# Introduction to Structural Equation Models

**Hugo Saiz** 

Institute of Plant sciences - University of Bern





## **BEF Research**

**Europe PMC Funders Group Author Manuscript** 

Adv Ecol Res. Author manuscript; available in PMC 2020 January 06.

Published in final edited form as:

Adv Ecol Res. 2019; 61: 1-54. doi:10.1016/bs.aecr.2019.06.001.

A multitrophic perspective on biodiversity-ecosystem functioning research

nature ecology & evolution

ARTICLES
https://doi.org/10.1038/s41559-020-1280-9

Check for updates

The results of biodiversity-ecosystem functioning experiments are realistic

### **PROCEEDINGS B**

rspb.royalsocietypublishing.org

The strength of the biodiversity – ecosystem function relationship depends on spatial scale



Biodiversity enhances ecosystem multifunctionality across trophic levels and habitats

# ECOLOGY LETTERS

Ecology Letters, (2020) 23: 757-776

doi: 10.1111/ele.13456

REVIEWS AND SYNTHESES

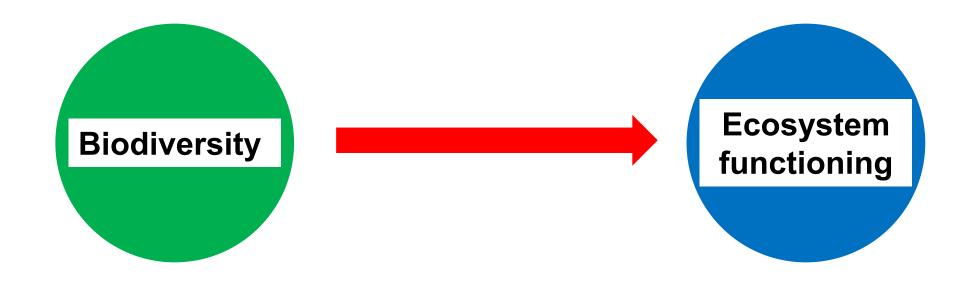
Scaling-up biodiversity-ecosystem functioning research



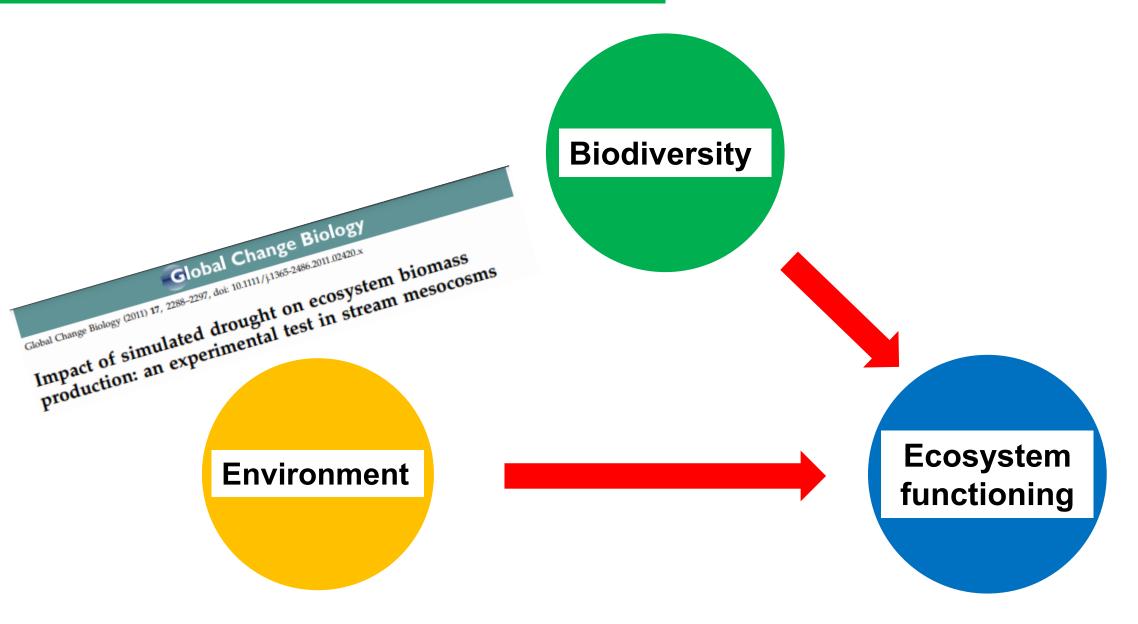


High plant diversity is needed to maintain ecosystem services

# **Biodiversity – Ecosystem Functioning**



# **Environment – Ecosystem Functioning**



# **Environment - Biodiversity**

### Global Change Biology

Global Change Biology (2016) 22, 2329-2352, doi: 10.1111/gcb.13160

### SPECIAL FEATURE

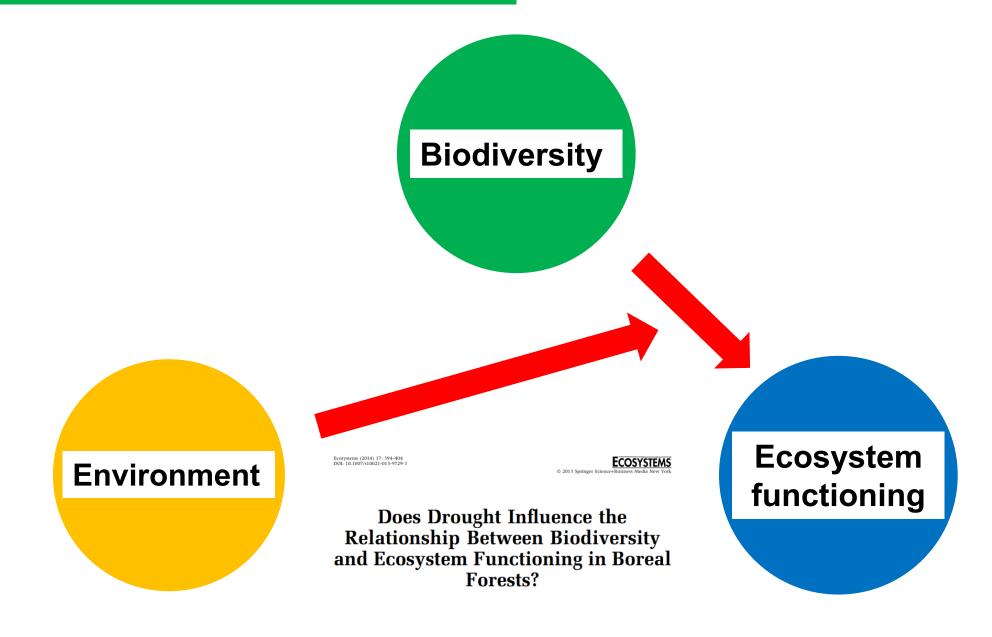
The impacts of increasing drought on forest dynamics, structure, and biodiversity in the United States



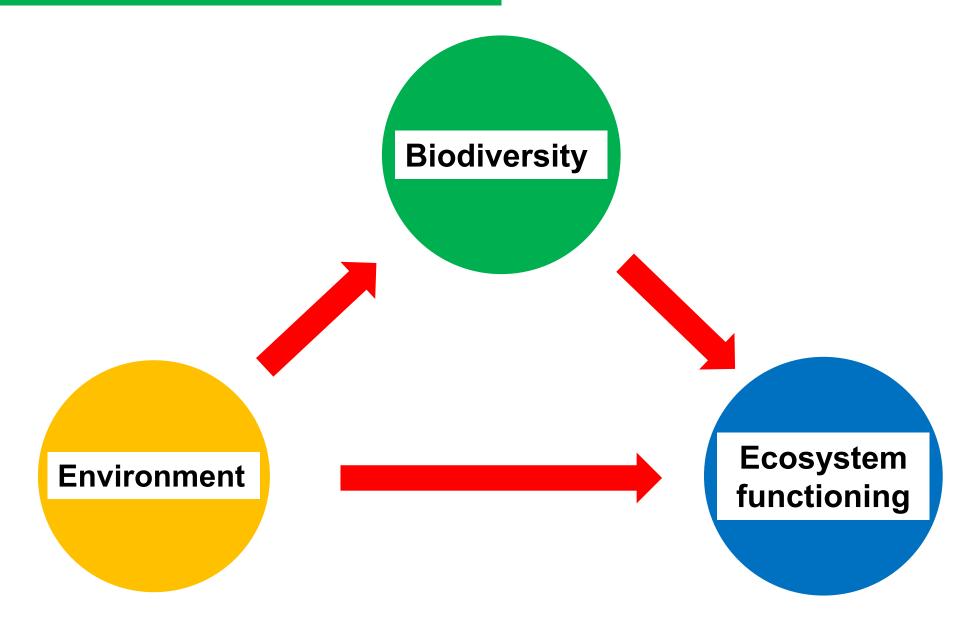
**Biodiversity** 

**Ecosystem** functioning

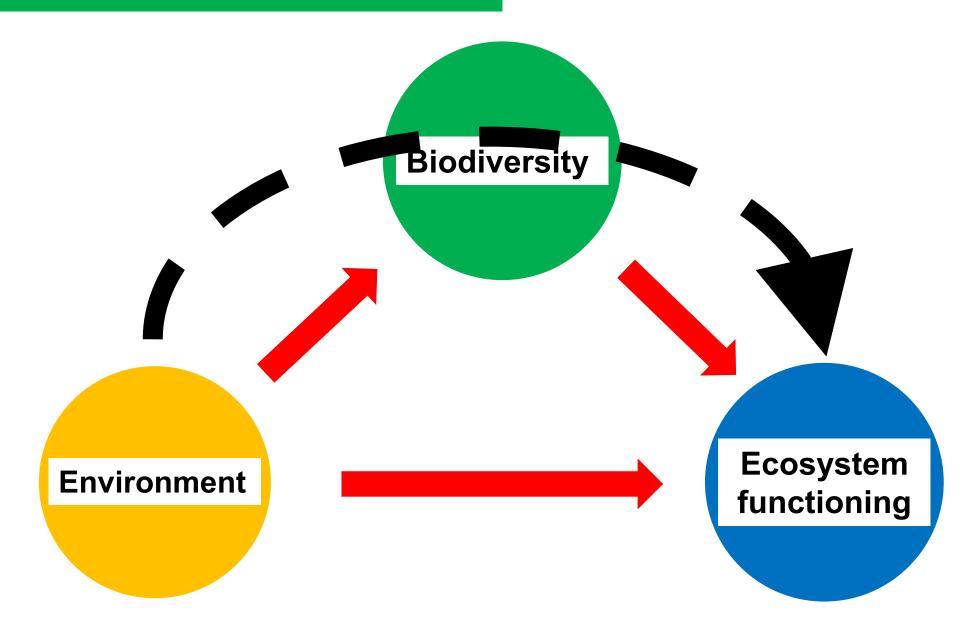
# **Environment \* Biodiversity**



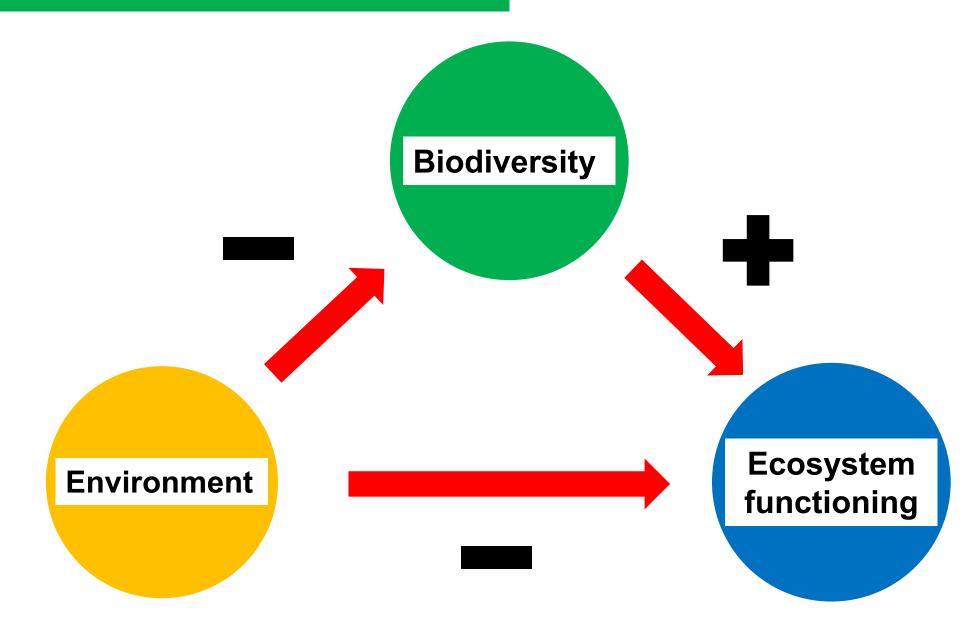
# **Environment – Biodiversity – Functioning**

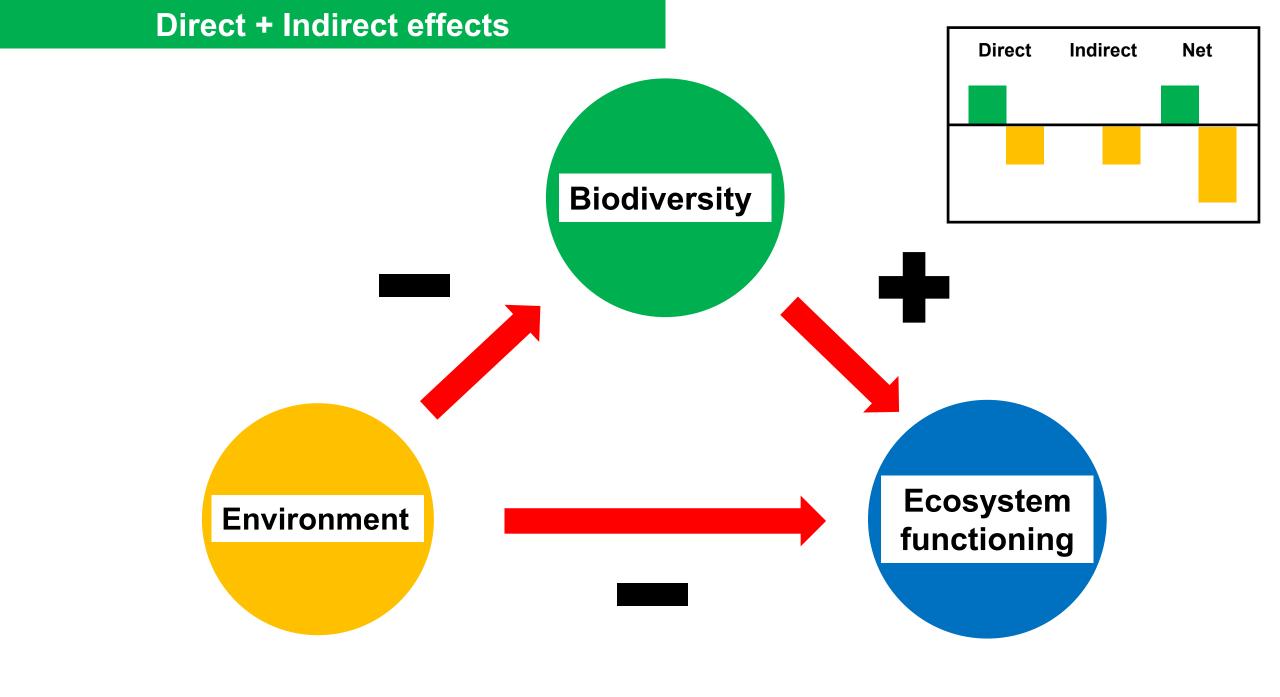


# **Indirect effects**



# **Direct + Indirect effects**





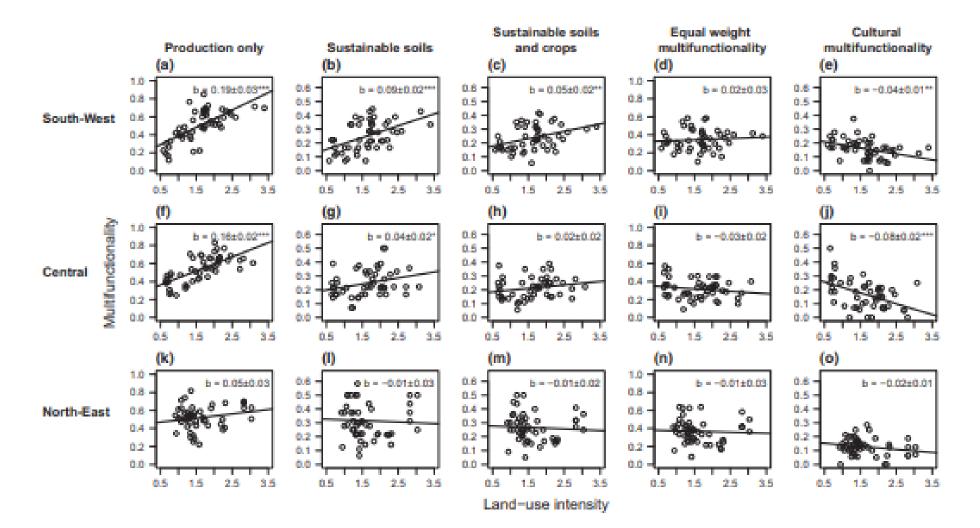
# ECOLOGY LETTERS

Ecology Letters, (2015) 18: 834-843

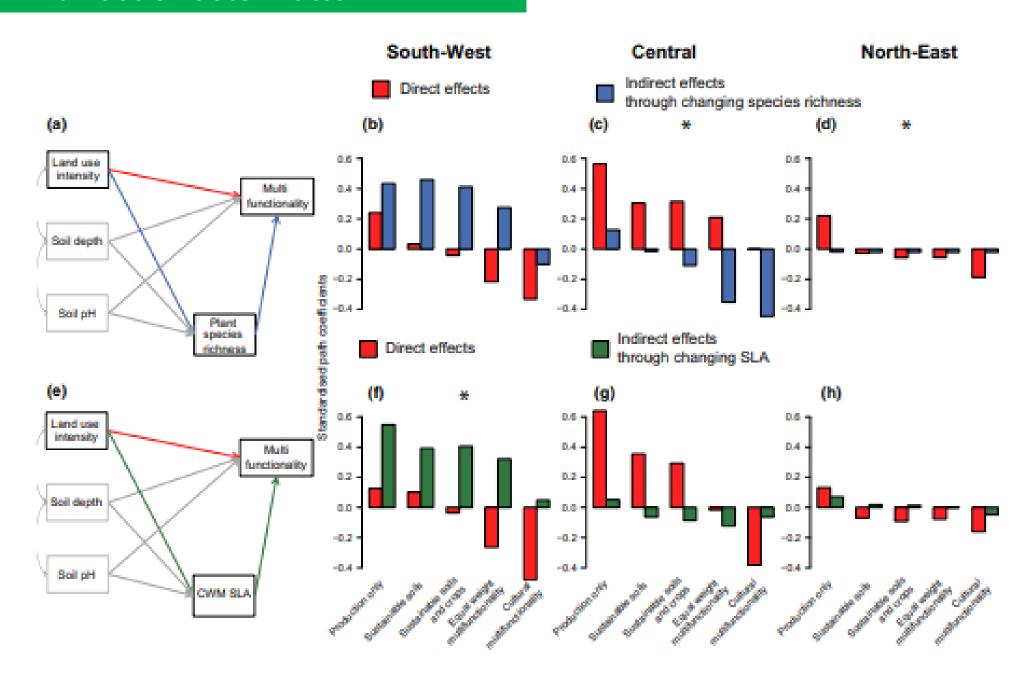
doi: 10.1111/ele.12469

LETTER

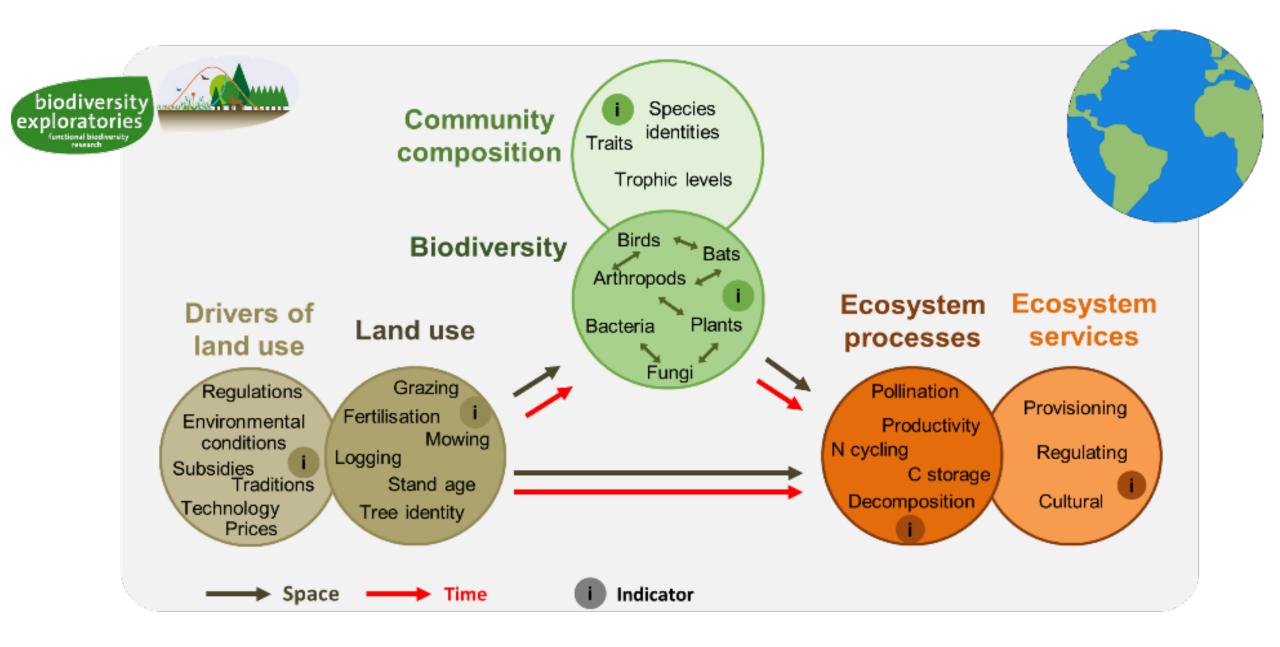
Land use intensification alters ecosystem multifunctionality via loss of biodiversity and changes to functional composition



# **Indirect effects matter**



# **Never-ending complexity**



# **Everything depends on everything**



Phylogenetic, functional, and taxonomic richness have both positive and negative effects on ecosystem multifunctionality

# **LETTER**

https://doi.org/10.1038/s41586-018-0627-8

Biodiversity increases and decreases ecosystem stability



Oikos 118: 1892-1900, 2009

doi: 10.1111/j.1600-0706.2009.17556.x, © 2009 The Authors. Journal compilation © 2009 Oikos Subject Editor: Ulrich Brose. Accepted 26 May 2009

Context dependency of relationships between biodiversity and ecosystem functioning is different for multiple ecosystem functions

# **Unraveling the complexity**

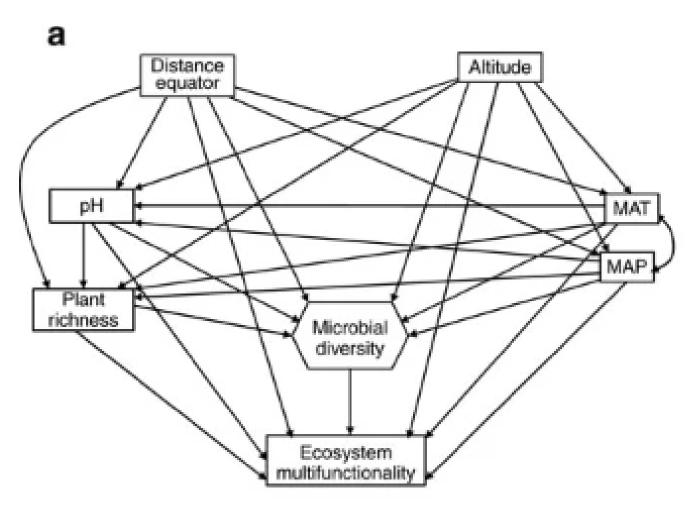
### **ARTICLE**

Received 25 Jun 2015 | Accepted 23 Dec 2015 | Published 28 Jan 2016

DOI: 10.1038/ncomms10541

OPE

Microbial diversity drives multifunctionality in terrestrial ecosystems



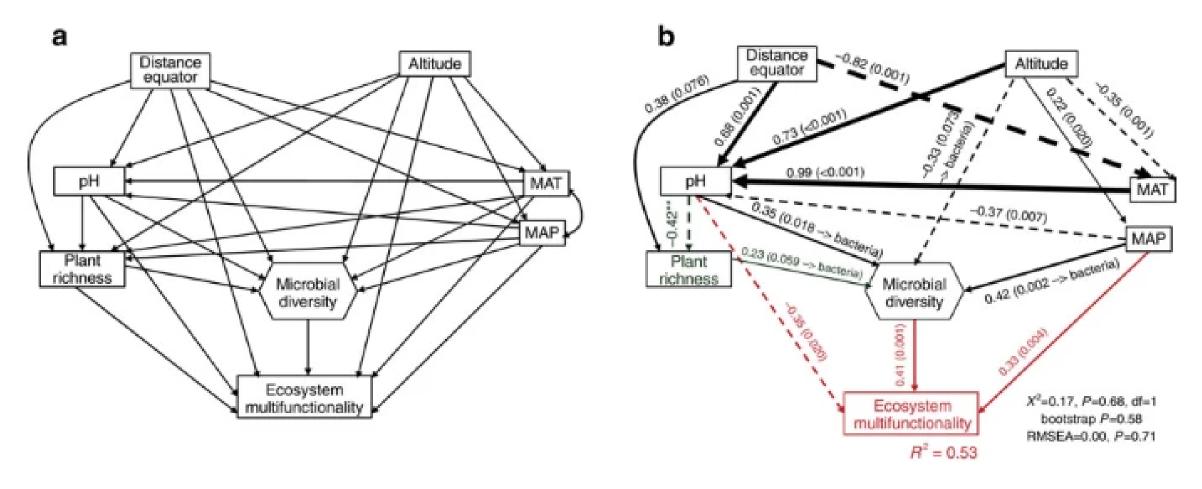
Change ecology for a more relaxing science

Or

Try to disentangle the ecological mess

# Unraveling the complexity

Figure 4: Direct and indirect effects of space, climate, soil pH, plant richness and microbial diversity on ecosystem multifunctionality in global drylands.



# Structural Equation Models

(SEM)

Special Section: Observational Studies

# Structural Equation Modeling for Observational Studies

JAMES B. GRACE, United States Geological Survey National Wetlands Research Center, 700 Cajundome Boulevard, Lafayette, LA 70506, USA



Jim Grace

Ecological Monographs, 80(1), 2010, pp. 67–87 © 2010 by the Ecological Society of America

## On the specification of structural equation models for ecological systems

James B. Grace, 1,4 T. Michael Anderson, 2,5 Han Olff, 2 and Samuel M. Scheiner 3

Ecology, 101(4), 2020, e02962Published 2019. This article is a U.S. Government work and is in the public domain in the USA.

Scientist's guide to developing explanatory statistical models using causal analysis principles





SCIENCE

**PRODUCTS** 

Maps, data,

NEWS Releases, CONNECT Contact, chat, ABOUT Organization, jobs, budget

Search C



Wetland and Aquatic Research Center

# Quantitative Analysis Using Structural Equation Modeling

Overview

Related Science

USGS scientists have been involved for a number of years in the development and use of Structural Equation Modeling (SEM). This methodology represents an approach to statistical modeling that focuses on the study of complex cause-effect hypotheses about the mechanisms operating in systems. SEM is increasingly used in ecological and environmental studies and this site seeks to provide educational materials related to that enterprise. This site serves up tutorials, exercises, and examples designed to help researchers learn and apply SEM. Please click on the "Science" tab to learn more.

### How to Use This Site

This site provides tutorials, examples, and exercises for those wishing to learn basic or specialized structural equation modeling methods. A description of what has been added and when can be found in the document **What's**New.

### Contact

Comments on existing tutorials and suggestions for additional tutorials can be sent to sem@usgs.gov. Please note that while emails to this address will be read, we cannot provide individual replies given time constraints. For this we apologize, but we do hope the materials provided will be helpful.



### Status - Active

### Contacts

### James Grace

Research Ecologist
Wetland and Aquatic Research Center

Email: gracej@usgs.gov Phone: 337-298-1671

### Explore More Science:

tutorials structural equation modeling SEM quantitative analysis statistics



Jon Lefcheck

# Methods in Ecology and Evolution

British Ecological Society

Methods in Ecology and Evolution 2015

doi: 10.1111/2041-210X.12512

### **APPLICATION**

PIECEWISESEM: Piecewise structural equation modelling in R for ecology, evolution, and systematics

Jonathan S. Lefcheck\*

# SAMPLE(ECOLOGY)

RANDOM THOUGHTS ON ECOLOGY, BIODIVERSITY, AND SCIENCE IN GENERAL

ABOUT ME PUBLICATIONS CV PHOTOS CONTACT BLOG

### TEACHING

### STRUCTURAL EQUATION MODELING

This introductory short course is designed to familiarize participants with the philosophy and practice of structural equation modeling / confirmatory path analysis. All course materials, including lectures, datasets, and R scripts, are available for download below. Thanks to Jarrett Byrnes for generous donation of lecture slides: visit his site for more materials!

### READINGS

Grace et al. (2013) "Guidelines for a graph-theoretic implementation of structural equation modeling." *Ecosphere*. PDF

Search ...

### - LATEST TWEETS -

RT @stefanako71: Imagine doing something so wacky that prompts Pauly and Hilborn to be in total agreement. 2 days ago

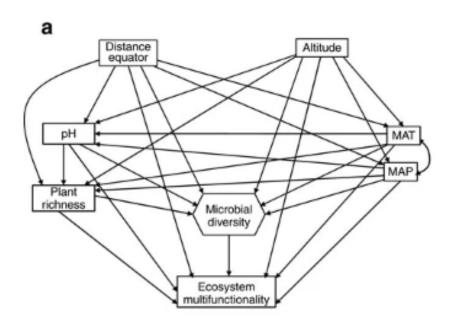
RT @bethaniey15: Don't forget to email me before 16 May if you would like to apply for this PhD opportunity on understanding the role of #s...3 days ago



# **Structural Equation Models**

Structural Equation Model is a **modelling framework** for building and evaluating hypotheses about cause-effect connections in systems

It is a **system level** approach to explore the interrelations among the components of the study system



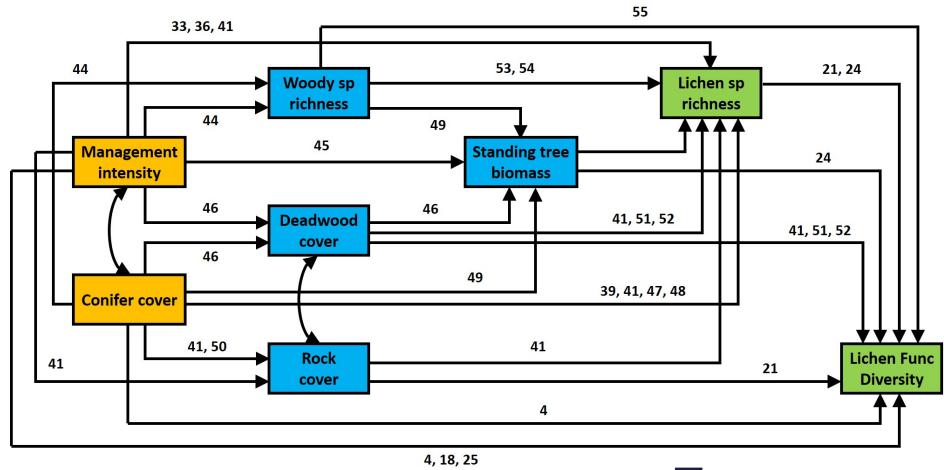


# Structural Equation Model is not a proof of causality



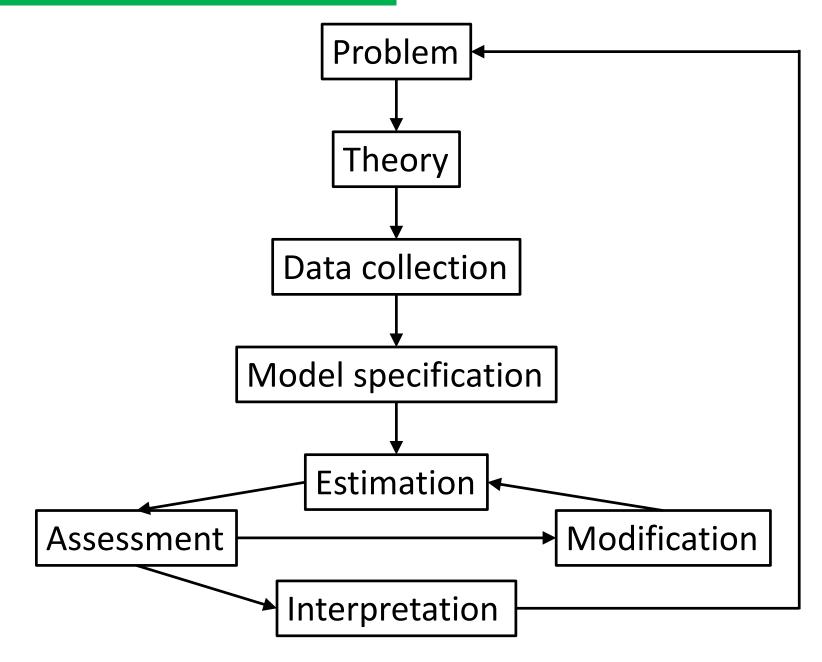
# Importance of ecological knowledge

All the causal relations must be built based on theoretical knowledge



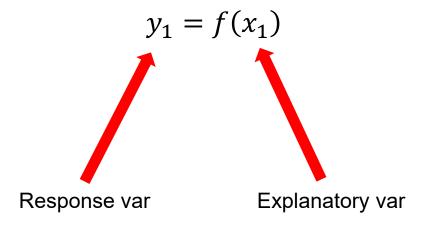


# Modelling process

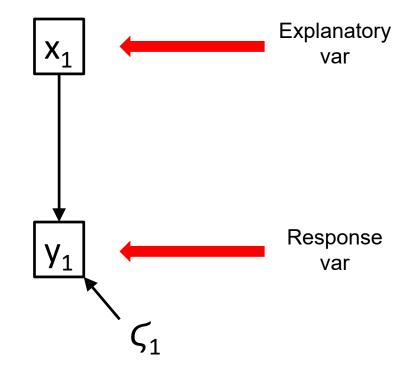


# **Graphical models**

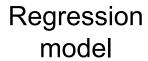
# **Equational form**

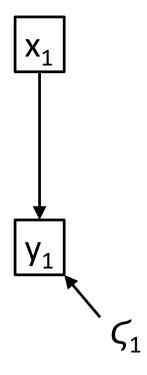


# Graphical form

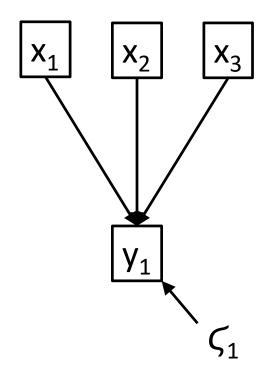


# **Graphical models for stats**





Mult Regression model





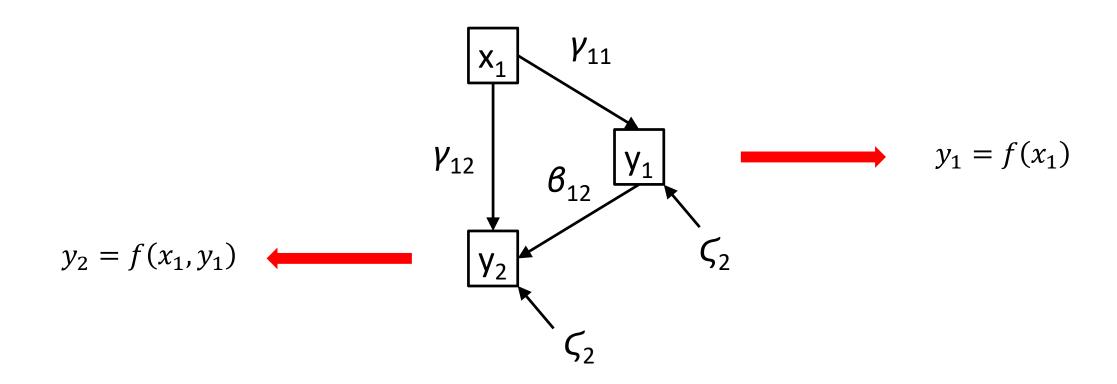
$$y_1 = a + x_1$$

$$y_1 = a + x_1 + x_2 + x_3$$

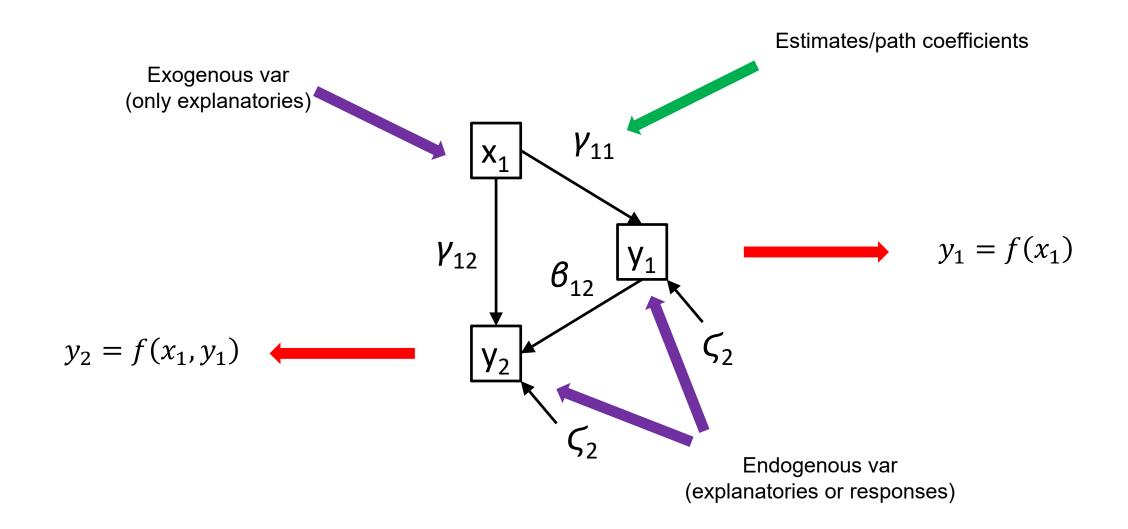
$$x_1 \sim x_2$$

# **SEM** test multiple equations

Linear-normal relations

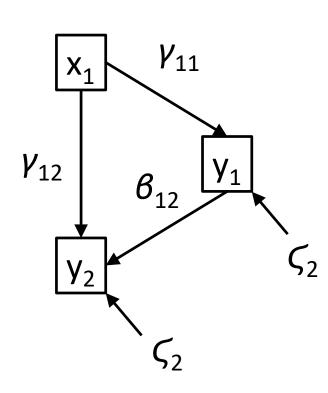


# **SEM** test multiple equations



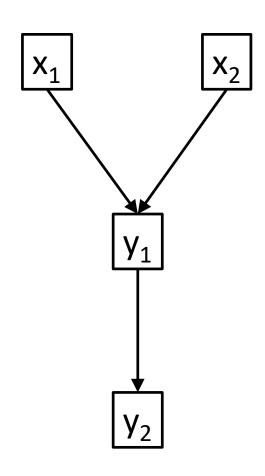
# **SEM** test multiple equations

Direct effect  $X_1 \rightarrow Y_2 = \gamma_{12}$ 

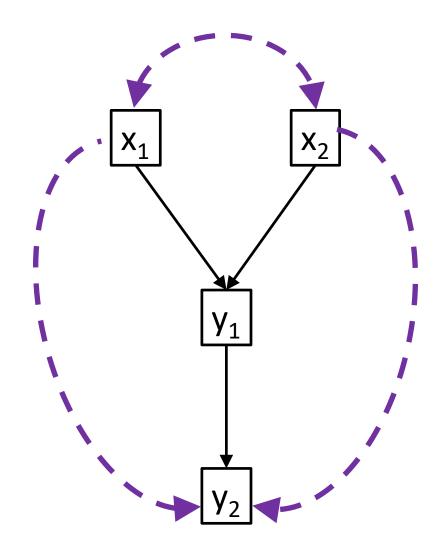


Indirect effect  $X_1 \rightarrow Y_2 = \gamma_{11} * \beta_{12}$ 

# **Relation + Absence of relations**



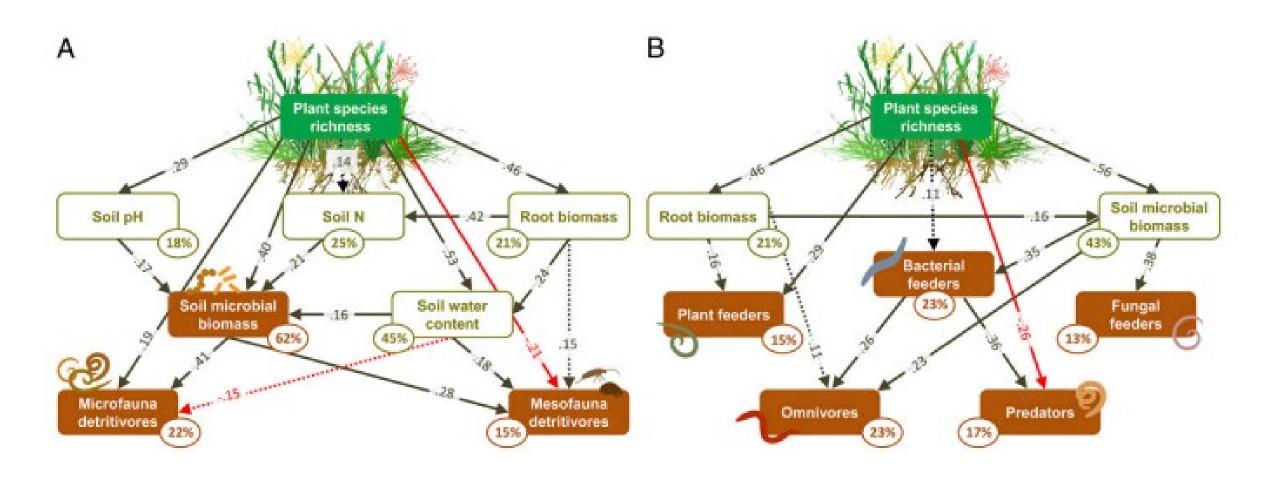
Non saturated



Saturated (if included)

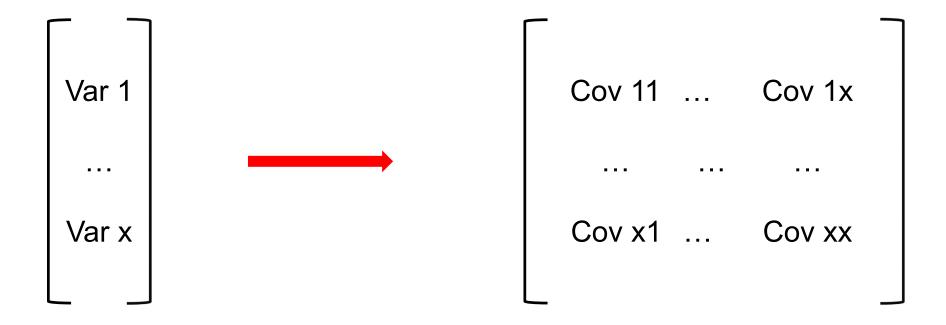
# Nice pictures for your work!

Plant diversity effects on soil food webs are stronger than those of elevated CO<sub>2</sub> and N deposition in a long-term grassland experiment

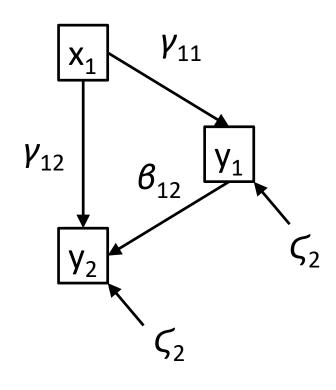


# **SEM** is based in covariance matrix

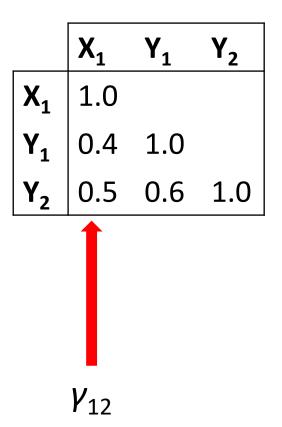
Covariance matrix is a square matrix including the covariance between elements in a vector



# **SEM** is based in covariance matrix

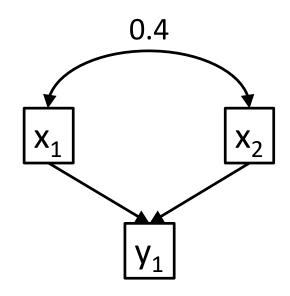


Standardized covariances (correlation)



# **Path calculation**

Correlations are just covariances

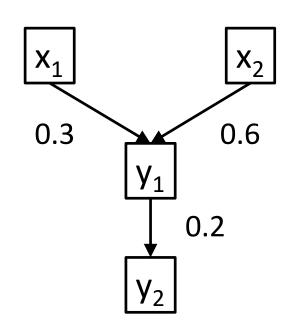


	X <sub>1</sub>	$X_2$	Y <sub>1</sub>
X <sub>1</sub>	1.0		
X <sub>2</sub>	0.4	1.0	
Y <sub>1</sub>	0.5	0.6	1.0

# **Path calculation**

Correlations are just covariances

Direct paths are just covariances



	$X_1$	$X_2$	$Y_1$	Y <sub>2</sub>
X <sub>1</sub>	1.0			
$X_2$	0	1.0		
Y <sub>1</sub>	0.3	0.6	1.0	
Y <sub>2</sub>	0.06	0.12	0.2	1.0

#### **Path calculation**

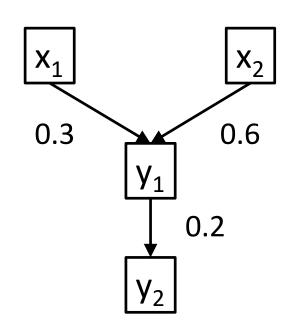
Correlations are just covariances

Direct paths are just covariances

Compound paths are the product of covariances

$$X_1 \rightarrow Y_2 = 0.3 * 0.2 = 0.06$$

$$X_2 \rightarrow Y_2 = 0.6 * 0.2 = 0.12$$

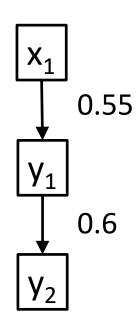


	$X_1$	$X_2$	$Y_1$	Y <sub>2</sub>
X <sub>1</sub>	1.0			
X <sub>2</sub>	0	1.0		
Y <sub>1</sub>	0.3	0.6	1.0	
Y <sub>2</sub>	0.06	0.12	0.2	1.0

# **Conditional independence**

Compound paths can be different from covariance

$$X_1 \rightarrow Y_2 = 0.55 * 0.6 = 0.33 \neq 0.5$$



	$X_1$	$Y_1$	Y <sub>2</sub>
X <sub>1</sub>	1.0		
Y <sub>1</sub>	0.55	1.0	
Y <sub>2</sub>	0.5	0.6	1.0

# **Conditional independence**

Compound paths can be different from covariance

$$X_1 \rightarrow Y_2 = 0.55 * 0.6 = 0.33 \neq 0.5$$

Partial effect 0.24

 $\begin{array}{c|c}
x_1 \\
0.55 \\
y_1 \\
\hline
y_2
\end{array}$ 

In this case,  $x_1$  and  $y_2$  are not conditional independent and additional paths are required

$$X_1 \rightarrow Y_2 = 0.55 * 0.47 + 0.24 = 0.26 + 0.24 = 0.5$$

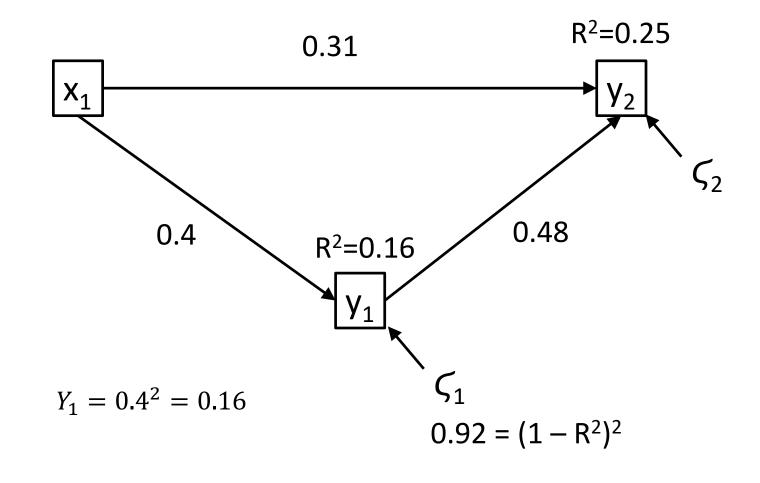
$$Y_1 \rightarrow Y_2 = 0.47 + 0.55 * 0.24 = 0.47 + 0.13 = 0.6$$

	X <sub>1</sub>	<b>Y</b> <sub>1</sub>	Y <sub>2</sub>
$X_1$	1.0		
Y <sub>1</sub>	0.55	1.0	
Y <sub>2</sub>	0.5	0.6	1.0

Also considers correlations

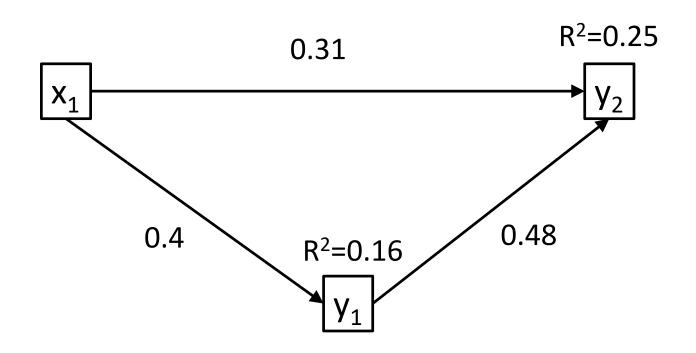
#### **Paths and errors**

Errors represent influence of unmodeled factors



#### **Total effects**

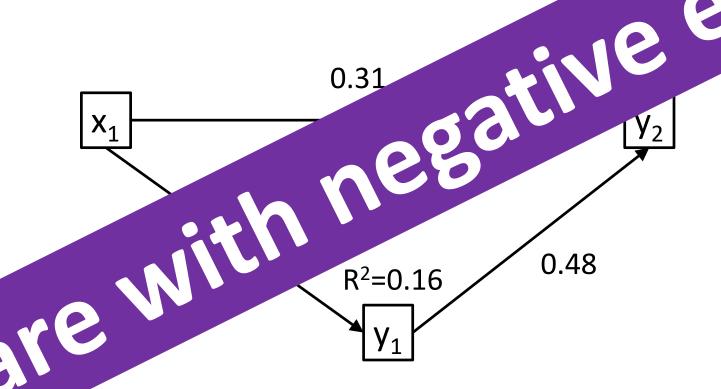
Total effects of one variable is the sum of all direct and indirect paths



$$Y_2 = 0.31 + 0.4 * 0.48 = 0.31 + 0.19 = 0.5$$

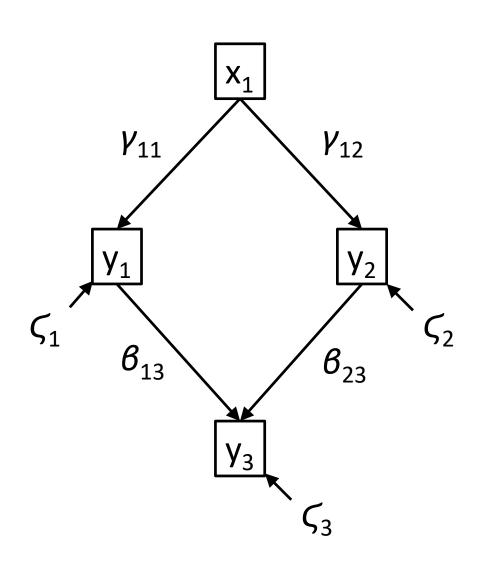
#### **Total effects**

Total effects of one variable is the sum of all direct and indirect



 $Y_2 = 0.31 + 0.4 * 0.48 = 0.31 + 0.19 = 0.5$ 

# **Calculating SEM**

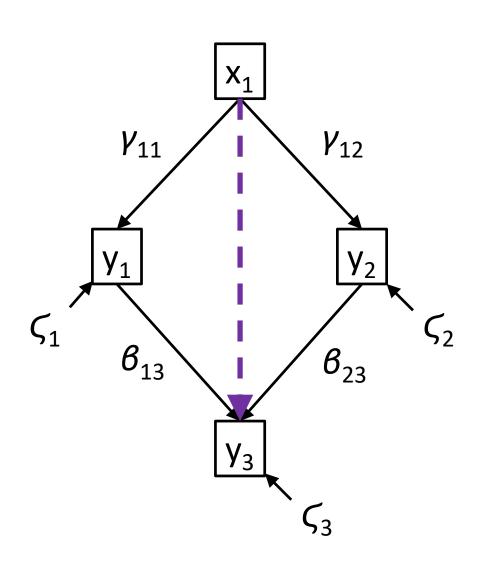


$$y_1 = a + \gamma_{11} x_1 + C_1$$

$$y_2 = a + \gamma_{12}x_1 + \mathcal{C}_2$$

$$y_3 = a + \beta_{13}y_1 + \beta_{23}y_2 + \zeta_3$$

# **Calculating SEM**



$$y_1 = a + \gamma_{11} x_1 + \varsigma_1$$

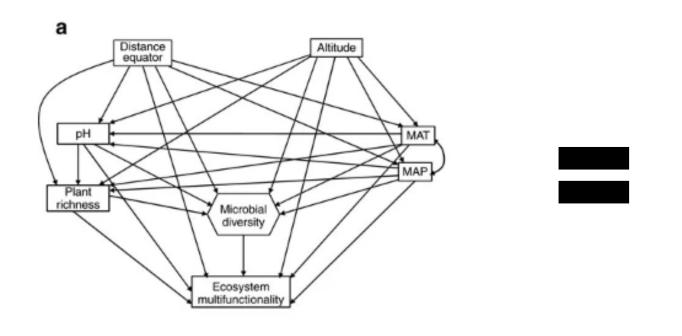
$$y_2 = a + \gamma_{12} x_1 + C_2$$

$$y_3 = a + \beta_{13}y_1 + \beta_{23}y_2 + \varsigma_3$$

$$X_1 \rightarrow Y_3 = \gamma_{11} * \beta_{13} + \gamma_{12} * \beta_{23}$$

# Hoping for no significance

# We accept SEM when p > 0.05





#### Global vs Local estimation





**Global estimation** 

Considers the model as a whole

Compares observed covariances to model covariances

Allows correlation between variables

lavaan R-package

#### **Local estimation**

Breaks the model in sub-models

Evaluates the importance of missing paths for the model

Allows generalized mixed models

piecewiseSEM R-package

#### Getting started with lavaan



All examples used here come from:

https://www.usgs.gov/centers/wetland-and-aquatic-research-center/science/quantitative-analysis-using-structural-equation?qt-science center objects=0#qt-science center objects

Big thank you to James Grace for open his teaching material to anyone interested in working with SEMs

# Getting started with lavaan

Working with *lavaan* requires R statistical software

Prior to SEM analysis:

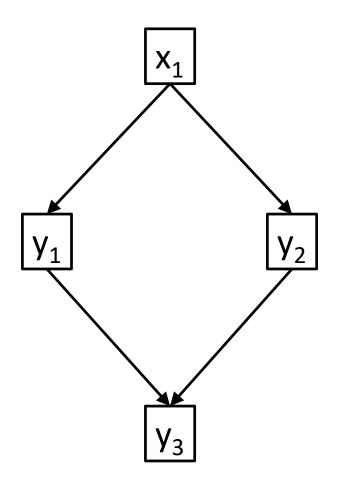
Install and load lavaan library

Load the data

Explore the data

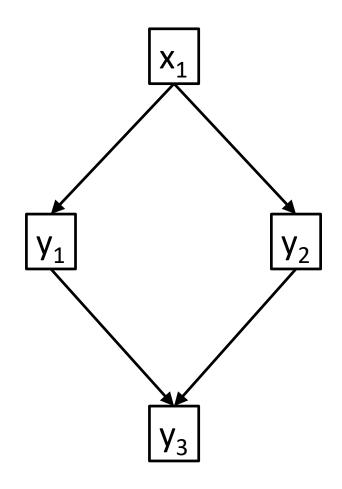
Standardize and transform the data when necessary to meet model assumptions (assumptions for linear modelling)

# Doing SEM in *lavaan*



# Step 1: Specify model

# Doing SEM in *lavaan*

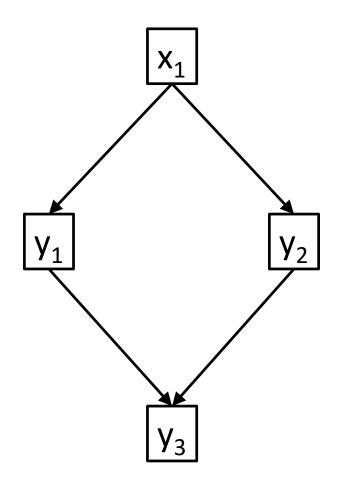


# Step 1: Specify model

# Step 2: Estimate model

mod.1.fit <- sem(mod.1, data=dat)</pre>

# Doing SEM in *lavaan*



# Step 1: Specify model

# Step 2: Estimate model

mod.1.fit <- sem(mod.1, data=dat)</pre>

# Step 3: Extract results

summary(mod.1.fit)

# **Interpreting SEM output**

lavaan 0.6-7 ende	d normally	after 27	iteration	s
Estimator Optimization me Number of free	thod	ur cc. 27		ML NLMINB 7
Number of observ	vations			90
Model Test User M	odel:			
Test statistic Degrees of free P-value (Chi-sq				17.729 2 0.000
Parameter Estimat	es:			
Standard errors Information Information sat	urated (h1)	model		Standard Expected ructured
Regressions:			_	
y1 ~	Estimate	Std.Err	z-value	P(> z )
x1	0.400	0.081	4.911	0.000
y2 ~ x1 y3 ~	0.875	0.367	2.381	0.017
y1	0.935			
y2	0.129	0.041	3.121	0.002
Variances:				
	Estimate			
. y1	0.005			
. y2	0.094			
. y3	0.015	0.002	6.708	0.000

# **Interpreting SEM output**

lavaan 0.6-7 ended normally after 27 iterations	
Estimator Optimization method Number of free parameters	ML NLMINB 7
Number of observations	90
Model Test User Model:	
Test statistic Degrees of freedom P-value (Chi-square)	17.729 2 0.000
Parameter Estimates:	
Information Ex	tandard xpected uctured

Regressions:				
	Estimate	Std.Err	z-value	P(> z )
y1 ~				
x1	0.400	0.081	4.911	0.000
y2 ~				
x1	0.875	0.367	2.381	0.017
y3 ~				
y1	0.935	0.171	5.475	0.000
y2	0.129	0.041	3.121	0.002
Variances:				
	Estimate	Std.Err	z-value	P(> z )
.y1	0.005	0.001	6.708	0.000
.y2	0.094	0.014	6.708	0.000
.y3	0.015	0.002	6.708	0.000
- ) -	3.025			

#### **Model info and evaluation**

Valid model: p-value > 0.05

# **Interpreting SEM output**

lavaan 0.6-7 ended normally after	27 iterations
Estimator Optimization method Number of free parameters	ML NLMINB 7
Number of observations	90
Model Test User Model:	
Test statistic Degrees of freedom P-value (Chi-square)	17.729 2 0.000
Parameter Estimates:	
Standard errors Information Information saturated (h1) model	Standard Expected l Structured

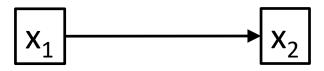
Regressions:	Estimate	Std.Err	z-value	P(> z )
y1 ~ x1	0.400	0.081	4.911	0.000
y2 ~ x1 y3 ~	0.875	0.367	2.381	0.017
y1 y2	0.935 0.129	0.171 0.041	5.475 3.121	0.000 0.002
Variances:		1 -		-6.1.15
.y1 .y2	Estimate 0.005 0.094	0.001 0.014	6.708 6.708	P(> z ) 0.000 0.000
. y3	0.015	0.002	6.708	0.000

#### **Paths info**

Regression estimate: equation parameters

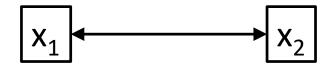
Variances estimates: error estimates

#### **SEM** definition in *lavaan*



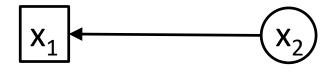
regression

 $x2 \sim x1$ 

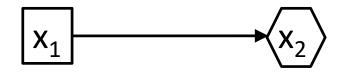


correlation

x2 ~~ x1



Latent variable x2 = x1 (unmeasured effect)



Composite variable x2 <~ x1 (caused by)

#### **SEM** model evaluation

When evaluating a SEM we asked two questions:

Are we missing important links?

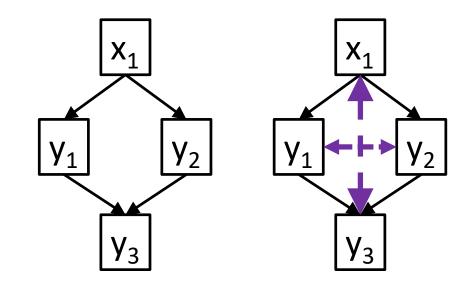
All included paths are supported by the data?

# Are we missing important links?

If model is significant, important paths are missing

We compare our model to a saturated model

Saturated model will be identical to data (p-value =1)

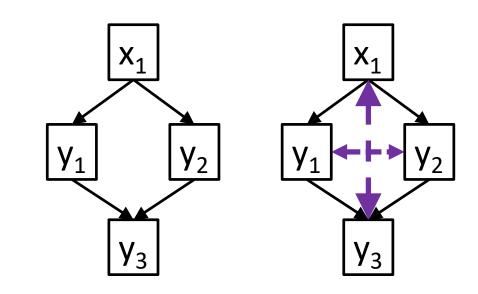


## Are we missing important links?

If model is significant, important paths are missing

We compare our model to a saturated model

Saturated model will be identical to data (p-value =1)



#### Modification Indices:

```
The op rhe
                     epc sepc.lv sepc.all sepc.nox
           0.000
                   0.000
                           0.000
                                    0.000
                                              0.000
       y2 0.014
                   0.000
                           0.000
                                    0.012
                                              0.012
       y3 16.119 -0.008
                          -0.008
                                    -0.943
                                             -0.943
       y3 16.119 -0.073
                          -0.073
                                    -1.945
                                             -1.945
       y2 0.014
                   0.003
                           0.003
                                    0.011
                                              0.011
       y3 10.215 -0.337
                          -0.337
                                    -0.662
                                             -0.662
           0.014
                                    0.014
                                             0.014
                   0.056
                           0.056
          2.107 -0.681
                          -0.681
                                    -0.324
                                             -0.324
                                              4.556
       x1 16.119
                   0.683
                           0.683
                                    0.400
                   0.000
                           0.000
                                    0.000
                                              0.000
           0.000
                   0.000
                           0.000
                                    0.000
                                              0.000
       y2 0.000
                   0.345
                           0.345
                                    0.590
                                              0.590
       v3 15.946
```

# Modification indices

summary(mod.1.fit, modindices = T)

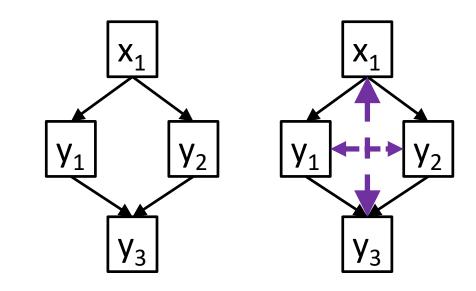
High mi (>3.64) indicates a missing path

# Are we missing important links?

We compare our model to a saturated model

Saturated model will be identical to data (p-value =1)

If model is significant, important paths are missing



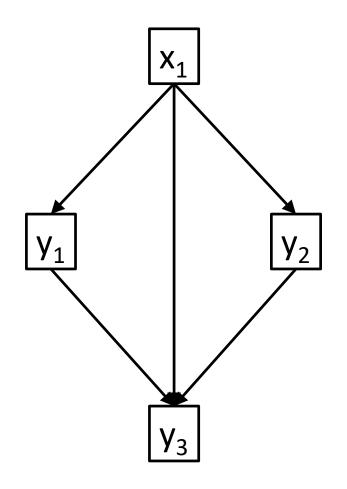
Modification Indices:									
8 9		•	rhs x1 v2	mi 0.000 0.014	epc 0.000 0.000	sepc.lv 0.000	sepc.all 0.000 0.012	sepc.nox 0.000 0.012	
10 11				16.119 16.119		-0.008 -0.073	-0.943 -1.945	-0.943 -1.945	
13	y1	~	y3	10.215	-0.337	-0.337	-0.662	-0.662	
14 15	y2	~	уı УI	0.014 2 107	0.036	0.036 0.681	0.014	0.014	
16	y3	~	x1	16.119	0.683	0.683	0.400	4.556	
1/	XI V1	~	yΣ	0.000	0.000	0.000	0.000	0.000	
19	<b>x1</b>	~	ý3	15.946	0.345	0.345	0.590	0.590	

# Modification indices

summary(mod.1.fit, modindices = T)

High mi (>3.64) indicates a missing path

# **Testing the improved model**



#### # Specify new model

#### Model Test User Model:

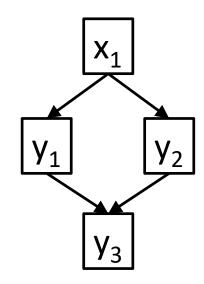
Test statistic	0.014
Degrees of freedom	1
P-value (Chi-square)	0.906

# All included paths are supported?

We evaluate each path independently

Some paths can be not supported (p-value > 0.05)

Regressions:				
	Estimate	Std.Err	z-value	P(> z )
y1 ~ x1	0.400	0.081	4.911	0.000
y2 ~	0.075	0.267	2 204	0.017
x1 y3 ~	0.875	0.367	2.381	0.017
y1	0.935	0.171	5.475	0.000
y2	0.129	0.041	3.121	0.002



#### How to select the best model

Statistical model selection is an important issue, not only for SEM

Some approaches:

Based on theory, model includes relations that make sense theoretically

Based on data, model includes relations that are statistically supported

#### How to select the best model

Statistical model selection is an important issue, not only for SEM

Some approaches:

Based on theory, model includes relations that make sense theoretically

Based on data, model includes relations that are statistically supported

# Not best option, Up to your philosophy

#### **Qualities of a good SEM**

Included paths make ecological sense

Model is not significant

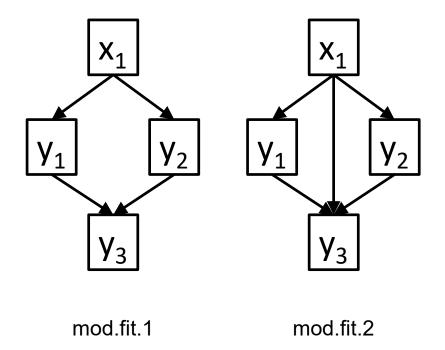
#### Options:

Keep as many paths as possible (highlights the importance of weak effects)

Remove paths to get the most parsimonious model (highlight important effects)

## Selecting the most parsimonious model

We can select models using AIC (lowest AIC = better model)



# Comparing models

anova(mod.fit.1,mod.fit.2)

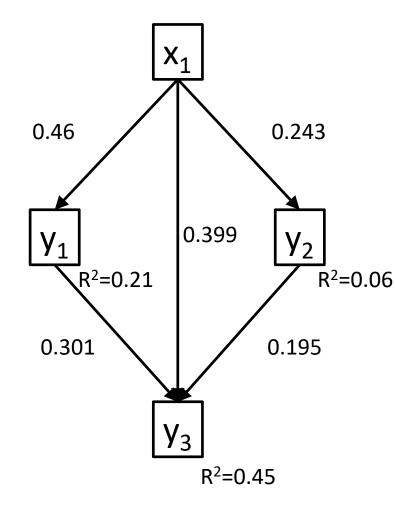
```
Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq) mod.2.fit 1 -310.38 -290.39 0.014 mod.1.fit 2 -294.67 -277.17 17.729 17.715 1 2.566e-05 *** --- Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '. ' 0.1 ' ' 1
```

#### **Interpreting results**

For interpreting the model is best to work with standardize estimates

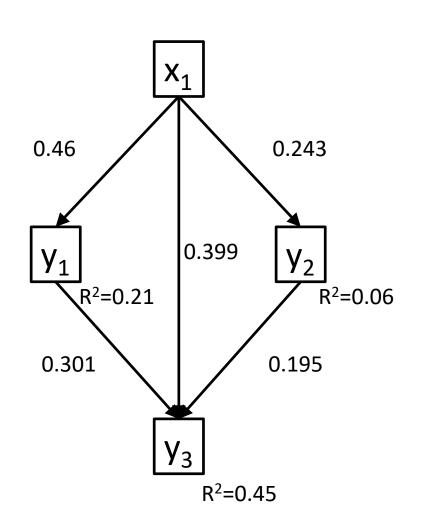
standardizedsolution(mod.2.fit,type="std.all") lavInspect(mod.2.fit,what="rsquare")

```
se z pvalue ci.lower ci.upper
lhs op rhs est.std
       x1
            0.460 0.079
                         5.848
                                0.000
                                         0.306
                                                  0.614
       x1
                         2.492
                                0.013
                                         0.052
                                                  0.435
            0.243 0.098
            0.301 0.086
                         3.508
                                0.000
                                         0.133
                                                  0.470
    ~ y1
                                         0.039
           0.195 0.080
                         2.442
                                0.015
                                                  0.352
                                         0.235
                         4.776
                                0.000
                                                  0.562
           0.399 0.083
                                         0.647
      y1
            0.789 0.072 10.911
                                0.000
                                                  0.930
          0.941 0.048 19.779
                               0.000
                                         0.848
                                                  1.034
          0.550 0.072
                         7.610
                                0.000
                                         0.409
                                                  0.692
            1.000 0.000
                                         1.000
                                                  1.000
                            NA
                                   NA
```



#### **Calculate the effects**

Direct, indirect and total effects can be calculated



Effects on  $y_1$ :

Effects on y<sub>2</sub>:

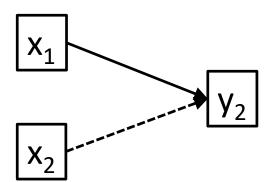
Direct = 0.46Indirect = NATotal = 0.46 Direct = 0.243 Indirect = NA Total = 0.243

Effects on y<sub>3</sub>:

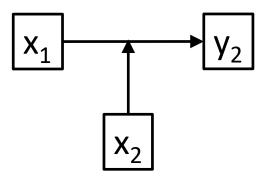
Direct = 0.301 + 0.399 + 0.195 = 0.895Indirect = 0.46 \* 0.301 + 0.243 \* 0.195 = 0.186Total = 0.895 + 0.186 = 1.081

#### Extras in lavaan

It is possible to work with factors (dummy variables)



It is possible to work with interactions



It is possible to work with non-linear (polynomial) relations

$$x_1^2 \rightarrow y_2$$

#### Where to look for nice info

#### <u>lavaan</u>

https://www.usgs.gov/centers/wetland-and-aquatic-research-center/science/quantitative-analysis-using-structural-equation?qt-science center objects=0#qt-science center objects

https://lavaan.ugent.be/tutorial/cfa.html

#### <u>piecewiseSEM</u>

https://jonlefcheck.net/teaching/

https://jslefche.github.io/sem\_book/index.html

#### Acknowledgements



UNIVERSITÄT BERN













