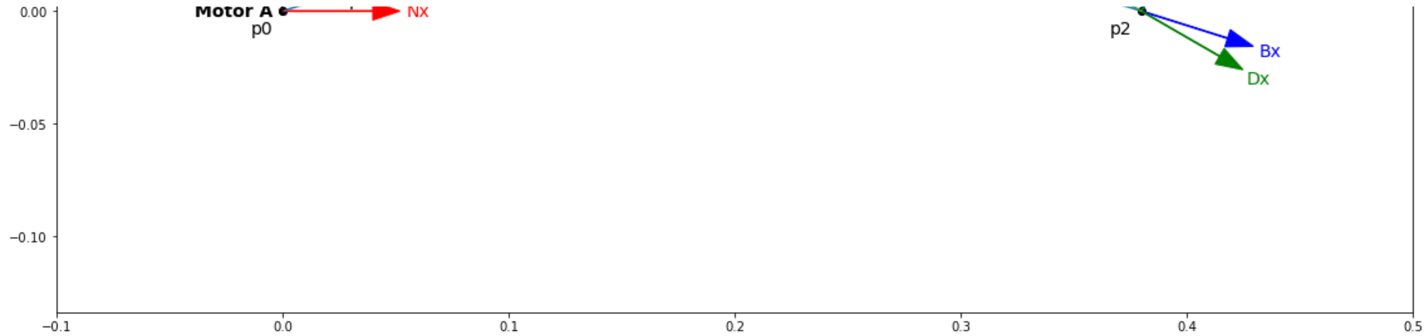
- **Names for each rotational reference frame. You will need one for each rigid body (that moves independently), as well as one for the Newtonian reference frame (N). Use the convention of a capital letter (A,B,C,...)**
- **Labeled joint locations**
- **A set of orthonormal basis vectors for each fram. It is best practice to align one of the basis vectors with each rigid link.**
- **variable names for each state variable**
- **geometric constants such as link lengths**

Save this figure for reuse later. You will need to add mass and inertial information as well as system stiffness information, so make sure you do your work in a way that permits reusing and modifying the figure.

In [4]:

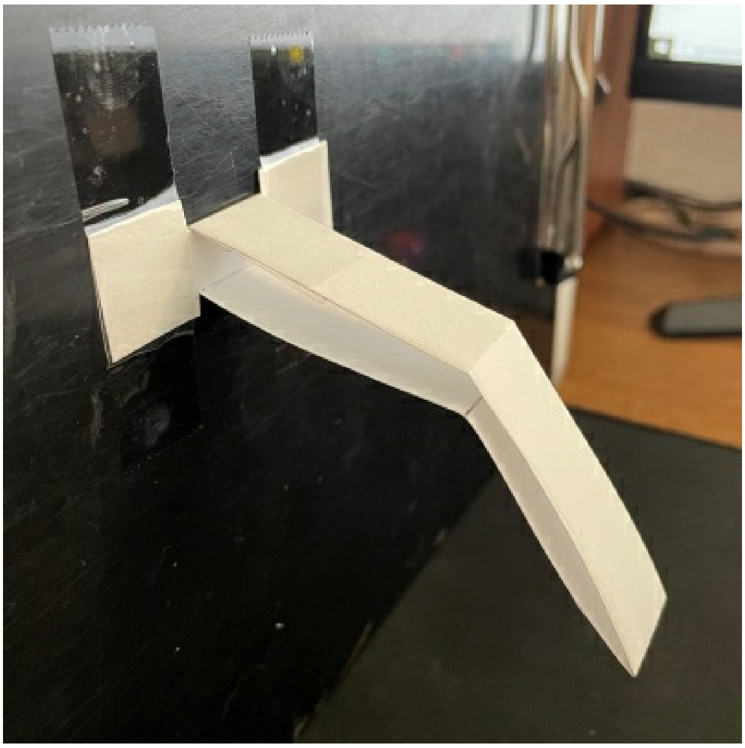
```
# Kinematics Demo Position
x = numpy.array([0, 0.19, 0.38, 0.19, 0, 0])
y = numpy.array([0, 0.06, 0, 0.11, 0.04, 0])
prettyplot(x,y)
```





2. Make the device in paper or cardboard.

You need an up-to-date model if it has changed from your individual assignments. The paper model should dimensionally match your code.



This is a 1/4 model of our prototype. This model is smaller but proportionally accurate to our intended prototype. The full prototype will be constructed after we acquire larger sheets of material.

3. Develop a Kinematic Model

Using a pynamics-based script, develop a kinematic model for your device. Following the triple pendulum example,

1. **Import packages**
2. **Define variables and constants (you may want to add, remove, or rename variables to match your figure)**
3. **Declare frames (you may need to add frames or rename them)**
4. **Define frame rotations (you may want to switch the axis about which frames rotate)**
5. **Compose kinematics (this depends entirely on the geometry of your system)**
6. **Take time-derivatives of position vectors**
7. **Assemble into a Jacobian that maps input velocities to output velocities.**

In [5]:

```
# Create a pynamics system
system = System()
pynamics.set_system(__name__,system)
```

In [6]:

```
# Declare constants
lA_num = 0.2
lB_num = 0.2
lC_num = 0.22
lD_num = 0.2
lE_num = 0.04

lA = Constant(lA_num,'lA',system)
lB = Constant(lB_num,'lB',system)
lC = Constant(lC_num,'lC',system)
lD = Constant(lD_num,'lD',system)
lE = Constant(lE_num,'lE',system)
```

In [7]:

```
# Create differentiable state variables
qA,qA_d,qA_dd = Differentiable('qA',system)
qB,qB_d,qB_dd = Differentiable('qB',system)
qC,qC_d,qC_dd = Differentiable('qC',system)
qD,qD_d,qD_dd = Differentiable('qD',system)
```

In [8]:

```
# initial values
# qC = 11.97 creates fully extended, horizontal wing
initialvalues = {}
initialvalues[qA]=0*pi/180
initialvalues[qA_d]=0*pi/180
initialvalues[qB]=0*pi/180
initialvalues[qB_d]=0*pi/180

initialvalues[qC]=11.97*pi/180
initialvalues[qC_d]=0*pi/180
initialvalues[qD]=0*pi/180
initialvalues[qD_d]=0*pi/180
```

In [9]:

```
# Retrieve state variables in the order they are stored in the system
# Create a list of initial values ini0 in the order of the system's state variables
statevariables = system.get_state_variables()
ini0 = [initialvalues[item] for item in statevariables]
```

In [10]:

```
# Create frames
N = Frame('N')
A = Frame('A')
B = Frame('B')
C = Frame('C')
D = Frame('D')
```

In [11]:

```
# Declare N as the Newtonian (fixed) frame
system.set_newtonian(N)
```

In [12]:

```
# Rotate other frames about their local Z axes.
A.rotate_fixed_axis_directed(N,[0,0,1],qA,system)
B.rotate_fixed_axis_directed(A,[0,0,1],qB,system)
C.rotate_fixed_axis_directed(N,[0,0,1],qC,system)
D.rotate_fixed_axis_directed(C,[0,0,1],qD,system)
```

In [13]:

```
# Define rigid body kinematics
# position vectors
```

```python
p0 = 0*N.x #Normal
p1 = p0 + lA*A.x #Link A
p2A = p1 + lB*B.x #Link B - OUTPUT
p4 = p0 + lE*N.y #Link E
p3 = p4 + lC*C.x #Link C
p2C = p3 + lD*D.x #Link D - OUTPUT
```

In [14]:

```python
# Declare a list of points that will be used for plotting
points = [p0, p1, p2A, p3, p4]
```

In [15]:

```python
# Constraints
eq_vector = p2A-p2C
eq = []
eq.append((eq_vector).dot(N.x))
eq.append((eq_vector).dot(N.y))
eq_d=[(system.derivative(item)) for item in eq]
```

In [16]:

```python
# Turn constraint equations into a vector
eq_d_sym = sympy.Matrix(eq_d)
```

In [17]:

```python
# Turn qi and qd into sympy vectors
qi_sym = sympy.Matrix([qA_d, qC_d])
qd_sym = sympy.Matrix([qB_d, qD_d])
```

In [18]:

```python
# Take partial derivative of constraints with respect to independent and dependent variab
les
AA = eq_d_sym.jacobian(qi_sym)
BB = eq_d_sym.jacobian(qd_sym)
```

In [19]:

```python
# Solve for internal input/output Jacobian
Jin = -BB.inv()*AA
Jin.simplify()
Jin
```

Out[19]:

$$\begin{bmatrix} \dfrac{\dfrac{lA \cdot \sin(-qA + qC + qD)}{\sin(qA + qB - qC - qD)} - lB}{lB} & \dfrac{-lC \cdot \sin(qD)}{lB \cdot \sin(qA + qB - qC - qD)} \\ \dfrac{lA \cdot \sin(qB)}{lD \cdot \sin(qA + qB - qC - qD)} & \dfrac{-\left(\dfrac{lC \cdot \sin(qA + qB - qC)}{\sin(qA + qB - qC - qD)} + lD\right)}{lD} \end{bmatrix}$$

In [20]:

```python
# Solving for the dependent variables
qd2_sym = Jin*qi_sym
qd2_sym
```

Out[20]:

$$\left[-\frac{lC \cdot qC\_d \cdot \sin(qD)}{\quad} + \frac{qA\_d \cdot \left(\dfrac{lA \cdot \sin(-qA + qC + qD)}{\sin(qA + qB - qC - qD)} - lB\right)}{\quad} \right]$$

$$
\left[\begin{matrix} \ldots & \ldots \\ lB\cdot\sin(qA + qB - qC - qD) & lB \\ \frac{lA\cdot \mathrm{qA\_d}\cdot\sin(qB)}{lD\cdot\sin(qA + qB - qC - qD)} - \frac{\mathrm{qC\_d}\cdot\left(\frac{lC\cdot\sin(qA + qB - qC)}{\sin(qA + qB - qC - qD)} + lD\right)}{lD} & \end{matrix}\right]
$$

In [21]:

```
# Substitution dictionary to replace all occurrances of qB_d and qD_d
subs_sym = dict([(ii,jj) for ii,jj in zip(qd_sym,qd2_sym)])
subs_sym
```

Out[21]:

$$
\left\{\mathrm{qB\_d}: -\frac{lC\cdot \mathrm{qC\_d}\cdot\sin(qD)}{lB\cdot\sin(qA + qB - qC - qD)} + \frac{\mathrm{qA\_d}\cdot\left(\frac{lA\cdot\sin(-qA + qC + qD)}{\sin(qA + qB - qC - qD)} - lB\right)}{lB},\ \mathrm{qD\_d}: \frac{lA\cdot \mathrm{qA\_d}\cdot\sin(qB)}{lD\cdot\sin(qA + qB - qC - qD)} - \frac{\mathrm{qC\_d}\cdot\left(\frac{lC\cdot\sin(qA + qB - qC)}{\sin(qA + qB - qC - qD)} + lD\right)}{lD}\right\}
$$

In [22]:

```
# End effector position
pend = p2A
pend
```

Out[22]:

```
lA*A.x + lB*B.x
```

In [23]:

```
# End effector velocity
vend = pend.time_derivative()
vend = vend.subs(subs_sym)
vend
```

Out[23]:

```
lA*qA_d*A.y + lB*B.y*(qA_d - lC*qC_d*sin(qD)/(lB*sin(qA + qB - qC - qD)) + qA_d*(lA*sin(-
qA + qC + qD)/sin(qA + qB - qC - qD) - lB)/lB)
```

In [24]:

```
# Turn end effector velocity equations into a vector
vend_sym = sympy.Matrix([vend.dot(N.x), vend.dot(N.y)])
```

In [25]:

```
# JACOBIAN
# Take partial derivative of end effector velocities with respect to independent variable
s
J = vend_sym.jacobian(qi_sym)
J.simplify()
J
```

Out[25]:

$$
\left[\begin{matrix} \frac{-lA\cdot\sin(qB)\cdot\sin(qC + qD)}{\sin(qA + qB - qC - qD)} & \frac{lC\cdot\sin(qD)\cdot\sin(qA + qB)}{\sin(qA + qB - qC - qD)} \\ \frac{lA\cdot\sin(qB)\cdot\cos(qC + qD)}{\sin(qA + qB - qC - qD)} & \frac{-lC\cdot\sin(qD)\cdot\cos(qA + qB)}{\sin(qA + qB - qC - qD)} \end{matrix}\right]
$$

4. Solve for an Initial Condition

Select or Solve for a valid initial condition that represents the system in the middle of a typical gait, when it is both moving and when forces are being applied to it (or to the world by it)

Despite the fact that you will be using a symbolic representation, you still need to solve for a valid initial condition if your device is a "parallel mechanism." This may be done using a nonlinear solver such as scipy.optimize.minimize

In [26]:

```
# Identify independent and dependent variables
qi = [qA, qC]
qd = [qB, qD]
```

In [27]:

```
# Create a copy of symbolic constants dictionary and add the initial value of qi to it
constants = system.constant_values.copy()
defined = dict([(item,initialvalues[item]) for item in qi])
constants.update(defined)
```

In [28]:

```
# Substitute constants in equation
eq = [item.subs(constants) for item in eq]
```

In [29]:

```
# Convert to numpy array
# Sum the error
error = (numpy.array(eq)**2).sum()
```

In [30]:

```
# Convert to a function that scipy can use.
# Sympy has a "labmdify" function that evaluates an expression, but scipy needs a slightl
y different format.
f = sympy.lambdify(qd,error)

def function(args):
    return f(*args)
```

In [31]:

```
# Take the derivative of the equations to linearize with regard to the velocity variables
guess = [initialvalues[item] for item in qd]
result = scipy.optimize.minimize(function,guess)
if result.fun>1e-3:
    raise(Exception("out of tolerance"))
ini = []
for item in system.get_state_variables():
    if item in qd:
        ini.append(result.x[qd.index(item)])
    else:
        ini.append(initialvalues[item])
points = PointsOutput(points, constant_values=system.constant_values)
points.calc(numpy.array([ini0,ini]))
#points.plot_time()
```

```
2021-02-28 14:00:15,505 - pynamics.output - INFO - calculating outputs
2021-02-28 14:00:15,506 - pynamics.output - INFO - done calculating outputs
```

Out[31]:

```
array([[[0.        , 0.        ],
        [0.2       , 0.        ],
        [0.4       , 0.        ],
```

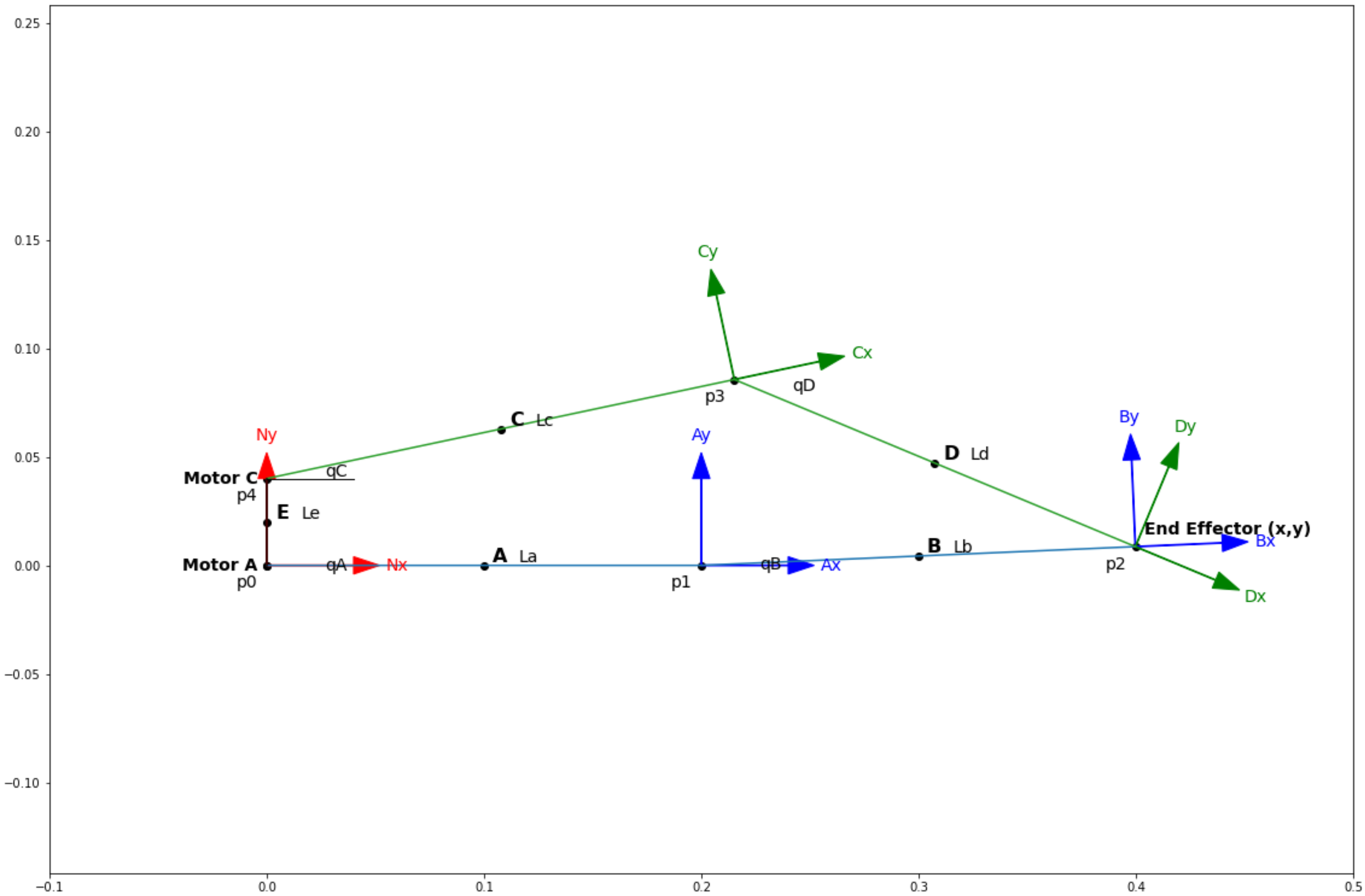
```
        [0.21521639, 0.08562789],
        [0.        , 0.04      ]],

       [[0.        , 0.        ],
        [0.2       , 0.        ],
        [0.39981349, 0.00863542],
        [0.21521639, 0.08562789],
        [0.        , 0.04      ]]])
```

5. Plot the system in this position

In [32]:

```
# Valid initial condition
x = numpy.array([points.y[1][0][0], points.y[1][1][0], points.y[1][2][0], points.y[1][3
][0], points.y[1][4][0],0])
y = numpy.array([points.y[1][0][1], points.y[1][1][1], points.y[1][2][1], points.y[1][3
][1], points.y[1][4][1],0])
prettyplot(x,y)
```



6. Define Force Vectors

From your biomechanics-based specifications, define one or more force vector estimates (one for each end effector) that the system should be expected to experience. Consider including, based on your research

1. **the force of gravity exerted by the mass of a “payload” or the main body of the robot.**
2. **the acceleration the system experiences during a typical gait**
3. **ground reaction forces measured from biomechanics studies.**

In [33]:

```
# End effector force
Fend_num = numpy.array([0, -2*0.5*9.81]).reshape([2,1])
Fend_num
```

Out[33]:

```
array([[ 0.  ],
       [-9.81]])
```

In [34]:

```
# End effector velocity
vend_num = numpy.array([0, -2*45*pi/180*0.4*2]).reshape([2,1])
vend_num
```

Out[34]:

```
array([[ 0.        ],
       [-1.25663706]])
```

7. Calculate Input Forces

Calculate the force or torque required at the input to satisfy the end-effector force requirements

In [35]:

```
# Input force symbolic
Fin = sympy.Matrix(J.transpose()*Fend_num)
Fin
```

Out[35]:

$$\left[\begin{matrix}\frac{-9.81\cdot lA\cdot\sin(qB)\cdot\cos(qC + qD)}{\sin(qA + qB - qC - qD)}\\ \frac{9.81\cdot lC\cdot\sin(qD)\cdot\cos(qA + qB)}{\sin(qA + qB - qC - qD)}\end{matrix}\right]$$

In [36]:

```
# Input force numeric
Fin_num = Fin.subs([(qA, ini[0]),(qB, ini[1]),(qC, ini[2]),(qD, ini[3]),(lA, lA_num),(lB
, lB_num),(lC, lC_num),(lD, lD_num)])
Fin_num
```

Out[36]:

$$\left[\begin{matrix}-0.184204845318843\\ -2.88544801602279\end{matrix}\right]$$

8. Calculate Input Velocity

Estimate the velocity of the end-effector in this configuration. Using the Jacobian, calculate the speed required by the input(s) to achieve that output motion.

This may not be directly solvable based on your device kinematics; an iterative guess-and-check approach is ok.

In [37]:

```
# Input velocity symbolic
vin = sympy.Matrix(J.inv()*vend_num)
vin
```

Out[37]:

$$\left[\begin{matrix}\frac{1.25663706143592\cdot\sin(qA + qB)\cdot\sin(qA + qB - qC - qD)}{-lA\cdot\sin(qB)\cdot\sin(qA + qB)\cdot\cos(qC + qD) + lA\cdot\sin(qB)\cdot\sin(qC + qD)\cdot\cos(qA + qB)}\\ \frac{1.25663706143592\cdot\sin(qC + qD)\cdot\sin(qA + qB - qC - qD)}{-lC\cdot\sin(qD)\cdot\sin(qA + qB)\cdot\cos(qC + qD) + lC\cdot\sin(qD)\cdot\sin(qC + qD)\cdot\cos(qA + qB)}\end{matrix}\right]$$

$$-lC \cdot \sin(qD) \cdot \sin(qA + qB) \cdot \cos(qC + qD) + lC \cdot \sin(qD) \cdot \sin(qC + qD) \cdot \cos(qA + qB)$$

In [38]:

```python
# Input velocity numeric
vin_num = vin.subs([(qA, ini[0]),(qB, ini[1]),(qC, ini[2]),(qD, ini[3]),(lA, lA_num),(lB
, lB_num),(lC, lC_num),(lD, lD_num)])
vin_num
```

Out[38]:

$$\begin{bmatrix} -6.28318530717959 \\ -3.87122427194665 \end{bmatrix}$$

9. Compute Required Power

Finally, using the two estimates about force and speed at the input, compute the required power in this configuration.

In [39]:

```python
# Input power symbolic
Pin = sympy.Matrix([[Fin[0]*vin[0]],[Fin[1]*vin[1]]])
Pin
```

Out[39]:

$$\begin{bmatrix} \dfrac{-12.3276095726863 \cdot lA \cdot \sin(qB) \cdot \sin(qA + qB) \cdot \cos(qC + qD)}{-lA \cdot \sin(qB) \cdot \sin(qA + qB) \cdot \cos(qC + qD) + lA \cdot \sin(qB) \cdot \sin(qC + qD) \cdot \cos(qA + qB)} \\ \dfrac{12.3276095726863 \cdot lC \cdot \sin(qD) \cdot \sin(qC + qD) \cdot \cos(qA + qB)}{-lC \cdot \sin(qD) \cdot \sin(qA + qB) \cdot \cos(qC + qD) + lC \cdot \sin(qD) \cdot \sin(qC + qD) \cdot \cos(qA + qB)} \end{bmatrix}$$

In [40]:

```python
# Input power numeric
Pin_num = Pin.subs([(qA, ini[0]),(qB, ini[1]),(qC, ini[2]),(qD, ini[3]),(lA, lA_num),(lB
, lB_num),(lC, lC_num),(lD, lD_num)])
Pin_num
```

Out[40]:

$$\begin{bmatrix} 1.15739317761864 \\ 11.1702163950677 \end{bmatrix}$$

Discussion

1. **How many degrees of freedom does your device have? How many motors? If the answer is not the same, what determines the state of the remaining degrees of freedom? How did you arrive at that number?**

Our system has two degrees of freedom. We would need two motors or servos to act as inputs to our system. It is possible to use one motor and add a constraint using a spring and damper or a gear to limit the motion created by our system. Constraining one of the inputs still allows movement from the other input, but only along one distinct trajectory. Adding one more constraint completely locks the mechanism. These two additional constraints indicate that there are two degrees of freedom in the mechanism.

1. **If your mechanism has more than one degree of freedom, please describe how those multiple degrees of freedom will work togehter to create a locomotory gait or useful motion. What is your plan for synchonizing, especially if passive energy storage?**

For our system the two degrees of freedom allows us to create a downward thrust of a birds wing. As the two

For our system the two degrees of freedom allows us to create a downward thrust of a birds wing. As the two inputs rotate at different rates this will allow a flapping motion where the wing is extended fully on downward flap and bent during the upward flap. To accurately control the two inputs, our system will need the use of stepper motors and microcontroller. Alternatively, the mechanism can be further constrained with a gear to relate the timing of the inputs.

1. How did you estimate your expected end-effector forces

Our model organism is a mallard duck. Ducks have a mass of 1-1.3kg [1]. Our prototype's mass targets the 500g mass of a typical racing quadrotor [4], which is half the mass of a duck. According to [3], smaller starling birds apply 4 times their body weight in thrust at takeoff. Our prototype will use an assisted takeoff to remain tractable but target 2 times body weight in thrust to accommodate sustained flight. To err on the side of applying more force than necessary, the force will be concentrated at the wing-tip end-effector.

In the fully extended and horizontal wing position, the end-effector applies the force in the negative y direction. The magnitude of the end-effector force becomes:

End-effector force = 2 *(0.5kg* 9.81m/s^2) = 9.81 N

This yields a maximum input torque of 2.9 Nm in one motor, which is achievable with some motors and servos, but likely not feasible within our mass constraint. This will require a change in geometry or motor selection.

1. How did you estimate your expected end-effector speeds

Ducks have a wingspan of 0.82-0.95m [1]. Our prototype's shape targets the low end of the wingspan to remain tractable with a fully extended wing radius of 0.4m. According to [2], birds flap their wings 10 times per second during sustained flight. To remain tractable, our prototype will target 5 wingbeats per second. Our prototype will target a wing range of motion of 45 degrees. One wingbeat consists of one upward arc through 45 degrees and one downward arc through 45 degrees.

In the fully extended and horizontal wing position, the end-effector travels only in the negative y direction. The magnitude of the end-effector velocity becomes:

End-effector velocity = 5hz *(45deg* pi / 180deg *r* 2arc) = 3.14 m/s

This yields a maximum input velocity of 15.7 rad/s and a maximum input power of 27.9 W in one motor, which is achievable with most motors and feasible within our mass constraint.

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[3] KATHLEEN D. EARLS, "KINEMATICS AND MECHANICS OF GROUND TAKE-OFF IN THE STARLING STURNIS VULGARIS AND THE QUAIL COTURNIX COTURNIX," Dec. 1999.

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